HOTS (Higher Order Thinking Skills)

Que 1. In Fig. 9.29, ABCD and AEFD are two parallelograms. Prove that ar (ΔPEA) = ar (ΔQFD).

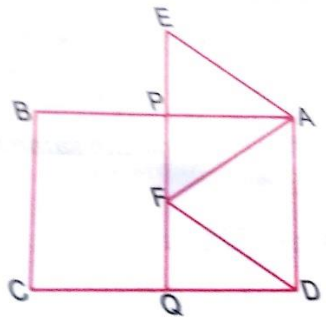

Fig. 9.29

Sol. In triangles PEA and QFD, we have

	$\angle APE = \angle DQF$	(Corresponding angles)
	$AE = DF$	(Opposite sides of $\|\|^{gm}$ AEFD)
	$\angle AEP = \angle DFQ$	(Corresponding angles)
$\therefore$	$\Delta PEA \cong \Delta QFD$	(AAS congruence criterion)

As congruent triangles have equal area.

$\therefore$ ar (ΔPEA) = ar (ΔQFD)

Que 2. In Fig. 9.30, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that ar(ABCDE) = ar(APQ).

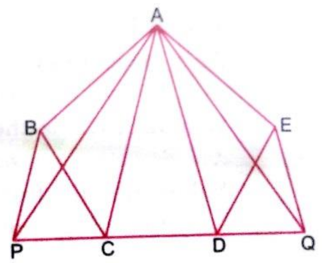

Fig. 9.30

Sol. Since, ΔABC and ΔAPC are on the same base AC and between the same parallels BP and AC

∴ ar (ΔABC) = ar (ΔAPC) …(i)

(Triangles on the same base and between the same parallels are equal in area)

Similarly, EQ || AD

∴ ar (ΔAED) = ar (AQD) …(ii)

Adding (i) and (ii), and then adding ar (ΔACD) to both the sides, we get

Ar (ΔABC) + ar (ΔAED) + ar (ΔACD) = ar (ΔAPC) + ar (ΔAQD) + ar (ΔACD)

⇒ ar (ABCDE) = ar (APQ).

Que 3. In Fig. 9.31, X and Y are the mid-point of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that ar (ΔABP) = ar (ΔACQ).

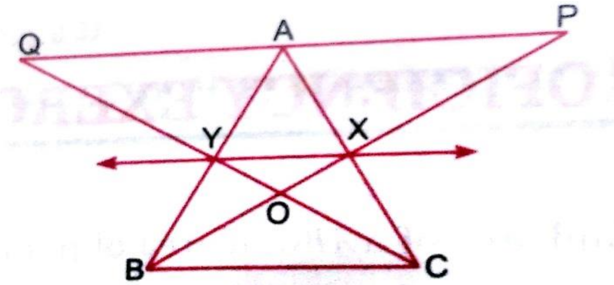

Fig. 9.31

Sol. As X and Y are the mid-point of AC and AB respectively.

∴ XY || BC

Since Δ BYC and Δ BXY are on the same base BC and between the same parallels XY and BC.

∴ ar (Δ BYC) = ar (Δ BXC)

⇒ ar (Δ BYC) – ar (Δ BOC) = ar (Δ BXC) – ar (Δ BOC)

⇒ ar (Δ BOY) = ar (Δ COX)

⇒ ar (Δ BOY) + ar (Δ XOY) = ar (Δ COX) + ar (Δ XOY)

⇒ ar (Δ BXY) = ar (Δ CXY) …(i)

Since quadrilaterals XYAP and XYQA are on the same base XY and between the same parallels XY and PQ.

∴ ar (XYAP) = ar (XYQA) …(ii)

Adding (i) and (ii), we get

Ar (Δ BXY) + ar (XYAP) = ar (Δ CXY) + ar (XYQA)

⇒ ar (Δ ABP) = ar (Δ ACQ)

Que 4. If the medians of a ΔABC intersect at G. Show that
Ar (ΔAGC) = ar (ΔAGB) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC)

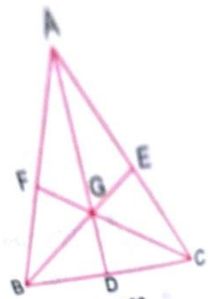

Fig. 9.32

Sol. Give: A Δ ABC in which medians AD, BE and CF intersect at G.

To prove:

Ar (ΔAGB) = ar (ΔBGC) = ar (ΔCGA) = $\frac{1}{3}$ ar (ΔABC)

Proof: In ΔABC, AD is the median.
As a median of a triangle divides it into two triangles of equal area.

∴ ar (ΔABD) = ar (ΔACD) …(i)
In ΔGBC, GD is the median
∴ Ar (ΔGBD) = ar (ΔGCD) …(ii)

Subtracting (ii) from (i), we get
Ar (ΔABD) – ar (ΔGBD) = ar (ΔACD) – ar (ΔGCD)
ar (AGB) = ar (ΔAGC) …(iii)

Similarly, ar (ΔAGB) = ar (ΔBGC) …(iv)
From (iii) and (iv), we get
ar (ΔAGB) = ar (ΔBGC) = ar (ΔAGC) …(v)
But, ar (ΔAGB) + ar (ΔBGC) + ar (ΔAGC) = ar (ΔABC) …(vi)
From (v) and (vi), we get
3ar (Δ AGB) = ar (Δ ABC)

⇒ ar (Δ AGB) = $\frac{1}{3}$ ar (Δ ABC)

Hence, ar (Δ AGB) = ar (Δ AGC) = ar (Δ BGC) = $\frac{1}{3}$ ar (Δ ABC)