Nyquist Plot :-

* Purpose:

⇒ To draw the Complete freq. response of OLTF.

⇒ To find the range of K value for System stability.

⇒ To find the no. of close loop poles in the Right of S-plane.

⇒ To find the Gain Margin, Phase Margin, Gain cross over freq & Phase cross over freq.

⇒ To find the Relative stability by using Gain Margin & Phase Margin.

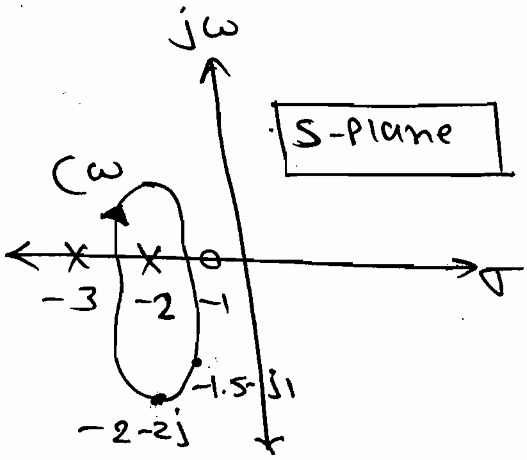
⇒ The Nyquist plot is developed by using a mathmatical principle called Principle of Arguments.

* Principle of Arguments:

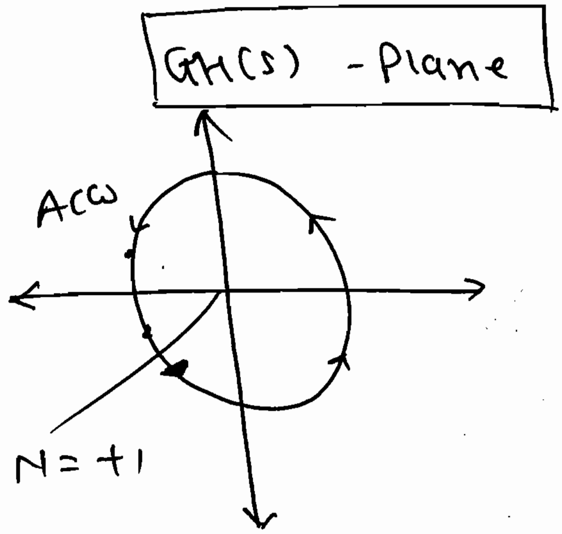
⇒ It states that if there are P poles and the Z zeros are enclosed by the random selected closed path than the corresponding $G(s) \cdot H(s)$ plane N-circles the origin with P-Z times i.e $N = P - Z$

$N = P - Z$.

e.g. $GH(s) = \dfrac{(s+1)}{(s+2)(s+3)}$.



⇒



$N = P - Z$

$\boxed{N = 1 - 0 = +1}$

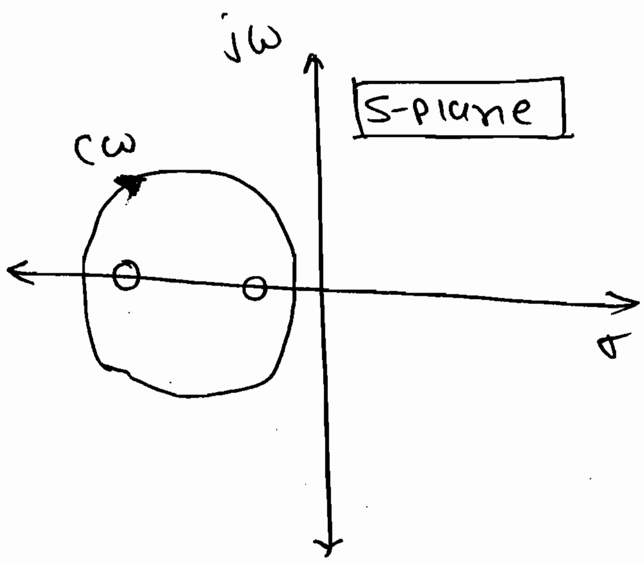
⇒

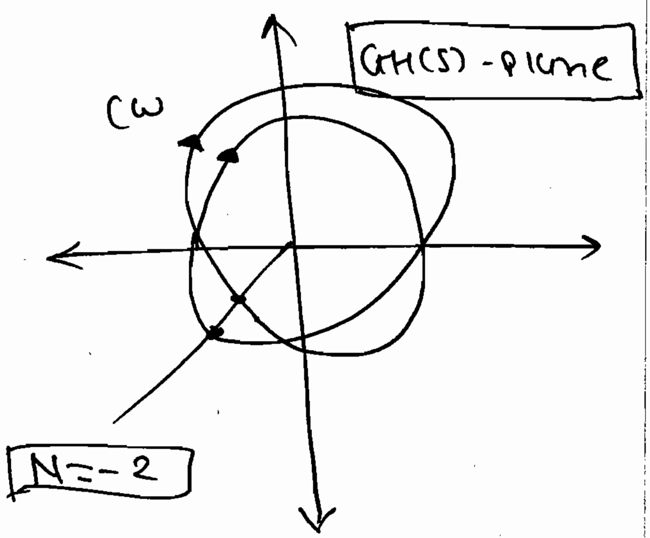
Pole → Change in direction.
Zero → No change in direction.

⇒

ACW	→	+ve
CW	→	−ve

⇒

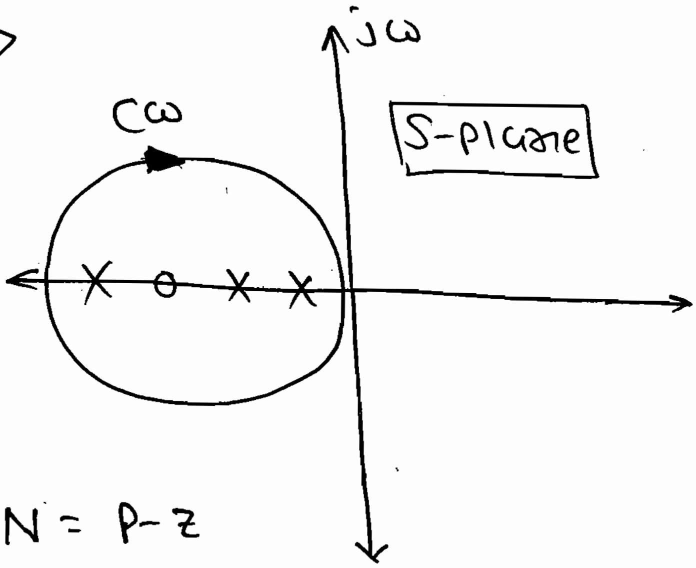




⇒ $N = P - Z$

$N = 0 - 2$ ⇒ $\boxed{N = -2}$

⇒



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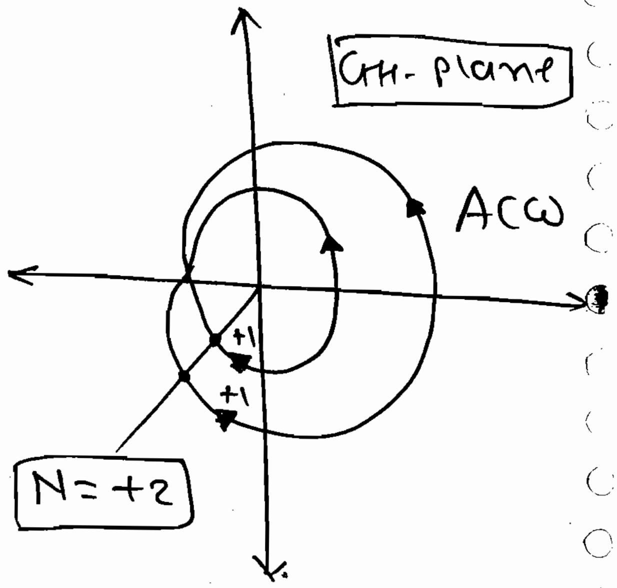
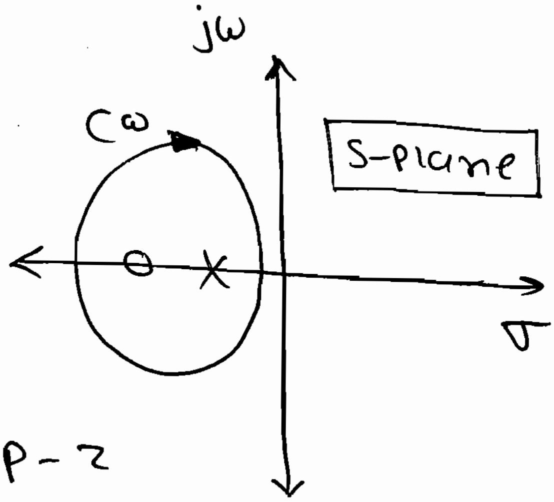


$N = P - Z$

$N = 3 - 1$

$\boxed{N = +2}$

⇒



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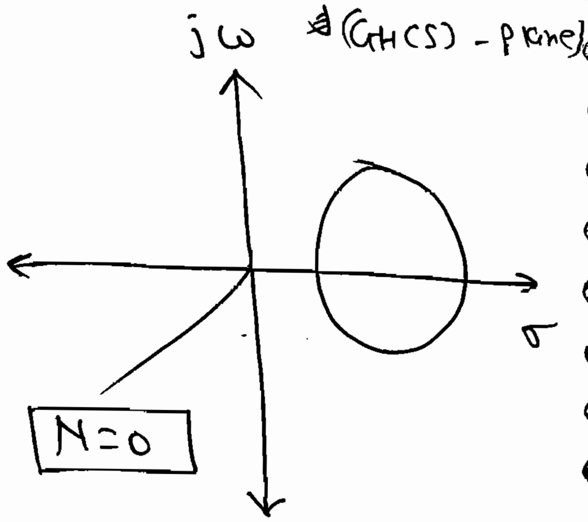
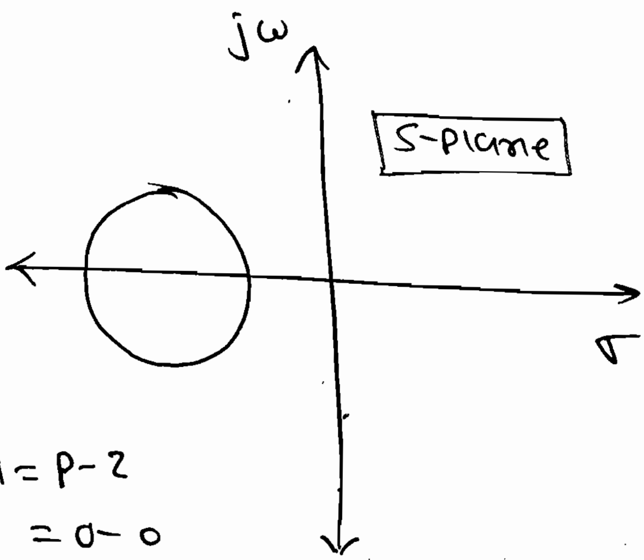


$N = P - Z$

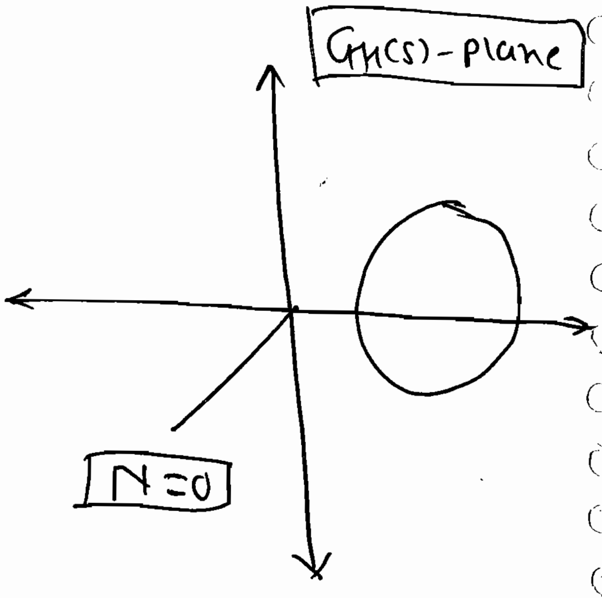
$= 1 - 1$

$N = 0$

⇒



⇒



$N = P - Z$

$= 0 - 0$

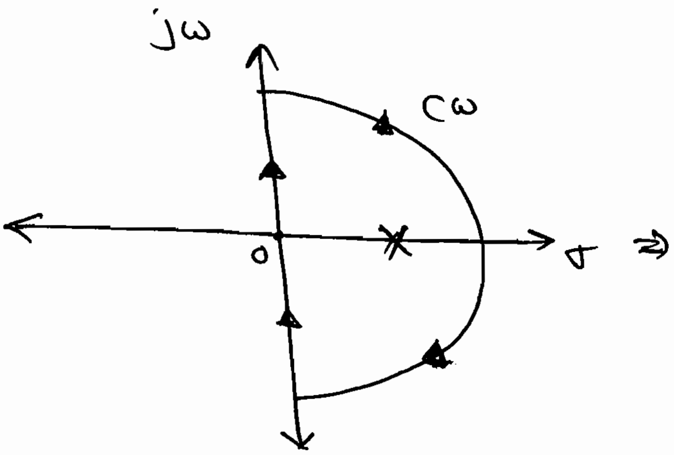
$\boxed{N = 0}$

⇒ The Random Selected closed path should not Pass through any Pole (or) zero.

⇒ The Principle of Arg. concept is applied to the total Right half of S-plane with radius of ∞.

⇒ The Nyquist Stability analysis is right of S-plane analysis.

⇒ $GH(s) = \dfrac{1}{(s-1)(s+2)(s+3)}$.



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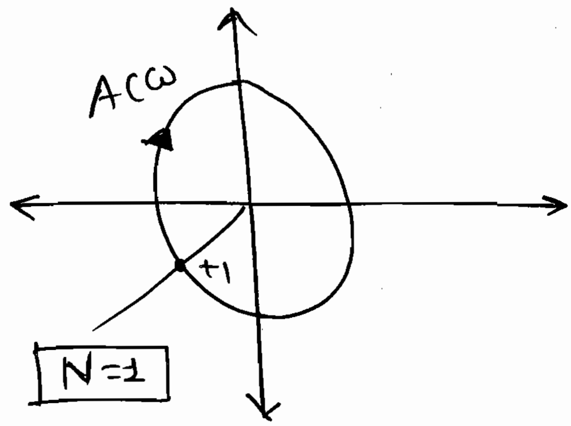


$N = P - Z$

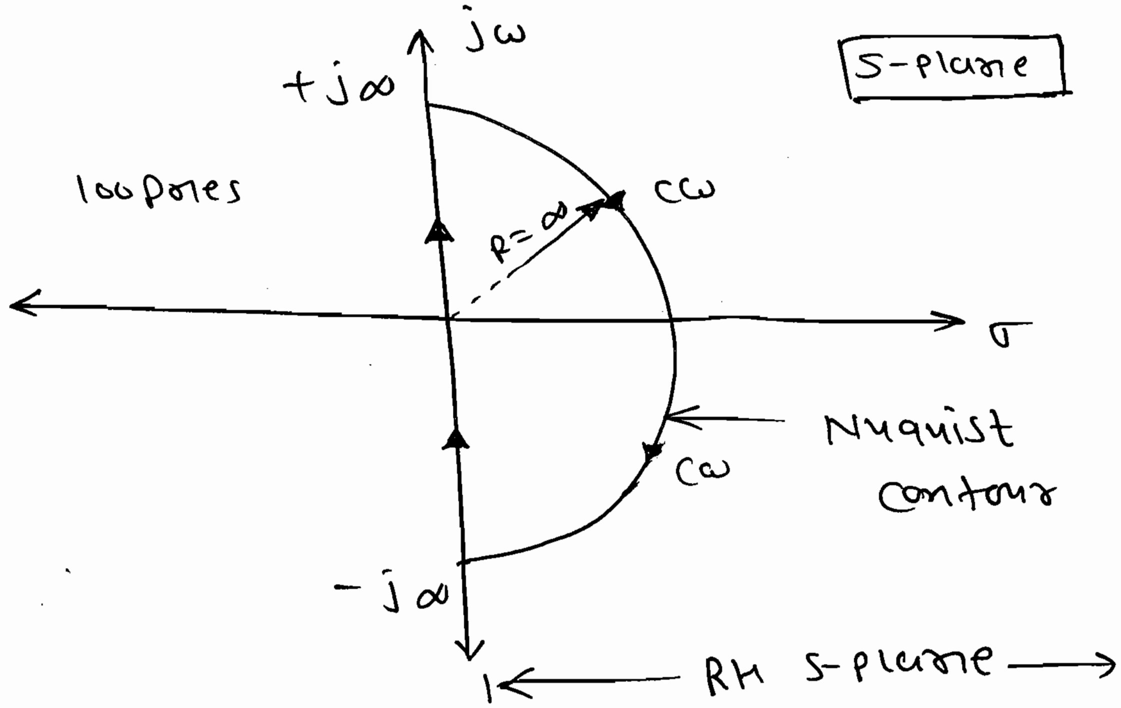
$= 1 - 0 = +1 \Rightarrow \boxed{N=+1}$

⇒ OL Sys. | OLTF

$N = P - Z$

- N ↓ No. of encirclements about origin
- P ↓ OL Pole RH
- Z ↓ OL zero RH.

⇒



⇒ To get about OLTF (or) OL sys.

Consider N as a No. of encirclements about origin.

⇒ P is no. of OL Poles on Right of the S-plane.

⇒ Z is no. of OL Zeros on Righ of the S-plane.

* Pole - Zero Configuration:-

⇒ The open loop transfer function (OLTF) is given by.

$$\boxed{GH(s) = K \frac{N(s)}{D(s)}} \quad \text{——— (1)}$$

⇒ The CLTF is given by.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}.$$

$$T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + k\frac{N(s)}{D(s)}}.$$

$$\therefore \boxed{T.F. = \frac{G(s).D(s)}{D(s) + k N(s)}} \quad \text{——— (2)}$$

⇒ The Closed loop stability is given by Char. eqn.

$$q(s) = 1 + G(s).H(s)$$

$$= 1 + k\frac{N(s)}{D(s)}.$$

$$\therefore \boxed{q(s) = \frac{D(s) + kN(s)}{D(s)}} \quad \text{——— (3).}$$

⇒ Compare eqn - 1 & 3.

$$\boxed{\text{Poles of CE = OLTF Poles.}}$$

⇒ Compare eqn 2 & 3.

$$\boxed{\text{Zeros of CE = CLTF Poles.}}$$ ***

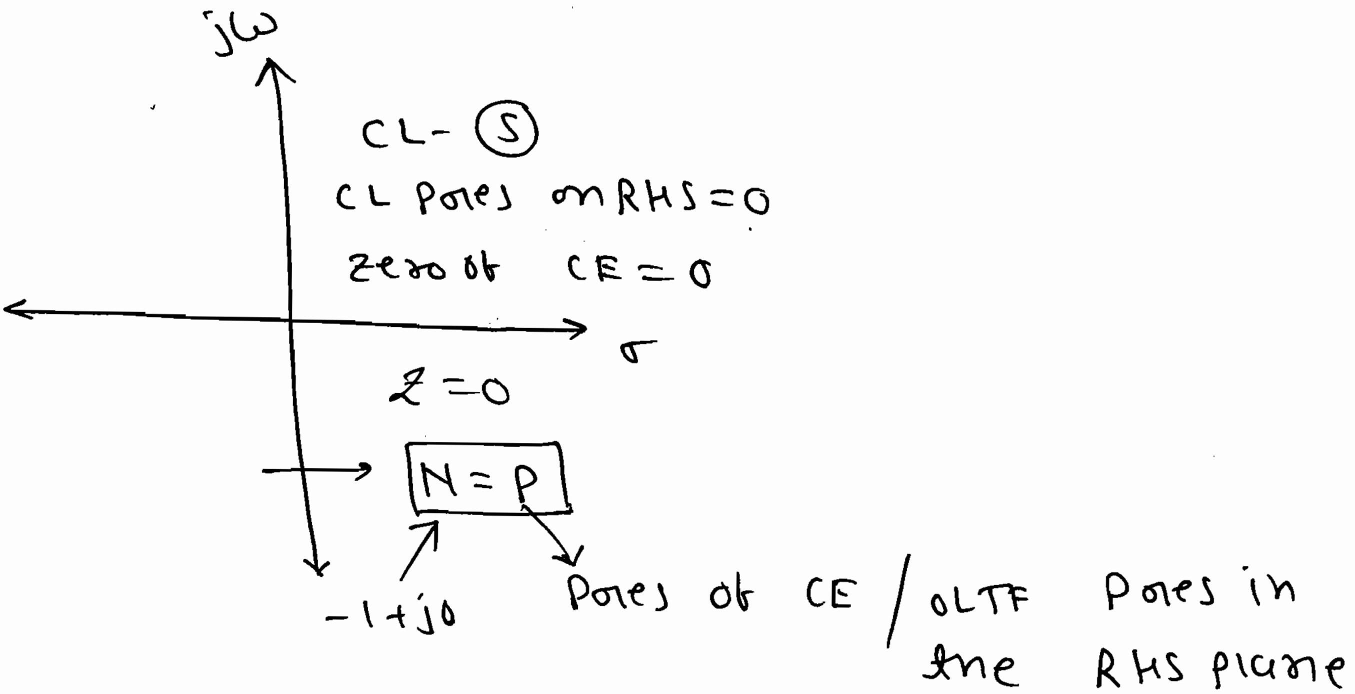
⇒ $N = P - Z$

N → No. of encirclements about critical point (-1+j0)

P → Poles of CE in the RH s-plane → OLTF poles in the RH s-plane

Z → Zeros of CE in the RHS plane → CLTF Poles in the RH s-plane

⟹



⟹ The Close-loop Pole is nothing but, zeros of CE which must be zero, in the right of S-plane that means $\boxed{Z=0}$ & $\boxed{N = P - Z = P}$

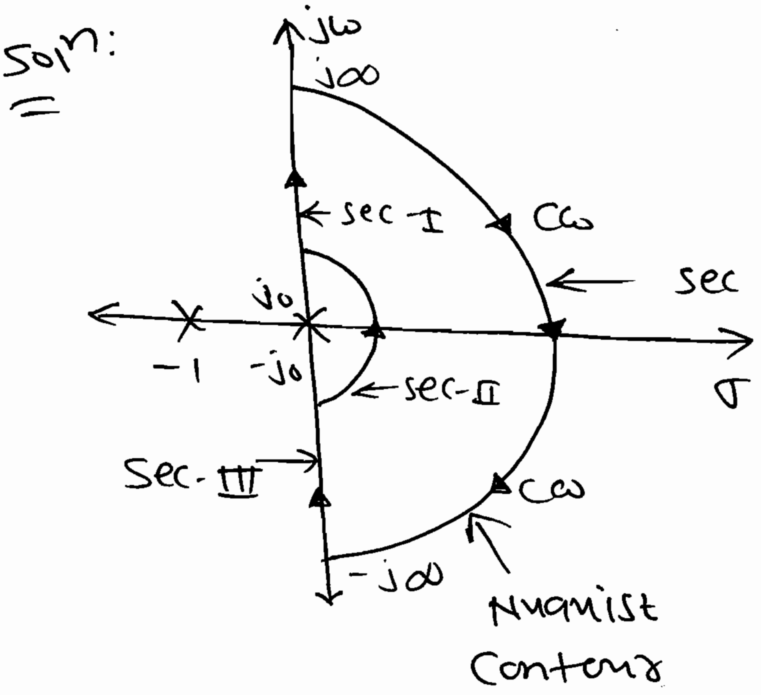
* Nyquist Stability Criteria:-

⟹ It states that the No. of ~~A~~- encirclements about the critical point must be equal to poles of chara. eqn which are nothing but OLTF poles in the Right of S-plane, i.e. $Z=0$, $N=P$.

Q Draw the Nyquist plot & find the sys. stability to the following sys.

$$G(s) \cdot H(s) = \frac{1}{s(s+1)}.$$

Soln:



$$M = \frac{1}{\omega\sqrt{\omega^2+1}}.$$

$$\phi = -90^\circ - \tan^{-1}(\omega).$$

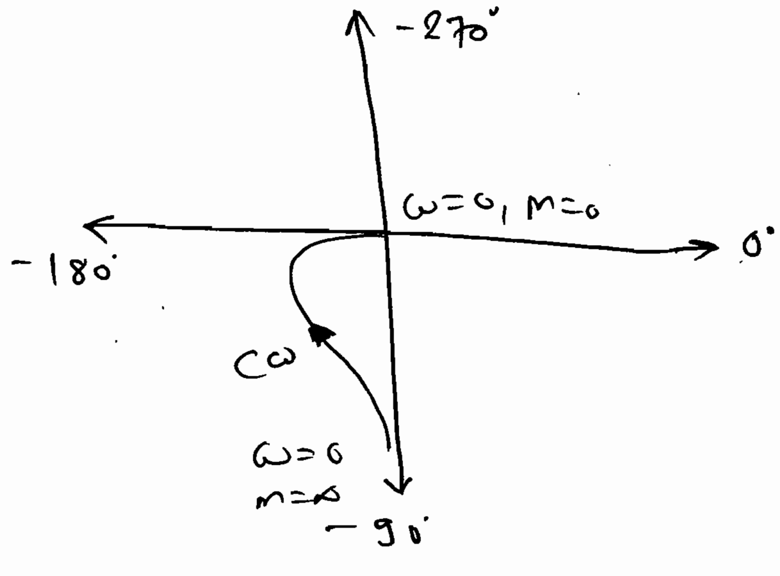
⇒ Sec-I:

→ $\omega = 0^+ \Rightarrow M = \infty$, $\phi = \angle -90^\circ$

$\omega = \infty^+ \Rightarrow M = 0$, $\phi = -180^\circ$

S.D. ⇒ fp ⇒ cω

E.D. ⇒ $\phi_1 - \phi_2 = +ve$ ⇒ ccω.



⇒ Sec-II:

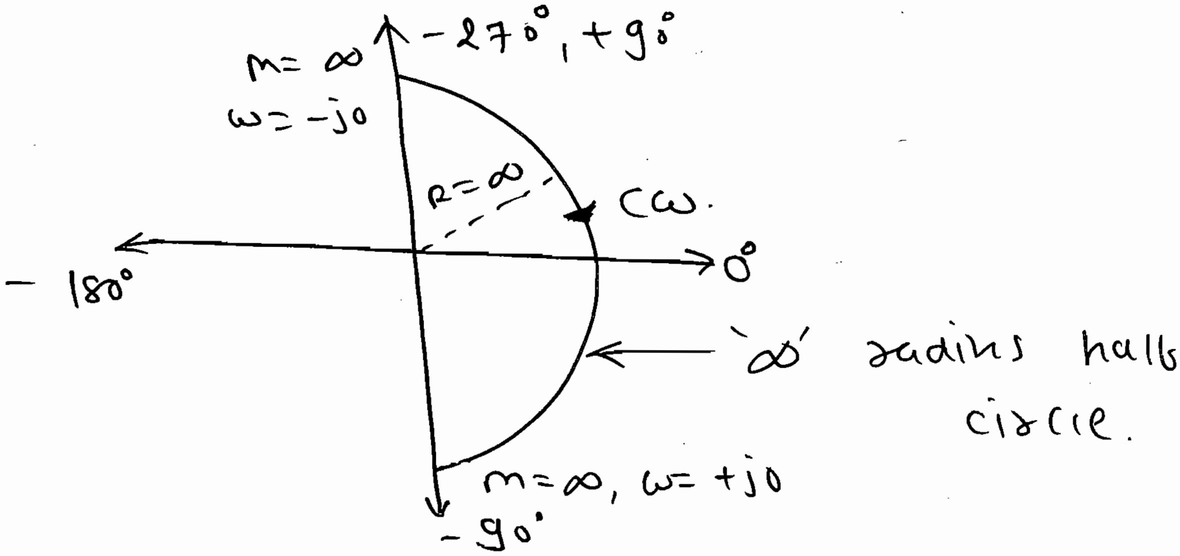
⇒ ~~ω=±0~~ $\omega = -j0 \Rightarrow M = \infty$, $\phi = -90^\circ + (180 - \tan^{-1}(0))$.

$$\phi = +90^\circ$$

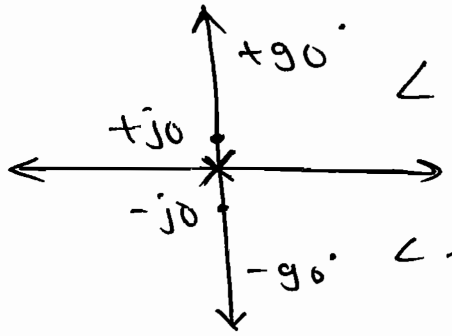
$\omega = +j0 \Rightarrow$ ~~ω=0~~ , $M = \infty$, $\phi = -90^\circ$

E.D. ⇒ $\phi_1 - \phi_2 = +ve \Rightarrow$ C.W.

S.D. ⇒ C.W.



⇒



$$\angle \frac{1}{s}\Big|_{s=+j0} = \frac{\angle 1}{\angle +j0} = -90^\circ$$

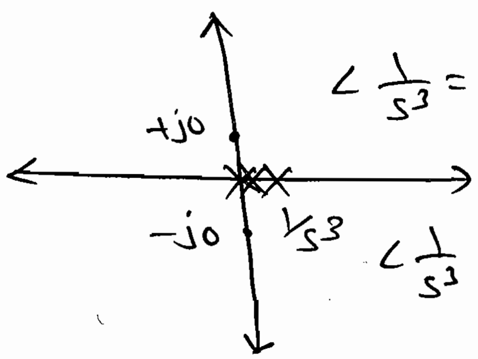
$$\angle \frac{1}{s}\Big|_{s=-j0} = \frac{\angle 1}{\angle -j0} = +90^\circ$$

$\boxed{180^\circ \times 1}$

$1/s$

1-pole at origin

⇒



$$\angle \frac{1}{s^3} = \frac{\angle 1}{3\angle +j0} = -270^\circ$$

$$\angle \frac{1}{s^3} = \frac{\angle 1}{3\angle -j0} = +270^\circ$$

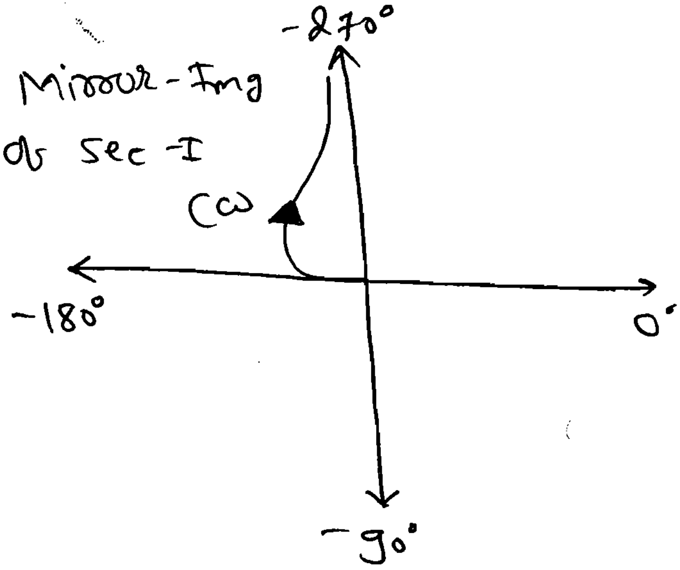
$540 \Rightarrow \boxed{180^\circ \times 3}$

$1/s^3$

3-pole at origin.

Note: No. of '∞' Radius half (180°) circles = No. of Poles at origin.

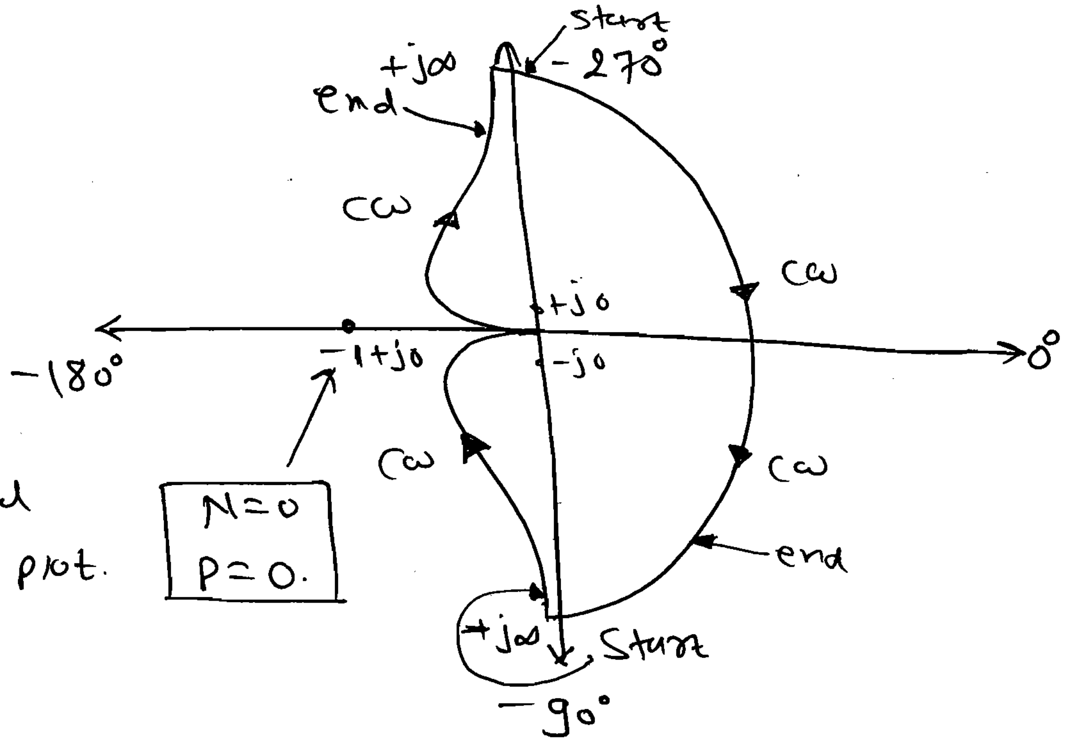
sec-III :-



⇒ sec-III is a Mirror-image of sec-I, about the Real axis but the direction is Continous.

⇒ Section- IV:

⇒ The sec-IV gives the magnitude of at $\omega = \infty$ ⇒ $m = \pm 0$. that means it is a point at origin. neglect the sec-IV.



Actual polar plot.

$\boxed{\begin{matrix} N=0 \\ P=0. \end{matrix}}$

Note:

⇒ The ∞ radius half circle should be start where the mirror img. end & the ∞ radius half circle end where the actual polar plot is start.

$\Rightarrow$ The ∞ radius half circle direction always CCW because it depends on nyquist contour direction.

$\boxed{Q}$ $GH(s) = \dfrac{10}{(s+1)}$.

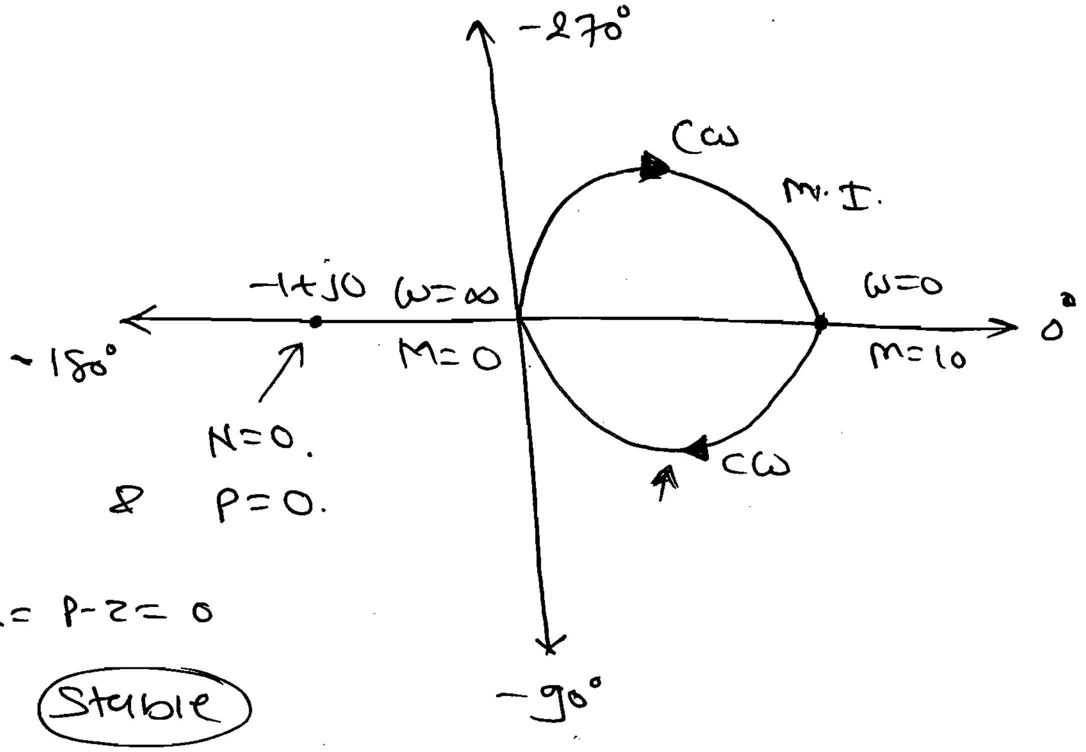
$\underline{Sol^n}$: $M = \dfrac{10}{\sqrt{\omega^2+1}}$. & $\phi = -\tan^{-1}(\omega)$.

$\omega = 0 \Rightarrow$ $M = 10$ & $\phi = 0^\circ$.

$\omega = \infty \Rightarrow$ $M = 0$ & $\phi = -90^\circ$

E.D. $\Rightarrow$ $\phi_1 - \phi_2 = +ve$ $\Rightarrow$ CW

S.D. $\Rightarrow$ fp = CW.



$N = P - Z = 0$

Stable

$\boxed{Q}$ $GH(s) = \dfrac{10}{(s+1)(s+2)}$.

$\underline{Sol^n}$: $M = \dfrac{10}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}$.

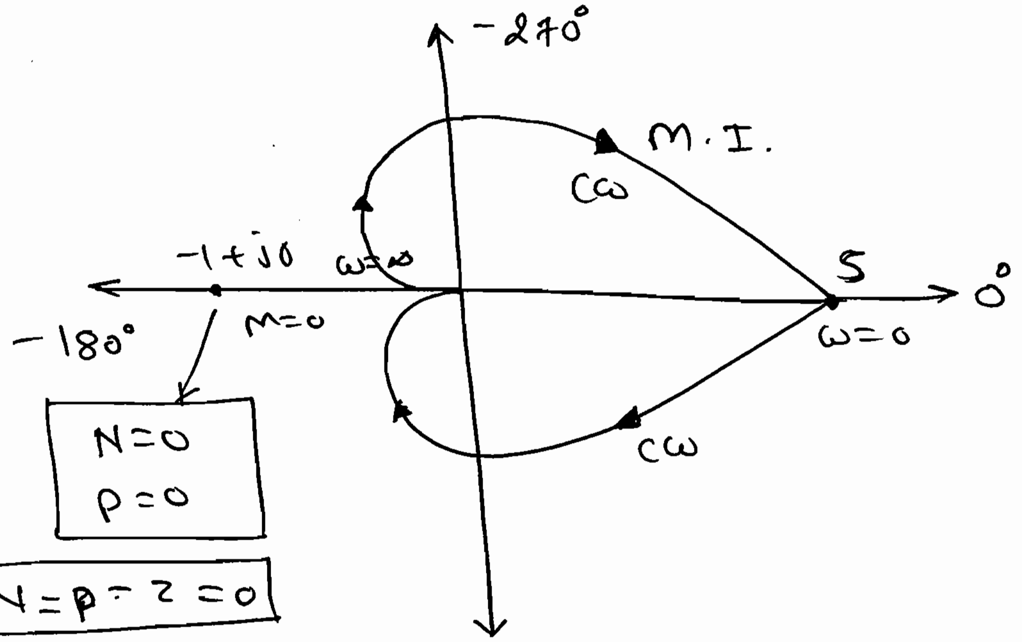
$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2)$.

$\Rightarrow \quad \omega = 0 \Rightarrow M = \frac{10}{2} = 5 \quad , \quad \phi_1 = 0^\circ$

$\omega = \infty \Rightarrow M = \infty \quad , \quad \phi_2 = -180^\circ.$

E.D. $\Rightarrow \phi_1 - \phi_2 = +ve = C\omega.$

S.D. $\Rightarrow$ bP $= C\omega.$



$\boxed{N = P = Z = 0}$

CL (S)

[a] $GH(s) = \dfrac{10}{s^2 (s+1)(s+2)}$.

Soln:

$$M = \frac{10}{\omega^2 \sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}.$$

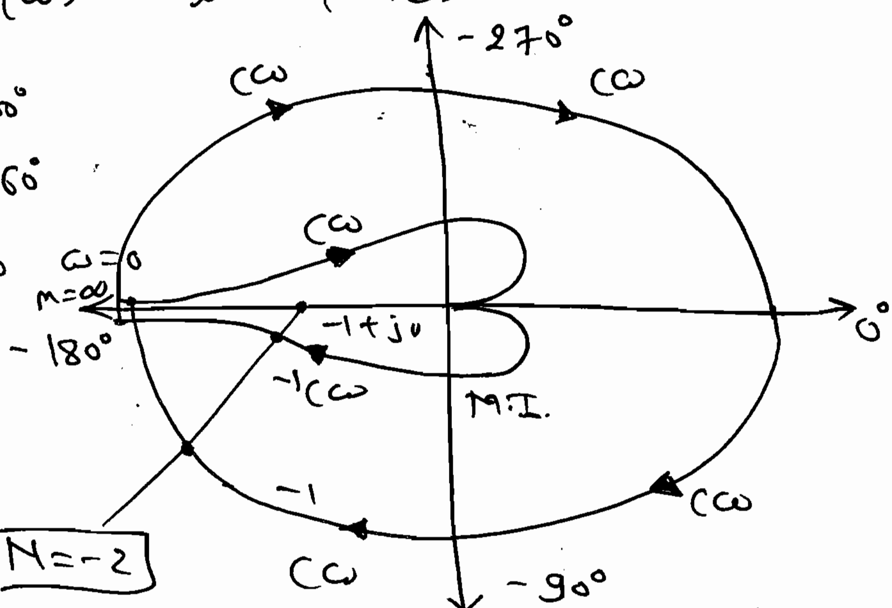
$$\phi = -180^\circ - \tan^{-1}(\omega) - \tan^{-1}(\omega/2).$$

$\omega = 0 \Rightarrow M = \infty, \ \phi_1 = -180^\circ$

$\omega = \infty \Rightarrow m = 0, \ \phi_2 = -360^\circ$

E.D. $\Rightarrow \phi_1 - \phi_2 = +ve = C\omega$

S.D. $\Rightarrow$ bP $= C\omega.$



$P = 0$

$Z = 0$ so, $\boxed{N \neq P}$

$\Rightarrow$ (US)

⇒ Here, $N = -2$, but $P = 0$

so, $N \neq P$ ⇒ CL US.

⇒ $N = P - Z$

$Z = P - N$

$Z = 0 - (-2)$

$\boxed{Z = 2}$ ⇒ 2 CL pole in the Right of S-plane.

Q $GH(s) = \dfrac{10}{s^3 (s+10)}$.

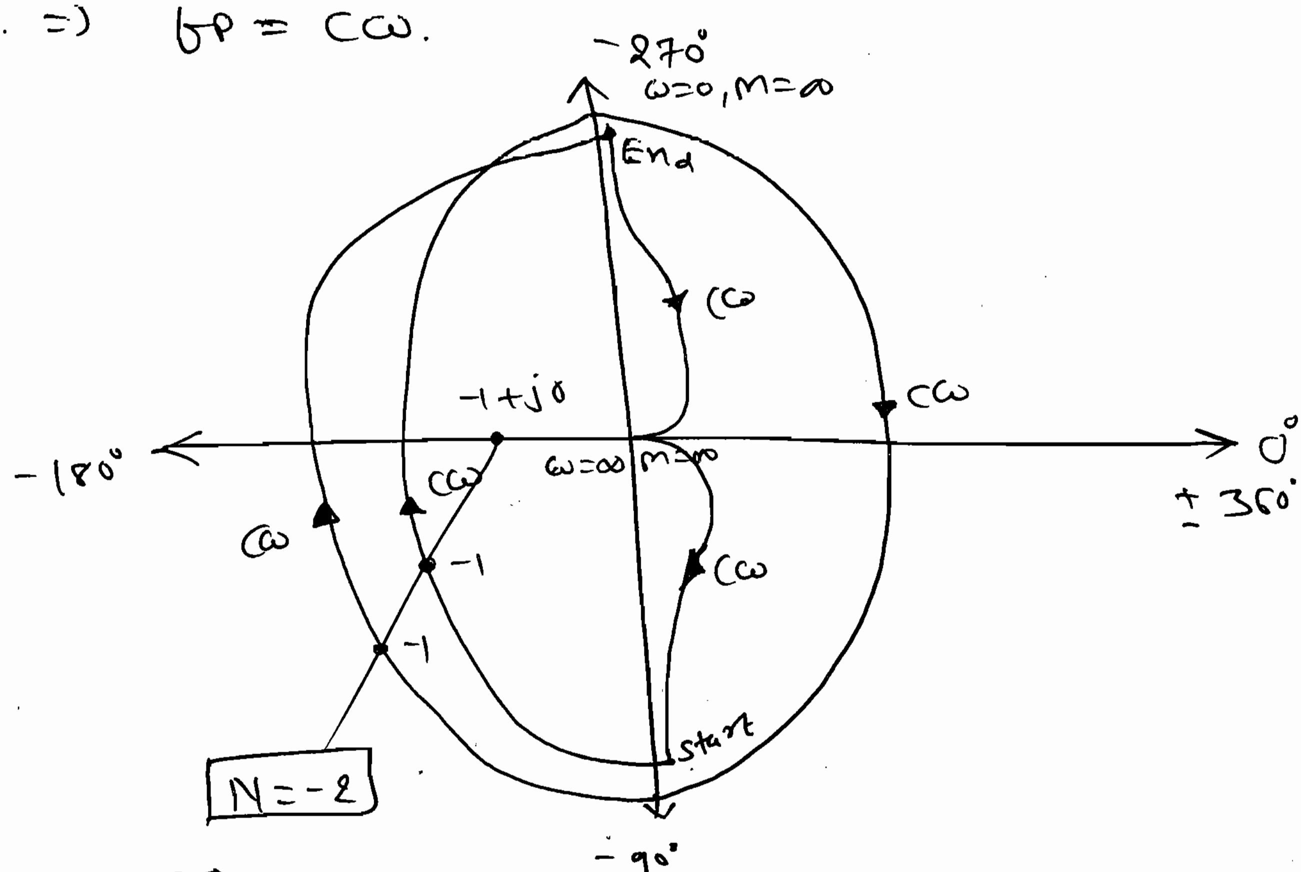
Soln: $M = \dfrac{10}{\omega^3 \sqrt{\omega^2 + 100}}$, $\phi = -270^\circ - \tan^{-1}(\omega/10)$

⇒ $\omega = 0$ ⇒ $M = \infty$, $\phi_1 = -270^\circ$.

⇒ $\omega = \infty$ ⇒ $M = 0$, $\phi_2 = -360^\circ$.

E.D. ⇒ $\phi_1 - \phi_2 = +ve = $ CW.

S.D. ⇒ FP = CW.



$P - Z = 0 - 0 = 0$

⇒ $\boxed{N \neq P}$ ⇒ US

⇒ The no. of CL Poles on RHS plane

$N = P - Z$

$\therefore -2 = 0 - Z \Rightarrow \boxed{Z = 2} \rightarrow$ CL Poles on RH s-plane.

$\boxed{Q}$ $G(s)H(s) = \dfrac{1}{s(1-s)}$.

Soln: $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$, $\phi = -90^\circ - (-\tan^{-1}\omega)$.

$\phi = -90^\circ + \tan^{-1}\omega$.

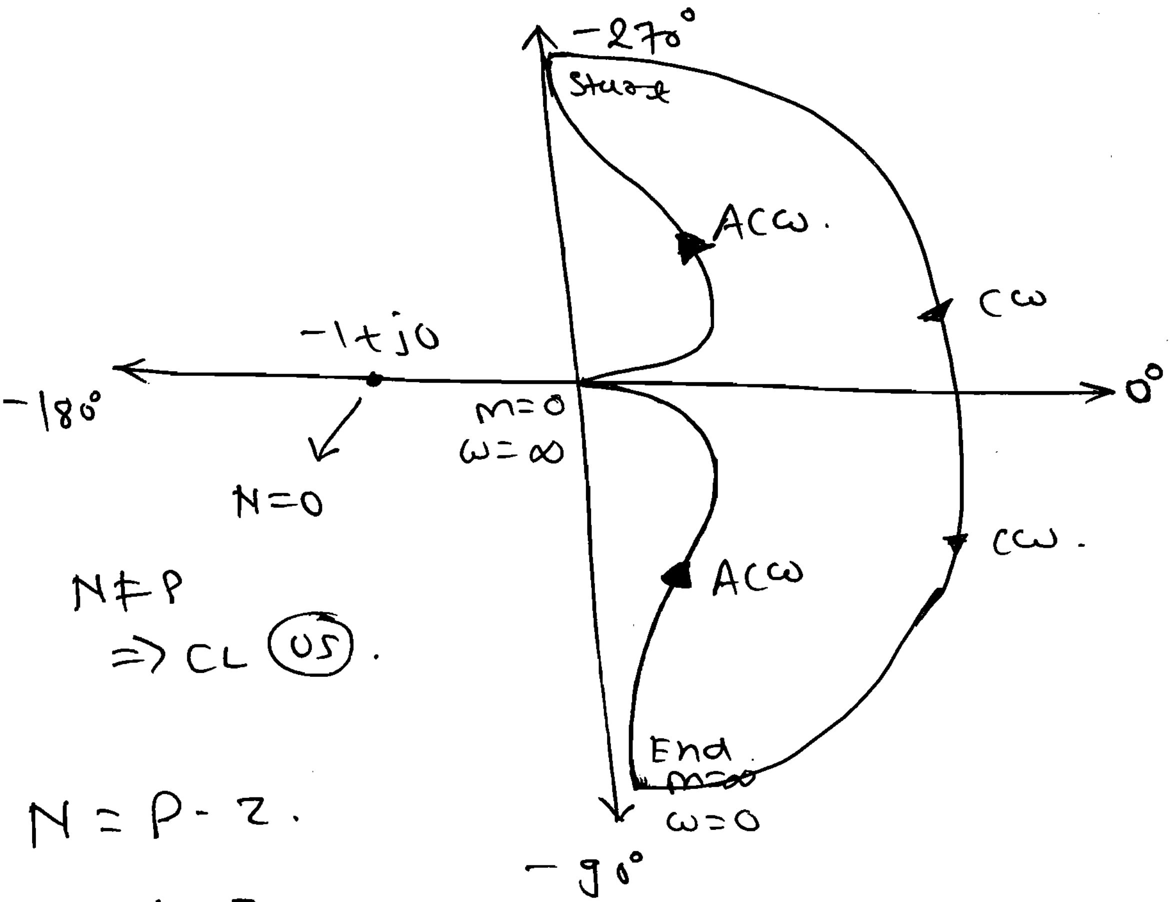
$\omega = 0 \Rightarrow M = \infty$, $\phi = -90^\circ$

$\omega = \infty \Rightarrow M = 0$, $\phi = 0^\circ$.

$\boxed{P = 1}$

F.D. ⇒ $\phi_1 - \phi_2 = -ve \Rightarrow$ ACω.

S.D. ✗ (∵ -ve sign in T.F.).



N ≠ P

⇒ CL US.

⇒ $N = P - Z$.

$\therefore 0 = 1 - Z$.

⇒ $\boxed{Z = 1}$ ⇒ 1 CL pole on RHS plane.

(OR) By RH- criterion

$\xrightarrow{CE}$ $1 + GH(s) = 0$

$$\Rightarrow 1 + \frac{1}{s(1-s)} = 0$$

$$\Rightarrow s - s^2 + 1 = 0.$$

$$\Rightarrow -s^2 + s + 1 = 0.$$

1 sign change

$$\begin{array}{c|cc} s^2 & -1 & 1 \\ s^1 & 1 & \\ s^0 & 1 & \end{array}$$

$\Rightarrow$ 1 sign chang

$\Rightarrow$ 1 CL pole on RHS plane.

✓

$\boxed{Q}$ Find the range of K value for close loop system stability by using Nyquist stability Analysis, to the following system.

$$G(s) \cdot H(s) = \frac{K}{(s+1)(s+2)(s+3)}.$$

Soln: $M = \dfrac{K}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+4} \times \sqrt{\omega^2+9}}.$

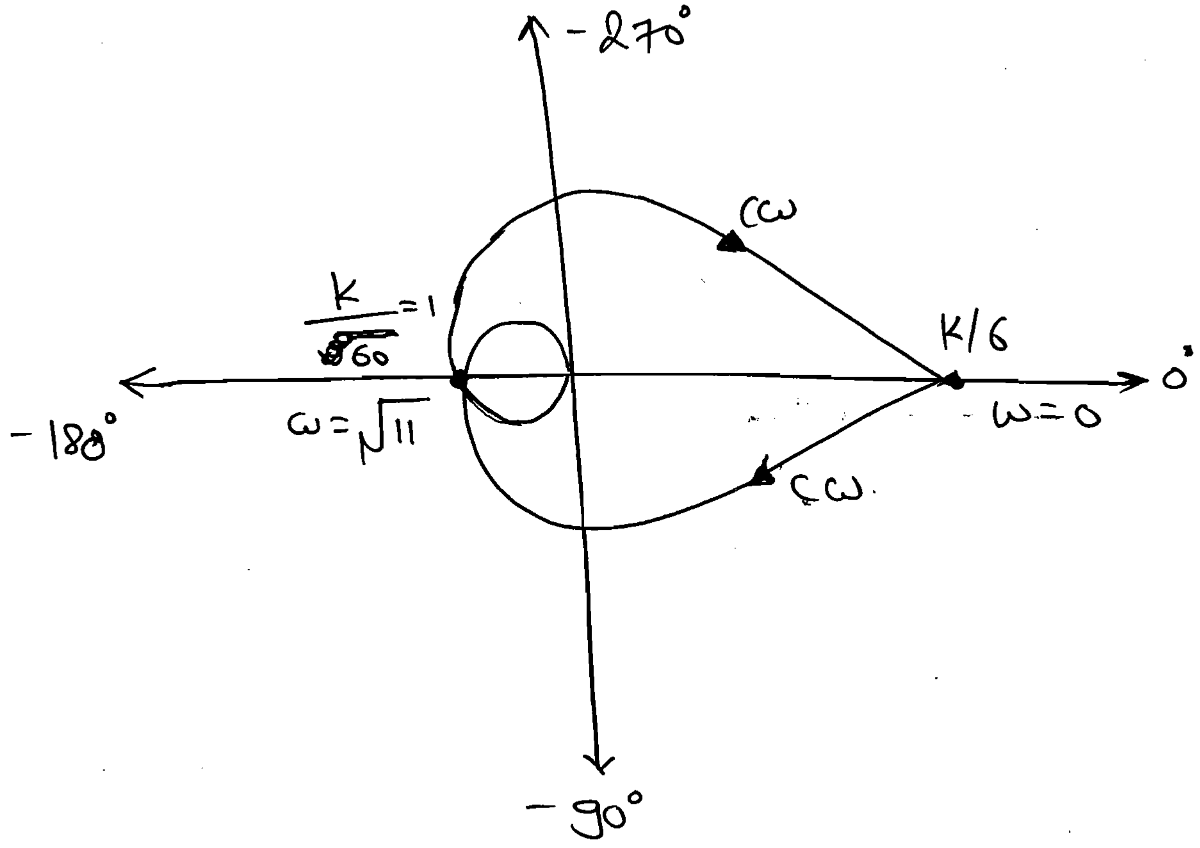
$$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3).$$

$\omega = 0 \Rightarrow M = \dfrac{K}{\sqrt{36}} = \dfrac{K}{6}, \quad \phi = 0^\circ.$

$\omega = \infty \Rightarrow M = 0, \quad \phi = -270^\circ.$

E.P $\Rightarrow$ $\phi_1 - \phi_2 = +ve = C.W.$

S.P $\Rightarrow$ FP = C.W.



$\Rightarrow$ I.P. with -180° axis is $\frac{K}{60} = \frac{K}{60}$.

* **Procedure to find the range of k values:-**

S1: Assume that the I.P. with -180° must be equal to the critical point, that means the mag. of I.P. = mag of critical point

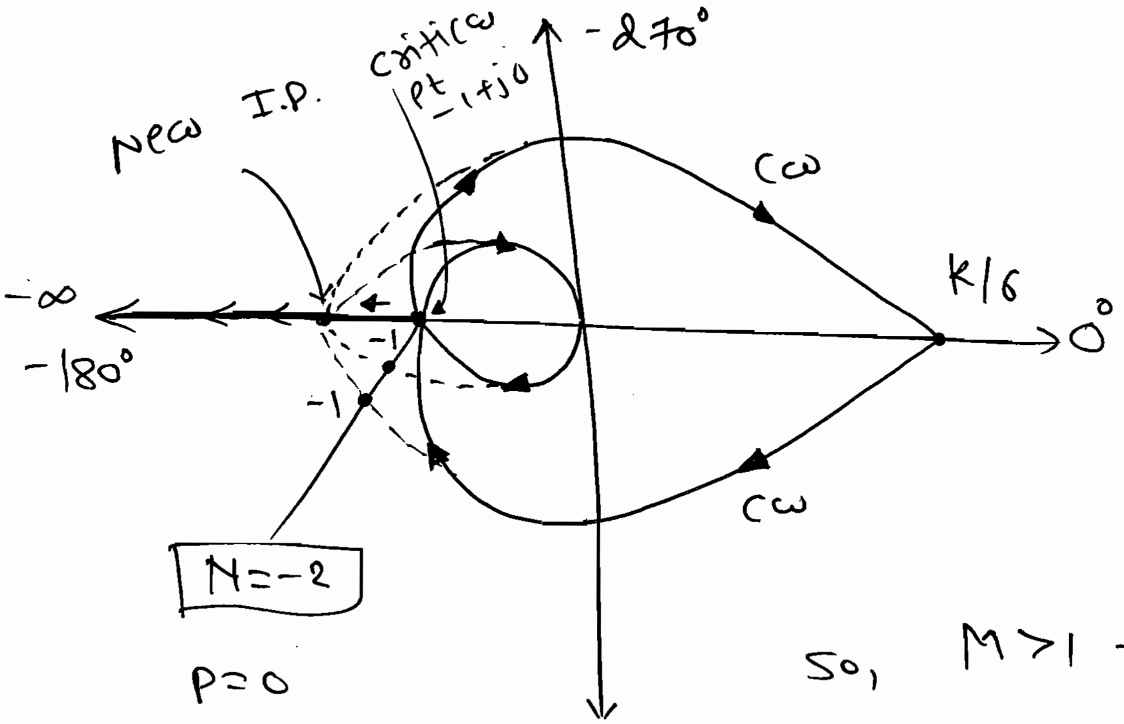
i.e. Mag. $\boxed{M=1}$

$\Rightarrow$ In the above case $\boxed{K/60=1.}$

S2: Shift the I.P. towards $-\infty$ by Considering $\boxed{M>1}$.

$\Rightarrow$ In this case, the critical point inside the loop. For this get the no. of encirclement & Condition for stability.

⇒



P=0

N ≠ P

CLS → US

So, M > 1 → US

⇒ $\frac{K}{60} > 1$

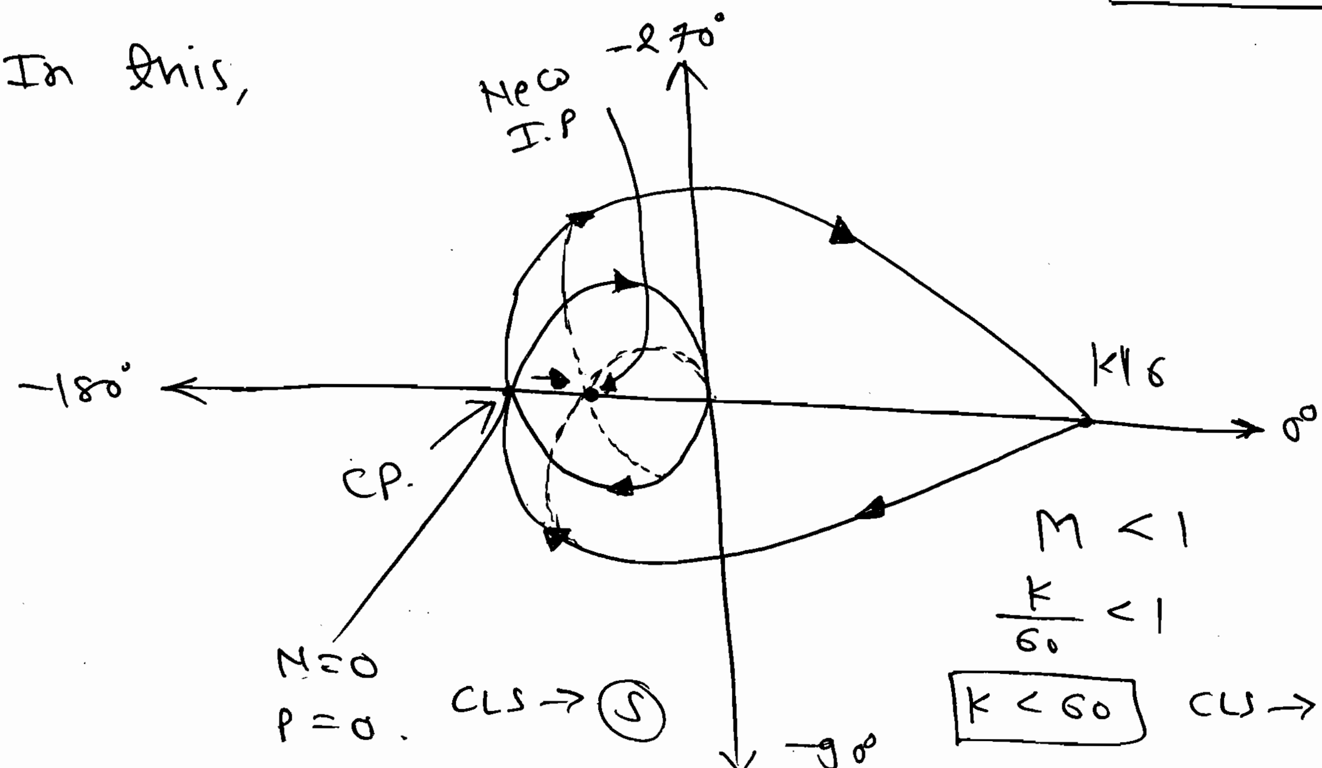
⇒ $\boxed{K > 60}$ ⇒ US.

N = P − Z ⇒ −2 = 0 − Z

⇒ $\boxed{Z = 2}$ → CL Poles on RHS plane.

$\boxed{S3}$: Shift the intersection point towards the origin by considering $\boxed{M < 1.}$

⇒ In this,



M < 1

$\frac{K}{60} < 1$

$\boxed{K < 60}$ CLS → S.

N=0

P=0. CLS → S

(54): Whenever the stability condition is less than certain value then the lower limit is desided by I.P. with 0°.

$\Rightarrow$ The intersection point with 0° must be greater than -1.

$\Rightarrow$ In the above problem $\frac{K}{6} > -1$.

$$\Rightarrow \boxed{K > -6}$$

So, $\boxed{-6 < K < 60.}$ $\Rightarrow$ Stable system.

Q $GH(s) = \dfrac{K(s+3)}{s(s-1)}$.

Soln: $M = \dfrac{K \times \sqrt{\omega^2+9}}{\omega \times \sqrt{\omega^2+1}}$.

$\Rightarrow \phi = -90° + \tan^{-1}(\omega/3) - 180° + \tan^{-1}(\omega)$.

$\phi = -270° + \tan^{-1}(\omega) + \tan^{-1}(\omega/3)$.

$\Rightarrow \omega = 0 \Rightarrow M = \infty, \quad \phi = -270°$.

$\Rightarrow \omega = \infty \Rightarrow M = 0, \quad \phi = -90°$.

E.D. $\Rightarrow \phi_1 - \phi_2 \Rightarrow -ve \Rightarrow$ ACW.

S.D X.

$\Rightarrow$ I.P. with $-180°$.

$\therefore -180° = -270° + \tan^{-1}\left(\dfrac{\omega + \omega/3}{1 - \omega^2/3}\right)$.

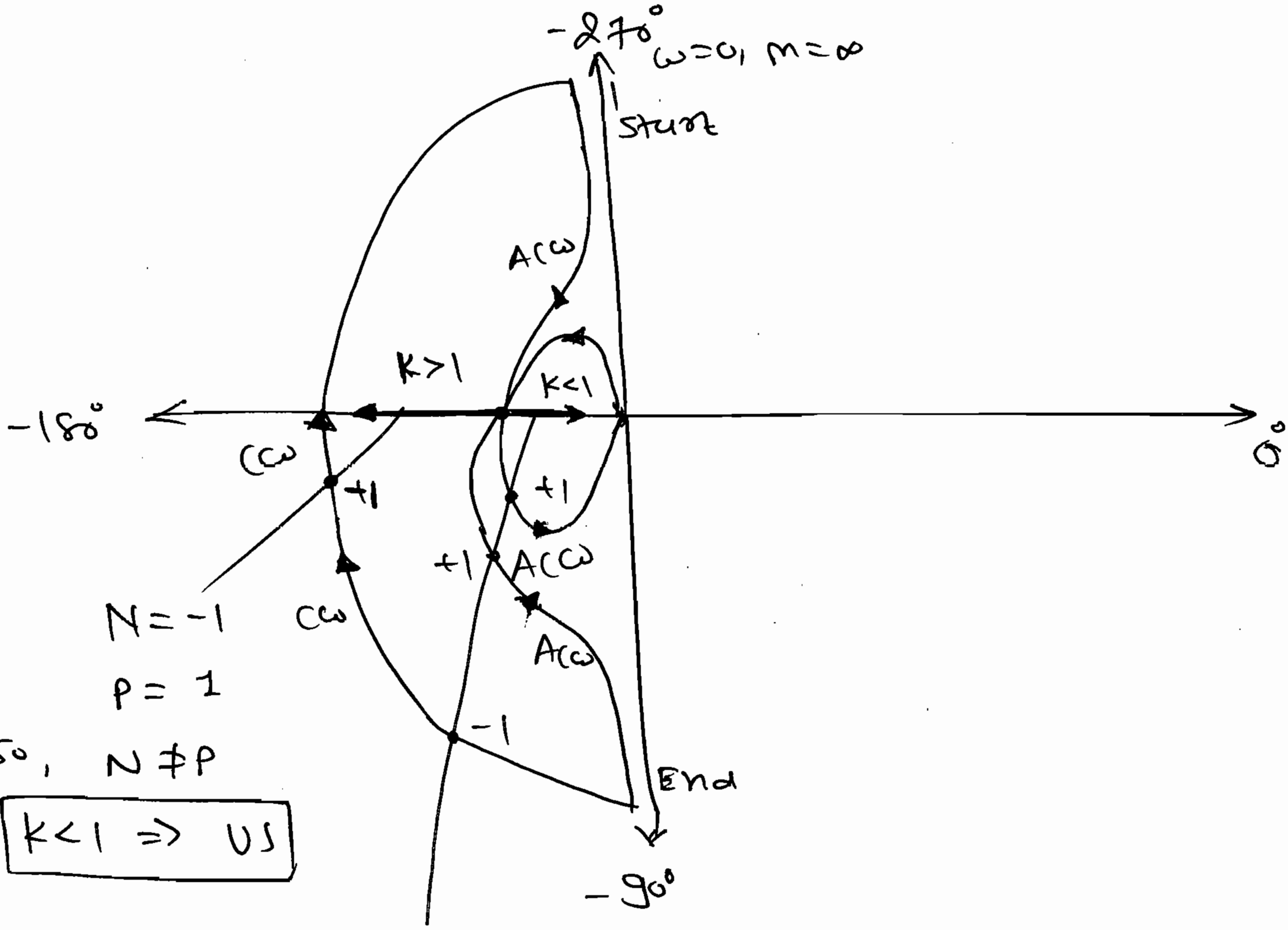
$\therefore 90° = \tan^{-1}\left(\dfrac{4\omega}{3-\omega^2}\right)$.

$\Rightarrow \quad \omega = \sqrt{3}$ rad/sec.

$$\Rightarrow \quad M\Big|_{\omega=\sqrt{3}} = \frac{K\sqrt{3+9}}{(\sqrt{3})^2 \times \sqrt{3+1}}.$$

$$= \frac{K\sqrt{12}}{\sqrt{3}\times 2} = \frac{K \times 2\cancel{\sqrt{3}}}{2\cancel{\sqrt{3}}} = K.$$

$$\therefore \quad \boxed{M\Big|_{\omega=\sqrt{3}} = K}$$



N = -1

P = 1

So, N ≠ P

$\boxed{K<1 \Rightarrow \text{US}}$

N = +1+1-1 = +1

N = +1, P = +1.

So, $\boxed{N=P}$ ⇒ CLS (S). $\boxed{K<1}$

No I.P. with 0°

So, $\boxed{0 < K < 1 \Rightarrow \text{CLS (S)}}$.

$\boxed{Q}$ $G(s)\cdot H(s) = \dfrac{K(s+2)}{(s+1)(s-1)} \Rightarrow \boxed{P=+1}$

Soln:

$$M = \frac{K\sqrt{\omega^2+4}}{\sqrt{\omega^2+1} \times \sqrt{\omega^2+1}}$$

$$\phi = -\cancel{\tan^{-1}\omega} + \tan^{-1}(\omega/2) - 180^\circ + \cancel{\tan^{-1}\omega}$$

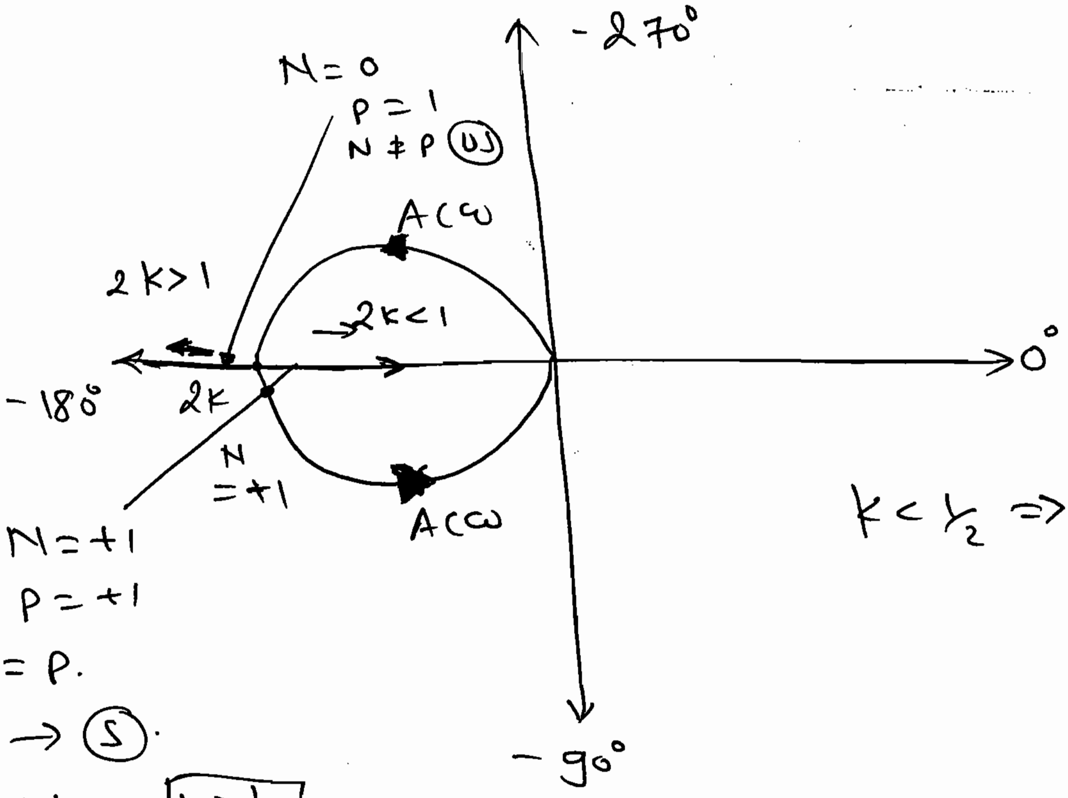
$$\phi = -180^\circ + \tan^{-1}(\omega/2).$$

$\omega = 0 \Rightarrow M = 2K, \quad \phi = -180^\circ.$

$\omega = \infty \Rightarrow M = 0, \quad \phi = -90^\circ.$

E.D. $\Rightarrow \phi_1 - \phi_2 = -ve \Rightarrow$ ACW

S.D. X.



$K < 1/2 \Rightarrow$ US

$N = P.$

CLS → S.

$2K > 1 \Rightarrow \boxed{K > 1/2}$.

S

$\boxed{Q}$ $G(s)\cdot H(s) = \dfrac{K(s-2)}{(s+2)}.$

Soln:

$$M = \frac{K\sqrt{\cancel{\omega^2+4}}}{\sqrt{\cancel{\omega^2+4}}} = K.$$

$\Rightarrow \quad \phi = -\tan^{-1}(\omega/2) + 180^\circ - \tan^{-1}(\omega/2)$

$\phi = 180^\circ - 2\tan^{-1}(\omega/2).$

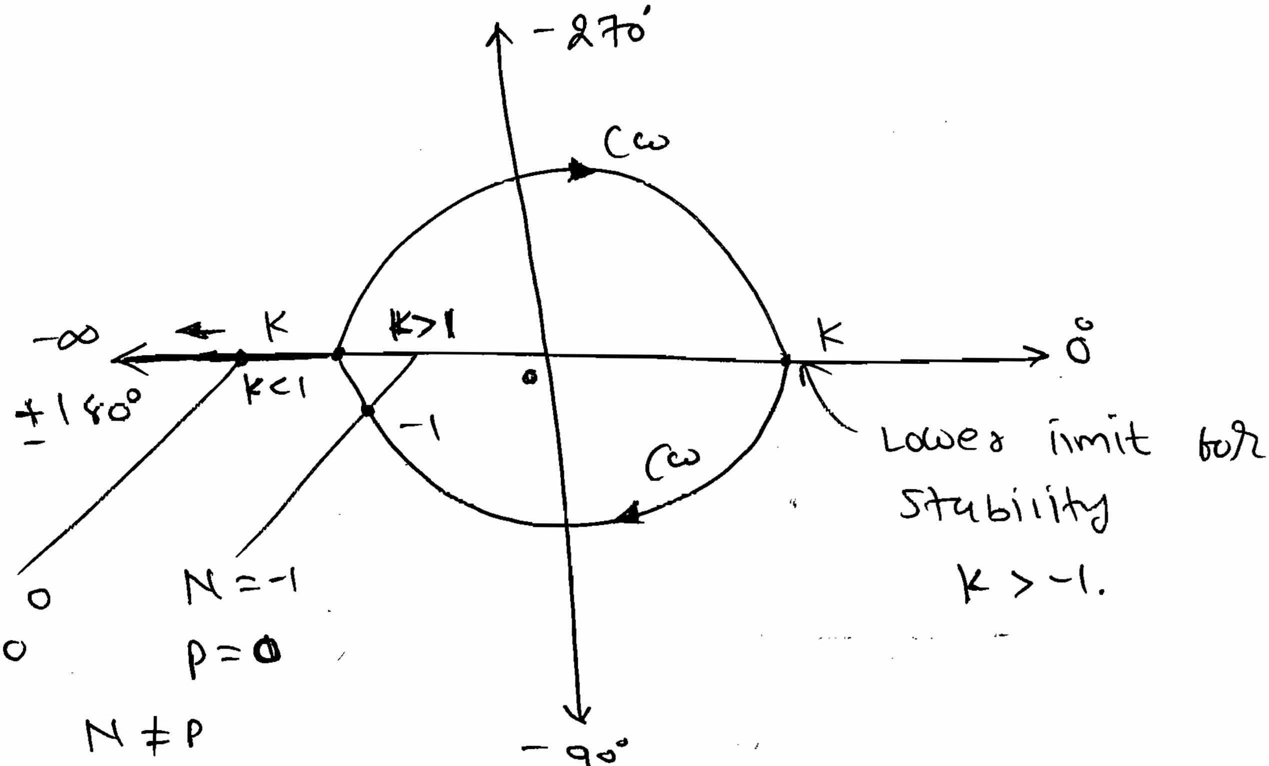
$\omega = 0 \Rightarrow M = K$, & $\phi_1 = +180^\circ$

$\omega = \infty \Rightarrow M = K$, & $\phi = 180^\circ - 2(90^\circ)$

$\phi_2 = 0^\circ$

$\Rightarrow$ E.D. $\Rightarrow \phi_1 - \phi_2 = 180^\circ - 0^\circ = +ve = CCW.$

S.D ✗.



N = 0
P = 0
⇓
CLSY: S
$\boxed{K<1}$

N = -1
P = 0

$N \neq P$
So, $K>1 \Rightarrow$ US.

So, $\boxed{-1 < K < 1} \Rightarrow$ CLS $\rightarrow$ S

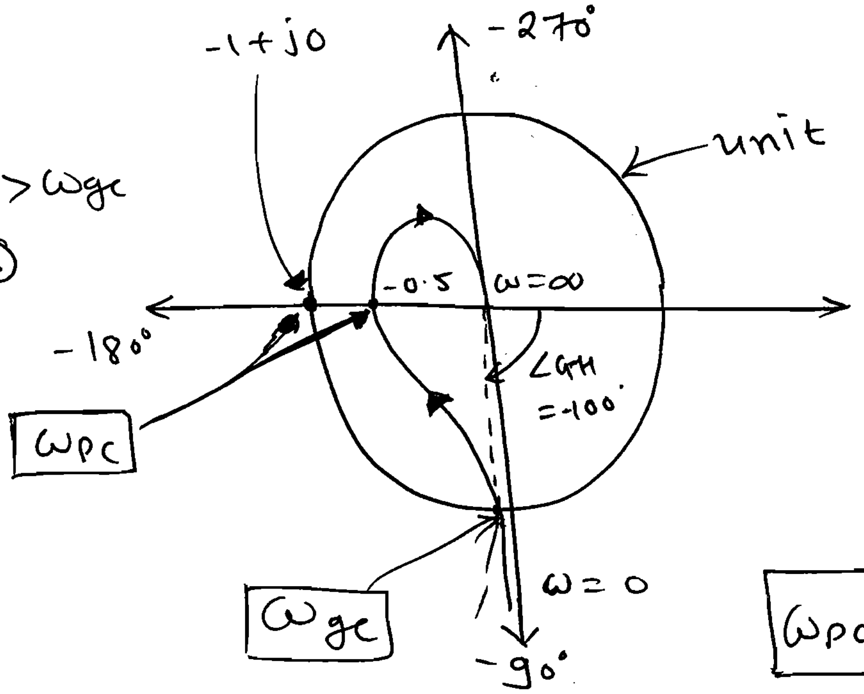
* GM & PM:

$\Rightarrow \quad GM = \dfrac{1}{M|_{\omega=\omega_{pc}}}$

$\Rightarrow \quad PM = 180^\circ + \angle GH|_{\omega=\omega_{gc}}.$

Q Identify the Stability to the following Polar plots:

① $\omega_{pc} > \omega_{gc}$ $\Rightarrow$ S



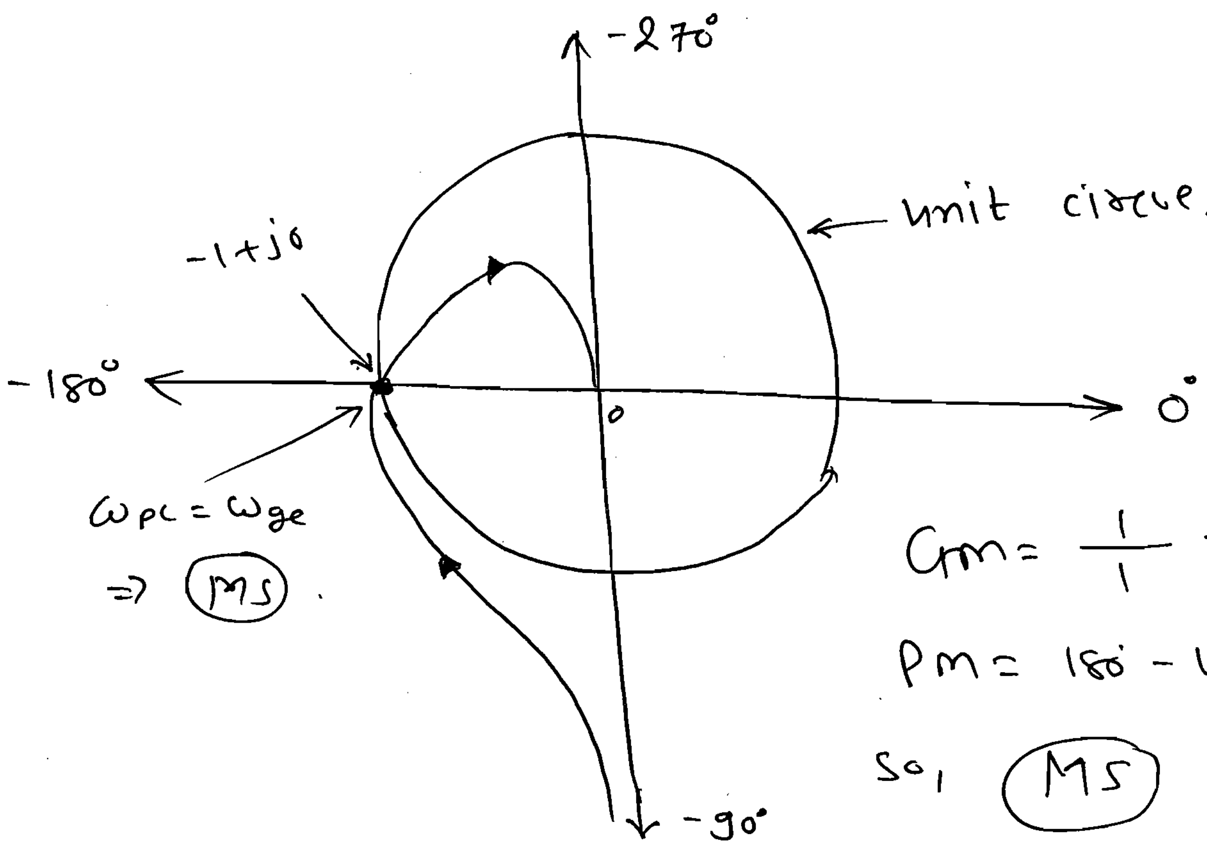
$\boxed{\omega_{pc} > \omega_{gc}} \rightarrow$ S

$$\Rightarrow GM = \frac{1}{M|_{\omega=\omega_{pc}}} = \frac{1}{0.5} = 2, > 1 \ (L)$$

$$PM = 180° + \angle GH = 180° + (-100°) = +80° > 0$$

So, CL system $\rightarrow$ S.

②



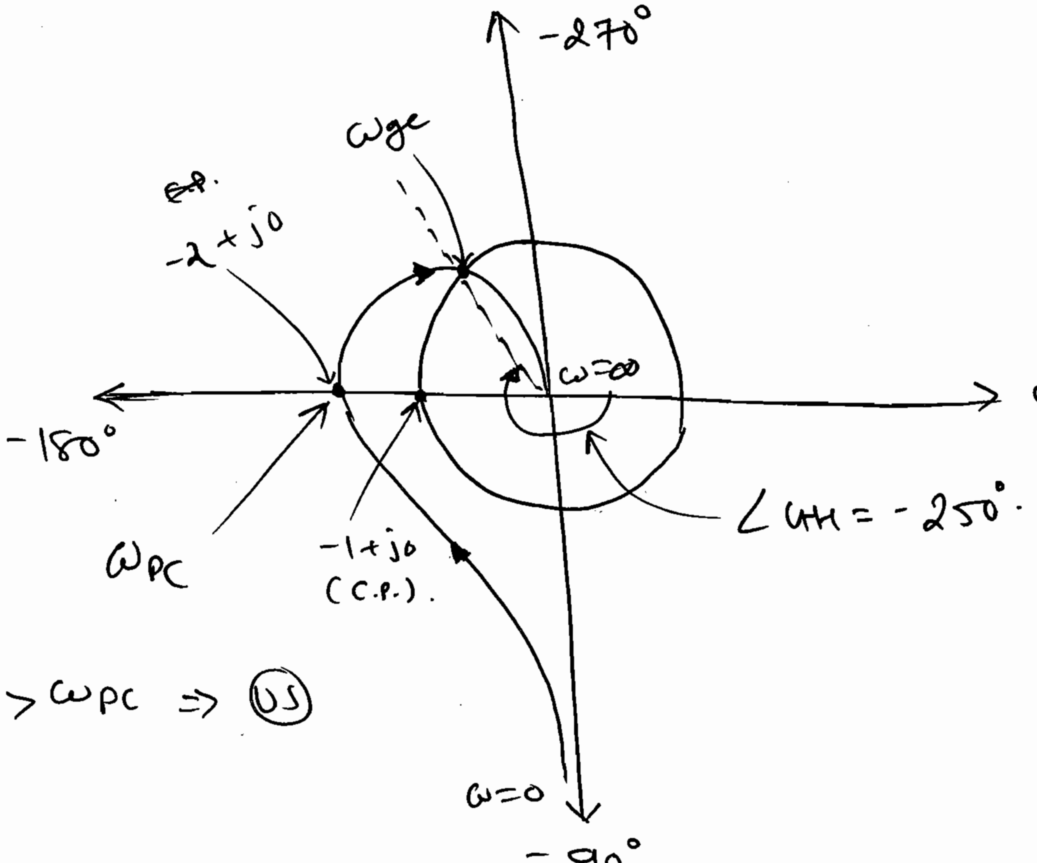
$\omega_{pc} = \omega_{gc}$ $\Rightarrow$ MS.

$$GM = \frac{1}{1} = 1.$$

$$PM = 180° - 180° = 0°$$

So, MS

③



$\omega_{gc} > \omega_{pc} \Rightarrow$ US

$$\Rightarrow GM = \frac{1}{M|_{\omega=\omega_{pc}}}$$

$$= \frac{1}{2} = 0.5 < 1 \quad \text{L}$$

$$\Rightarrow PM = 180° + \angle GH = 180° - 250° = -70° < 0$$

US.

So, CLS → US.

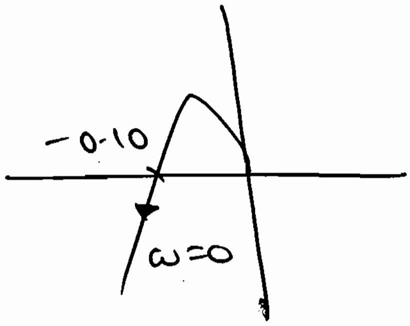
Note:

(i) Whenever the plot intersect −180° line with mag. less than 1 (i.e $\boxed{M<1}$), the sys. is $\boxed{\text{stable}}$ because here $\boxed{\omega_{pc} >> \omega_{gc}}$

(ii) Whenever the plot intersect −180° line with mag. $\boxed{M=1}$, then the sys. is $\boxed{\text{Marginal stable}}$ because here $\boxed{\omega_{pc} = \omega_{gc}}$

(iii) Whenever the plot intersect $-180°$ line with mag. $\boxed{M>1}$ then the syst. is $\boxed{\text{unstable}}$ because here $\boxed{\omega_{gc} > \omega_{pc}}$

Q The Polar plot of $G(s) \cdot H(s)$ for $K=10$ is given below. The range of 'K' for sys. stability is ?

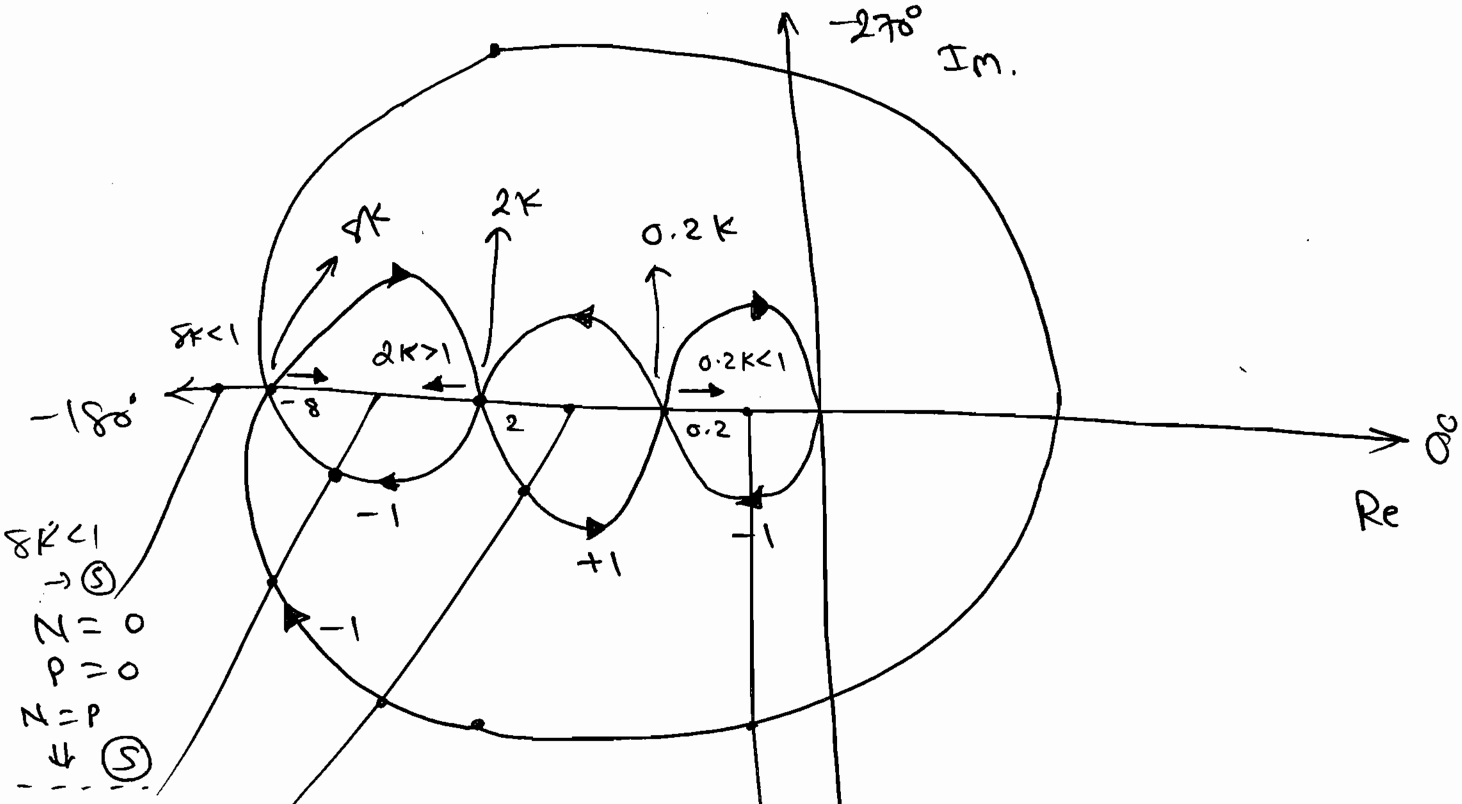


Soln: Note: To find the range of K value, product the K with given I.P. divided by given K value. (i.e here 10).

So, For CLS to be (S) $K \cdot \frac{0.10}{10} < 1.$

$\therefore$ $\boxed{K < 100}$

Q The Polar diagram of a Conditionally Stable sys. for open loop gain $K=1$ is given is shown in the fig. The OLTF of the sys. is known to be stable. The CL system stable for ?



$N = -2$

$N \neq P$

US

$N = -1 + 1 = 0$

$P = 0$

$N = P \Rightarrow$ S

$2K > 1$ &

$0.2K < 1 \rightarrow$ S

$N = -1 - 1 = -2$

$P = 0$

$N \neq P$ US

(1) OL S

$\xrightarrow{\text{OL RH}}$ $\boxed{P = 0}$

$0.2K < 1$ & $2K > 1$

$\frac{K}{5} < 1$ $\boxed{K > 0.5}$

$\boxed{K < 5}$

$\boxed{0.5 < K < 5} \Rightarrow$ S

(2) $\rightarrow 8K < 1. \Rightarrow \boxed{K < \frac{1}{8}} \Rightarrow$ S

So, Ans: $\boxed{K < 1/8 \ \& \ 0.5 < K < 5 \Rightarrow \text{S}}$

$\boxed{*}$ Consider the following Nyquist plots of loop T.F. over $\omega = 0$ to $\omega = \infty$, which of the following plots represents stable closed loop sys.?

* Calculation of Gain Margin & Phase Margin:

[a] Calculate the gain margin for

$$GH(s) = \frac{1}{s(s+1)(s+2)}.$$

Soln:

$$M = \frac{1}{\omega\sqrt{\omega^2+1} \times \sqrt{\omega^2+4}}.$$

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}(\omega/2).$$

Now, $$Gm = \frac{1}{M|_{\omega=\omega_{pc}}}$$

for ω_{pc}, $\phi|_{\omega=\omega_{pc}} = -180^\circ.$

$$\therefore -180^\circ = -90^\circ - \tan^{-1}\left(\frac{\omega_{pc} + \frac{\omega_{pc}}{2}}{1 - \frac{\omega_{pc}^2}{2}}\right).$$

$$\therefore \tan(90^\circ) = \frac{3\omega_{pc}}{2-\omega_{pc}^2}.$$

$$\therefore \boxed{\omega_{pc} = \sqrt{2} \text{ rad/sec.}}$$

Now, $$M|_{\omega=\omega_{pc}} = \frac{1}{\sqrt{2}\sqrt{3}\sqrt{6}} = 1/6.$$

So, $$Gm = \frac{1}{M|_{\omega=\omega_{pc}}} = \frac{1}{1/6} = 6.$$

$$\therefore \boxed{Gm = 6}$$

$$Gm_{dB} = 20\log 6 = \boxed{15.56\,dB.}$$

* Steps for finding GM:

$\boxed{S1}$: find ω_{pc} by using $\angle GH = -180^\circ$.

$\boxed{S2}$: find $M\big|_{\omega=\omega_{pc}}$.

$\boxed{S3}$: $GM = \dfrac{1}{M\big|_{\omega=\omega_{pc}}}$.

$\boxed{Q}$ Calculate the PM for $GH(s) = \dfrac{1}{s(s+1)}$

Sol^n: $M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$, $\phi = -90^\circ - \tan^{-1}(\omega)$.

* Steps for finding PM:

$\boxed{S1}$: find $\omega_{gc} \longrightarrow M = 1$.

$\boxed{S2}$: $PM = 180^\circ + \angle GH\big|_{\omega=\omega_{gc}}$.

$$\Rightarrow \quad M = 1 = \frac{1}{\omega\sqrt{\omega^2+1}}.$$

$$\Rightarrow \quad \omega^2(\omega^2+1) = 1$$

$$\omega^4 + \omega^2 - 1 = 0.$$

$$\therefore \omega^2 = 0.618 \checkmark, \quad \omega^2 = -1.618 \times$$

$$\therefore \boxed{\omega_{gc} = 0.786 \text{ rad/sec}}$$

$$\therefore \angle GH\big|_{\omega=\omega_{gc}} = -90 - \tan^{-1}(0.786).$$

$$\therefore \angle GH\big|_{\omega=\omega_{gc}} = -128.17^\circ.$$

$$\therefore PM = 180^\circ + \angle GH\big|_{\omega=\omega_{gc}} = 180^\circ - 128.17^\circ$$

$$\boxed{PM = 51.83}$$

[Q] find the K value to get the PM $= 30°$, $G(s)\cdot H(s) = \dfrac{K}{s(s+1)}$.

Soln:

$$PM = 180° + \angle GH|_{\omega=\omega_{gc}}$$

$$\therefore 30° = 180° + (-90° - \tan^{-1}\omega)$$

$$\therefore -60° = -\tan^{-1}(\omega_{gc})$$

$$\omega = \tan 60°$$

$$\therefore \boxed{\omega = \sqrt{3} \text{ rad/s.}}$$

at $\omega = \omega_{gc}$, $M = 1$.

$$\therefore M|_{\omega=\omega_{gc}} = 1.$$

$$\therefore \frac{K}{\omega_{gc}\sqrt{\omega_{gc}^2+1}} = 1.$$

$$\therefore K = \sqrt{3} \times \sqrt{3+1} = 2\sqrt{3}.$$

$$\therefore \boxed{K = 2\sqrt{3}.}$$

[Q] (i) find the K value for the PM $= 60°$.

Soln: $G(s)\cdot H(s) = \dfrac{K}{s(s+2)(s+4)}$.

$$\Rightarrow M = \frac{K}{\omega\sqrt{\omega^2+4} \times \sqrt{\omega^2+16}}.$$

$$\phi = -90° - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/4).$$

$$\Rightarrow PM = 180° + \angle GH|_{\omega=\omega_{gc}}.$$

$\therefore \quad 60^\circ = 180^\circ + \left(-90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\omega/4)\right).$

$\therefore \quad -30^\circ = -\tan^{-1}\left(\frac{\frac{\omega}{2} + \frac{\omega}{4}}{1 - \omega^2/8}\right).$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{6\omega}{8-\omega^2}.$

$8-\omega^2 = \cancel{6/\sqrt{3}\,\omega}.\ 6\sqrt{3}\omega$

$\therefore \quad \cancel{-\omega^2 + \frac{6}{\sqrt{3}}\omega + 8 = 0}.$

$\therefore \quad -\omega^2 - 6\sqrt{3}\omega + 8 = 0$

$\Rightarrow \quad \boxed{\omega_{gc} = 0.72 \text{ rad/sec}}.$

Now, $\quad M\Big|_{\omega=\omega_{gc}} = 1.$

$\therefore \quad \frac{K}{0.72\sqrt{(0.72)^2+4} \times \sqrt{(0.72)^2+16}} = 1.$

$\therefore \quad \boxed{K = 4.456}$

(ii) find the K value to get $GM = 20\,dB$.

Sol:

$GM = \frac{1}{M\big|_{\omega=\omega_{pc}}}$

given $\quad GM = 20\,dB \Rightarrow GM = 20 = 20\log(GM)$

$GM = 10.$

$\therefore$ for $\omega_{pc} \quad \phi = -180^\circ.$

$\therefore \quad -180^\circ = -90^\circ - \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{4}\right),$

$$\therefore \quad 90^\circ = \tan^{-1}\left(\frac{\frac{\omega}{2}+\frac{\omega}{4}}{1-\omega^2/8}\right)$$

$$\therefore \quad 1-\frac{\omega^2}{8} = 0$$

$$\boxed{\omega_{pc} = \sqrt{8} \text{ rad/sec}}$$

$$\Rightarrow \quad G_m = \frac{1}{M|_{\omega=\omega_{pc}}}$$

$$\therefore \quad 10 = \left(\frac{K}{\sqrt{8}\times\sqrt{8+4}\times\sqrt{8+16}}\right)^{-1}.$$

$$\therefore \quad 0.1 = \frac{K}{\sqrt{8}\times\sqrt{12}\times\sqrt{24}}.$$

$$\therefore \quad K = 48\times 0.1.$$

$$\therefore \quad \boxed{K = 4.8}$$

$\boxed{a}$ The OLTF of unity f/b sys. is $G(s) = \left(\frac{as+1}{s^2}\right)$, the value of 'a' to get the PM = 45°.

Soln:

$$M = \frac{\sqrt{(a\omega)^2+1}}{\omega^2}.$$

$$\phi = -180^\circ + \tan^{-1}(\omega a).$$

$$PM = 180^\circ + \angle GH|_{\omega=\omega_{gc}}.$$

$$\therefore \quad 45^\circ = \cancel{180} - \cancel{180} + \tan^{-1}(a\omega).$$

$$\therefore \quad 1 = a\omega \Rightarrow \quad \boxed{\omega_{gc} = 1/a.}$$

$$\Rightarrow \quad M\big|_{\omega=\omega_{gc}} = 1.$$

$$\Rightarrow \quad \frac{\sqrt{1 + a^2 \times \frac{1}{a^2}}}{a^2} = 1.$$

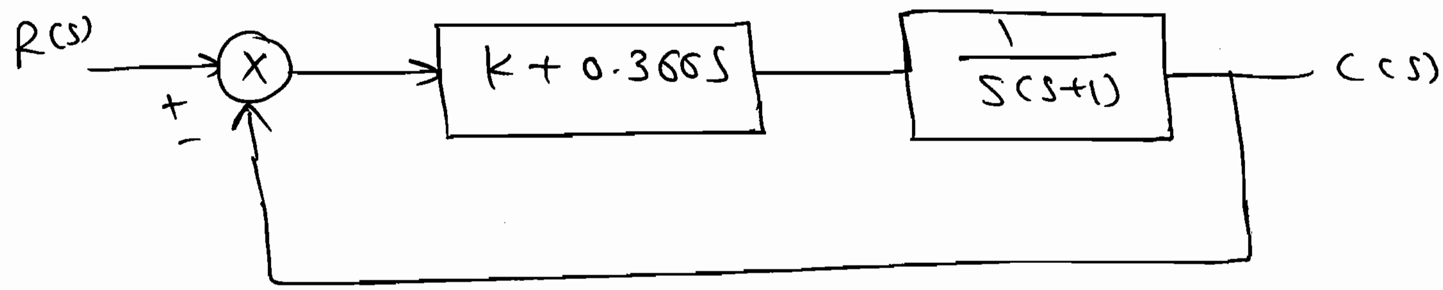
$$\therefore \quad a^2 = \frac{1}{\sqrt{2}}.$$

$$\therefore \quad a^4 = \frac{1}{2}.$$

$$a = (2)^{-1/4}.$$

$$\therefore \quad \boxed{a = 0.8408.}$$

$\boxed{Q}$ If the component connected sys as shown in fig. has pm of 60° at a cross over freq. of 1 rad/sec, the value of K is ______.



$\underline{\underline{\text{Sol}^n}}$: OLTF $\frac{\cancel{C(s)}}{\cancel{R(s)}} = \frac{(K + 0.366s)}{s(s+1)}.$

$G(s).H(s)$

$$PM = 180^\circ + \angle GH\big|_{\omega = 1\ \text{rad/sec}}.$$

$$\therefore \quad \cancel{45^\circ}\ 60^\circ = 180^\circ + \left(-90^\circ - \tan^{-1}(\omega_{gc}) + \tan^{-1}\left(\frac{0.366\omega}{K}\right)\right).$$

$\Rightarrow$ ~~Equating~~

$$\therefore \tan^{-1}(\omega) - \tan^{-1}\left(\frac{0.366\omega}{K}\right) = 30^\circ.$$

$$\therefore \tan^{-1}(1) - \tan^{-1}\left(\frac{0.366(1)}{K}\right) = 30^\circ.$$

$$\therefore 45^\circ - \tan^{-1}\left(\frac{0.366}{K}\right) = 30^\circ.$$

$$15^\circ = \tan^{-1}\left(\frac{0.366}{K}\right).$$

$$0.268 = \frac{0.366}{K}.$$

$$\boxed{K = 1.366}$$

<u>Note:</u> To calculate Gm & pm required OLTF of either unity (or) Non-unity f/b sys. i.e $G(s)$ (or) $G(s)\cdot H(s)$.

Q The loop gain of a Nyquist plot $GH(s) = \frac{\pi e^{-j0.25s}}{s}$ passes through the real axis at the point is ____.

Soln: Passing through the −ve real axis means it is a I.P. with −180°. i.e. mag. at ω_{pc}.

$$\therefore \angle GH(s) = -180^\circ = -90^\circ - 0.25\omega \times \frac{180}{\pi}$$

$$\therefore \cancel{0.25 \times \omega \times \frac{180}{\pi}} = \cancel{90^\circ}$$

$$\therefore 0.25 \times \omega \times \frac{180}{\pi} = 90$$

$\therefore$ ~~$\omega_{pc} =$~~ $\omega_{pc} = \frac{\pi}{0.5} = 2\pi$

$\therefore \boxed{\omega_{pc} = 6.28 \text{ rad/sec}}$

$\therefore M \Big|_{\omega = \omega_{pc}} = \frac{\pi}{2\pi} = \frac{1}{2} = 0.5.$

$\therefore$ I.Pt. Rec $(-0.5, j0)$.

Polar $\Rightarrow$ $0.5 \angle 0^\circ$.

Q Calculate the Gm & pm to the above sys.

Soln:

$$Gm = \frac{1}{M}\Big|_{\omega = \omega_{pc}}$$

$$= \frac{1}{0.5}$$

$$\Rightarrow \boxed{Gm = 2}$$

for ω_{gc} $M = 1$.

$$\therefore \frac{\pi}{\omega} = 1 \Rightarrow \boxed{\omega_{gc} = \pi}$$

$$\therefore Pm = 180^\circ + \left(-90^\circ - 0.25 \times \pi \times \frac{180^\circ}{\pi}\right).$$

$$\boxed{Pm = 45^\circ}$$

Q Calculate the Gm & pm for $GH(s) = \frac{e^{-s}}{s(s+1)}$.

Soln: $s \to j\omega$

$$GH(j\omega) = \frac{e^{-j\omega}}{j\omega(j\omega+1)}.$$

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}.$$

$$\phi = -90^\circ - \tan^{-1}(\omega) - \omega \times \cancel{\frac{\pi}{180}} \cdot \frac{180^\circ}{\pi}$$

$\angle GH = -180^\circ$ at $\omega = \omega_{pc}$.

$$\therefore -180^\circ = -90^\circ - \tan^{-1}(\omega) - \omega \times \frac{\pi}{180^\circ}$$

$$\therefore 90^\circ = \tan^{-1}(\omega) + \frac{\omega \times \pi}{180^\circ}$$

$$\therefore \tan^{-1}(\omega) + 57.3\omega - 90^\circ = 0.$$

$$\Rightarrow \boxed{\omega_{pc} = 0.88 \text{ rad/sec}}$$

In Calculator, write eqn i.e.

$\tan^{-1}(X) + 57.3(X) - 90$

then press shift + CALC.

$$Gm = \frac{1}{M|_{\omega=\omega_{pc}}}$$

$$= \frac{1}{\frac{1}{0.88\sqrt{(0.88^2+1)}}}$$

$$\boxed{Gm = 1.13}$$

$\Rightarrow$ Now, PM, $\quad M|_{\omega=\omega_{gc}} = 1.$

$$\therefore \frac{1}{\omega_{gc}\sqrt{\omega_{gc}^2+1}} = 1.$$

$$\therefore \omega_{gc}^2(\omega_{gc}^2+1) - 1 = 0$$

$$\Rightarrow \boxed{\omega_{gc} = 0.786 \text{ rad/sec.}}$$

$$\therefore PM = 180^\circ + \angle GH|_{\omega=\omega_{gc}}.$$

$$\therefore PM = 180^\circ + \left(-90^\circ - \tan^{-1}(0.786) - \left(0.786 \times \frac{180}{\pi}\right)\right)$$

$$\therefore \boxed{PM = 6.8^\circ}$$

$\boxed{Q}$ Calculate Gm & Pm $GH(s) = \frac{1}{(s+2)}$.

Sol^n: $M = \frac{1}{\sqrt{\omega^2+4}}$, $\phi = -\tan^{-1}(\omega/2)$.

$\Rightarrow$ $\cancel{-180^\circ = -180^\circ + \angle GH|_{\omega=\omega_{pc}}}$.

$\therefore$ $\cancel{-180^\circ = 180^\circ - \tan^{-1}(\omega_{pc}/2)}$.

$\therefore$ $\cancel{\tan^{-1}\left(\frac{\omega_{pc}}{2}\right) = 360^\circ}$.

$\therefore$ $\angle GH = -180^\circ$ at $\omega = \omega_{pc}$.

$\therefore$ $-180^\circ = -\tan^{-1}\left(\frac{\omega_{pc}}{2}\right)$.

$\therefore$ $\frac{\omega_{pc}}{2} = \tan(180^\circ)$.

$\therefore$ $\frac{\omega_{pc}}{2} = 0$ X.

**
* one Pole can give max angle 90°
i.e. ω varies from 0 to ∞ angle will varies, from 0 to 90°.

So, $\boxed{\omega_{pc} = \infty}$ rad/s.

$\Rightarrow$ $Pm = 180^\circ - \tan^{-1}(\omega/2)$.

$\therefore$ $M = \left.\frac{1}{\sqrt{\omega^2+4}}\right|_{\omega=\omega_{pc}}$

$= 1/\infty$

$\therefore$ $\boxed{M = 0}$

$\Rightarrow$ $Gm = \frac{1}{M|_{\omega=\omega_{pc}}} = 1/0 \Rightarrow \boxed{Gm = \infty}$

$\Rightarrow \quad M\big|_{\omega=\omega_{gc}} = 1.$

$\therefore \quad \dfrac{1}{\sqrt{\omega^2+4}} = 1. \quad \Rightarrow \quad \omega_{gc} = -3$

$\omega_{gc} = \pm\sqrt{3}$ X In valid.

$\rightarrow \quad \omega = 0 \longrightarrow$ $\underline{m}$ 0.5

$\omega = \infty \longrightarrow 0.$

So, one pole give max mag. 0.5.

$\xrightarrow{\underline{M<1}}$ ω_{gc} doesnot exist.

$\boxed{PM = \infty}$

Note:

$\Rightarrow$ whenever the plot (OR) T.F. gives less mag. than 1 (i.e) $M<1$ & -ve phase angle than -180° at all the freq. range then the $GM = PM = \infty$. (ω_{pc}, ω_{gc} does not exist).

$\boxed{a}$ $GH = 1/s.$

Soln: $M = 1/\omega, \quad \phi = -90^\circ.$

$\xrightarrow{GM}$ $\omega_{pc} \quad \angle GH = -180^\circ$

$-90^\circ = -180^\circ$ X

$\xrightarrow{\angle -180^\circ}$ $\omega_{pc} = \infty.$

$m\big|_{\omega=\omega_{pc}} = 0 \quad \Rightarrow \quad \boxed{GM = \infty}$

$PM \rightarrow \omega_{gc} \Rightarrow$ $M = 1$

$\frac{1}{\omega_{gc}} = 1 \Rightarrow \omega_{gc} = 1$ rad/sec

$\therefore PM = 180^\circ - 90^\circ$

$\boxed{PM = 90^\circ}$ $\boxed{\text{Stable}}$

Note:

$M < 1$, $\phi < -180^\circ$, ω_{gc} & ω_{pc} = doesn't exist

$GM = PM = \infty$

options: ① $PM = \infty$ → 1st priority.

② None → 2nd priority.

③ $\omega_{gc} = 0$ & calculate PM → last priority.

$\boxed{Q}$ $GH = \frac{1}{s^2}$.

Soln: $M = \frac{1}{\omega^2}$, $\phi = -180^\circ$.

$\therefore \angle GH = -180^\circ$

$\angle GH|_{\omega=\omega_{pc}} = -180^\circ$

$\underbrace{-180^\circ = -180^\circ}$

$\boxed{\omega_{pc} = \omega_{gc}}$

$M = 1$

$\frac{1}{\omega_{gc}^2} = 1$

$\boxed{\omega_{gc} = \omega_{pc} = 1 \text{ rad/sec}}$

$\Rightarrow M|_{\omega=\omega_{pc}} = 1/1 = 1$. $\Rightarrow$ MS

$\boxed{GM = 1}$ ①

$GM_{in\,dB} = 0$ dB

$PM = 180^\circ + \angle GH|_{\omega=\omega_{gc}}$

$PM = 180^\circ - 180^\circ$

$\boxed{PM = 0^\circ}$

$\boxed{Q}$ $GH(s) = \frac{1}{s^3}$.

Sol^n: $M = \frac{1}{\omega^3}$, $\angle GH = \phi = -270°$.

$\xrightarrow{GM}$ $\angle GH|_{\omega=\omega_{pc}} = -180°$

$\therefore$ $-270° \neq 180°$

$> \xrightarrow{-180°} \omega_{pc} = 0$.

$GM = \frac{1}{M|_{\omega=\omega_{pc}}} = \frac{1}{\infty} = 0$.

$\boxed{GM = 0 < 1}$ (US)

$\xrightarrow{PM}$ ω_{gc} $M = 1$,

$\Rightarrow \omega_{gc} = 1$ rad/sec.

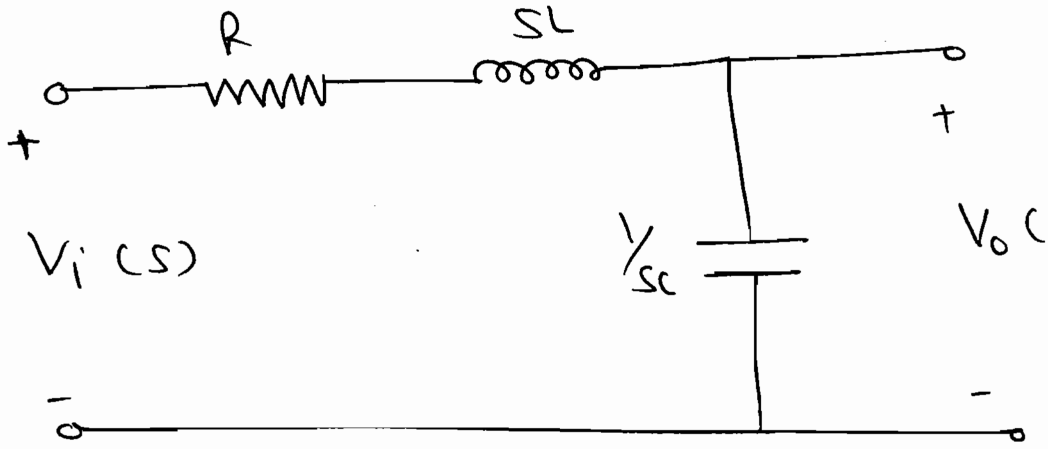
$PM = 180° - 270°$

$\boxed{PM = -90° < 0} \Rightarrow$ (U.S).

So, CLS (US).

* Frequency Domain Specification:-

⇒ The general freq. response of RLC ckt is shown in fig.



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sL}}.$$

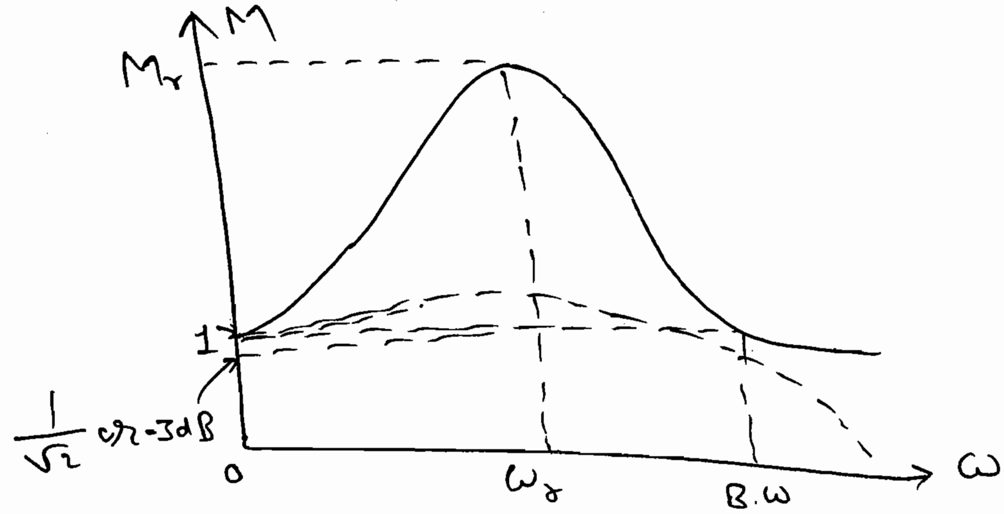
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

$$\therefore \boxed{\omega_n = \frac{1}{\sqrt{LC}} \text{ rad/sec.}}$$

$$2\xi\omega_n = R/L.$$

$$\therefore \boxed{\xi = \frac{R}{2} \times \sqrt{\frac{C}{L}}}$$

$$\boxed{Q = \frac{1}{2\xi} = \frac{1}{R} \times \sqrt{\frac{L}{C}}}$$



* **Resonant freq:-**

⇒ It is a freq at which max. magnitude occurs so,

$$\boxed{\omega_r = \omega_n \sqrt{1-2\xi^2} \quad \text{rad/sec}}$$

* **Resonant Peak:-**

⇒ It is a max. magnitude occurs at resonant freq.

$$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$

⇒ $\boxed{\xi < \frac{1}{\sqrt{2}}}$ * then freq. domain specification is valid otherwise not valid.

⇒ when $\xi \geq 1/\sqrt{2}$, No resonant Peak & No Resonant freq. exist.

* **Band-width:-**

⇒ It is the range of freq. at which the mag. dropped by $-3\,dB$ (or) $1/\sqrt{2}$ from the maximum value at the low freq.

$\Rightarrow$ Bw. for 1st order

$$\boxed{Bw = \frac{1}{T}} \leftarrow \text{Bw for 1st order.}$$

Hz

$\Rightarrow$ Bw for 2nd order.

$$\boxed{B.\omega. = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}}} \text{ Hz}$$