Chapter – 15

Probability

Introduction to Probability

In our day-to-day life, we generally use the statements such as

- It will probably rain today.
- Indian cricket team has good chances of winning the world cup.
- You are probably right.
- I doubt that he will pass the test.
- Chances are high that the prices of petrol will go up.
- There are 50-50 chances of India winning the match today.
- Most probably, Ram will stand the first rank in the board examination.

In such statements, we are using words like probably, chances, most probably and doubt, etc. These words mean that the event is not certain to take place. For example: in the first statement above, probably rain will mean it may rain or may not rain today. We make these statements or predict based on experience or the information available to us.

This uncertainty is measured numerically by calculating probability.

Probability is simply how likely something is to happen.

The best example for understanding probability is by flipping a coin:

When we toss a coin, there are two possible outcomes—heads or tails. What's the probability of the coin landing on heads?

You may already know that the possibility is half-half or 50%. But, how do we calculate the probability?

Probability of getting heads = $\dfrac{number\ of\ ways\ heads\ can\ come}{total\ number\ of\ outcomes}$

$= 1/2 = 0.5$

Probability of getting tails = $\dfrac{nnumber\ of\ ways\ tails\ can\ come}{total\ number\ of\ outcomes}$

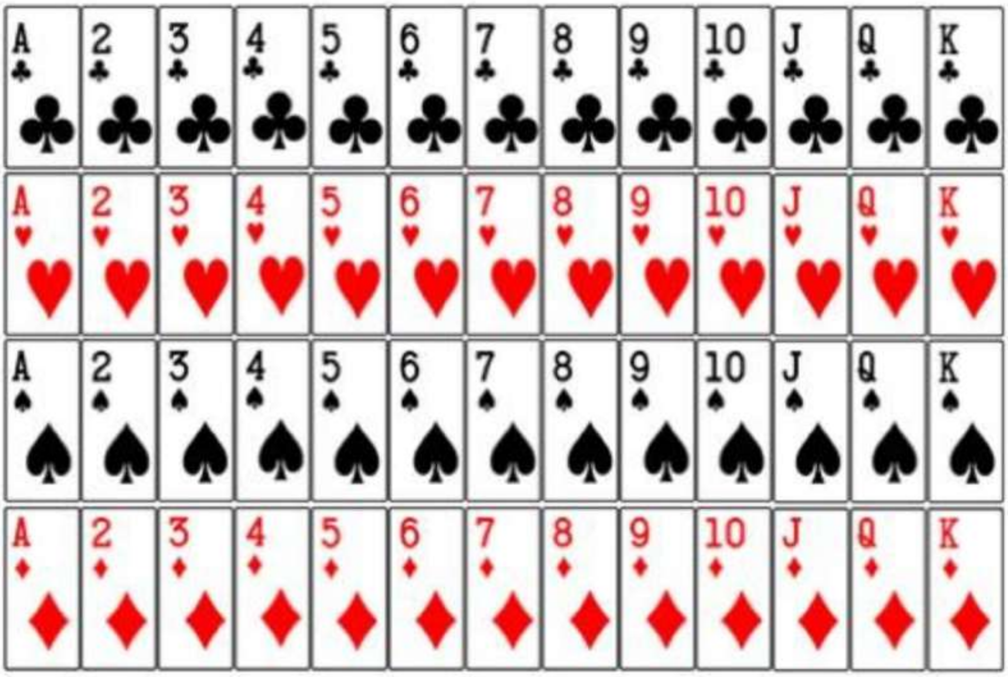
$= 1/2 = 0.5$

Probability of an event = (number of ways it can happen) / (total number of outcomes)

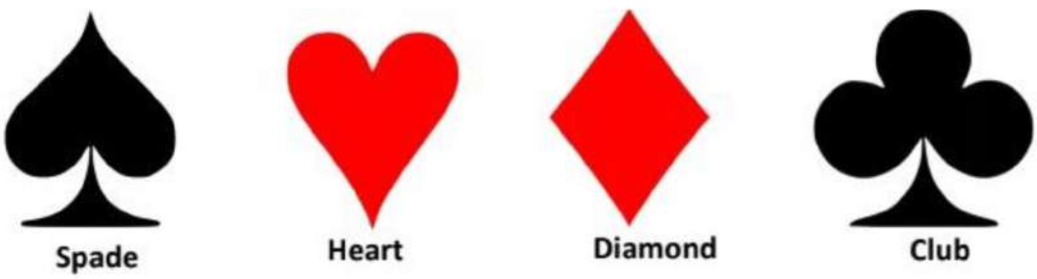
P(A) = (number of ways A can happen) / (Total number of outcomes)

Playing cards: Playing cards have a total of 52 cards. We are going to learn to solve the probability problems based on a well-shuffled pack of these 52 playing cards.

A brief about playing cards is given below.

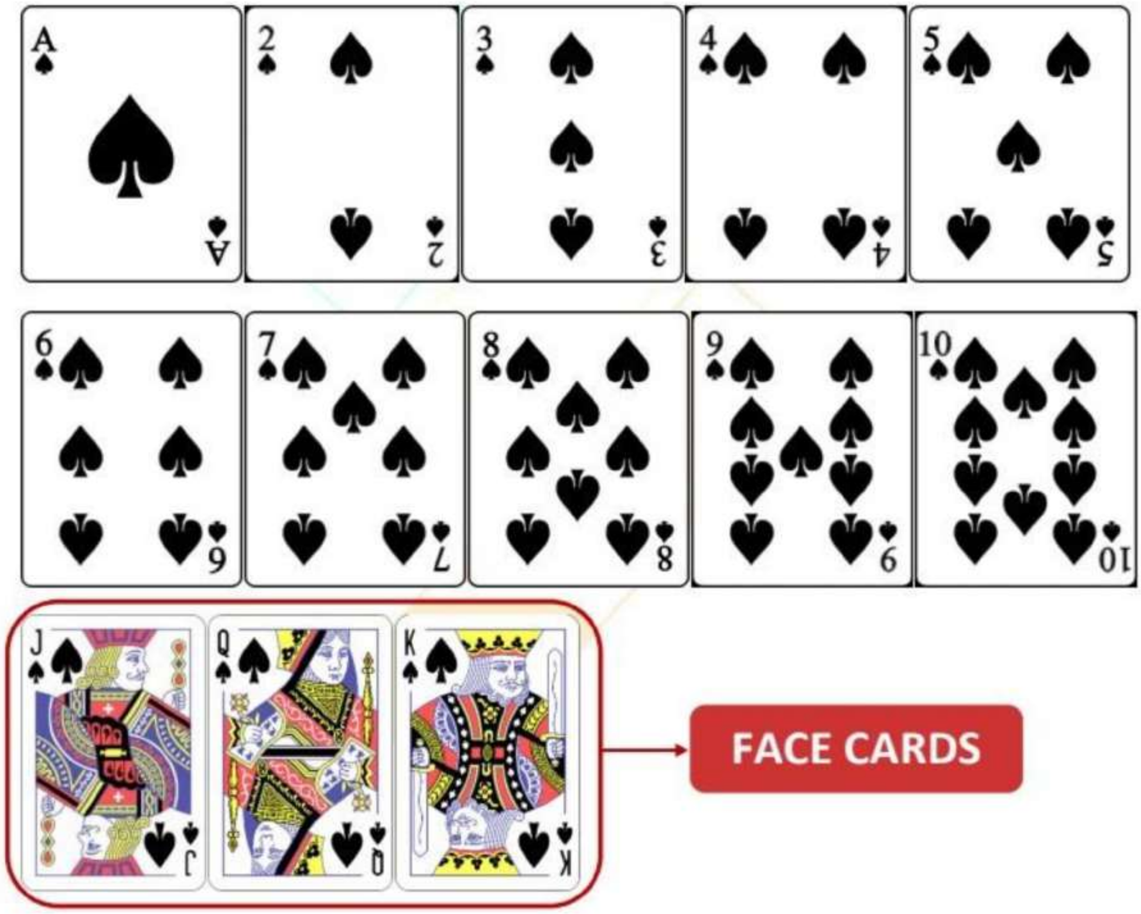


Spades, Hearts, Diamonds, and Clubs are four different types of cards are shown in the picture given below. Each type is called a suit.



Each suit has a total of 13 cards named ace (A), 2, 3, 4, 5, 6, 7, 8, 9, Jack (J), Queen(Q) and King (K).

The suits of spades cards are shown in the picture given below.



Card	Number	Card	Number
Spades	13	Hearts	13
Clubs	13	Diamonds	13
Black	26	Ace	04
Red	26	Jack	04
Queen	04	King	04
Face	12	Numeric	36

Probability an experimental approach

Terms related to Probability

Trial: When we perform an experiment, it is called a trial. For example, if a coin is tossed 5 times, then each toss is called a trial. If a dice is thrown 4 times, then each throw is called a trial. As shown in the figure below.

Outcome: It is a possible result of an experiment or trial. For example, when you flip a coin, the outcome is either head or tail. Similarly, when you roll a dice, the outcome is either 1, 2, 3, 4, 5, or 6 as shown in the figure below.

Event: The process of getting an outcome is called an event. When we perform an experiment, getting a possible result is called the Event. For example,

- Getting a Tail when tossing a coin is an event.
- Getting 6 when a die is rolled, is an event.

An event can have several outcomes:

• Choosing a "Queen" from the deck of cards (any of the 4 Queens) is also an event.
• Rolling an "odd number" (1, 3, 5) is an event.

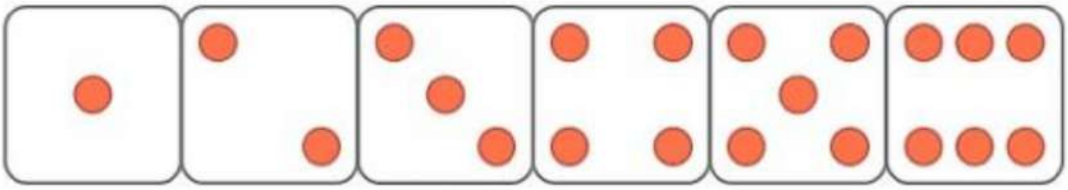
Random experiments: It is an experiment for which the total possible outcomes are known but the outcome cannot be controlled.
For example,

• when we toss a coin, the outcome of this experiment can be either head or tail showing up.
• The rolling of a die can yield six possible outcomes, 1, 2, 3, 4, 5 and 6.

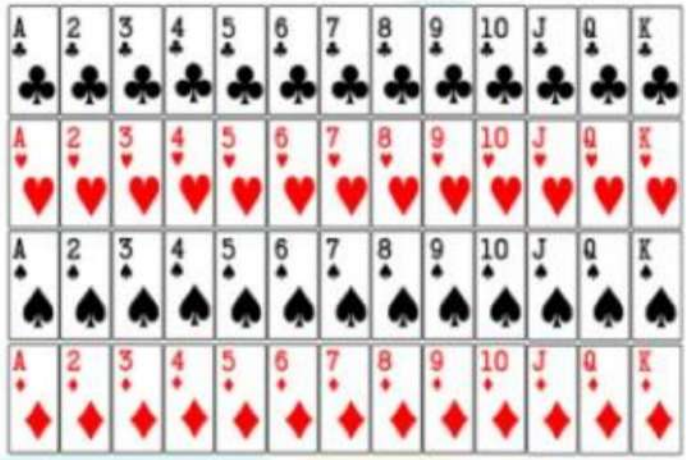
Sample space: The sample space of an experiment is the set of all possible outcomes of its experiment.

• Tossing a coin: the sample space is {Head, Tail} as shown in the figure below.

• Rolling a die: the sample space is the six outcomes {1, 2, 3, 4,5, 6} as shown in the figure below.



• Drawing a card from a deck: the sample space has 52 outcomes as shown in the figure below.



Suppose, a coin is tossed 5 times and tail appears 2 times.

So, the probability of getting the tail is $\frac{2}{5}$.

Hence, the probability of occurrence of an event

$$= \frac{Number\ of\ trials\ in\ which\ an\ event\ occures}{Total\ number\ of\ trials}.$$

It is also known as empirical probability or experimental probability and it is denoted by P(E).

$$\therefore P(E) = \frac{Number\ of\ trials\ in\ which\ an\ event\ occures}{Total\ number\ of\ trials}$$

We know that there are only six possible outcomes in a single throw of a die. These outcomes are 1, 2, 3, 4, 5 and 6. Since no face of the die is marked 7, i.e., the number of such outcomes is zero. In other words, getting 7 in a single throw of a die is impossible.

$$\therefore P\ (\text{getting } 7) = \frac{0}{6} = 0.$$

And also, sunrise from the west is impossible. So, the probability of sun rising from the west is zero.

∴ Probability of getting 6 in a throw of a die is $\frac{1}{6} = 0.166$

When a coin is tossed one time then the probability of getting head is $\frac{1}{2} = 0.5$.

∴ P (getting a number less than 7 in a throw of a die) = $\frac{6}{6} = 1$

We know that the next day Sun will rise from the east. It is a sure event. So, the probability of sun rising from the east is 1.

Thus, the probability of happening of an event always lies between 0 and 1, i.e. $0 \leq P(E) \leq 1$.

If $P(E) = 1$, then E is called a certain event or sure event and E is known as an impossible event, if $P(E) = 0$.

Example 1: A coin is tossed 200 times with the following frequencies of two outcomes: Head = 70 times, tail = 130 times. Find the probability of occurrence of each of these events.

Solution: It is given that the coin is tossed 200 times.

∴ Total number of trials = 200.

Suppose that the event of getting a head and of getting a tail by $E_1$and E^2 respectively. Then,

Number of trials in which the event E_1 happens = 70.

∴ $P(E_1) = \frac{Number\ of\ trials\ in\ which\ the\ event\ E1\ occures}{Total\ number\ of\ trials}$

$= \frac{70}{200}$

= 0.35.
And, Number of trials in which the event E2 happens = 130.

∴ $P(E2) = \frac{Number\ of\ trials\ in\ which\ the\ event\ E2\ occures}{Total\ number\ of\ trials}$

$= \frac{130}{200}$

= 0.65.

Hence, the probability of occurrence of head events and tail events is 0.35 and 0.65 respectively.

Example 2: A die is thrown 500 times with the following frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given below table.

Outcome	1	2	3	4	5	6
Frequency	130	50	120	70	30	100

Find the probability of happening of each outcome.

Solution: Suppose that Ei denoted the event of getting the outcome

i, where i = 1, 2, 3, 4, 5, 6. Then,

E_1 = Event of getting outcome 1.

$\therefore P(E1) = \frac{Number\ of\ trials\ in\ which\ the\ event\ E1\ occures}{Total\ number\ of\ trials}$

$= \frac{Frequency\ of\ E1}{Total\ number\ of\ trials.}$

$= \frac{130}{500}$

= 0.26.
E2 = Event of getting outcome 2.

$\therefore P(E2) = \frac{Number\ of\ trials\ in\ which\ the\ event\ E2\ occurs}{Total\ number\ of\ trials}$

$= \frac{Frequency\ of\ E2}{Total\ number\ of\ trials.}$

$= \frac{50}{500}$

= 0.1.

E3 = Event of getting outcome 3.

$\therefore$ P(E3) = $\frac{Number\ of\ trials\ in\ which\ the\ event\ E3\ occurs}{Total\ number\ of\ trials}$

$= \frac{Frequency\ of\ E3}{Total\ number\ of\ trials}$

$= \frac{120}{500}$

= 0.24.

E4 = Event of getting outcome 4.

$\therefore$ P(E4) = $\frac{Number\ of\ trials\ in\ which\ the\ event\ E4\ occurs}{Total\ number\ of\ trials}$

$= \frac{Frequency\ of\ E4}{Total\ number\ of\ trials}$

$= \frac{70}{500}$

= 0.14.

E5 = Event of getting outcome 5.

$\therefore$ P(E5) = $\frac{Number\ of\ trials\ in\ which\ the\ event\ E5\ occurs}{Total\ number\ of\ trials}$

$= \frac{Frequency\ of\ E5}{Total\ number\ of\ trials}$

$= \frac{30}{500}.$

$= 0.06.$

E6 = Event of getting outcome 6.

$\therefore$ P(E6) =

$$\frac{Number\ of\ trials\ in\ which\ the\ event\ E6\ occurs}{Total\ number\ of\ trials}$$

$$= \frac{Frequency\ of\ E6}{Total\ number\ of\ trials}$$

$$= \frac{100}{500}$$

$= 0.2.$

Example 3: A card is drawn at random from a well-shuffled pack of 52 cards. What is the probability that the drawn card is king?

Solution: Suppose that E1 be the event of drawing a king card.

There are 4 king cards in total.

$$\therefore P(E1) = \frac{Number\ of\ trials\ in\ which\ the\ event\ E1\ occurs}{Total\ number\ of\ trials}$$

$$= \frac{Frequency\ of\ E1}{Total\ number\ of\ trials}$$

$$= \frac{4}{52}$$

$= 0.076.$

Hence, the probability of getting a king card is 0.076.

If we have two or more groups which are performed experiments like the tossing of coins separately. To calculate the combined probability of all groups we have to need the cumulative frequency.

Cumulative frequency is the sum of frequencies of the current class interval and the class interval that comes before it.

Score	Frequency	Cumulative frequency
1	3	3
2	5	8
3	3	11
4	2	13

Therefore, the cumulative frequency for score 3 is 11.

Example 4: An insurance company selected 1800 drivers at random in a particular city to find a relationship between age and accidents.

The data are given in the following table:

Age of drivers (in years)	Accidents in one year			
	0	1	2	3
18-29	400	160	110	61
30-50	505	125	60	22
Above 50	262	45	35	15

Find the probability of the following events for a driver chosen at random form the life city:

1. Being 18-29 years of age and having exactly 3 accidents in one year.

2. Being 30-50 years of age and having one and two accidents in a year.

3. Having no accidents in one year.

Solution: Total number of drivers = 1800.

1. The number of drivers who are 18-29 years old and have exactly 3 accidents in one year is 61.

∴ Probability of drivers being 18-29 years of age and has exactly

three accidents = $\frac{Frequency\ of\ exactly\ 3\ accidents}{Total\ number\ of\ drivers.}$

$= \frac{61}{1800} = 0.0338$

2. The number of drivers who are 30-50 years old and having one and two accidents in one year is $125 + 60 = 185$.

∴ Probability of drivers being 30-50 years of age and having one and two accidents

$= \frac{Frequency\ of\ one\ and\ two\ accidents}{Total\ number\ of\ drivers}$.

$= \frac{185}{1800} = 0.102$

3. The number of drivers having no accidents in one year is $400 + 505 + 262 = 1167$.

∴ Probability of drivers having no accidents = $\frac{Frequency\ of\ no\ accidents}{Total\ number\ of\ drivers}$.

$= \frac{1167}{1800} = 0.6483$.

Summary of Probability

Playing cards have a total 52 cards. These are distributed as below:

Card	Number	Card	Number
Spades	13	Hearts	13
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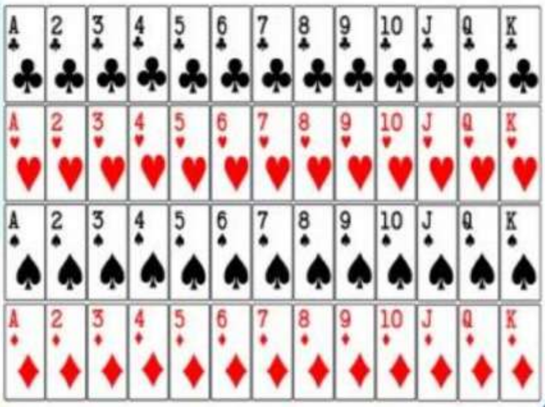
Red	26	Jack	04
Queen	04	King	04
Face	12	Numeric	36



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Outcome: It is a possible result of an experiment or trial. For example, when you flip a coin, the outcome is either head or tail.

Event: The process of getting an outcome is called an event. For example, getting a Tail when tossing a coin is an event.

Random experiments: It is an experiment for which the total possible outcomes are known but the outcome cannot be controlled. For example, the rolling of a die can yield six possible outcomes, 1, 2, 3, 4, 5 and 6.

Sample space: The sample space of an experiment is the set of all possible outcomes of its experiment. For example, tossing a coin: the sample space is {Head, Tail}.

The experimental or empirical probability of an event E is given by

$$\therefore P(E) = \frac{\text{Number of trials in which an event occures}}{\text{Total number of trials}}$$

The probability of an event lies between 0 and 1.

If $P(E) = 1$, then E is called a certain event or sure event and E is known as an impossible event, if $P(E) = 0$.