Class: XII
SESSION : 2022-2023
SUBJECT: Mathematics
SAMPLE QUESTION PAPER - 5
with SOLUTION

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment (**4 marks each**)** with sub parts.

Section A

1. $\int(\sin(\log x)+\cos(\log x))$ dx is equal to **[1]**

a) log (sin x - cos x) + c

b) x sin (log x) +C

c) sin (log x) – cos (log x) + C

d) x cos (log x) + C

2. The angle between the lines $\vec{r}=(3\hat{i}+\hat{j}-2\hat{k})+\ \lambda(\hat{i}-\hat{j}-2\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-5\hat{k})\ +\mu(3\hat{i}-5\hat{j}-4\hat{k})$ is **[1]**

a) $\cos^{-1}\left(\frac{5\sqrt{3}}{8}\right)$

b) $\cos^{-1}\left(\frac{6\sqrt{2}}{5}\right)$

c) $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

d) $\cos^{-1}\left(\frac{5\sqrt{2}}{6}\right)$

3. The area between the hyperbola $xy=c^2$, then x – axis and the ordinates at a and b with a > b is : **[1]**

a) $c^2\log\left(\frac{b}{a}\right)$

b) $c^2\log(ab)$

c) none of these

d) $c^2\log\left(\frac{a}{b}\right)$

4. The vector $\vec{b}=3\hat{i}+4\hat{k}$ is to be written as the sum of a vector $\vec{\alpha}$ parallel to $\vec{a}=\hat{i}+\hat{j}$ and a vector $\vec{\beta}$ perpendicular to $\vec{a}$. Then $\vec{\alpha}=$ **[1]**

a) $\frac{2}{3}(i+j)$

b) $\frac{3}{2}(i+j)$

c) $\frac{1}{2}(i+j)$

d) $\frac{1}{3}(i+j)$

5. If P(A) = 0.4, P(B) = 0.8 and P(B | A) = 0.6, then P(A ∪ B) is equal to **[1]**

a) 0.48

b) 0.96

c) 0.3

d) 0.24

6. The area bounded by the curves y = sin x between the ordinates x = 0, x = π and the x-axis is **[1]**

a) 2 sq. units
b) 4 sq. units
c) 3 sq. units
d) 1 sq. units

7. Maximize Z = 5x+3y , subject to constraints x + y ≤ 300 , 2x + y ≤ 360, x ≥ 0, y ≥ 0. **[1]**

a) 1020
b) 1050
c) 1040
d) 1030

8. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is **[1]**

a) $\frac{1}{28}$
b) $\frac{3}{28}$
c) $\frac{167}{168}$
d) $\frac{2}{21}$

9. Evaluate the product of $\left(3\vec{a}-5\vec{b}\right).\left(2\vec{a}+7\vec{b}\right)$ **[1]**

a) $6|\vec{a}|^2+11\vec{a}.\vec{b}-35\left|\vec{b}\right|^2$
b) $7|\vec{a}|^2+13\vec{a}.\vec{b}-45\left|\vec{b}\right|^2$
c) $6|\vec{a}|^2+11\vec{a}.\vec{b}-30\left|\vec{b}\right|^2$
d) $6|\vec{a}|^2+13\vec{a}.\vec{b}-35\left|\vec{b}\right|^2$

10. The differential equation $\frac{dy}{dx}+Py=Qy''$, n > 2 can be reduced to linear form by substituting **[1]**

a) $z = y^{1 - n}$
b) $z = y^{n - 1}$
c) $z = y^{n + 1}$
d) $z = y^{n}$

11. $\int\frac{\sin^{-1}x}{(1-x^2)^{3/2}}$ dx = ? **[1]**

a) $\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\frac{1}{2}$ log |1 - x^2| + C
b) None of these
c) x $\sin^{-1}$ x + $\frac{1}{2}$ log |1 - x^2| + C
d) $\frac{\sin^{-1}x}{\sqrt{1-x^2}}-\frac{1}{2}$ log |1 - x^2| + C

12. The general solution of the DE $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$ is **[1]**

a) $x^2 + y^2 = C_1x$
b) $x^2 + y^2 = C_1y$
c) none of these
d) $x^2 - y^2 = C_1x$

[1]

13. Which of the following is the integrating factor of (x log x) $\frac{dy}{dx}$ + y = 2 logx?

a) e^x

b) x

c) log x

d) log (log x)

14. Let f(x) =|x - 1|, then **[1]**

a) f (x + y) = f (x) + f(y)

b) f(|x|) = |f(x)|

c) $f(x^2) = (f(x))^2$

d) f (x) is not derivable at x = 1.

15. The general solution of the differential equation $\left(\frac{d^4y}{dx^4}\right)^{\frac{3}{5}} - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 5$ = 0? **[1]**

a) 2

b) 5

c) 3

d) 4

16. The roots of the equation det. $\begin{vmatrix} 1-x & 2 & 3 \\ 0 & 2-x & 0 \\ 0 & 2 & 3-x \end{vmatrix} = 0$ are **[1]**

a) None of these

b) 2 and 3

c) 1 , 2 and 3

d) 1 and 3

17. The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is **[1]**

a) $\frac{4\pi}{3}$

b) $\frac{2\pi}{3}$

c) $\frac{\pi}{3}$

d) $\frac{-\pi}{3}$

18. **Assertion (A):** The maximum value of Z = 11x + 7y subject to the constraints 2x + y $\leq$ 6; x $\leq$ 2; x $\geq$ 0, y $\geq$ 0 **[1]**
occurs at the comer point (0, 6).
Reason (R): If the feasible region of the given LPP is bounded, then the maximum and minimum value of the objective function occurs at corner points.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$ is **[1]**

a) 45°

b) 60°

c) 30°

d) 90°

20. $f(x) = \begin{cases} x+\pi, & \text{for } x \in [-\pi, 0) \\ \pi \cos x, & \text{for } x \in \left[0, \frac{\pi}{2}\right] \\ \left(x-\frac{\pi}{2}\right)^2, & \text{for } x \in \left(\frac{\pi}{2}, \pi\right] \end{cases}$ **[1]**

Consider the following statements

Assertion (A): The function f(x) is continuous at x = 0.

Reason (R): The function f(x) is continuous at x = $\frac{\pi}{2}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve the initial value problem: $x\frac{dy}{dx}$ - y + x $\sin\left(\frac{y}{x}\right)$ = 0, $y(2) = \pi$ **[2]**

22. The vector equation of a line is $\vec{r} = (2\hat{i} + \hat{j} - 4\hat{k}) + \lambda(\hat{i} - \hat{j} - \hat{k})$. Find its Cartesian equation. **[2]**

OR

The equations of a line are given by $\frac{4-x}{3} = \frac{y+3}{3} = \frac{z+2}{6}$. Write the direction cosines of a line parallel to this line.

23. Determine P(E|F): A dice is thrown three times.E : 4 appears on the third toss, F : 6 and 5 appears respectively on first two tosses. **[2]**

24. Write the value of $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$ **[2]**

25. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. **[2]**

Section C

26. Draw the rough sketch of $y^2 + 1$ = x, x $\leq$ 2. Find the area enclosed by the curve and the line x = 2 **[3]**

OR

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

27. The feasible region for a LPP is shown in Figure. Evaluate Z = 4x + y at each of the corner points of this region. Find the minimum value of Z, if it exists. **[3]**

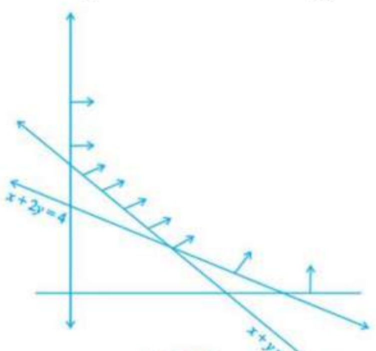


Fig. 12.10

28. Find the value of λ for which the points A(2, 5,1), B(1, 2, -1) and C(3, λ, 3) are collinear. **[3]**

OR

Find the foot of the perpendicular drawn from the point $\hat{i}+6\hat{j}+3\hat{k}$ to the line $\vec{r}=\hat{j}+2\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$. Also, find the length of the perpendicular.

29. Find the area of the region $\{(x,y): x\geq 0, x+y\leq 3, x^2\leq 4y \quad \text{and } y\leq 1+\sqrt{x}\}$ **[3]**

30. Write the derivative of sin x with respect to cos x. **[3]**

31. Evaluate: $\int \sin^7 x dx$ **[3]**

OR

Evaluate $\int_0^1 \frac{x^4+1}{x^2+1}dx$.

Section D

32. Find the area of the Δ with vertices A (1, 1, 2) B (2, 3, 5) and C (1, 5, 5). **[5]**

33. Let A and B be two sets. Show that f: $A\times B\rightarrow B\times A$ such that f(a, b) = (b, a) is **[5]**
(i) injective
(ii) bijective

OR

Let A = R – {3} and B = R- {1}. Consider the function of f: A $\rightarrow$ B defined by f(x) = $\frac{x-2}{x-3}$ is one – one and onto.

34. The cost of 4kg onion, 3kg wheat and 2kg rice is Rs. 60. The cost of 2kg onion, 4kg wheat and 6kg rice is Rs. 90. The cost of 6kg onion 2kg wheat and 3kg rice is Rs. 70. Find the cost of each item per kg by matrix method. **[5]**

OR

For the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence find A^{-1}.

35. Evaluate: $\int \frac{3+2\cos x+4\sin x}{2\sin x+\cos x+3}dx$ **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

On her birthday, Seema decided to donate some money to the children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in

₹).

(i) Represent given information in matrix algebra.

(ii) Find the adjoint of Matrix containing information about of number of children and amount she paid?

(iii) Find the number of children who were given some money by Seema?

OR

How much amount does Seema spend in distributing the money to all the students of the Orphanage?

37. **Read the text carefully and answer the questions:** **[4]**

The Government declare that farmers can get ₹300 per quintal for their onions on 1st July and after that, the price will be dropped by ₹3 per quintal per extra day. Govind's father has 80 quintals of onions in the field on 1st July and he estimates that the crop is increasing at the rate of 1 quintal per day.

(i) If x is the number of days after 1^{st} July, then express price and quantity of onion and the revenue as a function of x.

(ii) Find the number of days after 1st July, when Govind's father attains maximum revenue.

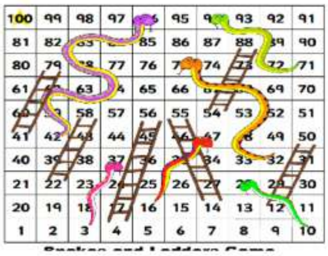
(iii) On which day should Govind's father harvest the onions to maximize his revenue?

OR

Find the maximum revenue collected by Govind's father.

38. **Read the text carefully and answer the questions:** **[4]**

Akshat and his friend Aditya were playing the snake and ladder game. They had their own dice to play the game. Akshat was having red dice whereas Aditya had black dice. In the beginning, they were using their own dice to play the game. But then they decided to make it faster and started playing with two dice together.



Aditya rolled down both black and red die together.
First die is black and second is red.

(i) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(ii) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

SOLUTION

Section A

1. **(b)** x sin (log x) +C

   Explanation: $\int(sin(log\ x) + cos(log\ x))dx$
   (Use By Part, Take 1 as II function)
   $= \int \sin(log\ x).1dx + \int\cos(\log\ x)dx$
   $= (\sin(\log x)).\ x - \int\cos(\log x)\frac{1}{x}.\ xdx + \int\cos(log\ x)dx.$
   $= x\sin(\log x) + C$

2. **(c)** $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

   Explanation: Let $\vec{a} = \hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} - 4\hat{k}$ and $|\vec{a}| = \sqrt{1+1+2^2} = \sqrt{6}$

   $|\vec{b}| = \sqrt{3^2+5^2+4^2} = 5\sqrt{2}$

   $\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}| \times |\vec{b}|}$

   $\Rightarrow \cos\alpha = \frac{(3\hat{i}-5\hat{j}-4\hat{k})\cdot(\hat{i}-\hat{j}-2\hat{k})}{5\sqrt{2}\times\sqrt{6}}$

   $\Rightarrow \cos\alpha = \frac{3+5+8}{5\sqrt{12}}$

   $\Rightarrow \cos\alpha = \frac{8\sqrt{3}}{15}$

3. **(d)** $c^2\log\left(\frac{a}{b}\right)$

   Explanation: Required area :

   $= \int_b^a \frac{c^2}{x}dx = c^2[\log x]_b^a = c^2(\log a - \log b) = c^2 log\left(\frac{a}{b}\right).$

   Which is the required solution.

4. **(b)** $\frac{3}{2}(i+j)$

   Explanation: Let $\vec{a} = a_1\hat{i} + \alpha_2\hat{j} + a_3\hat{k}, \vec{\beta} = \beta_1\hat{i} + \beta_2\hat{j} + \beta_3\hat{k}$

   $\vec{b} = 3\hat{i} + 4\hat{k}$

   $\vec{\alpha} + \vec{\beta} = 3\vec{i} + 4\vec{k}$

   $(\alpha_1+\beta_1)i + (\alpha_2+\beta_2)j + (\alpha_3+\beta_3)\hat{k} = \bar{3} + 4\hat{k}$

   $\Rightarrow \alpha_1 + \beta_1 = 3$

$\alpha_2 + \beta_2 = 0$

$\alpha_3 + \beta_3 = 4$

Given that $\vec{a}$ is parllel to $\vec{a}$

$\vec{\alpha} \times \vec{a} = 0$

$$\begin{vmatrix} i & \hat{j} & \hat{k} \\ \alpha_1 & \alpha_2 & \alpha_3 \\ 1 & 1 & 0 \end{vmatrix} = 0 \ \{\text{Given } \vec{a} = \vec{i} + \vec{j}\}$$

$-\alpha_3\hat{i} + \alpha_3\hat{j} + \left(\alpha_1 - \alpha_2\right)\hat{k} = 0$

$\alpha_3 = 0, \alpha_1 - \alpha_2 = 0$

$\alpha_3 = 0, \alpha_1 = \alpha_2$

Given $\vec{\beta}$ is perpendicular to $\vec{a}$

$\vec{\beta} \cdot \vec{a} = 0$

$\left(\beta_1\tilde{i} + \beta_2\tilde{j} + \beta_3\tilde{k}\right) \cdot (\tilde{i} + \tilde{j}) = 0$

$\beta_1 + \beta_2 = 0$

$\beta_1 = -\beta_2$

Solving $\alpha_3 = 0, \alpha_1 = \alpha_2, \alpha_1 + \beta_1 = 3$

$\alpha_2 + \beta_2 = 0, \alpha_3 + \beta_3 = 4, \beta_1 = -\beta_2$

$\Rightarrow \alpha_1 = \alpha_2 = \frac{3}{2}, \alpha_3 = 0$

$\vec{\alpha} = \alpha_1\hat{i} + \alpha_2\hat{j} + \alpha_3\hat{k}$

$\vec{\alpha} = \frac{3}{2}(i + \vec{j})$

5. **(b)** 0.96

Explanation: Here, $P(A) = 0.4$, $P(B) = 0.8$ and $\boldsymbol{P(B \mid A) = 0.6}$

$\because P(B/A) = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow P(B \cap A) = P(B/A) \cdot P(A)$

$= 0.6 \times 0.4 = 0.24$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.8 - 0.24$

$= 1.2 - 0.24 = 0.96$

6. **(a)** 2 sq. units

Explanation: $\int_0^{\pi} y dx = \int_0^{\pi} \sin x dx$

$\int_0^{\pi} y dx = -[\cos x]_0^{\pi}$

$\int_0^{\pi} y dx = -[-1 - 1]$

$\int_0^{\pi} y dx = 2$

7. **(a)** 1020

Explanation: Here , Maximize Z = 5x+3y , subject to constraints $x + y \leq 300$, $2x + y \leq 360$, $x \geq 0$, $y \geq 0$.

Corner points	Z = 5x +3 y
P(0, 300)	900
Q(180, 0)	900
R(60, 240)	1020…………………..(Max.)
S(0, 0)	0

Hence, the maximum value is 1020

8. **(b)** $\frac{3}{28}$

Explanation: Probability of drawing 2 green balls and one blue ball

$= P_G \cdot P_G \cdot P_B + P_B \cdot P_G \cdot P_G + P_G \cdot P_B \cdot P_G$

$= \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{7} + \frac{2}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{6}$

$= \frac{1}{28} + \frac{1}{28} + \frac{1}{28} = \frac{3}{28}$

9. **(a)** $6\left|\vec{a}\right|^2 + 11\vec{a}.\,\vec{b} - 35\left|\vec{b}\right|^2$

Explanation: $\left(3\vec{a} - 5\vec{b}\right).\left(2\vec{a} + 7\vec{b}\right) =$

$6\vec{a} \cdot \vec{a} + 21\bar{a} \cdot \bar{b} - 10\vec{b} \cdot \vec{a} - 35\vec{b} \cdot \vec{b} = 6|\vec{a}|^2 + 11\vec{a}\vec{b} - 35|\vec{b}|^2$

10. **(a)** $z = y^{1 - n}$

Explanation: We have,

$\frac{dy}{dx} + Py = Qy^n$

$\Rightarrow y^{-n}\frac{dy}{dx} + Py^{1-n} = Q$

$\Rightarrow y^{-n}\frac{dy}{dx} + Py^{1-n} = Q$

Let $y^{1-n} = v$

$\Rightarrow \frac{dv}{dx} = (1-n)v\frac{dy}{dx}$

$\Rightarrow \frac{1}{1-n}\frac{dv}{dx} + Pv = Q$

$\Rightarrow \frac{dv}{dx} + Pv(1-n) = Q(1-n)$

which is linear form.

$z = y^{(1-n)}$ can be reduced to linear form.

11. **(a)** $\frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{1}{2}\log|1 - x^2| + C$

Explanation: Put x = sin t so that dx = cos t dt and t = $\sin^{-1}x$

$$\therefore I = \int \frac{t\cos t}{\left(1-\sin^2 t\right)^{3/2}}dt = \int \frac{t\cos t}{\cos^3 t}dt = \int tI\ secII^2 tdt$$

After solving the above equation we get,

$$I = \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{1}{2}\log|1 - x^2| + C$$

12. **(a)** $x^2 + y^2 = C_1x$

Explanation: We have, $\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{x^2v^2-x^2}{2vx^2} = v + x\frac{dv}{dx}$$

$$\frac{v^2-1}{2v} - v = x\frac{dv}{dx}$$

$$\frac{-v^2-1}{2v} = x\frac{dv}{dx}$$

$$\frac{dx}{x} + \frac{2vdv}{v^2+1} = 0$$

On integrating on both sides, we obtain

$\log x + \log(v^2 + 1) = C$

$\log(x(v^2 + 1) = C$

$$x\left(\frac{y^2}{x^2} + 1\right) = c$$

$y^2 + x^2 = Cx$

13. **(c)** log x

Explanation: We have,

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x}$$

Comparing with $\frac{dy}{dx}$ = Py = Q

$P = \frac{1}{x\log x}, Q = \frac{2}{x}$

I.F. = $\int \frac{1}{x\log x}dx = e^{\log(\log x)} = \log x$

14. **(d)** f (x) is not derivable at x = 1.

Explanation: Here, $f(x) = |x - 1|\ x \in R$. So $f(x)$ is not derivable when x - 1 = 0 i.e. at x = 1

15. **(c)** 3

Explanation: Given differential equation is $\left(\frac{d^4y}{dx^4}\right)^{\frac{3}{5}} - 5\left(\frac{d^3y}{dx^3}\right) + 6\left(\frac{d^2y}{dx^2}\right) - 3\left(\frac{dy}{dx}\right) + 5 =$ 0

Since the highest exponent of the highest derivative is called the degree of a differential equation provided exponent of each derivative and the unknown variable appearing in the differential equation is a non-negative integer.

$$\therefore \left(\frac{d^4y}{dx^4}\right)^{\frac{3}{5}} = 5\left(\frac{d^3y}{dx^3}\right) - 6\left(\frac{d^2y}{dx^2}\right) + 8\left(\frac{dy}{dx}\right) - 5$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^3$$

$$= \left\{5\left(\frac{d^3y}{dx^3}\right) - 6\left(\frac{d^2y}{dx^2}\right) + 8\left(\frac{dy}{dx}\right) - 5\right\}^5$$

$\therefore$ Required degree = 3

16. **(c)** 1 , 2 and 3

Explanation: Expanding along C_1

$$\begin{vmatrix} 1-x & 2 & 3 \\ 0 & 2-x & 0 \\ 0 & 2 & 3-x \end{vmatrix} = 0 \Rightarrow (1 - x)(2 - x)(3 - x) = 0 \Rightarrow x = 1, 2, 3.$$

17. **(b)** $\frac{2\pi}{3}$

Explanation: Let the principle value be given by x

Now, let x = $\cos^{-1}\left(\frac{-1}{2}\right)$

$\Rightarrow \cos x = \frac{-1}{2}$

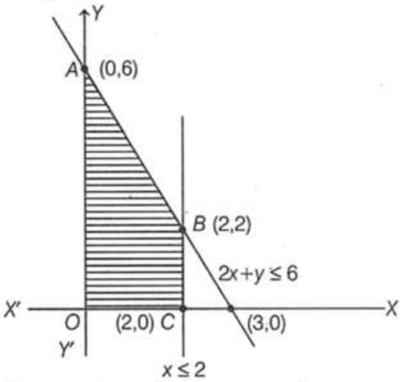
$\Rightarrow \cos x = -\cos\left(\frac{\pi}{3}\right)\left(\because \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}\right)$

$\Rightarrow \cos x = \cos\left(\pi - \frac{\pi}{3}\right)(\because -\cos(\theta) = \cos(\pi - \theta))$

$\Rightarrow x = \frac{2\pi}{3}$

18. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: The corresponding graph of the given LPP is



From the above graph, we see that the shaded region is the feasible region OABC which is bounded.

∴ The maximum value of the objective function Z occurs at the corner points. The corner points are O(0, 0), A(0, 6), B(2, 2), C(2, 0).

The values of Z at these comer points are given by

Comer point	Corresponding value of Z = 11x + 7y
(0, 0)	0
(0, 6)	42 ← Maximum
(2, 2)	36
(2, 0)	22

Thus, the maximum value of Z is 42 which occurs at the point (0, 6).

19. **(d)** 90°

Explanation: We have,

$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$

$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$

The direction ratios of the given lines are proportional to 2, 5, 4 and 1, 2, -3.

The given lines are parallel to the vectors $\vec{b_1} = 2\hat{i} + 5\hat{j} + 4\hat{k}$ and $\vec{b_2} = \hat{i} + 2\hat{j} - 3\hat{k}$

Let θ be the angle between the given lines.

Now,

$$\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$$

$$= \frac{(2\hat{i}+5\hat{j}+4\hat{k}) \cdot (\hat{i}+2\hat{j}-3\hat{k})}{\sqrt{2^2+5^2+4^2}\sqrt{1^2+2^2+(-3)^2}}$$

$$= \frac{2+10-12}{\sqrt{45}\sqrt{14}}$$

$$= 0$$

$$\Rightarrow \theta = 90^\circ$$

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: LHL = $\lim_{x \to 0^-} f(x)$

$= \lim_{x \to 0} (x + \pi) = \pi$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{x \to 0} \pi \cos \pi$

$= \pi \cos(0) = \pi$

Also, $f(0) = \pi \cos(0) = \pi$

Hence, f(x) is continuous at x = 0.

∴ Assertion is true.

Reason: Now, for $x = \frac{\pi}{2}$

LHL = $\lim_{x \to \frac{\pi}{2}^-} f(x) = = \lim_{x \to \frac{\pi}{2}} \pi \cos x$

$= \pi \cos \frac{\pi}{2} = 0$

RHL = $\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}} \left(x - \frac{\pi}{2}\right)^2$

$= \left(\frac{\pi}{2} - \frac{\pi}{2}\right)^2 = 0$

Also, $f\left(\frac{\pi}{2}\right) = \pi \cos \frac{\pi}{2} = 0$

Hence, f(x) is continuous at $x = \frac{\pi}{2}$.

∴ Reason is true.

Section B

21. Here we have

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0,\ y(2) = \pi$$

It is a homogeneous equation

Put y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So, $v + x\frac{dv}{dx} = \frac{vx}{x} - \sin\left(\frac{vx}{x}\right)$

$$x\frac{dv}{dx} = -\sin v$$

$$\frac{dv}{\sin v} = -\frac{dx}{x}$$

$$\text{cosec}(v)\,dv = -\frac{dx}{x}$$

Integraing both sides we get,

log (cosec (v) - cot (v)) = -log x + log c

$$\log\left(\text{cosec}\left(\frac{y}{x}\right) - \cot\left(\frac{y}{x}\right)\right) = -\log x + \log c$$

Putting the values x = 2 and y = π

$$\log\left(\text{cosec}\left(\frac{\pi}{2}\right) - \cot\left(\frac{\pi}{2}\right)\right) = -\log 2 + \log c$$

c = 2

$$\log\left(\text{cosec}\left(\frac{y}{x}\right) - \cot\left(\frac{y}{x}\right)\right) = 0$$

22. We know that the vector equation of the line is given by

$\vec{r} = p\vec{i} + 9\vec{j} + r\vec{k} + \lambda(\vec{l} + m\vec{j} + n\vec{k})$ then its Cartesian equation is given by

$$\frac{x-p}{1} = \frac{y-q}{m} = \frac{z-r}{n}$$

Thus,The vector equation of a line is

$$\vec{r} = (2\hat{i} + \hat{j} - 4\hat{k}) + \lambda(\hat{i} - \hat{j} - \hat{k})$$

Here p = 2, q = 1, r = -4 and l = 1,m = -1, n = -1

Thus,Carte equation is given by

$$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z-(-4)}{-1}$$

$$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z+4}{-1}$$

Cartesian equation of the line is given by $\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z+4}{-1}$

OR

We have

$$\frac{4-x}{3}=\frac{y+3}{3}=\frac{z+2}{6}$$

The equation of the given line can be re-written as

$$\frac{x-4}{-3}=\frac{y+3}{3}=\frac{z+2}{6}$$

The direction ratios of the line parallel to the given line are proportional to -3, 3, 6.

Hence, the direction cosines of the line parallel to the given line are proportional to

$$\frac{-3}{\sqrt{(-3)^2+3^2+6^2}}, \frac{3}{\sqrt{(-3)^2+3^2+6^2}}, \frac{6}{\sqrt{(-3)^2+3^2+6^2}}$$

$$=\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$$

23. Since a dice has six faces. Therefore $n(S) = 6 \times 6 \times 6 = 216$

E = (1, 2, 3, 4, 5, 6) x (1, 2, 3, 4, 5, 6) x (4)

F = (6) x (5) x (1, 2, 3, 4, 5, 6)

$\Rightarrow n(F) = 1 \times 1 \times 6 = 6$

$$P(F) = \frac{n(F)}{n(S)} = \frac{6}{216}$$

$\therefore E \cap F = (6, 5, 4)$

$n(E \cap F) = 1$

$$\therefore P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{216}$$

$$\text{And } P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/216}{6/216} = \frac{1}{6}$$

24. Given $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$$= \sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$$

$$= \sin^{-1}\left(\frac{1}{3}\right) - \pi + \cos^{-1}\left(\frac{1}{3}\right)$$

$$= \sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right) - \pi$$

$$= \frac{\pi}{2} - \pi$$

$$= -\frac{\pi}{2}$$

Therefore we have,

$$\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$$

25. Let x = a $\cos^3 \theta$, y = a$\text{Sin}^3\theta$

Then $\frac{dx}{d\theta}$ = -3a $\cos^2 \theta \sin \theta$

and $\frac{dy}{d\theta}$ = 3a $\sin^2 \theta \cos \theta$

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{dx}}{\frac{dx}{d\theta}} = \frac{3a\sin^2 \theta\cos \theta}{-3a\cos^2 \theta\sin \theta} = -\tan\theta = -\sqrt[3]{\frac{y}{x}}$

Section C

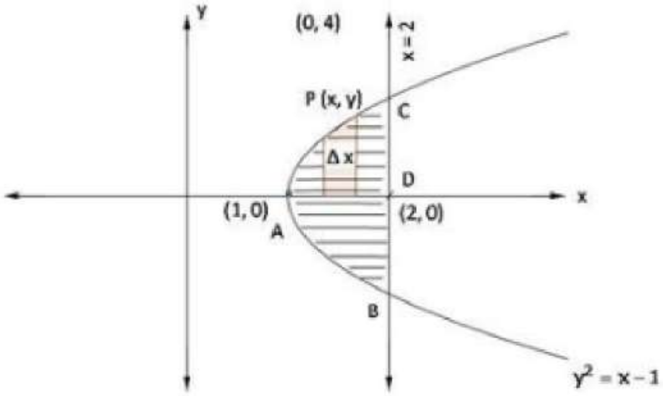
26. We have to find area enclosed by the given curve

$y^2 = x - 1$ ...(i)

and x = 2 ...(ii)

Equation (i) is a parabola with vertex at (1, 0) and axis as x-axis,. Equation (ii) represents a line parallel to y-axis passing through (2, 0).

A rough sketch of curves is as below in the Figure.



Shaded region shows the required area, We slice it in appropriation rectangle with its Width $\triangle x$ and length = y - 0 = y

Area of the rectangle $= y\triangle x$

This rectangle can slide from x = 1 to x = 2, so

the Required area = Region AB CA= twice area of region AOCA

= 2 (Region AOCA)

$= 2\int_1^2 y dx$

$= 2\int_1^2 \sqrt{x-1} dx$

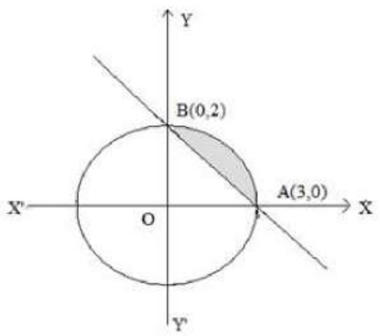
$= 2\left(\frac{2}{3}(x-1)\sqrt{x-1}\right)_1^2$

$= \frac{4}{3}[((2-1)\sqrt{2-1}) - ((1-1)\sqrt{1-1})]$

$= \frac{4}{3}(1 - 0)$

Required area $= \frac{4}{3}$ square units

OR



$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\frac{x}{3} + \frac{y}{2} = 1$

$\Rightarrow \frac{x^2}{(3)^2} + \frac{y^2}{(2)^2} = 1$ is the equation of ellipse and

$\frac{x}{3} + \frac{y}{2} = 1$ is the equation of intercept form of line

Area $= \frac{2}{3}\int_0^3 \sqrt{9 - x^2}dx - \frac{2}{3}\int_0^3 (3 - x)dx$

$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}sin^{-1}\frac{x}{3} - 3x + \frac{x^2}{3}\right]_0^3$

$= \frac{2}{3}\left[(0 + \frac{9}{2}(sin^{-1}(-1) - 3(3) + \frac{9}{2}) - 0\right]$

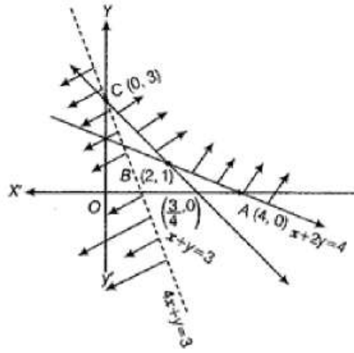
$= \frac{2}{3}[\frac{9\pi}{4} - \frac{9}{2}]$

$= \frac{3}{2}(\pi - 2)$sq. unit

27. From the shaded region, it is clear that feasible region is unbounded with the corner points A(4, 0), B(2, 1) and C(0, 3).

Also, we have Z = 4x + y.

[Since, x + 2y = 4 and x + y = 3 $\Rightarrow$ y = 1 and x = 2]



Corner Points	Corresponding value of Z
(4, 0)	16
(2, 1)	9
(0, 3)	3 (minimum)

Now, we see that 3 is the smallest value of Z at the corner point (0, 3). Note that here we see that the region is unbounded, therefore 3 may or may not be the minimum value of Z. To decide this issue, we graph the inequality $4x + y < 3$ and check whether the resulting open half plan has no point in common with feasible region otherwise, Z has no minimum value.

From the shown graph above, it is clear that there is no point in common with feasible region and hence Z has minimum value of 3 at (0, 3).

28. Here, it is given that

A = (2,5,1), B(1, 2, -1) and c=(3, λ, 3)

The direction ratios of the line AB can be given by

((1 - 2),(2 - 5),(-1 - 1))

= (-1, -3, -2)

Similarly, the direction ratios of the line BC can be given by

((3 - 1),(λ - 2),(3 +1))

$= (2, \lambda - 2, 4)$

We know that – If it is shown that direction ratios of AB = α times that of BC, where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

Thus, d.r. of AB

=(-1, -3, -2)

$(\frac{-1}{2}) \times (2, \lambda - 2, 4)$

Since, A, B and C are collinear.

$\therefore\ -\frac{1}{2}(\lambda - 2) = -3$

$\Rightarrow \lambda - 2 = 6$

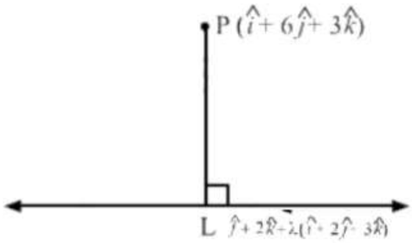
$\Rightarrow \lambda = 8$

OR

Suppose L be the foot of the perpendicular drawn from the point $P(\hat{i} + 6\hat{j} + 3\hat{k})$ to the line

$\vec{r} = \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

Let the position vector L be $\vec{r} = \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}) = \lambda\hat{i} + (1 + 2\lambda)\hat{j} + (2 + 3\lambda)\hat{k}$



Now,

$\overrightarrow{PL}$ = Position vector of L - Position vector of P

$\Rightarrow \overrightarrow{PL} = \{\lambda\hat{i} + (1 + 2\lambda)\hat{j} + (2 + 3\lambda)\hat{k}\} - (\hat{i} + 6\hat{j} + 3\hat{k})$ ....(i)

$\Rightarrow \overrightarrow{PL} = (\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}$ ......(ii)

Since $\overrightarrow{PL}$ is perpendicular to the given line, which is parallel to $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ we have,

$\overrightarrow{PL} \cdot \vec{b} = 0$

$\Rightarrow \{(\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}\} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$

$\Rightarrow 1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\Rightarrow \lambda = 1$

put $\lambda = 1$ in (i)

we get the position vector of L as $\hat{i} + 3\hat{j} + 5\hat{k}$

put $\lambda = 1$ in (ii)

we get, $\overrightarrow{PL} = -3\hat{j} + 2\hat{k}$

$= \sqrt{(-3)^2 + 2^2}$

$= \sqrt{13}$

Therefore,the length of the perpendicular from point P on PL is $\sqrt{13}$ units.

29. Required area $= \int_0^1 (1 + \sqrt{x})dx + \int_1^2 (3 - x)dx - \int_0^2 \frac{x^2}{4} dx$

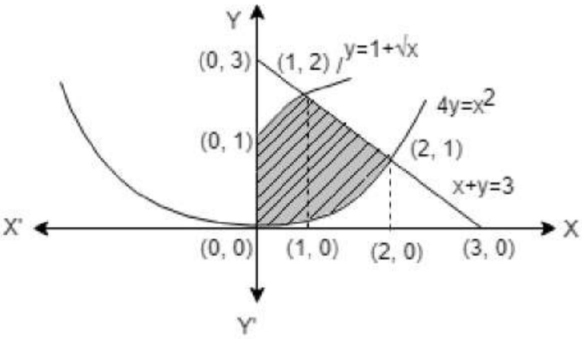
$= \left(x + \frac{x^{3/2}}{3/2}\right)_0^1 + \left(3x - \frac{x^2}{2}\right)_1^2 - \left(\frac{x^3}{12}\right)_0^2$

$= \left(1 + \frac{2}{3}\right) + \left(6 - 2 - 3 + \frac{1}{2}\right) - \left(\frac{8}{12}\right)$

$= \frac{5}{3} + \frac{3}{2} - \frac{2}{3}$

$= 1 + \frac{3}{2}$

$=\frac{5}{2}$ sq.units



30. Let u = sinx

On differentiating both sides w.r.t x, we get

$\frac{du}{dx} = \cos x$........(i)

Also, let v = cosx

On differentiating both sides w.r.t x, we get

$\frac{dv}{dx} = -\sin x$.......(ii)

Now, $\frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv} = -\frac{\cos x}{\sin x}$ [from eq(i) and (ii)]

$\therefore \quad \frac{du}{dv} = -\cot x$

31. Let the given integral be,

$l = \int \sin^7 x dx$. Then,

$I = \int \sin^6 x \sin x dx$

$= \int (\sin^2 x)^3 \sin x dx$

$= \int (1 - \cos^2 x)^3 \sin x dx$

$= \int (1 - \cos^6 x + 3\cos^4 x - 3\cos^2 x)^3 \sin x dx$

$\Rightarrow \; I = \int \sin x\, dx - \int \cos^6 x \sin x dx + 3\int \cos^4 x \sin x dx - 3\int \cos^2 x \sin x\, dx$

Putting cos x = t and - sin xdx = dt in 2ne, 3rd and 4th integral, we get

$I = \int \sin x dx - \int t^6(-dt) + 3\int t^4(-dt) - 3\int t^2(-dt)$

$= -\cos x + \frac{t^7}{7} - \frac{3}{5}t^5 + \frac{3}{3}t^3 + c$

$= -\cos x + \frac{\cos^7 x}{7} - \frac{3}{5}\cos^5 x + \cos^3 x + c$

$\therefore \; I = -\cos x + \cos^3 x - -\frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

OR

Given, $I = \int_0^1 \frac{x^4+1}{x^2+1} dx \Rightarrow I = \int_0^1 \frac{\left(x^4-1\right)+2}{x^2+1} dx$

$= \int_0^1 \frac{\left(x^2-1\right)\left(x^2+1\right)+2}{x^2+1} dx$

[$\because (a^2 - b^2) = (a - b)(a + b)$

$= \int_0^1 \left[\frac{\left(x^2-1\right)\left(x^2+1\right)}{x^2+1} + \frac{2}{x^2+1}\right] dx$

$\Rightarrow \quad I = \int_0^1 \left[x^2 - 1 + \frac{2}{x^2+1}\right] dx$

$\Rightarrow I = \left[\frac{x^3}{3} - x + 2\tan^{-1}x\right]_0^1$

$\therefore I = \frac{1}{3} - 1 + 2\tan^{-1}1 - 0 = -\frac{2}{3} + 2 \times \frac{\pi}{4} = \frac{3\pi-4}{6}$

Section D

32. A (1, 1, 2) B(2, 3, 5) C (1, 5, 5)

$\overrightarrow{OA} = \hat{i} + \hat{j} + 2\hat{k}$

$\overrightarrow{OB} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$\overrightarrow{OC} = \hat{i} + 5\hat{j} + 5\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 4\hat{j} + 3\hat{k}$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$

$= -6\hat{i} - 3\hat{j} + 4\hat{k}$

Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \frac{1}{2}\sqrt{(-6)^2 + (-3)^2 + 4^2}$

$= \frac{1}{2}\sqrt{61}$ sq. unit

33. i. Let $(a_1\ b_1)$ and $(a_2, b_2) \in A \times B$ such that

$f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

$\Rightarrow a_1 = a_2$ and $b_1 = b_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

Therefore, f is injective.

ii. Let (b, a) be an arbitrary

Element of $B \times A$. then $b \in B$ and $a \in A$

$\Rightarrow (a, b)) \in (A \times B)$

Thus for all $(b, a) \in B \times A$ their exists $(a, b)) \in (A \times B)$ such that

$f(a, b) = (b, a)$

So $f: A \times B \to B \times A$

is an onto function.

Hence f is bijective.

OR

For all $x_1\ x_2 \in A$

if $f(x_1) = f(x_2)$ implies $x_1 = x_2$ then f is one one

Now $f(x_1) = f(x_2)$

$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

Cross multiplying and solving, we get

$x_1 = x_2$

f is one – one

$$y = \frac{(x-2)}{(x-3)}$$

$(x-3)y = x-2$

$xy - 3y = x - 2$

$xy - x = 3y - 2$

$$x = \frac{(3y-2)}{(y-1)}$$

$$f\left(\frac{3y-2}{y-1}\right) = y$$

Hence f is onto.

34. Let cost of 1kg onion = x

cost of 1kg wheat = y

cost of 1kg rise = z

By the question ,we have,

$4x + 3y + 2z = 60$

$2x + 4y + 6z = 90$

$6x + 2y + 3z = 70$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 50 \neq 0$$

$Now, A_{11} = 0, A_{12} = 30, A_{13} = -20$
$A_{21} = -5, A_{22} = 0, A_{23} = 10$
$A_{31} = 10, A_{32} = -20, A_{33} = 10$

$$\therefore adjA = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

x = 5, y = 8, z = 8

OR

$$\text{Given: } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

Now $A^3 = A^2 A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 4+2+2 & 4+4+1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

L.H.S. $= A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5 & 7-12+5 & 1-6+5 \\ -23+18+5 & 27-48+10 & -69+84-15 \\ 32-42+10 & -13+18-5 & 58+84+15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

Now, to find A^{-1}, multiplying $A^3 - 6A^2 + 5A + 11I = 0$ by A^{-1}

$\Rightarrow A^3 A^{-1} - 6A^2 A^{-1} + 5AA^{-1} + 11I.A^{-1} = 0.A^{-1}$

$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = 0$

$\Rightarrow 11A^{-1} = 6A - 5I - A^2$

$$\Rightarrow 11A^{-1} = 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} 6-5-4 & 6-2 & 6-1 \\ 6+3 & 12-5-8 & -18+14 \\ 12-7 & -6+3 & 18-5-14 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

35. Let the given integral be

$$I = \int \frac{3+2\cos x+4\sin x}{2\sin x+\cos x+3}dx$$

Let, 3 + 2 cos x + 4 sin x = A(2 sin x + cos x + 3) + B(2 cos x - sin x) + C

$\Rightarrow$ 3 + 2 cos x + 4 sin x =(2A - B) sin x + (A + 2B) cos x + 3A + C

Comparing the coefficients of like terms

2A - B = 4 ...(i)

A + 2B = 2 ...(ii)

3A + C = 3 ...(iii)

Multiplying equation (i) by (ii) and then adding it to equation (ii) we get,

$\Rightarrow$ 4A - 2B + A + 2B = 8 + 2

$\Rightarrow$ 5A = 10

$\Rightarrow$ A = 2

Putting value of A = 2 in eq (i)

$\Rightarrow$ 2 × 2 - B = 4

$\Rightarrow$ B = 0

Putting value of A in eq (iii)

$\Rightarrow$ 3 × 2 + C = 3

$\Rightarrow$ C = -3

$$\therefore I = \int \left[\frac{2(2\sin x+\cos x+3)-3}{2\sin x+\cos x+3}\right]dx$$

$$= \int dx - 3\int \frac{1}{2\sin x + \cos x + 3}dx$$

Substituting $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\therefore\ I = 2\int dx - 3\int \frac{1}{2\times\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + 3}dx$$

$$I = 2\int dx - 3\int \frac{\left(1+\tan^2\frac{x}{2}\right)}{4\tan\left(\frac{x}{2}\right) + 1 - \tan^2\left(\frac{x}{2}\right) + 3\left(1+\tan^2\frac{x}{2}\right)}dx$$

$$I = 2\int dx - 3\int \frac{\sec^2\left(\frac{x}{2}\right)}{2\tan^2\left(\frac{x}{2}\right) + 4\tan\left(\frac{x}{2}\right) + 4}dx$$

$$I = 2\int dx - \frac{3}{2}\int \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 2\tan\left(\frac{x}{2}\right) + 2}dx$$

Putting $\tan\left(\frac{x}{2}\right) = t$

$$\Rightarrow \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx = dt$$

$$\Rightarrow \sec^2\left(\frac{x}{2}\right)dx = 2dt$$

$$\therefore I = 2\int dx - \frac{3}{2}\int \frac{2}{t^2+2t+2}dt$$

$$= 2\int dx - 3\int \frac{1}{t^2+2t+1+1}dt$$

$= 2\int dx - 3\int \frac{1}{(t+1)^2+(1)^2}dt$

$= 2x - \frac{3}{1}\tan^{-1}\left(\frac{t+1}{1}\right) + C$

$= 2x - 3\tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C\left[\because t = \tan\frac{x}{2}\right]$

Section E

36. **Read the text carefully and answer the questions:**

On her birthday, Seema decided to donate some money to the children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹).

(i) 5x - 4y = 40

5x - 8y = -80

$$\begin{bmatrix}5 & -4\\5 & -8\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}40\\-80\end{bmatrix}$$

(ii)

$A = \begin{bmatrix}5 & -4\\5 & -8\end{bmatrix}, X = \begin{bmatrix}x\\y\end{bmatrix}$ and $B = \begin{bmatrix}40\\-80\end{bmatrix}$

$|A| = -40 + 20 = -20 \neq 0$

Cofactor matrix $A = \begin{bmatrix}-8 & -5\\4 & 5\end{bmatrix}$ adj $A = \begin{bmatrix}-8 & 4\\-5 & 5\end{bmatrix}$

(iii)

$A = \begin{bmatrix}5 & -4\\5 & -8\end{bmatrix}, X = \begin{bmatrix}x\\y\end{bmatrix}$ and $B = \begin{bmatrix}40\\-80\end{bmatrix}$

$|A| = -40 + 20 = -20 \neq 0$

Cofactor matrix $A = \begin{bmatrix}-8 & -5\\4 & 5\end{bmatrix}$, adj $A = \begin{bmatrix}-8 & 4\\-5 & 5\end{bmatrix}$

$X = A^{-1}B$ ...(i)

$A^{-1} = \frac{1}{|A|}\cdot adjA$

$$A^{-1} = \frac{1}{-20} \cdot \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$

From (i)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-20} \cdot \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-20} \begin{bmatrix} -320 - 320 \\ -200 - 400 \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

x = 32 and y = 30

OR

There are 32 Children, and each child is given ₹30.

Total money spent by Seema = 32 × 30 = ₹960

Hence Seema spends ₹960 in distributing the money to all the students of the Orphanage.

37. **Read the text carefully and answer the questions:**

The Government declare that farmers can get ₹300 per quintal for their onions on 1st July and after that, the price will be dropped by ₹3 per quintal per extra day. Govind's father has 80 quintals of onions in the field on 1st July and he estimates that the crop is increasing at the rate of 1 quintal per day.

(i) Let x be the number of extra days after 1st July.

∴ Price = ₹(300 - 3 × x) = ₹(300 - 3x)

Quantity = 80 quintals + x(1 quintal per day) = (80 + x) quintals

Revenue = R(x) = Quantity × Price = (80 + x) (300 - 3x) = 24000 - 240x + 300x - $3x^2$

R(x) = 24000 + 60x - $3x^2$

(ii) We have, R(x) - 24000 + 60x - $3x^2$

⇒ R'(x) = 60 - 6x ⇒ R''(x) = -6

For R(x) to be maximum, R'(x) = 0 and R''(x) < 0

⇒ 60 - 6x = 0 ⇒ x = 10

(iii) Govind's father will attain maximum revenue after 10 days.

So, he should harvest the onions after 10 days of 1st July i.e., on 11th July.

OR

Maximum revenue is collected by Govind's father when x = 10

∴ Maximum revenue = R(10)

= 24000 + 60(10) - $3(10)^2$ = 24000 + 600 - 300 = ₹24,300

38. **Read the text carefully and answer the questions:**

Akshat and his friend Aditya were playing the snake and ladder game. They had their own dice to play the game. Akshat was having red dice whereas Aditya had black dice. In the beginning, they were using their own dice to play the game. But then they decided to make it faster and started playing with two dice together.


Aditya rolled down both black and red die together.
First die is black and second is red.

(i) Let A represents obtaining a sum greater than 9 and B represents black die resulted in a 5.

n(S) = 36

n(A) = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)} = 6

n(B) = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5)} = 6

n(A ∩ B) = {(5, 5), (5, 6)} = 2

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$$

(ii) Let A represents obtaining a sum 8 and B represents red die resulted in number less than 4.

n(S) = 36

n(A) = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) = 5

n(B) = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)} = 18

n(A ∩ B) = {(5, 3), (6, 2)} = 2

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$$