29. Electric Field and Potential

Short Answer

1. Question

The charge on a proton is +1.6 × 10^{-19} C and that on an electron is −1.6 × 10^{-19} C. Does it mean that the electron has a charge 3.2 × 10^{-19}C less than the charge of a proton?

Answer

No, it does not mean that the electron has a charge 3.2 × 10^{-19}C less than the charge of a proton. Electrons and Protons have same amount of charge just the nature of charge is different. Electron has negative charge and Proton has positive charge. If we keep an electron and a proton at a distance apart then attractive force will be observed as their nature of charge is different.Hence, magnitude of charge of both electron and a proton is1.6 × 10^{-19} C but nature is opposite.

2. Question

Is there any lower limit to the electric force between two particles placed at a separation of 1 cm?

Answer

Yes, there is a lower limit to the electric force between two particles.We know that electric force is given as:$F = \frac{kq_1q_2}{r^2}$This is Coulomb's Law. Here, k is a constant .k = $\frac{1}{4\pi\epsilon_0}$ = 9× 10^9 Nm^2C^{-2}, q_1 and q_2 are the charges of two particles and r is the distance between two charges.The smallest possible charge would be that of an electron.r=1 cm = 0.01 mThus,$F = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{0.01^2}$

$\therefore F = 2.3 \times 10^{-24}\ N$Hence, lower limit to the electric force between two particles placed at a separation of 1 cm would be 2.3× 10^{-24} N.

3. Question

Consider two particles A and B having equal charges and placed at some distance. The particle A is slightly displaced towards B. Does the force on B increase as soon as the particle A is displaced? Does the force on the particle An increase as soon as it is displaced?

Answer

Yes, the force on particle B as well as particle A will increase when particle is A is displaced towards particle B.By Coulomb's Law, Electric force is inversely proportional to the square of the distance between the two charges.F α $1/r^2$Thus, when A is displaced towards B, the distance between them decreases and hence the force on both the particles will increase.

4. Question

Can a gravitational field be added vectorially to an electric field to get a total field?

Answer

No, gravitational field cannot be added vectorially to an electric field to get a total field. This is because Electric field comprises of influence due to electric charges whereas gravitational field comprises of influence due to masses of the bodies and thus, electric field and gravitational field have different dimensions. We can obtain net Force by adding Gravitational force and electric force.Hence, gravitational field cannot be added to electric field vectorially.

5. Question

Why does a phonograph-record attract dust particles just after it is cleaned?

Answer

When a phonograph record is cleaned, electric charges are produced on the record due to induction. This happens as we rub a cloth on the record, friction causes the induction of charges on the record. This phenomenon is similar to a glass rod being rubbed by a cloth. Now, these deposited charges on the record attract dust particles having neutral charge.Hence, a phonograph-record attract dust particles just after it is cleaned.

6. Question

Does the force on a charge due to another charge depend on the charges present nearby?

Answer

No, electric force between two charges or on any one of the two charges does not depends on the charges present nearby.According to Coulomb's Law,Electric force is given as: $F = \frac{kq_1q_2}{r^2}$Here, k is a constant and k = $\frac{1}{4\pi\epsilon_0}$ = 9×10^9 Nm^2C^{-2}, q_1 and q_2 are the two charges and r is the distance between two charges. As we see, the force on one charge depends only on the magnitude of the second charge and vice versa.Third charge won't affect the force between first two charges.The only thing that would affect the force on the charge is the **net force** due to superposition of all the individual forces.

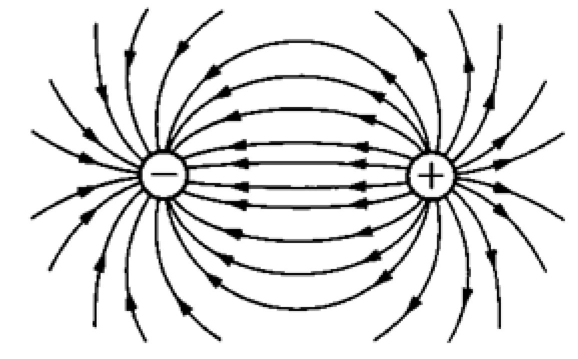
Hence, the force on a charge due to another charge does not depend on the charges present nearby.

7. Question

In some old texts it is mentioned that 4π lines of force originate from each unit positive charge. Comment on the statement in view of the fact that 4π is not an integer.

Answer

Here 4π is considered as a solid angle. Meaning the lines of force can subtend radially outward as shown in the figure below:



It does not mean that 4π lines will come out of a point charge, it means lines of force will move radially outwards from a positive charge having radial angle of 4π.

8. Question

Can two equipotential surfaces cut each other?

Answer

No, two equipotential surfaces cannot cut each other. When two equipotential surfaces intersect at a point, the potential at that point will have two values which is not possible. Also, electric field is perpendicular to the equipotential surface, when two surfaces intersect there will be two directions of the electric field (one of each equipotential surface) which is not possible either.Hence, two equipotential surfaces cannot cut each other.

9. Question

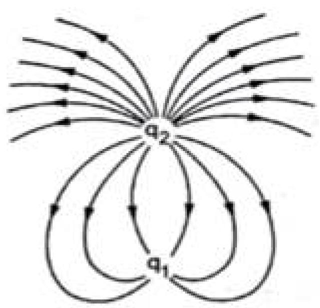
If a charge is placed at rest in an electric field, will its path be along a line of force? Discuss the situation when the lines of force are straight and when they are cured.

Answer

Yes, when a charge is placed at rest in an electric field its path will be tangential the line of force. When the lines of forces are straight, the path of the charge would be in a straight line. When the lines of force are curved, the path of the charge would be tangential to the curve.

10. Question

Consider the situation shown in figure. What are the signs of q_1 and q_2? If the lines are drawn in proportion to the charge, what is the ratio q_1/q_2?



Answer

Lines of force are moving outwards from q_2, hence q_2 is positively charged: $+q_2$. Lines of force are going into q_1, hence q_1 is negatively charged : $-q_1$.From the diagram, 18 lines of electric field come out of q_2 and 6 lines of electric field get into q_1.Ratio of $\frac{q_1}{q_2} = \frac{6}{18} = \frac{1}{3} = 1:3$Hence, If the lines are drawn in proportion to the charge then q_1:q_2= 1:3

11. Question

A point charge is taken from a point A to a point B in an electric field. Does the work done by the electric field depend on the path of the charge?

Answer

No, the work done by the electric field does not depend on the path of the charge.Work done is given as: W = Potential difference × charge.= $(V_A - V_B) \times q$Here, V_A and V_B is the potential at Points A and B respectively. And q is the charge.Thus, the position of the charge is important as potential difference will vary with position and so the work done will change.Hence, when a point is taken from A to a point B the work done on the charge due to electric field will not depend on the path.

12. Question

It is said that the separation between the two charges forming an electric dipole should be small. Small compared to what?

Answer

The distance between the two charges forming the dipole must be small compared to the distance between the point of influence and the center of the dipole.

13. Question

The number of electrons in an insulator is of the same order as the number of electrons in a conductor. What is then the basic difference between a conductor and an insulator?

Answer

The basic difference between a conductor and an insulator is that conductor has many free electrons in its outer shell and an insulator has no free electrons. Free electrons are responsible for conduction of electricity in a material as they are not bound to the atom and are free to move. Whereas in an insulator electrons are tightly bound to the atom, thus no free electrons.

14. Question

When a charged comb is brought near a small piece of paper, it attracts the piece. Does the paper become charged when the comb is brought near it?

Answer

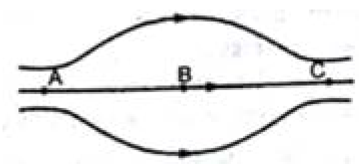
When a charged comb is brought near a small piece of paper, it attracts the piece because of induction. Induction is the process of redistribution of the charges on one body due to the presence of a charged body. Similarly, the charges are distributed on the paper from the comb. Thus when a charged comb is brought near a piece, the charges at the pointing end of the comb attracts the opposite charge on the piece of the paper which were introduced due to induction.Whole paper does not become charged, just the area near the charged comb contains induced electrons.

Hence When a charged comb is brought near a small piece of paper, it attracts the piece.

Objective I

1. Question

Figure shows some of the electric field lines corresponding to an electric field. The figure suggests that



A. $E_A > E_B > E_C$

B. $E_A = E_B = E_C$

C. $E_A = E_C > E_B$

D. $E_A = E_C < E_B$

Answer

At any given point in the electric field region, the electric flux (i.e. the number of field lines per unit area) determines the intensity of the field. The points in the region where the flux is more are associated with strong electric fields.

For a given area, field lines are dense and equal in number at point A and C.

Therefore, $E_A = E_C$

And at point B, the field lines are far away which indicates lesser intensity of electric field at point B than at point A and C.

2. Question

When the separation between two charges is increased, the electric potential energy of the charges

A. increases

B. decreases

C. remains the same

D. may increase or decrease

Answer

Electric potential energy between two charges Q and Q' kept at a distance R is given by:

$$U = \frac{1}{4\pi\epsilon}\frac{(Q)(Q')}{R}$$

If R is increased, then energy may decrease or increase depending on the charges involved.

∴ The potential energy may increase or decrease.

3. Question

If a positive charge is shifted from low-potential region to a high-potential region, the electric potential energy

A. increases

B. decreases

C. remains the same

D. may increase or decrease.

Answer

Electric potential energy of a system of point charges is defined as the work required in assembling this system of charges by bringing them close together, as in the system from infinite distance.

As we know,

$E=-\nabla V$

Electric field flows from high potential to low potential and a positive charge moves in the direction of electric field. In this process, it loses its energy.

But when a positive charge is moved from low potential to high potential i.e. in the direction opposite to the electric field, its energy increases as some work needs to be done to the particle. This work done gets stored as energy in the particle.

4. Question

Two equal positive charges are kept at points A and B. The electric potential at the points between A and B (excluding these points) is studied while moving from A to B. The potential

A. continuously increases

B. continuously decreases

C. increases then decreases

D. decrease then increases

Answer

Potential due to a point charge at a distance r is given by:

$$V = \frac{1}{4\pi\epsilon}\frac{(Q)}{r}$$

Therefore, at a point very nearby to A and B, r → 0 ∴ V → ∞

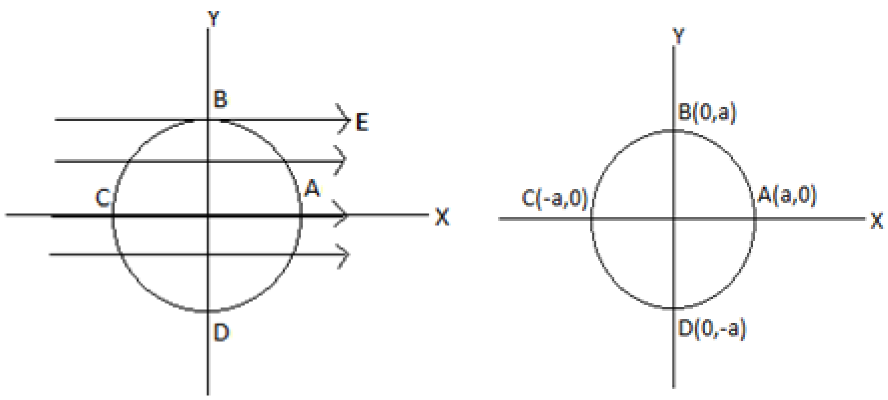
i.e.

At a very nearby point right to A and a very nearby point left to B, potential will tend to ∞ (maximum possible value) as separation will tend to zero. That means at a point somewhere in the middle of path AB, the potential will acquire its minimum value.

Therefore, while moving from A to B, potential will first decrease and then increase.

5. Question

The electric field at the origin is along the positive x-axis. A small circle is drawn with the center at the origin cutting the axes at points A, B, C and D having coordinates (α, 0), (0, α), (−α, 0), (0, −α) respectively. Out of the points on the periphery of the circle, the potential is minimum at



A. A

B. B

C. C

D. D

Answer

As the electric field is in +X direction, that is field lines will flow from point C to point A. And as we already know that field lines flow from high potential to low potential, that means point A needs to be at a low potential region and point C needs to be at a high potential region.

Points B and D are equipotential.

The order of potentials is:

$V_C > V_B = V_D > V_A$

Therefore, potential is minimum at A.

6. Question

If a body is charged by rubbing it, its weight

A. remains precisely constant

B. increases slightly

C. decreases slightly

D. may increase slightly or may decrease slightly.

Answer

On charging a body by rubbing it, there are two possibilities to happen: Either the body acquires a positive charge or a negative charge.

As q=ne (where q is the amount of charge acquired on a body, n is the number of electrons transferred from or on the body and e is the charge on one electron.)

In case the body acquires a positive charge, there is loss of electrons from that body, therefore, its mass decreases.

In case the body acquires a negative charge, there is gain of electrons from that body, therefore, its mass increases.

So, on charging a body, its mass may increase or may decrease slightly.

7. Question

An electric dipole is placed in a uniform electric field. The net electric force on the dipole

A. is always zero

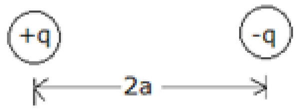
B. depends on the orientation of the dipole

C. can never be zero

D. depends on the strength of the dipole

Answer

An electric dipole consists of equal and opposite charges placed at some distance.



The dipole moment of this configuration is given as **p**=q(2**a**)

When the dipole is kept in an external electric field E, both its charges experience some force. Let the force on the positive charge(q) and negative charge(-q) be $\mathbf{F}_P$ and $\mathbf{F}_N$ respectively.

Therefore, $\mathbf{F}_P = q\mathbf{E}$ and $\mathbf{F}_N = -q\mathbf{E}$.

I.e. $\mathbf{F}_P = -\mathbf{F}_N$

that is force on the dipole is equal in magnitude but opposite in direction irrespective of its orientation.

Therefore, force on an electric dipole placed in an electric field is always zero.

8. Question

Consider the situation of figure. The work done in taking a point charge from P to A is W_A, from P to B is W_B and from P to C is W_C.

A. $W_A < W_B < W_C$

B. $W_A > W_B > W_C$

C. $W_A = W_B = W_C$

D.None of these

Answer

Since electric field is a conservative field and therefore electric force is also a conservative force. Therefore, work done will not depend in the path taken.

As for a conservative force field, work done on a charge does not depend on the path taken by the charge but depends only on the initial and final points. Therefore, in this situation,

i.e. $W_A = W_B = W_C$

∴ The work done in taking a point charge from P to all the points A, B and C is the same.

9. Question

A point charge q is rotated along a circle in the electric field generated by another point charge Q. The work done by the electric field on the rotating charge in one complete revolution is

A. zero

B. positive

C. negative

D. zero if the charge Q is at the center and nonzero otherwise.

Answer

If a force **F** applies on a body and it allows it to move an infinitely small distance d, then work done is given by dW=**F**.d**s** and for a path, integrate it putting lower limits (starting position) and upper limits (ending position).

Here, in the given question,

since a complete rotation is made i.e. net displacement is zero. i.e. $ds = 0$

$$\therefore\ w = \oint F.ds\ = 0\ (for\ one\ complete\ revolution)$$

i.e. The work done by the electric field on the rotating charge in <u>one complete revolution</u> is zero.

Objective II

1. Question

Mark out the correct options.

A. The total charge of the universe is constant.

B. The total positive charge of the universe is constant.

C. The total negative charge of the universe is constant.

D. The total number of charged particles in the universe is constant.

Answer

Option (a) is correct because total charge of the universe is constant. It just gets transferred from one particle to another.

Option (b) is incorrect as the total positive charge of the universe is not constant. It is because the positive charges get converted into negative charges and vice-versa.

Option (c) is incorrect as the total negative charge of the universe is not constant. It is because the negative charges get converted into positive charges and vice-versa.

Option (d) is incorrect as in universe, the total number of charged particles is not a constant as a pair of positive and negative charges appear at the time of pair production and a pair of positive and negative charge combine to form a neutral particle at the time of pair annihilation.

2. Question

A point charge is brought in an electric field. The electric field at a nearby point.

A. will increase if the charge is positive

B. will decrease if the charge is negative

C. may increase if the charge is positive.

D. may decrease if the charge is negative.

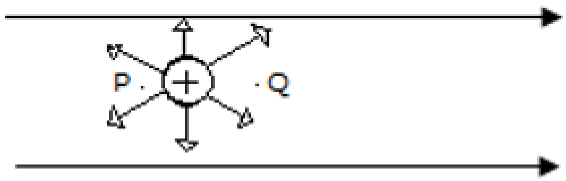
Answer

Option (c) is correct:

In an electric field, if a positive charge is placed, then the charge itself will generate some electric field on its own diverging from the point.

At a nearby point Q (towards the direction of external field), the field will increase as the field lines from the charge will also add to external field lines.

At a nearby point P (away from the direction of external field), the field will decrease as the field lines from the charge will cancel some of the external field lines.



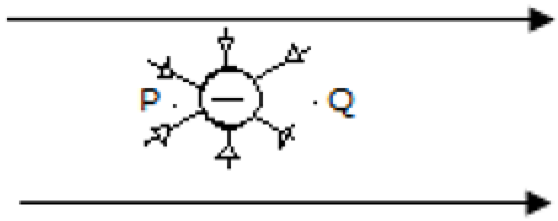
Therefore, at a nearby point, field can either increase or decrease.

Option (d) is correct:

In an electric field, if a negative charge is placed, then the charge itself will generate some electric field on its own converging to the point.

At a nearby point Q (towards the direction of external field), the field will decrease as the field lines from the charge will cancel some of the external field lines.

At a nearby point P (away from the direction of external field), the field will increase as the field lines from the charge will add on to the external field lines.



Therefore, at a nearby point, field can either increase or decrease.

3. Question

The electric field and the electric potential at a point are E and V respectively.

A. If E = 0, V must be zero.

B. If V = 0, E must be zero.

C. If E ≠ 0, V cannot be zero

D. If V ≠ 0, E cannot be zero.

Answer

We know that electric field is negative of the space rate of change of electric potential in a region. i.e.

$$\boldsymbol{E} = -\nabla V$$

Option (a) is incorrect as for E =0, V=0 is not the only condition. As we can see from the above equation, electric field is the negative of vector derivative of V that means

if V=constant, then also, E=0.

(As $\frac{d(constant)}{dr} = 0$)

Option (b) is incorrect as:

If at a point, the electric potential is zero, it doesn't imply electric field to be also zero.

Take an example of a dipole where somewhere between the two charges on its axial line, there exists a point where potential is zero but electric field is always non zero and directs in the direction from positive charge to negative charge.

Option (c) is incorrect as if E≠0, it doesn't mandate the potential to be non-zero. We can take again the example of a dipole where somewhere at a point on its axial line, there exists a point where E≠0 but V=0. So the statement "If E ≠ 0, V cannot be zero" is incorrect.

Option (d) is incorrect as:

If V ≠ 0 (let's say V= non-zero constant)

Then E=0

Which is contradicted in the given statement.

Thus, none of the options is correct.

4. Question

The electric potential decreases uniformly from 120V to 80V as one moves on the x-axis from x = −1 cm to x = +1 cm. The electric field at the origin

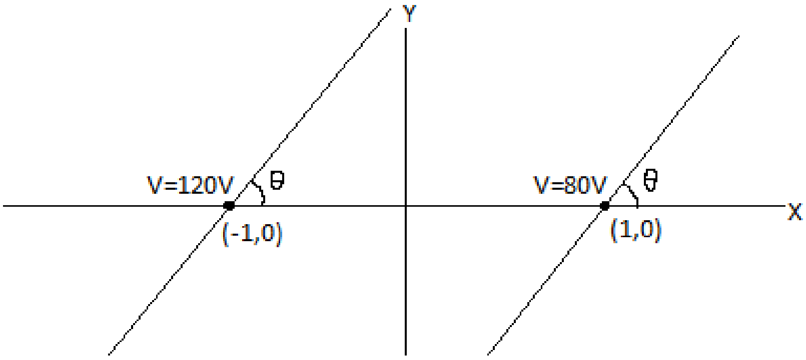
A. must be equal to 20 V cm^{-1}

B. may be equal to 20 V cm^{-1}

C. may be greater than 20 V cm^{-1}

D. may be less than 20 V cm^{-1}.

Answer



$$V = -\int \boldsymbol{E.dr}$$

Here also, $\Delta V = \int_{-1}^{1} \boldsymbol{E}.\,\boldsymbol{dx}$ and $\Delta V = 120 - 80 = 40\ V$

Upon solving, we get $E = 20\ V/cm$ but it will also depend on the angle that equipotential surface makes with the X axis.

Therefore, the value of electric field may greater than 20V/cm (if the potential decreases uniformly) or equal to 20 V/cm (when equipotential surfaces are at right angles from X axis.)

5. Question

Which of the following quantities do not depend on the choice of zero potential or zero potential energy?

A. Potential at a point

B. Potential difference between two points

C. Potential energy of a two-charge system

D. Change in potential energy of a two-charge system.

Answer

Option (a) is incorrect because potential at a point depends on our choice of zero potential. Normally infinity is taken as reference point where potential is assumed to be zero.

If choice of zero potential is changed, then the potential at a point will also change.

Option (b) is correct as difference in potential between the two points doesn't depend on the choice of zero potential.

Option (c) is incorrect as the potential energy of the two-charge system also depends on the choice of zero potential in order to calculate the potential at the point where the two charges are kept.

Option (d) is correct as the change in potential energy of the two-charge system is independent of our choice of the zero-potential point.

Thus, we conclude that potential difference between two points and change in potential energy of a two-charge system don't depend on the choice of zero potential point.

6. Question

An electric dipole is placed in an electric field generated by a point charge.

A. The net electric force on the dipole must be zero.

B. The net electric force on the dipole may be zero.

C. The torque on the dipole due to the field must be zero.

D. The torque on the dipole due to the field may be zero.

Answer

The field due a point charge is always radial (radially outwards for positive charge and radially inwards for negative charge). A dipole placed in that electric field will never feel equal and opposite forces due to the radial nature of the field. Therefore, option (a) and (b) are incorrect.

Torque on the dipole may be zero in the case when the dipole is placed along the electric field (as in that only case, $\boldsymbol{F}$ and $\boldsymbol{r}$ will be parallel or antiparallel, thus torque will be zero.)

7. Question

A proton and an electron are placed in a uniform electric field.

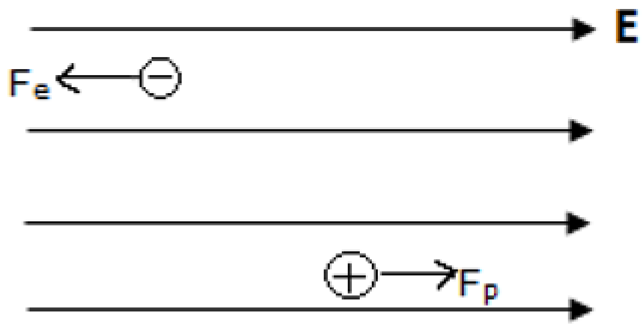
A. The electric forces acting on them will be equal.

B. The magnitudes of the forces will be equal.

C. Their accelerations will be equal.

D. The magnitudes of their accelerations will be equal

Answer



Let the force on the electron and proton is F_e (F) and F_p (F') respectively.

Then,

$F = -eE \text{ and } F' = eE$

Option (a) is incorrect as the forces are equal only in magnitude but not in direction.

Option(b) is correct as magnitude of the forces are equal.

Option (c) is incorrect as the magnitude of accelerations for both particles is not equal and also the direction is also not the same.

Option (d) is incorrect as the magnitude of acceleration of both will be different as the masses of both are different.

8. Question

The electric field in a region is directed outward and is proportional to the distance r from the origin. Taking the electric potential at the origin to be zero,

A. it is uniform in the region

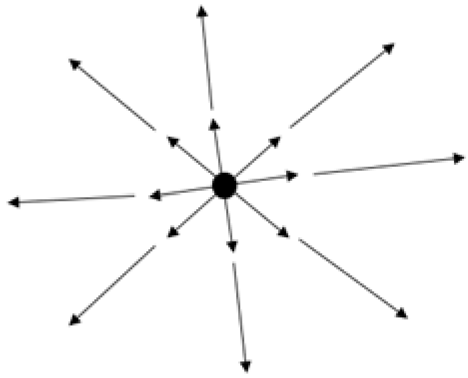
B. it is proportional to r

C. it is proportional to r^2

D. it increases as one goes away from the origin

Answer

The field around the origin will be like:



Where the intensity of the field will increase as we will move away from the charge placed at origin (as field is proportional to the distance r from the origin.)

Option (a) is incorrect as the field is not uniform in the region (can be perceived by the given situation and the diagram)

Field is given as:

$\boldsymbol{E} \propto \boldsymbol{r}$

$\boldsymbol{E} = k\boldsymbol{r}$ (where k is some positive constant)

Now,

$$V = -\int \boldsymbol{E}.d\boldsymbol{r} = \int Edr\cos\,0$$

$$V = -\int krdr$$

$$V = -k\frac{r^2}{2} + c\;(c\; is\; constant\; of\; integration)$$

Option (b) is incorrect since V is not proportional to r.

Option (c) is correct since V is proportional to r.

Option (d) is incorrect. From the expression for V, we can say that the potential decreases as we move away. Alternatively, the field lines always flow from high potential to low potential. Therefore, we can say that the potential decreases with increase in r.

Exercises

1. Question

Find the dimensional formula of ϵ_0.

Answer

We know that,

Electrostatic force, $F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

Where

ϵ_0 is the permittivity of vacuum

q_1, q_2 is the magnitude of the charges on the charge particles

r is the distance between the two charge particles.

Taking the dimensions,

$$[F_e] = \frac{1}{[4\pi][\epsilon_0]}\frac{[q_1][q_2]}{[r^2]}$$

Note that Charge = Current × Time. Hence, its dimensional formula is $[Current][Time] = [A][T]$

Note that 4π is a dimensionless constant. Note that $F = ma$. Hence, $[F] = [m][a] = [MLT^{-2}]$. Also, r is just distance or length. Thus, $[r] = [L]$

$$[MLT^{-2}] = \frac{[AT][AT]}{[\epsilon_0][L^2]}$$

Rearranging, we get

$$[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$$

2. Question

A charge of 1.0 C is placed at the top of your college building and another equal charge at the top of your house. Take the separation between the two charges to be 2.0 km. Find the force exerted by the charges on each other. How many times of your weight is this force?

Answer

Given:

$$q_1 = q_2 = 1.0C$$

Distance between the charges, $r = 1.0km = 10^3m$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

By Coulomb's law,

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

$$F_e = \frac{9 \times 10^9 NC^{-2}m^2 \times 1.0C \times 1.0C}{(2 \times 10^3 m)^2}$$

$= 2.25 \times 10^3 N$

Formula used:

The weight of an object is given by

$F_w = mg$

where g is the acceleration due to gravity

and m is the mass of the object.

Let the mass of my body, m be 70 kg.

$F_w = mg = 70kg \times 9.8ms^{-2} = 686N$

Thus, dividing the Electrostatic force and the body weight, we get

$$\frac{F_e}{F_w} = \frac{2.25 \times 10^3 N}{686N} \approx 3.3$$

The electric force between the two charges is 3.3 times my weight.

3. Question

At what separation should two equal charges, 1.0C each, be placed so that the force between them equals the weight of a 50 kg person?

Answer

Given:

Mass of the person , $m = 50kg$

Weight of the person, $w = mg = 50kg \times 9.8ms^{-2} = 490N$

Charges: $q_1 = q_2 = 1.0C$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

The magnitude of electric force is given by:

$$F_e = k\frac{q_1q_2}{r^2}$$

Rearranging, we get

$$r = \sqrt{k\frac{q_1q_2}{F_e}}$$

Note that $F_e = w = 490N$. Substituting all the values, we get

$$r = \sqrt{\frac{9 \times 10^9 NC^{-2}m^2 \times 1.0C \times 1.0C}{490N}}$$

$$\approx 4.3 \times 10^3 m$$

4. Question

Two equal charges are placed at a separation of 1.0 m. What should be the magnitude of the charges so that the force between them equals the weight of a 50 kg person?

Answer

Given:

Mass of the person , $m = 50kg$

Weight of the person, $w = mg = 50kg \times 9.8ms^{-2} = 490N$

Let the magnitude of charge on both be q.

Here, $q_1 = q_2 = q$

Distance of separation, $r = 1.0m$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \mathrm{k}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

$$F_e = \mathrm{k}\frac{q_1 q_2}{r^2}$$

$$= k\frac{q^2}{r^2}$$

Rearranging, we get

$$q = \sqrt{\frac{F_e r^2}{k}}$$

Substituting the corresponding values, we get

$$q = \sqrt{\frac{490N \times (1.0m)^2}{9 \times 10^9 NC^{-2}m^2}}$$

$$\approx 2.3 \times 10^{-4}\ C$$

5. Question

Find the electric force between two protons separated by a distance of 1 fermi (1 fermi = 10^{-15} m). The protons are a nucleus remain at a separation of this order.

Answer

<u>Given:</u>

Charge on each proton, q_1=q_2=1.6×10^{-19} C

Distance of separation, $r = 1\ fermi\ m = 10^{-15}m$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = k\frac{q_1q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

The magnitude of electric force is given by:

$$F_e = k\frac{q_1q_2}{r^2}$$

$$= \frac{9 \times 10^9 NC^{-2}m^2 \times 1.6 \times 10^{-19}C \times 1.6 \times 10^{-19}C}{(10^{-15}m)^2}$$

$$\approx 230\ N$$

6. Question

Two charges 2.0 × 10^{-6} C and 1.0 × 10^{-6} C are placed at a separation of 10 cm. Where should a third charge be placed such that it experiences no net force due to these charges?

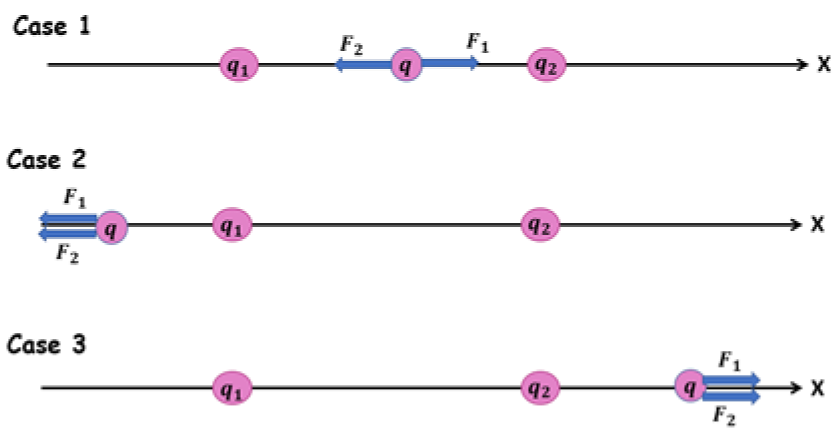
Answer

Given:

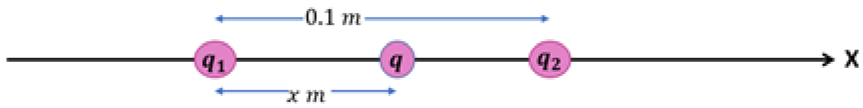
Charges: $q_1 = 1.0 \times 10^{-6}C, q_2 = 2.0 \times 10^{-6}C$

Distance between the charge is 10 cm

Let the third charge be q. There are three possible scenarios:



As we can see from the picture, the only possible location for q where net force on q is zero is between the two charges. Let it be x m away from q_1, $x > 0$.



Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = k\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

Force on q due to q_1 is given by:

$$F_1 = k\frac{q_1 q}{x^2}$$

Force on q due to q_2 is given by:

$$F_2 = k\frac{q_2 q}{(0.1 - x)^2}$$

Now,

$$F_1 = F_2$$

$$\frac{(0.1 - x)^2}{x^2} = \frac{q_2}{q_1} = 2$$

$$\frac{(0.1 - x)}{x} = \pm\sqrt{2}$$

$$(0.1 - x) = \pm\sqrt{2}x$$

$$x = \frac{0.1}{1 \pm \sqrt{2}}$$

$$= -0.241 \; or \; 0.041$$

As x > 0. The correct option is 0.041.

Distance from charge q_2 = 0.1 – 0.041 = 0.059 m

Hence, the charge q should be placed 5.9 cm from q_2.

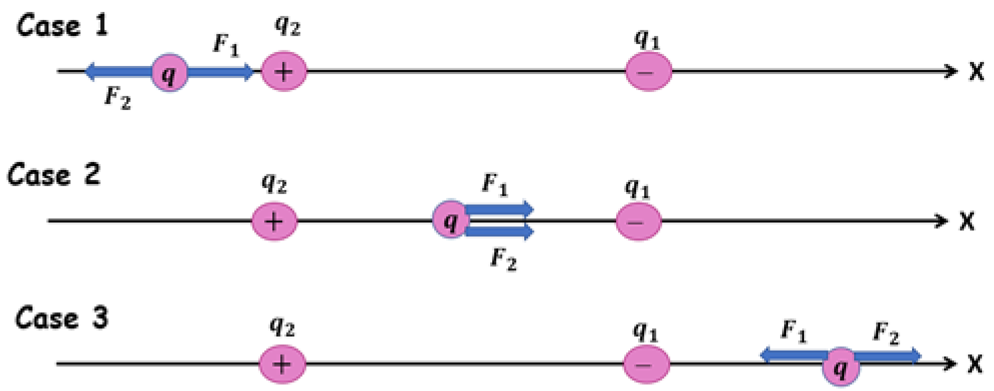
7. Question

Suppose the second charge in the previous problem is -1.0×10^{-6} C. Locate the position where a third charge will not experience a net force.

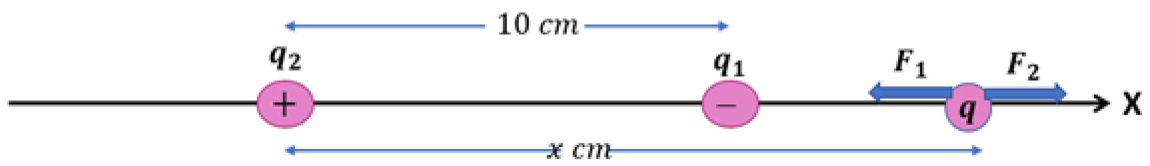
Answer

<u>Given:</u>

Charges: $q_1=-1.0\times10^{-6}$ C, $q_2=2.0\times10^{-6}$ C



Let the third charge q be at a distance x cm from q_2. As the forces must cancel, case II is not possible. Moreover, for forces to cancel, q must be near the smaller charge. Hence, only Case III is possible.



Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = k\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

Force on q due to q_1 is given by:

$$F_1 = \mathrm{k}\frac{|q_1|q}{((x-10)cm)^2}$$

Force on q due to q_2 is given by:

$$F_2 = k\frac{|q_2|q}{(x\ cm)^2}$$

Now,

$$F_1 = F_2$$

$$\frac{x^2}{(x-10)^2} = \left|\frac{q_2}{q_1}\right| = 2$$

$$\frac{x}{x-10} = \pm\sqrt{2}$$

$$x = \pm\sqrt{2}(x - 10)$$

$$x = \frac{10\sqrt{2}}{\sqrt{2}\pm 1}$$

$$= 5.86, 34.1$$

For Case III, x must be greater than 10. Hence, x = 34.14 cm

Thus, q should be placed *34.1 cm* from the larger charge on the side of the smaller charge.

8. Question

Two charged particles are placed at a distance 1.0 cm apart. What is the minimum possible magnitude of the electric force acting on each charge?

Answer

<u>Given: missing</u>

For the electric force to be minimum, the magnitude of charge on the particles must be minimum. As both the particles are charged, they must have non-zero minimum charge.

Hence, they both must carry charge having magnitude of fundamental unit of charge i.e. 1.6×10^{19} C.

Magnitude of charges, $q_1 = q_2 = 1.6 \times 10^{19}C$

Distance between the charged particles, $r = 1.0cm = 0.01\,m$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \mathrm{k}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

The magnitude of electric force is:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 NC^{-2}m^2 \times 1.6 \times 10^{-19}C \times 1.6 \times 10^{-19}C}{(0.01m)^2} \approx 2.3 \times 10^{-24}N$$

9. Question

Estimate the number of electrons in 100g of water. How much is the total negative charge on these electrons?

Answer

<u>Given:</u>

Mass of water , $m = 100g$

Molar mass of water $M = 18\ gmol^{-1}$

Moles of water in 100g of water = $\frac{100g}{18gmol^{-1}}$

Number of electrons in 1 molecule of water = 2 times one electron each in hydrogen atom + 8 electrons in oxygen atom = 10 electrons

---

Formulas used:

The relation between molar mass M, mass of sample m and number of moles of sample n is given by:

$$n = \frac{m}{M}$$

1 mol of "something" contains N_A number of "something".

Number of electrons in 1 mol of electrons $= 10\ electrons \times N_A$= $10\ electrons \times 6.022 \times 10^{23} mol^{-1}$

Number of electrons in 1 g of water

= 10 electrons × N_a × $\frac{1g}{18\ g\ mol^{-1}}$

Number of electrons in 100 g of water

$$= 10\ electrons \times 6.022 \times 10^{23} mol^{-1} \times \frac{100g}{18gmol^{-1}}$$

$$= 3.34556 \times 10^{25} electrons$$

$$\approx 3.35 \times 10^{25} electrons$$

Negative charge on one electron = $1.6 \times 10^{-19} C$

Total negative charge in 100g of water
$= 3.34556 \times 10^{25} \times 1.6 \times 10^{-19} C \ \approx 5.35\ \times 10^{6} C$

10. Question

Suppose all the electrons of 100g water are lumped together to form a negatively charged particle and all the nuclei are lumped together to form a positively charged particle. If these two particles are placed 10.0 cm away from each other, find the force of attraction between them. Compare it with your weight.

Answer

Given:

Mass of water , $m \ = \ 100g$

Molar mass of water $M \ = 18gmol^{-1}$

Moles of water in 100g of water = $\frac{100g}{18gmol^{-1}}$

Number of electrons in 1 molecule of water = 2 times one electron each in hydrogen atom + 8 electrons in oxygen atom = 10 electrons

---

Formulas used:

The relation between molar mass M, mass of sample m and number of moles of sample n is given by:

$$n = \frac{m}{M}$$

1 mol of "something" contains N_A number of "something".

Number of electrons in 1 mol of electrons $= 18\ electrons \times N_A$= $10\ electrons \times 6.022 \times 10^{23} mol^{-1}$

Number of electrons in 100 g of water

$$= 10\ electrons \times 6.022 \times 10^{23} mol^{-1} \times \frac{100g}{18gmol^{-1}}$$

$$= 3.34556 \times 10^{25} electrons$$

$$\approx 3.35 \times 10^{25} electrons$$

Negative charge on one electron = $1.6 \times 10^{-19} C$

Total negative charge $= 3.34556 \times 10^{25} \times 1.6 \times 10^{-19} C \approx 5.35 \times 10^{6} C$

We have,

Charges: $q_1 = -5.35 \times 10^{6} C, q_2 = 5.35 \times 10^{6} C$

Distance of separation, $r = 10cm = 0.1\ m$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \mathrm{k}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

The magnitude of electric force is:

$$F_e = \frac{k|q_1||q_1|}{r^2}$$

$$= \frac{9 \times 10^9 NC^{-2} m^2 \times (5.35 \times 10^6 C)^2}{(0.1m)^2}$$

$$\approx 2.57 \times 10^{25} N$$

Let my mass m be 70 kg.

$$F_w = mg = 70kg \times 9.8ms^{-2} = 686N$$

$$\frac{F_e}{F_w} = \frac{2.57 \times 10^{25} N}{686N} \approx 3.7 \times 10^{22}$$

The electric force between the two charges is about 10^{22} times my weight!

11. Question

Consider a gold nucleus to be a sphere of radius 6.9 fermi in which protons and neutrons are distributed. Find the force of repulsion between two protons situated at largest separation. Why do these protons not fly apart under this repulsion?

Answer

Given:

Largest Distance of Separation = Diameter of the nucleus ,
$d = 13.8 fermi = 13.8 \times 10^{-15} m$

Charge on each proton , $q_1 = q_2 = 1.6 \times 10^{-19} C$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \mathrm{k}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

The electric force is repulsive and is given by:

$F = k\frac{q_1 q_2}{d^2}$

$F = \frac{9 \times 10^9 NC^{-2} m^2 \times (1.6 \times 10^{-19} C)^2}{(13.8 \times 10^{-15} m)^2}$

$\approx 1.21\, N$

The protons do not fly apart because there is also strong nuclear force which is attractive and balances the repulsive electric force.

12. Question

Two insulating small spheres are rubbed against each other and placed 1 cm apart. If they attract each other with a force of 0.1N, how many electrons were transferred from one sphere to the other during rubbing?

Answer

Given,

Electric force between the two spheres, $F_e = 0.1N$

Distance between the two spheres, $r = 1cm = 0.01m$

We know that, magnitude of charge on electron, $e = 1.6 \times 10^{-19}$

Let n electrons be transferred from one sphere to another. Then, charge on one sphere is +ne and the other is -ne.

$q_1 = q_2 = ne$

Formula used:

By Coulomb's law, the electric force is given by:

$F_e = k\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

The magnitude of electric force is given by:

$$F_e = k\frac{q_1 q_2}{r^2}$$

$$F_e = k\frac{(ne)^2}{r^2}$$

$$n = \frac{r}{e}\sqrt{\frac{F_e}{k}}$$

$$= \frac{0.01m}{1.6 \times 10^{-19}C}\sqrt{\frac{0.1N}{9 \times 10^9 NC^{-2}m^2}}$$

$$\approx 2.0 \times 10^{11}$$

Hence, about 2.0×10^{11} electrons are transferred.

13. Question

NaCl molecule is bound due to the electric force between the sodium and the chlorine ions when one electron of sodium is transferred to chlorine. Taking the separation between the ions to be 2.75×10^{-8} cm, find the force of attraction between them. State the assumptions (if any) that you have made.

Answer

Given:

Charge on Na^+ ion = $q = 1.6 \times 10^{-19}$

Charge on Cl^- ion = $-q = -1.6 \times 10^{-19}$

Distance between the ions, $r = 2.75 \times 10^{-8} cm = 2.75 \times 10^{-10} m$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = k\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

We assume that the distance between the transferred electron and the sodium nucleus is the distance between the two ions.

$$F_e = k\frac{q \times q}{r^2}$$

$$= \frac{9 \times 10^9 NC^{-2}m^2 \times (1.6 \times 10^{-19}C)^2}{(2.75 \times 10^{-10}m)^2}$$

$$\approx 3.05 \times 10^{-9}N$$

14. Question

Find the ratio of the electric and gravitational forces between two protons.

Answer

Given:

Mass of proton, $m = 1.67 \times 10^{-27}$

Charge on proton, $q = 1.6 \times 10^{-19}$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \mathrm{k}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

Formula used:

By Newton's law of gravitation, the gravitational force is given by:

$$F_g = \mathrm{G}\frac{m_1 m_2}{r^2}$$

Where G is the Gravitational constant

m_1 and m_2 are the magnitude of charges r is the distance of separation between the masses

---

The electric force is given by:

$$F_e = k\frac{q^2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The gravitational force is given by:

$$F_g = G\frac{m^2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Dividing (2) by (1), we get

$$\frac{F_e}{F_g} = \frac{kq^2}{Gm^2}$$

$$= \frac{9 \times 10^9 NC^{-2}m^2 \times (1.6 \times 10^{-19}C)^2}{6.67 \times 10^{-11}Nkg^{-2}m^2 \times (1.67 \times 10^{-27}kg)^2}$$

$$\approx 1.23 \times 10^{36}$$

Hence the ratio of electric force to gravitational force between two protons is about $1.23 \times 10^{36.}$

15. Question

Suppose an attractive nuclear force acts between two protons which may be written as $F = CE^{-kx}/r^2$.

(a) Write down the dimensional formulae and appropriate SI units of C and κ.

(b) Suppose that κ = 1 fermi^{-1} and that the repulsive electric force between the protons is just balanced by the attractive nuclear force when the separation is 5 fermi. Find the value of C.

Answer

Given:

The attractive nuclear force between two protons is, $F = \frac{CE^{-kx^2}}{r^2}$

(a)The power of e must be dimensionless.

Thus,

$[k][r] = 1$

$[k] = \frac{1}{[r]} = [L^{-1}]$

Hence, SI unit of k is m.

Now,

$[F] = \frac{[C][e^{-kx}]}{[r^2]}$

$[MLT^{-2}] = [C][1][L^{-2}]$

$[C] = [ML^3T^{-2}]$

Let us replace the formula with SI units

$N = \frac{C}{m^2}$

Hence, SI unit of C is Nm^2

(b)

Given,

Separation between the protons, r = 5fermi = 5×10^{-15} m.

We know that the charge on a proton is q = 1.6×10^{-19}C.

Now,

Formula used:

By Coulomb's law, the electric force is given by:

$F_e = \mathrm{k}\frac{q_1q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

The electric force between the two protons is given by:

$$F_e = k\frac{q^2}{r^2}$$

Also, the nuclear force between the protons is given by:

$$F_n = C\frac{e^{-kr}}{r^2}$$

These two forces balance each other.

$F_e = F_n$

$kq^2 = Ce^{-kr}$

$$C = \frac{kq^2}{e^{-kr}}$$

$$= \frac{9 \times 10^9 NC^{-2} m^2 \times (1.6 \times 10^{-19})^2\ C}{e^{-1 fermi^{-1} \times 5 fremi}}$$

$$\approx 3.4 \times 10^{-26} Nm^2$$

16. Question

Three equal charges, 2.0 × 10^{-6} C each, are held fixed at the three corners of an equilateral triangle of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest two.

Answer

Given:

Let the force due to charge at corner B, C be $\mathbf{F_B}$, $\mathbf{F_C}$.

Charge at each corner, $q = 2.0 \times 10^{-6}$.

Note that the length of side of the triangle is, $a = 5cm = 0.05\ m$.

Formula used:

By Coulomb's law, the electric force is given by:

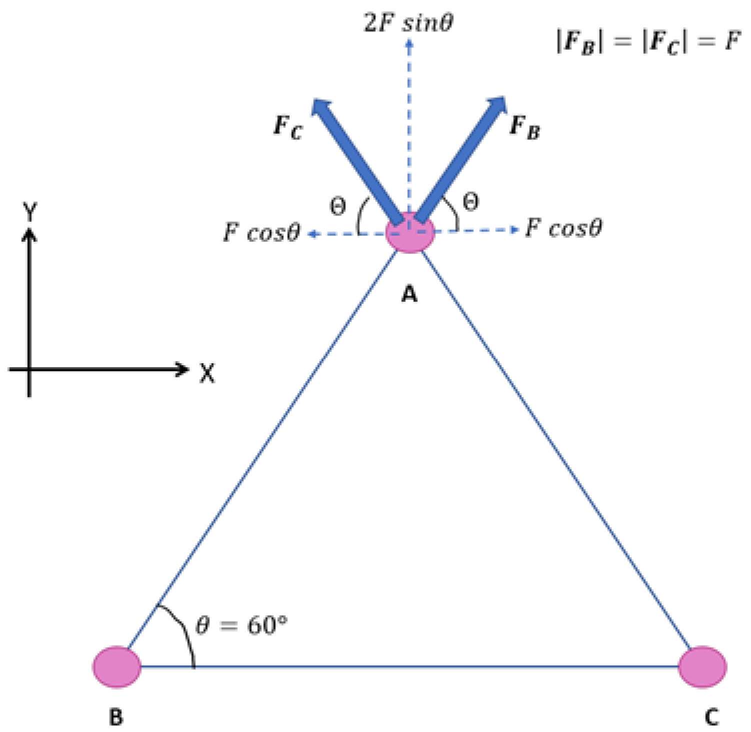
$$F_e = k\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges



$$F = |\vec{F_B}| = |\vec{F_C}| = k\frac{q^2}{a^2}$$

Putting the values in the above formula, we get

$$= \frac{9 \times 10^9 NC^{-2}m^2 \times (2.0 \times 10^{-6}C)^2}{(0.05m)^2} \approx 14.4N$$

We can see from the diagram that $\mathbf{F_C}$and $\mathbf{F_B}$cab be resolved into theirs x and y components.

$F_{c,x} = F\cos 60°$

$F_{c,y} = F\sin 60°$

$F_{b,x} = F\cos 60°$

$F_{b,y} = F\sin 60°$

The y's are in the same direction and add up but the x's are in the opposite direction, hence they cancel out.

Hence,

$$\left|\overrightarrow{F_{net}}\right| = F_{c,x} + F_{c,y} = 2F\, sin\, 60°$$

$$\left|\overrightarrow{F_{net}}\right| = 2\ \times 14.4\, sin(60°)N \approx 24.9N$$

17. Question

Four equal charges 2.0 × 10^{-6} C each are fixed at the four corners of a square of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest three.

Answer

Given:

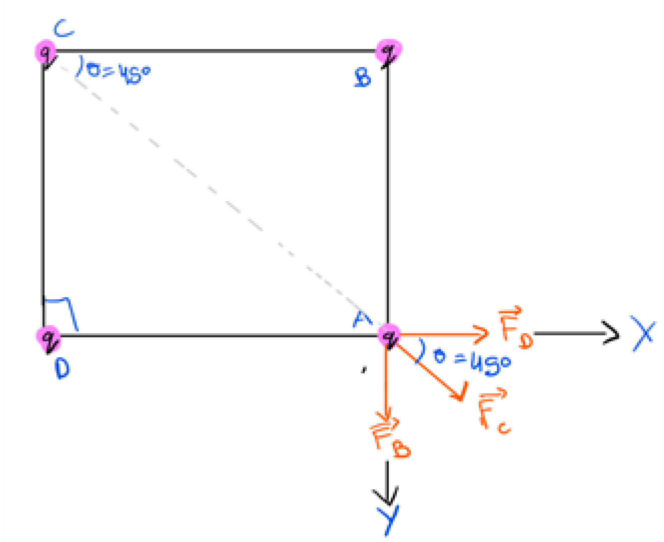
Let the force due to charge at corner B, C and D be $\mathbf{F_B}$, $\mathbf{F_C}$, $\mathbf{F_D}$.

Charge at each corner, $q = 2.0 \times 10^{-6}$.

The side of square is, $a = 5cm = 0.05\ m$.

By Pythagoras theorem,

The length of diagonal $= a\sqrt{2} = 0.05\sqrt{2}\ m$.



Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = k\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

k is the electrostatic constant

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

$$\overrightarrow{F_D} = k\frac{q^2}{a^2}\hat{\imath} = \frac{9\times10^9NC^{-2}m^2\times(2\times10^{-6}C)^2}{(0.05m)^2}\hat{\imath} = 14.4N\hat{\imath}$$

$$\overrightarrow{F_B} = k\frac{q^2}{a^2}\hat{\jmath} = \frac{9\times10^9NC^{-2}m^2\times(2\times10^{-6}C)^2}{(0.05m)^2}\hat{\jmath} = 14.4N\hat{\jmath}$$

$$|\overrightarrow{F_C}| = k\frac{q^2}{a^2} = \frac{9\times10^9NC^{-2}m^2\times(2\times10^{-6}C)^2}{\left(0.05\sqrt{2}m\right)^2} = 7.2N$$

Resolving $\mathbf{F_C}$, we get

$$F_x = |\overrightarrow{F_c}|\cos\theta = 7.2N\cos(45°) = \frac{7.2N}{\sqrt{2}}$$

$$F_y = |\overrightarrow{F_c}|\sin\theta = 7.2\text{N}\sin(45°) = \frac{7.2N}{\sqrt{2}}$$

$$\overrightarrow{F_c} = F_x\hat{\imath} + F_y\hat{\jmath}$$

$$\overrightarrow{F_c} = \frac{7.2\text{N}}{\sqrt{2}}(\hat{\imath}+\hat{\jmath}) = \frac{14.4}{2\sqrt{2}}(\hat{\imath}+\hat{\jmath})\text{N}$$

The net force is:

$$\overrightarrow{F_{net}} = \overrightarrow{F_b} + \overrightarrow{F_c} + \overrightarrow{F_d}$$

$$\overrightarrow{F_{net}} = 14.4\left(1+\frac{1}{2\sqrt{2}}\right)(\hat{\imath}+\hat{\jmath})N \approx 19.49(\hat{\imath}+\hat{\jmath})N$$

We know that

$$|\overrightarrow{F_{net}}| = \sqrt{F_x^2+F_y^2}$$

Hence, the magnitude of net force is

$$|\overrightarrow{F_{net}}| \approx \sqrt{19.49^2+19.49^2}N \approx 27.5N$$

18. Question

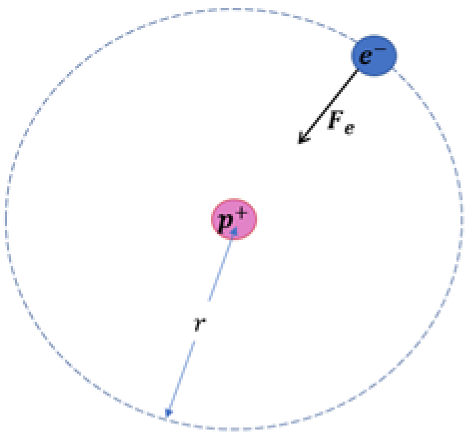
A hydrogen atom contains one proton and one electron. It may be assumed that the electron revolves in a circle of radius 0.53 angstrom (1 angstrom = 10^{-19} m and is abbreviated as Å) with the proton at the center. The hydrogen atom is said to be in the ground sate in this case. Find the magnitude of the electric force between the proton and the electron of a hydrogen atom in its ground state.

Answer

Given:

Magnitude of charge on both proton and electron, $e = 1.6 \times 10^{-19} C$

The radius of the circular orbit, $r = 0.53 \times 10^{-10} m$



The radius of the circular orbit is the distance between the electron and the proton.

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

(Here, $q_1 = q_2 = e$)

The magnitude of electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

Putting the values in the above formula, we get

$$F_e = 9 \times 10^9 NC^{-2}m^2 \times \frac{(1.6 \times 10^{-19} C)^2}{(0.53 \times 10^{-10} m)^2} \approx 8.2 \times 10^{-8} N$$

19. Question

Find the speed of the electron in the ground state of a hydrogen atom. The description of ground state is given in the previous problem.

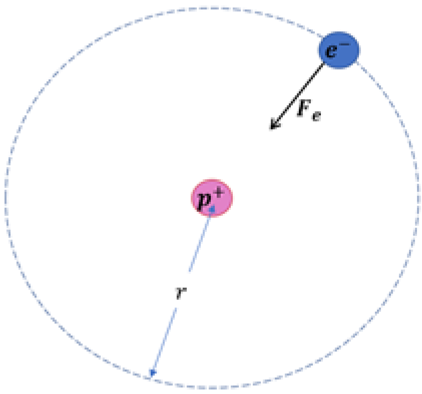
Answer

Given:

Magnitude of charge on both proton and electron, $e = 1.6 \times 10^{-19} C$

The radius of the circular orbit, $r = 0.53 \times 10^{-10} m$

Mass of electron , $m = 9.11 \times 10^{-31} kg$



The radius of the circular orbit is the distance between the electron and the proton.

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

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r is the distance of separation between the charges

(Here, $q_1 = q_2 = e$)

The magnitude of electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

$$F_e = 9 \times 10^9 NC^{-2}m^2 \times \frac{(1.6 \times 10^{-19} C)^2}{(0.53 \times 10^{-10} m)^2} \approx 8.2 \times 10^{-8} N$$

Formula used:

Centripetal force is given by

$$F_c = \frac{mv^2}{r}$$

Where m is the mass if the object, v is the speed of the object and r is the radius of the circular path.

Let F_c be the centripetal force on the electron.

$$F_c = \frac{mv^2}{r} \qquad ...(1)$$

Now, the electric force acts as centripetal force.

Hence,

$$F_e = F_c$$

Substituting (1) and rearranging, we get

$$v = \sqrt{\frac{F_e r}{m}}$$

Putting the values in the above formula, we get

$$v = \sqrt{\frac{8.2 \times 10^{-8} N \times 0.53 \times 10^{-10} m}{9.11 \times 10^{-31} kg}} \approx 2.18 \times 10^6 \ m/s$$

20. Question

Ten positively charged particles are kept fixed on the x-axis at points x = 10 cm, 20 cm 30 cm, …., 100 cm. The first particle has a charge 1.0×10^{-8} C, the second 8×10^{-8} C, the third 27×10^{-8} C and so on. The tenth particle has a charge 1000×10^{-8} C. Find the magnitude of the electric force acting on a 1C charge placed at the origin.

Answer

Given:

Charge on the i^{th} particle ,$q_i = i^3 \times 10^{-8} C$

Distance of i^{th} particle from origin, $r_i = 10i\text{cm} = 0.1i\text{m}$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

Magnitude of force on the 1C charge due to i^{th}particle is given by:

(Here, $q_1 = 1C \times 10^{-8}$, $q_2 = q_i$ and $r = r_i$)

$$F_i = \frac{1}{4\pi\epsilon_0}\frac{1C \times 10^{-8} \times q_i}{(r_i)^2}$$

Substituting the values, we get

$$F_i = 9000N \times i$$

All the forces are in negative-x direction. Hence, we can simply add them up to get the magnitude of net force.

$$F_{x,net} = \sum_{i=1}^{10} 9000 \times i\ N$$

$$F_{x,net} = 9000N \sum_{i=1}^{10} i$$

$$F_{x,net} = 9000N \times 55N = 4.95 \times 10^5 N$$

21. Question

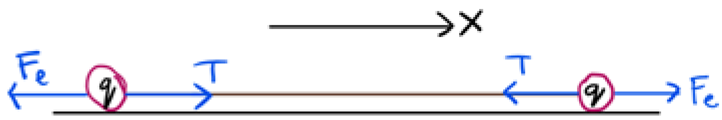
Two charged particles having charge 2.0 × 10^{-8} C each are joined by an insulating string of length 1m and the system is kept on a smooth horizontal table. Find the tension in the string.

Answer

Given,

Charge on each particle, $q = 2.0 \times 10^{-8} C$

Distance between the two charges, $r = 1m$



The tension will adjust such that the net force is zero.

$$\sum F_x = F_e - T = 0$$

Formula used:

By Coulomb's law, the electric force id given by:

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

(Here, $q_1 = q_2 = q$.)

$$T = F_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

Putting the values in the above formula, we get

$$T = \frac{9 \times 10^9 NC^{-2} m^2 \times (2 \times 10^{-8} C)^2}{(1m)^2} = 3.6 \times 10^{-6} N$$

22. Question

Two identical balls, having a charge of 2.00×10^{-7} C and a mass of 100g, are suspended from a common point by two insulating strings each 50 cm long. The balls are held at a separation 5.0 cm apart and then released. Find

(a) the electric force on one of the charged balls

(b) the components of the resultant force on it along and perpendicular to the string

(c) the tension in the string

(d) the acceleration of one of the balls. Answers are to be obtained only for the instant just after the release.

Answer

Given,

Mass, $\mathrm{m} = 100\mathrm{g} = 0.1\mathrm{kg}$

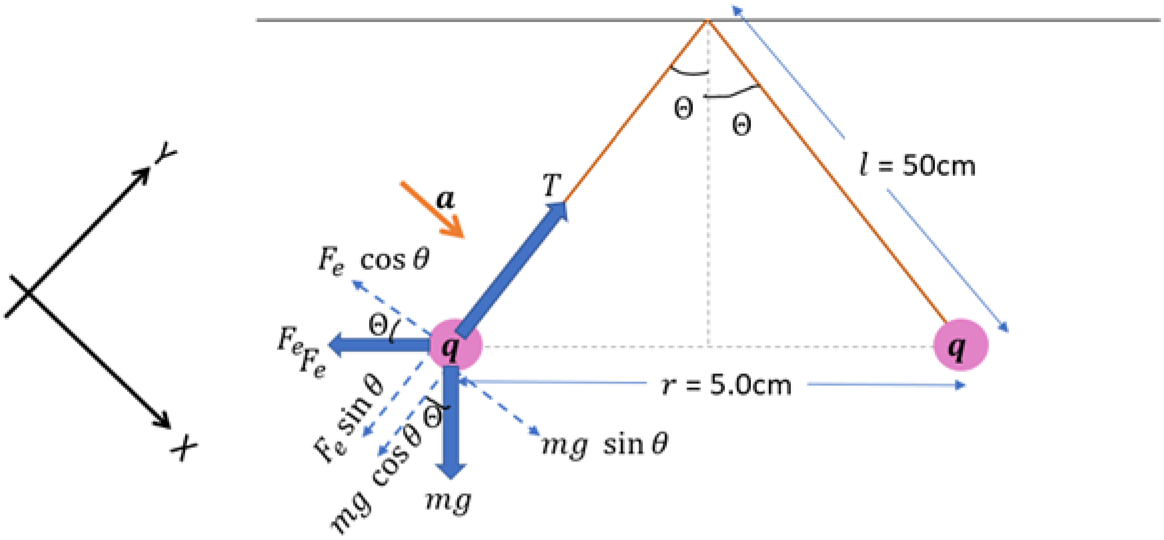
Length of strings, l = 50cm = 0.5 m

Distance between spheres, $\mathrm{r} = 5\mathrm{cm} = 0.05\mathrm{m}$

Magnitude of charge on each ball, $q = 2 \times 10^{-7} C$

Let the magnitude of Tension be T. Let the magnitude of electric force between the spheres be F_e.

Note that tension will adjust itself so that there is no acceleration along its direction.



(a)

Formula used:

By Coulomb’s law, the electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

(Here, $q_1 = q_2 = e$)

The magnitude of the electric force, F_e is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} \quad ...(1)$$

$$F_e = \frac{9 \times 10^9 NC^{-2}m^2 \times (2 \times 10^{-7}C)^2}{(0.05m)^2} = 0.144\, N$$

Hence, the electric force is 0.144 N along the line joining the charges and away from the other charge.

(b)

By trigonometry,

$$sin\, \theta = \frac{12.5cm}{50cm} = 0.05$$

$$cos\, \theta = \sqrt{1 - sin^2\theta} \approx 0.99875$$

Now, along the string, there is component of acceleration. Hence,

$$\sum F_y = ma_y = m(0) = 0$$

Now, consider the direction perpendicular to the string,

$$\sum F_x = F_e\, cos\, \theta - mg\, sin\, \theta$$

$$= 0.144N \times 0.99875 - 0.1kg \times 9.8ms^{-2} \times 0.05$$

$$= 0.09482\, N \approx +0.095\, N$$

Hence, the component of net force perpendicular to the string is 0.095N and away from the other charge (indicated by positive sign)

(c)From the free body diagram,

$$\sum F_y = T - F_e\, sin\, \theta - mg\, cos\, \theta = 0$$

Rearranging to get the above expression, we get

$$T = F_e \sin\theta + mg\cos\theta$$

$$T = 0.144N \times 0.05 + 0.1kg \times 9.8ms^{-2} \times 0.99875 \approx 0.986\ \text{N}$$

(d)

Applying Newton's second law along the y-direction,

$$\sum F_y = ma_y$$

$$a_y = \frac{\sum F_y}{m} \approx \frac{0.095N}{0.1kg} = 0.95ms^{-2}$$

$$\& \ a_x = 0$$

$$\therefore a = 0.95ms^{-2}$$

Thus the acceleration of one of the balls is $0.95ms^2$ perpendicular to the string and going away from the other charge.

23. Question

Two identical pith balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length20 cm each, the separation between the suspension points being 5 cm. In equilibrium, the separation between the balls is 3 cm. Find the mass of each ball and the tension in the strings. The charge on each ball has a magnitude 2.0×10^{-8} C.

Answer

Given,

Length of strings, $l = 20cm = 0.2\ m$

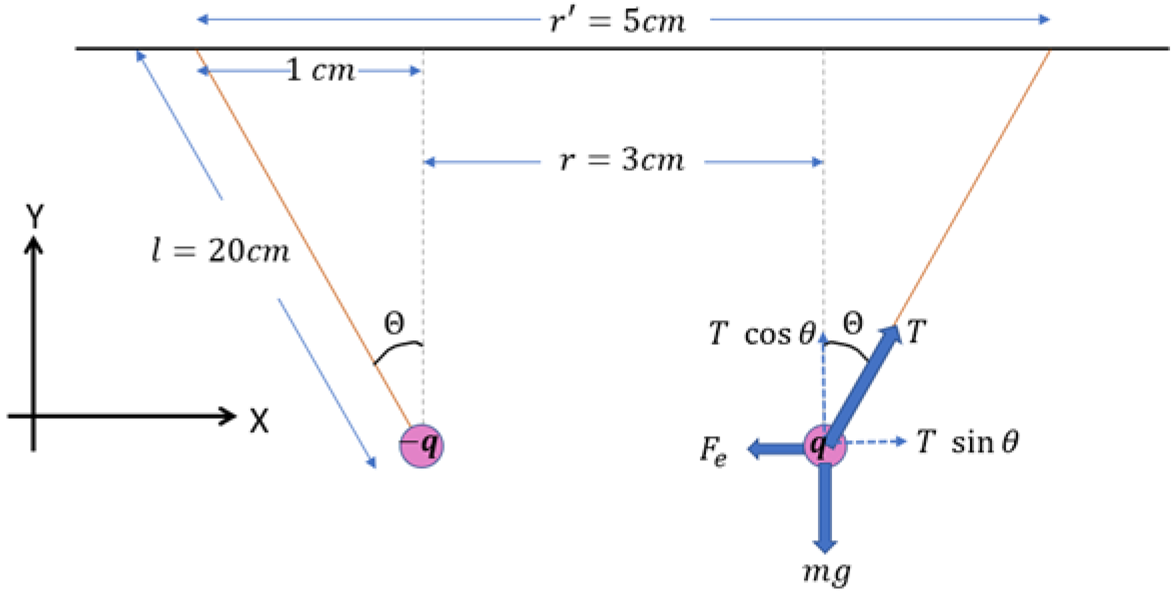
Distance between suspension points, $r' = 5cm = 0.05m$

Magnitude of charge on each ball, $q = 2 \times 10^{-8}C$

Distance between the two balls, $r = 3cm = 0.03m$

Let the mass of each ball be m. Let the magnitude of Tension be T and the electric force between the balls be F_e.

To be in accordance with the fact, $r' > r$, the balls must be attracted to each other. Hence, they are oppositely charged.



By trigonometry,

$$sin\ \theta = \frac{1\ cm}{20cm} = 0.05$$

$$cos\ \theta = \sqrt{1 - sin^2\theta} \approx 0.99875$$

At equilibrium, all forces must cancel out in accordance with newton's first law. Hence,

$$\sum F_x = T\ sin\ \theta\ - F_e = 0 \qquad \text{..............................(1)}$$

$$\sum F_y = T\cos\theta - mg\ = 0 \qquad \text{..............................(2)}$$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

(Here, $q_1 = q_2 = e$)

The magnitude of the electric force, F_e is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} \quad ....(3)$$

Substituting values in (3), we get

$$F_e = \frac{9 \times 10^9 NC^{-2}m^2 \times (2 \times 10^{-8}C)^2}{(0.03m)^2} = 0.004\,N$$

From equation (1), we get

$$T = \frac{F_e}{\sin\theta}$$

$$T \approx \frac{0.004N}{0.05} = 0.08\,N$$

From equation (2), we get

$$m = \frac{T\cos\theta}{g}$$

$$m \approx \frac{0.08\,N \times 0.99875}{9.8m/s^2} \approx 0.008153\,kg \approx 8.2\,g$$

Hence, mass of each of the balls is 8.2 grams and Tension in each of the ropes is 0.08N.

24. Question

Two small spheres, each having a mass of 20g, are suspended from a common point by two insulating strings of length 40 cm each. The spheres are identically charged and the separation between the balls at equilibrium is found to be 4 cm. find the charge on each sphere.

Answer

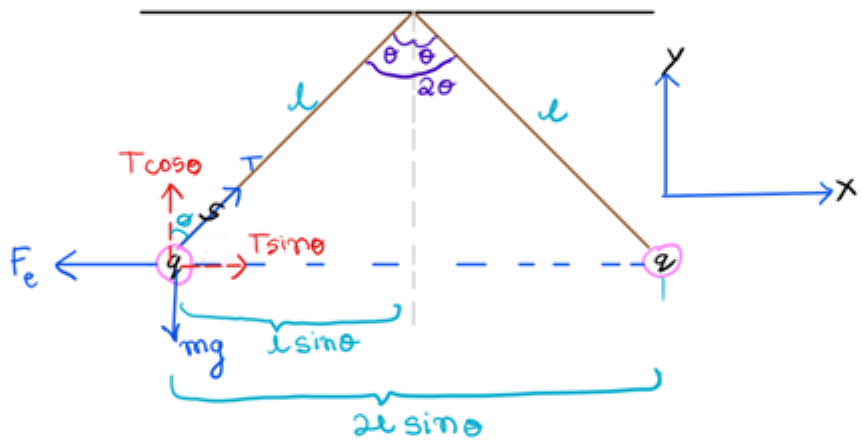
Given,

Mass, $m = 20g = 0.02kg$

Length of strings, l = 40cm = 0.4 m

Distance between spheres, $r = 4cm = 0.04m$

Let the magnitude of Tension be T. Let the electric force between the spheres be F_e. Let the magnitude of charge on each pitch ball be q.



By trigonometry,

$$sin\,\theta = \frac{2cm}{40cm} = 0.05$$

$$cos\,\theta = \sqrt{1 - sin^2\theta} \approx 0.99875$$

At equilibrium, all forces must cancel out in accordance with newton's first law. Hence,

$$\sum F_x = F_e - T\,sin\,\theta = 0 \qquad \text{..............................(1)}$$

$$\sum F_y = T\cos\theta - mg = 0 \qquad \text{..............................(2)}$$

Formula used:

By Coulomb's law, the electric force is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

(Here, $q_1 = q_2 = e$)

The magnitude of the electric force, F_e is given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

From (2), we get,

$$T = \frac{mg}{\cos\theta} = \frac{0.02kg \times 9.8m/s^2}{0.99875} \approx 0.19625\ N$$

From (3) and (1), we get

$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = T\sin\theta$$

$$q^2 = 4\pi\epsilon_0 T\sin\theta\, r^2$$

$$q^2 \approx \frac{0.19625\ N \times 0.05 \times (0.04\ m)^2}{9 \times 10^9\ Nm^2C^{-2}}$$

$$q \approx 4.17 \times 10^{-8} C$$

25. Question

Two identical pith balls, each carrying a charge q, are suspended from a common point by two strings of equal length ℓ. Find the mass of each ball if the angle between the strings is 20 in equilibrium.

Answer

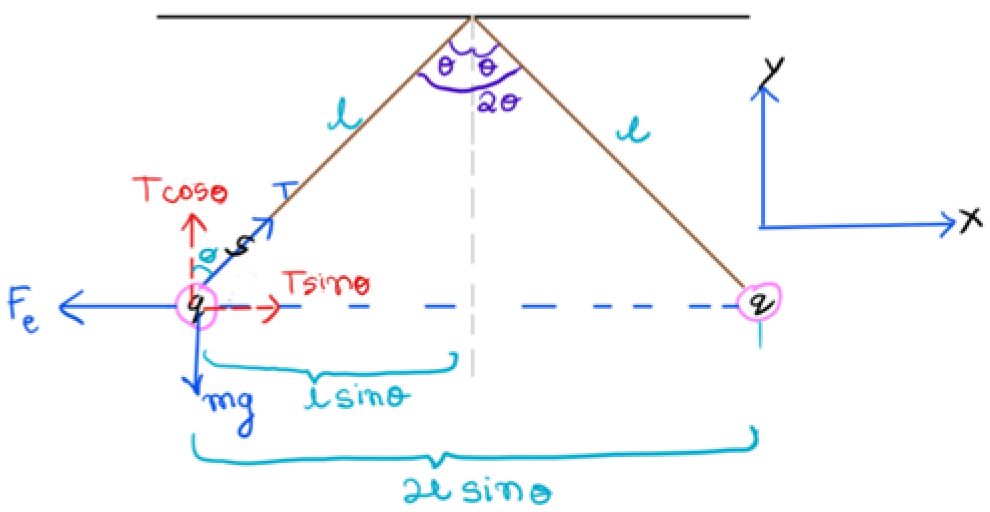
Given:

Charges on ball = q

Angle between the balls = 2θ

Length of the strings = l

Let the magnitude of Tension be T. Let the electric force between the pitch balls be F_e. Let the mass of each pitch ball be m. The free body diagram is as follows:



Now, let the distance between the two charges be r.

By trigonometry,

$$sin(\theta) = \frac{r/2}{l}$$

$$r = 2l\,sin\,\theta \quad ...(1)$$

By Coulomb's law, the electric force id given by:

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Where ϵ_0 is the permittivity of free space

q_1 and q_2 are the magnitude of charges

r is the distance of separation between the charges

Here, $q_1 = q_2 = q$.

Now, let us find the magnitude of the electric force, F_e :

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q \cdot q}{r^2}$$

Substituting eq (1), we get

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{4l^2 \sin^2(\theta)} \quad ...(2)$$

As the system is in equilibrium, all the forces in each direction must sum up to zero.

$$\sum F_x = T \sin\theta - F_e = 0 \quad ...(3)$$

$$\sum \mathrm{F_y} = \mathrm{T} \cos\theta - \mathrm{mg} = 0 \quad ...(4)$$

Substituting (2) in (3), we get

$$T = \frac{1}{4\pi\epsilon_0} \frac{q^2}{4l^2 \sin^3\theta} \quad ...(5)$$

Substituting (4), we get

$$m = \frac{T \cos\theta}{g} \quad ...(6)$$

Substituting (5) in (6), we get

$$m = \frac{q^2 \cot\theta}{16\pi\epsilon_0 g l^2 \sin^2\theta}$$

26. Question

A particle having a charge of 2.0×10^{-4} C is placed directly below and at a separation of 10 cm from the bob of a simple pendulum at rest. The mass of the bob is 100g. What charge should the bob be given so that the string becomes loose?

Answer

Given: Charge of the particle : q = 2.0×10^{-4} CDistance between charged particle and the bob:r =10 cm=0.1mMass of the bob : m = 100 g = 100×10^{-3} kg.**Formula used:**T = mgWhere T is the tension in the string and g is the acceleration due to gravity.∴ T = 0.1 × 9.8 ∴ T = 0.98 NNow let the charge on the bob be : q'Now the electrostatic force between the bob and the particle is given as:

$F = \left(\frac{1}{4\pi\epsilon_0} \times \left(\frac{qq'}{r^2}\right)\right)$This is the Coulomb's Law. Where ϵ_0 is the permittivity of free space and it's value is : $\epsilon_0 = 8.85418782 \times 10^{-12}$ m^{-3} kg^{-1} s^4 A^2,q is the charge of the particle and q' is the charge of the bob, r is the distance between charged particle and the bob.Now, in order to have a loose string the tension in the string should be zero: T=0For tension to be zero, the particle must repel the bob along the direction of the tension.Also, weight of the bob is in opposite direction to the string.Thus the equation is:T + F = mg.∴ $0 + \left(\frac{1}{4\pi\epsilon_0} \times \left(\frac{qq'}{r^2}\right)\right) = 0.1 \times 9.8$

$\therefore \left(\frac{1}{4 \times \pi \times 8.85418782 \times 10^{-12}} \times \left(\frac{qq'}{0.1^2}\right)\right) = 0.98$

$\therefore q' = \frac{0.98 \times 4 \times \pi \times 8.85418782 \times 10^{-12} \times 0.1^2}{2.0 \times 10^{-4}}$∴ $\boldsymbol{q' = 5.4 \times 10^{-9}\ C}$

Hence, the charge on the bob should be 5.4×10^{-4} C so that the string would become loose.

27. Question

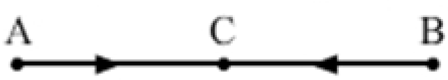
Two Particles A and B having charges q and 2q respectively are placed on a smooth table with a separation d. A third particle C is to be clamped on the table in such a way that the particles A and B remain at rest on the table under electrical forces. What should be the charge on C and where should it be clamped?

Answer

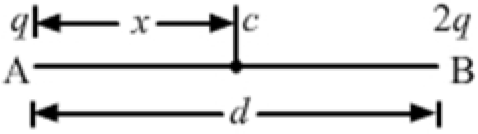
Given: Charge of the particle A : qCharge of the particle B : 2q

Distance between A and B : d**Formula used:**We will be using Coulomb's Law:

$F = \left(\frac{1}{4\pi\epsilon_0} \times \left(\frac{qq'}{r^2}\right)\right)$Where ϵ_0 is the permittivity of free space and it's value is :

$\epsilon_0 = 8.85418782 \times 10^{-12}$ m^{-3} kg^{-1} s^4 A^2 and F is the electrostatic force between two charges. And r is the distance between two charges.The diagram below shows the given conditions: 

C is the third particle clamped on the table. Let charge of point C be q'For A and B to remain at rest, the electrostatic forces from A and B must cancel out each other. Which means magnitude of force from A to C : $\bar{F}_{AC}$ should be equal and opposite to the force from B to C : $\bar{F}_{AB}$ as Point B has twice the charge of A.Thus the equation of rest is:

$\bar{F}_{AC} + \bar{F}_{AB} = 0$ 

Here x is the distance between charge A and C.From Coulomb's Law,

$F_{AC} = \left(\frac{1}{4\pi\epsilon_0} \times \left(\frac{qq'}{x^2}\right)\right) F_{AB} = \left(\frac{1}{4\pi\epsilon_0} \times \left(\frac{2qq}{(d)^2}\right)\right)$ Substituting we get,

$\left(\frac{1}{4\pi\epsilon_0} \times \left(\frac{qq'}{x^2}\right)\right) + \left(\frac{1}{4\pi\epsilon_0} \times \left(\frac{2q^2}{(d)^2}\right)\right) = 0 \ldots\ldots..(1)$ Since we know that the magnitudes of F_{AC} and F_{BC} the forces are equal,

$\frac{1}{4\pi\epsilon_0} \times \left(\frac{qq'}{x^2}\right) = \frac{1}{4\pi\epsilon_0} \times \left(\frac{2qq'}{(d-x)^2}\right)$

$\frac{1}{x^2} = \frac{2}{(d-x)^2} \therefore (d-x)^2 = 2x^2 \therefore \sqrt{2}\,x = d - x \therefore \sqrt{2}\,x + x = d$

$\therefore (\sqrt{2}+1)x = d \therefore x = \left(\frac{d}{(\sqrt{2}+1)}\right)$ Rationalizing the denominator we get,

$\boldsymbol{x = (\sqrt{2}-1)d}$ Substituting value of x in the equation (1) we get,

$$\left(\frac{1}{4\pi\epsilon_0} \times \left(\frac{qq'}{\left((\sqrt{2}-1)d\right)^2}\right)\right) + \left(\frac{1}{4\pi\epsilon_0} \times \left(\frac{2q^2}{(d)^2}\right)\right) = 0$$

$$\therefore \frac{1}{4\pi\epsilon_0} \times \left(\frac{qq'}{\left((\sqrt{2}-1)d\right)^2}\right) + \frac{1}{4\pi\epsilon_0} \times \left(\frac{2q^2}{d^2}\right) = 0 \therefore \frac{q'}{\left((\sqrt{2}-1)d\right)^2} + \frac{2q}{d^2} = 0$$

$$\therefore \frac{q'}{\left((\sqrt{2}-1)d\right)^2} = -\left(\frac{2q}{d^2}\right) \therefore q' = -\left(\frac{2q}{d^2}\right) \times (\sqrt{2}-1)^2 d^2$$

$\therefore q' = -q \times 2(2 - 2\sqrt{2} + 1) \therefore q' = -q(4 - 4\sqrt{2} + 1) \therefore \boldsymbol{q' = -q(6 - 4\sqrt{2})C}$

Hence, the charge on the C is -q(6-4√2) C and it should be clamped at a distance of (√2-1)d .

28. Question

Two identically charged particles are fastened to the two ends of a spring of spring constant 100 N m^{-1} and natural length 10 cm. The system resets on a smooth horizontal table. If the charge on each particle is 2.0 × 10^{-8} C, find the extension in the length of the spring. Assume that the extension is small as compared to the natural length. Justify this assumption after you solve the problem.

Answer

Given: Natural Length of the spring: l = 10cm = 0.1mSpring Constant : K = 100 N m^{-1}Charge on each particle: q = 2.0 × 10^{-8} CSeparation between two charges =

l**Formula used:**Let the extension be ‘x’ m.We use Coulomb’s Law:

$\boldsymbol{F_e} = \left(\boldsymbol{k} \times \left(\frac{\boldsymbol{qq}}{\boldsymbol{r^2}}\right)\right)$ Where F_e is the electrostatic force, k is a constant .k = $\frac{1}{4\pi\epsilon_0}$ = 9× 10^9 Nm^2C^{-2} and r is the distance between two charges.Since the electrostatic force is repulsive in nature, the spring will exert a restoring spring force F_r.F= -KxHere, K is the spring constant and x is the extension.Negative sign is because the restoring spring force us opposite to the applied force. The system would be in equilibrium when the Electrostatic force of repulsion between the two charges is equal to the spring force

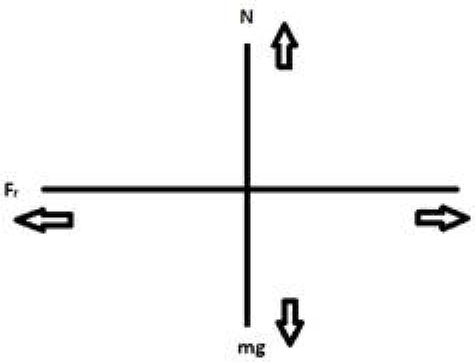
$F_e + F_r = 0$∴ $\left(k \times \left(\frac{qq}{r^2}\right)\right) - Kx = 0$∴ $\left(k \times \left(\frac{qq}{r^2}\right)\right) = Kx$∴ $k \times \left(\frac{q^2}{r^2}\right) = Kx$ ∴ $x = k \times \left(\frac{q^2}{r^2 K}\right)$∴ $x = \left(9 \times 10^9 \times \frac{(2.0 \times 10^{-8})^2}{(0.1^2) \times 100}\right)$∴ $\boldsymbol{x = 3.6 \times 10^{-6}\, m}$ Yes, the assumption is justified. When two similar charges are present at two ends, they will exert repulsive force on each other. The spring will extend due to its elastic nature.The repulsive force will have an opposite force called as restoring force in the spring. This force is directly proportional to the extension of the spring and depends on the elasticity if the material. If the extension is large compared to the natural length, then the restoring force would be proportional to the high powers of the extension.

29. Question

A particle A having a charge of 2.0 × 10^{-6} C is held fixed on a horizontal table. A second charged particle of mass 80g stays in equilibrium on the table at a distance of 10 cm from the first charge. The coefficient of friction between the table and this second particle is μ = 0.2. Find the range within which the charge of this second particle may lie.

Answer

Given: Charge of the particle A : q_1 = 2.0 × 10^{-6} C.Mass of the second charged particle i.e. B:m=80 g=80×10^{-3} kg.Separation between both the charged particles:r = 10 cm = 0.1 mCoefficient of friction between the table and this second particle: μ = 0.2



Formula used: Let the charge of the particle B be q_2We use Coulomb’s law:$\boldsymbol{F_e} = \boldsymbol{k} \times \left(\frac{\boldsymbol{q_1 q_2}}{\boldsymbol{r^2}}\right)$ Where F_e is the electrostatic force on b due to A, k is a constant .k = $\frac{1}{4\pi\epsilon_0}$ = 9× 10^9 Nm^2C^{-2} and r is the distance between two charges.The Friction force is given as:F_r = μN= μmgHere, μ is the coefficient of friction, N is the normal reaction of table to the particle. N=mg.It is given that particle B is at equilibrium with the table,Thus$F_e = F_r$

$\therefore k \times \left(\frac{q_1 q_2}{r^2}\right) = \mu mg \therefore q_2 = \frac{\mu \times m \times g \times r^2}{k \times q_1}$

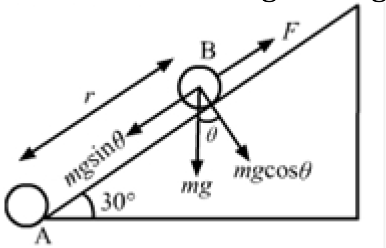
$\therefore q_2 = \frac{0.2 \times 80 \times 10^{-3} \times 9.8 \times (0.1)^2}{9 \times 10^9 \times 2.0 \times 10^{-6}} \therefore \boldsymbol{q_2 = 8.71 \times 10^{-8} C}$ Hence, the range within which the charge of the second particle lies is ± 8.71× 10^{-8} C. Since, charge can be positive or negative, thus ± .

30. Question

A particle A having a charge of 2.0 × 10^{-6} C and a mass of 100g is placed at the bottom of a smooth inclined plane of inclination 30°. Where should another particle B, having same charge and mass, be placed on the incline so that it may remain in equilibrium?

Answer

Given: Charge of the particle A and B: $q_1=q_2= q = 2.0 \times 10^{-6}$ C.Mass of the particles A and B : m = 100 g =0.1 kg.Inclination of the inclined plain : θ = 30°



Formula used: The particle A and B will exert electrostatic repulsive forces on each other.This can be given by Coulomb's Law:We use Coulomb's law:$\boldsymbol{F_e = k \times \left(\frac{q_1 q_2}{r^2}\right)}$ Where F_e is the electrostatic force on b due to A, k is a constant .k = $\frac{1}{4\pi\epsilon_0}$ = 9× 10^9 Nm^2C^{-2} and r is the distance between two charges.Particle B would face a friction force opposite to the rolling force due to inclination. It is given as:F= mgsinθFor particle B to remain in equilibrium with the inclination and particle A. The electrostatic force of repulsion and the friction force must be equal.F_e = F∴ $k \times \left(\frac{q_1 q_2}{r^2}\right) = mgsin\theta \therefore r^2 = \left(\frac{k}{mgsin\theta}\right) \times q^2$

$\therefore r^2 = \frac{9 \times 10^9}{0.1 \times 9.8 \times \sin(30)} \times (2.0 \times 10^{-6})^2 \therefore r^2 = 0.073\ m$

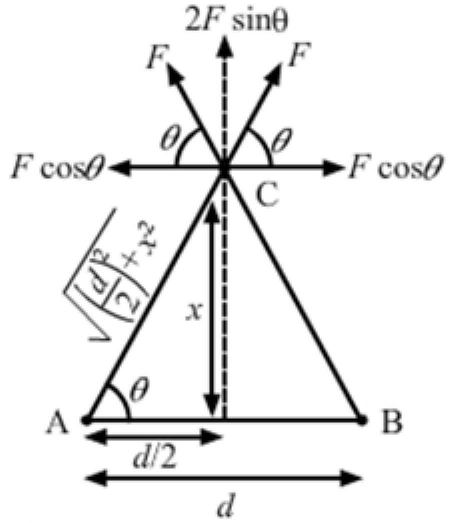
$\boldsymbol{\therefore r = \sqrt{0.073} = 0.2701\ m}$ Hence, particle b must be place at a distance of 0.2701m from particle A to remain in equilibrium.

31. Question

Two particles A and B, each having a charge Q, are placed a distance d apart. Where should a particle of charge q be placed on the perpendicular bisector of AB so that it experiences maximum force? What is the magnitude of this maximum force?

Answer

Given: Charge on particles A and B: $q_1 = q_2 = Q$Separation between A and B : d



here 'x' is the distance at which particle C of charge 'q' is place on the perpendicular bisector of AB.**Formula used:**From the figure we can find sinθ : $sin\theta = \frac{x}{\sqrt{\left(\frac{d}{2}\right)^2 + x^2}}$The horizontal components of force cancel each other.Total vertical component of force is: F'= 2Fsinθ We use Coulomb's Law: $\boldsymbol{F = k \times \left(\frac{q_1 q_2}{r^2}\right)}$ Where F is the electrostatic force on b due to A, k is a constant .k $= \frac{1}{4\pi\epsilon_0}$ = 9× 10^9 Nm^2C^{-2} and r is the distance between the two charges.Here r = $\sqrt{\left(\frac{d}{2}\right)^2 + x^2}$Substituting we get,$F' = 2 \times k \times \left(\frac{qQ}{\left(\frac{d}{2}\right)^2 + x^2}\right) \times \frac{x}{\sqrt{\left(\frac{d}{2}\right)^2 + x^2}}$

$\therefore F' = 2 \times k \times qQ \times \left(\frac{x}{\left(\left(\frac{d}{2}\right)^2 + x^2\right)^{\frac{3}{2}}}\right)$Using maxima ,

For maximum force $\frac{\mathrm{dF'}}{\mathrm{dx}} = 0$.Thus,

$$\frac{\mathrm{dF'}}{\mathrm{dx}} = 2kqQ \times \left[\left(\left(\frac{d}{2}\right)^2 + x^2\right)^{-\frac{3}{2}} - \left(\frac{3}{2}x\right)\left[\left(\left(\frac{d}{2}\right)^2 + x^2\right)^{\frac{-5}{2}} \times 2x\right]\right] = 0$$

$$\therefore \left(\left(\frac{d}{2}\right)^2 + x^2\right)^{-\frac{3}{2}} = (3x^2)\left[\left(\left(\frac{d}{2}\right)^2 + x^2\right)^{\frac{-5}{2}}\right] \therefore \frac{\left(\left(\frac{d}{2}\right)^2 + x^2\right)^{-\frac{3}{2}}}{\left(\left(\frac{d}{2}\right)^2 + x^2\right)^{\frac{-5}{2}}} = 3x^2$$

$$\therefore \left(\frac{d}{2}\right)^2 + x^2 = 3x^2 \therefore \left(\frac{d}{2}\right)^2 + x^2 - 3x^2 = 0 \therefore \left(\frac{d}{2}\right)^2 - 2x^2 = 0 \therefore \frac{d}{2} = \sqrt{2}\,x$$

$\boldsymbol{\therefore x = \frac{d}{2\sqrt{2}}}$ The magnitude of maximum Force is:

$$F'_{Max} = 2kqQ\left(\frac{\frac{d}{2\sqrt{2}}}{\left(\left(\frac{d}{2}\right)^2+\left(\frac{d}{2\sqrt{2}}\right)^2\right)^{\frac{3}{2}}}\right)$$ Hence, the particle C must be placed at a distance of $\frac{d}{2\sqrt{2}}$ from the perpendicular bisector of AB so as to experience a maximum force of magnitude $F'_{Max} = 2kqQ\left(\frac{\frac{d}{2\sqrt{2}}}{\left(\left(\frac{d}{2}\right)^2+\left(\frac{d}{2\sqrt{2}}\right)^2\right)^{\frac{3}{2}}}\right)$

32. Question

Two particles A and B, each carrying a charge Q, are held fixed with a separation d between them. A particle C having mass m and charge q is kept at the middle point of the line AB.

(a) If it is displaced through a distance x perpendicular to AB, what would be the electric force experienced by it.

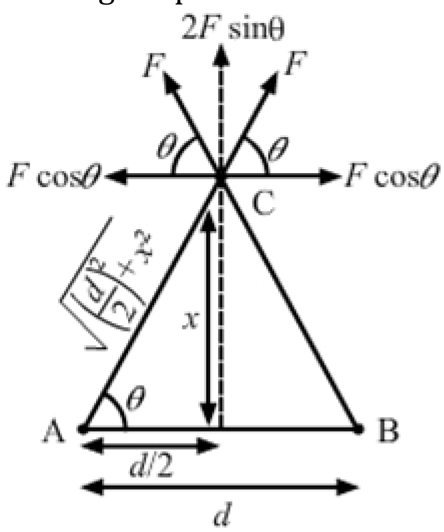
(b) Assuming x << d, show that this force is proportional to x.

(c) Under what conditions will the particle C execute simple harmonic motion if it is released after such a small displacement?

Find the time period of the oscillations if these conditions are satisfied.

Answer

Given: Charge of particles A and B : q = QSeparation between A and B : d(a)



here 'x' is the distance at which particle C of mass m and charge 'q' is place on the perpendicular bisector of AB.**Formula used:**From the figure we can find sinθ : $sin\theta = \frac{x}{\sqrt{\left(\frac{d}{2}\right)^2+x^2}}$ The horizontal components of force cancel each other.Total vertical component of force is: F’= 2Fsinθ We use Coulomb’s Law: $\boldsymbol{F} = \boldsymbol{k} \times \left(\frac{\boldsymbol{q_1 q_2}}{\boldsymbol{r^2}}\right)$ Where F is the electrostatic force on b due to A, k is a constant .k = $\frac{1}{4\pi\epsilon_0}$ = 9× 10^9 Nm^2C^{-2} and r is the distance between the two charges.Here r = $\sqrt{\left(\frac{d}{2}\right)^2 + x^2}$ Substituting we get,

$$F' = 2 \times k \times \left(\frac{qQ}{\left(\frac{d}{2}\right)^2 + x^2}\right) \times \frac{x}{\sqrt{\left(\frac{d}{2}\right)^2 + x^2}}$$

$$\therefore F' = 2 \times k \times qQ \times \left(\frac{x}{\left(\left(\frac{d}{2}\right)^2 + x^2\right)^{\frac{3}{2}}}\right)$$

F’ is the net electric force experience by particle C of charge q.(b)When x ≪ d, $\therefore x^2 \ll \left(\frac{d}{2}\right)^2$ Thus, x^2 can be neglected.Substituting we get, $\therefore F' = 2kqQ \times \left(\frac{x}{\left(\frac{d}{2}\right)^3}\right) \therefore F' = 16kqQ \times \left(\frac{x}{d^3}\right)$

$\therefore F' \alpha x$ Thus, force is proportional to x.(c)The condition for Simple harmonic Motion of a particle is: $F' = m\omega^2 x$ Here, m is the mass of the particle C, ω is the angular frequency.Thus comparing two equations of F’, we get

$\therefore 16kqQ \times \left(\frac{x}{d^3}\right) = m\omega^2 x$ We know that $\omega = \frac{2\pi}{T}$. Where t is the Time Period.

$$\therefore 16kqQ \times \left(\frac{x}{d^3}\right) = m \times \left(\frac{2\pi}{T}\right)^2 \times x \therefore T^2 = \frac{m \times 4\pi^2 \times x}{16kqQ \times \left(\frac{x}{d^3}\right)} \therefore T^2 = \frac{m\pi^2 d^3}{4kqQ}$$

$$\therefore \boldsymbol{T} = \sqrt{\frac{\boldsymbol{m\pi^2 d^3}}{\boldsymbol{4kqQ}}}$$

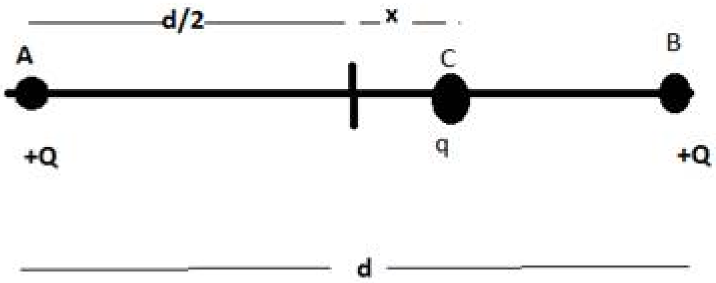
Hence time period when the particle is released after a small displacement under SHM is $\boldsymbol{T} = \sqrt{\frac{\boldsymbol{m\pi^2 d^3}}{\boldsymbol{4kqQ}}}$

33. Question

Repeat the previous problem if the particle is displaced through a distance x along the line AB.

Answer

Given: Charge of particles A and B : q = QSeparation between A and B : d



Here x is the displacement of particle C along AB.Distance between A and C is : $r_{AC} = \frac{d}{2} + x$ Distance between B and C is : $r_{BC} = \frac{d}{2} - x$**Formula used:**For part (a), we will be using Coulomb's Law,$\boldsymbol{F = k \times \left(\frac{q_1 q_2}{r^2}\right)}$ Where F is the electrostatic force on B due to A, k is a constant .k = $\frac{1}{4\pi\epsilon_0}$ = 9× 10^9 Nm^2C^{-2} and r is the distance between the two charges.q_1=q_2=Q.now, the net force acting on C is:$F_{net} = F_{BC} - F_{AC}$

$$\therefore F_{net} = \frac{kqQ}{r_{BC}^2} - \frac{kqQ}{r_{AC}^2} \therefore F_{net} = kqQ\left[\left(\frac{1}{\left(\frac{d}{2} - x\right)^2}\right) - \left(\frac{1}{\left(\frac{d}{2} + x\right)^2}\right)\right]$$

$$\therefore F_{net} = kqQ\left[\frac{\left(\frac{d}{2} + x\right)^2 - \left(\frac{d}{2} - x\right)^2}{\left(\frac{d}{2} + x\right)^2 \times \left(\frac{d}{2} - x\right)^2}\right]$$

$$\therefore F_{net} = kqQ\left[\frac{\frac{d^2}{4} + dx + x^2 - \frac{d^2}{4} + dx - x^2}{\left(\left(\frac{d}{2}\right)^2 - x^2\right)^2}\right] \therefore \boldsymbol{F_{net} = kqQ \times \frac{2dx}{\left(\left(\frac{d}{2}\right)^2 - x^2\right)^2}}$$

F_{net} is the net electric force experience by particle C of charge q.(b)When x≪d,x^2≪$(d/2)^2$Thus x^2 can be neglected.We get,$F_{net} = kqQ \times \frac{2dx}{\frac{d^4}{16}}$

$$\therefore \boldsymbol{F_{net} = \frac{32kqQx}{d^3}} \therefore F_{net} \,\alpha\, x$$

Thus, when x≪d force is proportional to x(c)The condition for Simple harmonic Motion of a particle is:F'=mω^2xHere, m is the mass of the particle C, ω is the angular frequency.Let F_{net} = F'Thus comparing two equations of F', we get

$\frac{32kqQx}{d^3} = m\omega^2 x$ We know that $\omega = \frac{2\pi}{T}$. Where t is the Time Period.

$$\therefore \frac{32kqQx}{d^3} = m\left(\frac{2\pi}{T}\right)^2 x \therefore T^2 = \frac{m\pi^2 d^3}{8kqQ} \therefore \boldsymbol{T = \sqrt{\frac{m\pi^2 d^3}{8kqQ}}}$$

Hence time period when particle is displaced along AB is

$$T = \sqrt{\frac{m\pi^2 d^3}{8kqQ}}$$

34. Question

The electric force experienced by a charge of 1.0×10^{-6} C is 1.5×10^{-3} N. Find the magnitude of the electric field at the position of the charge.

Answer

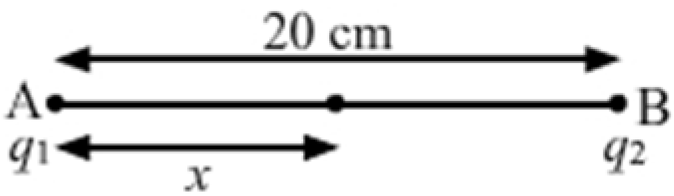
Given: Electric Force exerted by a charge: F = 1.5×10^{-3} NCharge: q = 1.0×10^{-6} C**Formula used:**Here we use:$F = qE$Where, F is the electric force, q is the charge and E is the electric field at the position of the charge.Substituting we get, $1.5 \times 10^{-3}\ \mathrm{N} = 1.0 \times 10^{-6}\ \mathrm{C} \times \mathrm{E}$$\therefore \mathrm{E} = \dfrac{1.5 \times 10^{-3}\ \mathrm{N}}{1.0 \times 10^{-6}\ \mathrm{C}}$$\therefore E = 1500\ NC^{-1}$ $\therefore \boldsymbol{E = 1.5 \times 10^{3}\ NC^{-1}}$Hence, the magnitude electric field at the position of the charge is 1.5×10^{3} N/C

35. Question

Two particles A and B having charges of $+2.00 \times 10^{-6}$ C and of -4.00×10^{-6} C respectively are held fixed at a separation of 20.0 cm. Locate the point(s) on the line AB where (a) electric field is zero (b) the electric potential is zero.

Answer

Given: Charge on particle A: $q_A = +2 \times 10^{-6}$ CCharge on particle B : $q_B = -4 \times 10^{-6}$ CSeparation between A and B : r = 20.0 cm=0.2m**Formula used:**Electric Field given as:$E = \dfrac{kq}{r^2}$Here, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =$9 \times 10^{9}\ \mathrm{Nm^2C^{-2}}$. q is the point charge and r is the distance between two charges.Also,Electric potential is given as: $V = \dfrac{kq}{r}$Here, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =$9 \times 10^{9}\ \mathrm{Nm^2C^{-2}}$ .q is the point charge and r is the distance between two charges.Now,



Let there be a point at distance 'x' from A where net electric field is zero.Distance between A and the point be : r_A = x mDistance between B and the point be : r_B = (0.2-x) m

$E_{net} = \dfrac{kq_A}{x^2} + \dfrac{kq_B}{(0.2-\mathrm{x})^2} = 0$$\therefore E_{net} = \dfrac{2\times 10^{-6}}{x^2} - \dfrac{4\times 10^{-6}}{(0.2-\mathrm{x})^2} = 0$ $\therefore \dfrac{2\times 10^{-6}}{x^2} = \dfrac{4\times 10^{-6}}{(0.2-\mathrm{x})^2}$$\therefore \dfrac{(2\times 10^{-6})}{4\times 10^{-6}} = \dfrac{\mathrm{x}^2}{(0.2-\mathrm{x})^2}$$\therefore 0.5 = \dfrac{\mathrm{x}^2}{(0.2-\mathrm{x})^2}$ $\therefore \sqrt{0.5} = \dfrac{\mathrm{x}}{0.2-\mathrm{x}}$$\therefore \sqrt{0.5} \times (0.2-\mathrm{x}) = \mathrm{x}$$\therefore 0.1414 - 0.707\mathrm{x} = \mathrm{x}$ $\therefore 0.1414 = \mathrm{x} - 0.707\mathrm{x}$$\therefore \mathrm{x} = \dfrac{0.1414}{0.293}$$\therefore \boldsymbol{\mathrm{x} = 0.4825\ \mathrm{m}}$Hence, electric field is zero at a point 0.4825 m from A along AB.Now,Zero Net potential at the point is $V_{net} = \dfrac{kq_A}{r_A} + \dfrac{kq_B}{r_B} = 0$$\therefore \dfrac{2\times 10^{-6}}{x} - \dfrac{4\times 10^{-6}}{0.2 \pm x} = 0$$\therefore \dfrac{2\times 10^{-6}}{x} = \dfrac{4\times 10^{-6}}{0.2 \pm x}$

$\therefore \dfrac{2\times10^{-6}}{4\times10^{-6}}=\dfrac{x}{0.2\pm x}\therefore 0.5=\dfrac{x}{0.2\pm x}\therefore 0.1\pm 0.5x=x\therefore 0.1=1.5x\therefore x=\dfrac{0.1}{1.5}$

$\therefore \boldsymbol{x=0.0666\ m}$ OR$\therefore 0.1=0.5x\therefore \boldsymbol{x=0.2\ m}$Hence, potential can be zero at 0.0666 m from A along AB and at 0.2 m from B along AB.

36. Question

A point charge produces an electric field of magnitude 5 N C^{-1} at a distance of 40 cm from it. What is the magnitude of the charge?

Answer

Given: Electric field due to a point charge: E = 5 N C^{-1}Distance between the point charge and the point at which electric field is produced : r = 40 cm = 0.4 m**Formula used:**

Electric field is given as:E=kq r^2Here , k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ = 9× 10^9 Nm^2C^{-2} .

q is the point charge .Substituting we get,$5=\dfrac{9\times10^9\times q}{0.4^2}\therefore q=\dfrac{5\times0.4^2}{9\times10^9}$

$\therefore \boldsymbol{q=8.89\times10^{-11}\ C}$ Hence, magnitude of the charge is 8.89× 10^{-11} C.

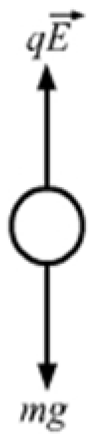
37. Question

A water particle of mass 10.0 mg and having a charge of 1.50 × 10^{-6} C stays suspended in a room. What is the magnitude of electric field in the room? What is its direction?

Answer

Given: Mass of the water particle : m= 10.0 mg = 10× 10^{-6} kgCharge of the particle:

q = 1.5× 10^{-6} C



The suspended particle will be under the influence of gravity.Hence , gravity will act in downward direction as shown in the figure.**Formula used:** We know that,F_e = qEWhere F is the electrostatic force, q is the charge and E is the electric field produced by the charge.But, Force due to gravity : is F_G= mgHere, m is the mass of the particle and g is the acceleration due to gravity.Now, for the particle to stay suspended in the room, the downward gravitational force must be equal and opposite to the electric force. $\therefore F_e=F_G$

$\therefore$ qE=mg$\therefore E=\dfrac{mg}{q}\therefore E=\dfrac{10\times10^{-6}\times9.8}{1.5\times10^{-6}}\therefore \boldsymbol{E=65.33\ NC^{-1}}$

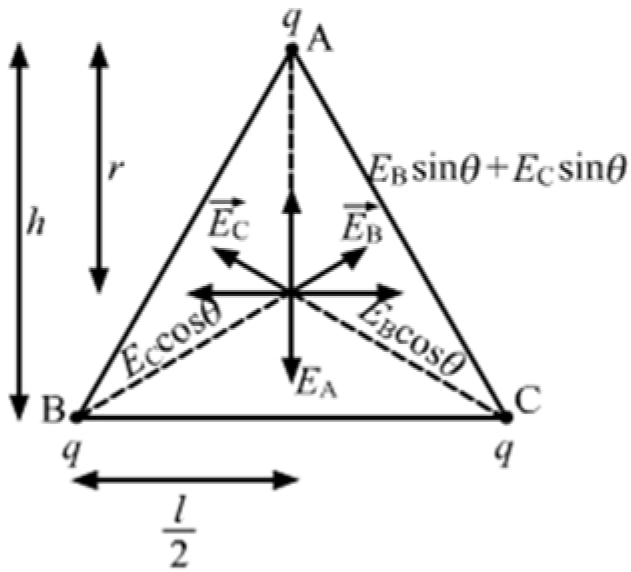
Thus, the magnitude of electric field due to the charged water molecule suspended in the room is 65.33 NC^{-1} and it is in upwards direction opposite to the gravitational force.

38. Question

Three identical charges, each having a value 1.0×10^{-8} C, are placed at the corners of an equilateral triangle of side 20 cm. Find the electric field and potential at the center of the triangle.

Answer

Given: Value of three identical charges: $q = 1.0 \times 10^{-8}$ C Side of the equilateral triangle: l = 20cm = 0.2m



From the diagram,

A,B and C are the three vertices having equal charge q.E_A,E_B and E_c are the electric fields at the center of the triangle due to charges A,B and C respectively.h is the height of the equilateral triangleand r is the distance from the center of the triangle to it's all three vertices.

Formula used: Formula for potential at a point is: $V = \frac{kq}{r}$

Where k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} .q is the point charge and r is the distance between the centre of the triangle and the vertex.

Since charges are equal at A,B and C: $E_A = E_B = E_C = E$ The Field from B and C are resolved into horizontal and vertical components as seen from the figure.Here θ is 30° as every angle of an equilateral triangle is 60° .The horizontal components balance each other.Therefore net electric field ,$E_{net} = E_A - (E_B \sin\theta + E_C \sin\theta)$ $\therefore E_{net} = E - (E\sin\theta + E\sin\theta)$ $\therefore E_{net} = E(1 - \sin(30) - \sin(30))$ $\therefore E_{net} = E(1 - 0.5 - 0.5)$ $\therefore E_{net} = 0$ Thus, the electric field at the center of the given equilateral triangle is zero.Now, using Pythagoras theorem to find value of h, $l^2 = \left(\frac{l}{2}\right)^2 + h^2 \therefore h^2 = 0.2^2 - 0.1^2$

$\therefore h = 0.173\ m$We know that, in an equilateral triangle$r = \frac{2}{3} \times h$Thus we get, $r = \frac{2}{3} \times 0.173 \therefore r = 0.1153\ m$

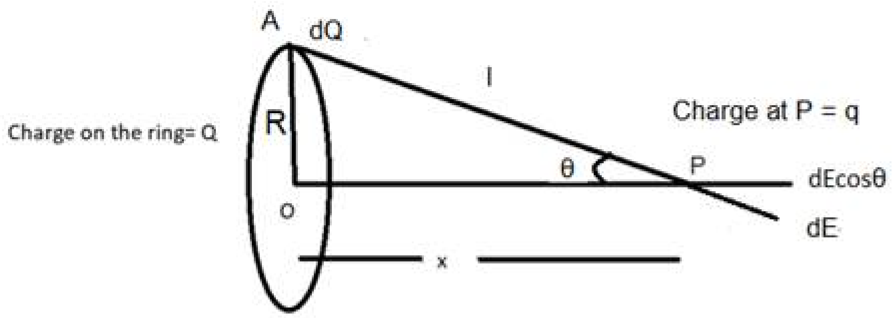
Since Electric field is same for all three points: $V_A=V_B=V_C$The potential at the center is :$V = V_A + V_B + V_C \therefore V = 3V_A \therefore V = 3 \times k \times \frac{q}{r}$

$\therefore V = 3 \times 9 \times 10^9 \times \frac{1.0 \times 10^{-8}}{0.1153} \therefore \boldsymbol{V = 2341\ V}$Hence , potential at the centre of the triangle is 2341 V and Electric field at the center is zero.

39. Question

Positive charge Q is distributed uniformly over a circular ring of radius R. A particle having a mass m and a negative charge q, is placed on its axis at a distance x from the centre. Find the force on the particle. Assuming x <<R, find the time period of oscillation of the particle if it is released from there.

Answer

Given: 

Charge on the ring: QRadius of the ring : RCharge of the particle at point P : qMass of the particle : mDistance of P from the centre of the ring: xDistance of P from the element A : l**Formula used:**Electric force is given as:$F = qE \ldots .(1)$Where F is the electric force, q is the charge and E is the electric field.

Newton's Law gives : $F = ma \therefore a = \frac{F}{m} = \frac{kqQx}{mR^3} \ldots ..(2)$Time period is given as: $T = 2\pi\sqrt{\frac{l}{a}} \ldots \ldots ..(3)$Where, l is the length. In this case OP=x and a is the acceleration.

Where F is the electric force, q is the charge and E is the electric field.Consider an element of charge dQ on the ring at A.Electric field at P due to the element A is given as:$\boldsymbol{dE = \frac{kdQ}{l^2}}$Here, dE is the electric field due to element A. k=$\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2}, dQ is the charge of the element A and l is the distance between A and P.Now, $l^2 = x^2 + R^2$ And $cos\theta = \frac{x}{\sqrt{x^2 + R^2}}$

As the ring is symmetric, the electric field at P will be along OP axis: Ecosθ

$\therefore dE\cos\theta = \frac{kdQ}{l^2} \times \frac{x}{\sqrt{x^2+R^2}} \therefore dE\cos\theta = \frac{kdQx}{(x^2+R^2)^{\frac{3}{2}}}$ Therefore, the net electric field at P due to the entire ring is: $E = \int dE\cos\theta \therefore E = \int \frac{kdQx}{(x^2+R^2)^{\frac{3}{2}}}$

$\therefore E = \frac{kx}{(x^2+R^2)^{\frac{3}{2}}} \int dQ \therefore \boldsymbol{E} = \frac{\boldsymbol{kQx}}{(\boldsymbol{x^2}+\boldsymbol{R^2})^{\frac{3}{2}}}$ Therefore substituting in equation (1) we get, $\boldsymbol{F} = \frac{\boldsymbol{kqQx}}{(\boldsymbol{x^2}+\boldsymbol{R^2})^{\frac{3}{2}}}$ Here, F is the electric force on the particle due to entire charged ring. Now the condition given in the question is x≪R. Thus, x^2 can be neglected. $F = \frac{kqQx}{(R^2)^{\frac{3}{2}}} \therefore F = \frac{kqQx}{R^3}$ Hence using equation (2) and (3) we get,

$T = 2\pi\sqrt{\left(\frac{x}{\frac{kqQx}{mR^3}}\right)} \therefore \boldsymbol{T} = \boldsymbol{2\pi}\sqrt{\frac{\boldsymbol{mR^3}}{\boldsymbol{kqQ}}}$ Putting the value of k: $\boldsymbol{T} = \left(\frac{\boldsymbol{16\pi^3\epsilon_0 mR^3}}{\boldsymbol{Qq}}\right)^{\frac{1}{2}}$

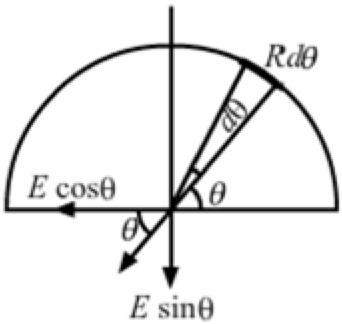
Hence time period of oscillation of the particle is $\left(\frac{16\pi^3\epsilon_0 mR^3}{Qq}\right)^{\frac{1}{2}}$

40. Question

A rod of length L has a total charge Q distribute uniformly along its length. It is bent in the shape of a semicircle. Find the magnitude of the electric field at the centre of curvature of the semicircle.

Answer

Given: Length of the rod: L Uniformly distributed charge: Q When the rod is bent into a semicircle, let R be its radius.



Here, dθ is the width of the angular element on the circumference. The displacement of the element with width dθ is Rdθ. Let the displacement of the element be dl. The Electric field is divided in to horizontal and vertical components. Horizontal components are cancelled out. **Formula Used:** We know that: $\lambda = \frac{Q}{L}$ Here, λ is the linear charge density of the rod, Q is the charge of the rod and L is the length of the rod. for a charge dq of element dl we have $dq = \lambda \times dl$ We know what dl =rdθ

$\therefore dq = \frac{Q}{L} \times Rd\theta$ The formula for electric field is: $\boldsymbol{E} = \frac{\boldsymbol{kq}}{\boldsymbol{r^2}}$ Here, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} . q is the point charge and r is the distance between the charge and the point of influence. here r=R. The net electric field due to vertical component is:$E_{net} = \int dEsin\theta\ d\theta$Here, dE is the electric field due to element dl having charge dq and the limits of the integral would be from 0 to π as it is a semicircle.$\therefore\ E_{net} = \int_0^{\pi} \frac{kdq}{r^2}$$\therefore\ E_{net} = \frac{kQ}{L}\int_0^{\pi} \frac{Rd\theta}{R^2} \times sin\theta$$\therefore\ E_{net} = \frac{kQ}{LR}\int_0^{\pi} sin\theta d\theta$

$\therefore\ E_{net} = \frac{kQ}{lR}[-cos\theta]_0^{\pi}$$\therefore\ E_{net} = \frac{kQ}{LR}[-(-1-1)]$$\therefore\ E_{net} = \frac{2kQ}{LR}$

If we substitute value of k we get,$\therefore\ E_{net} = \frac{2Q}{4\pi\epsilon_0 LR}$$\therefore\ \boldsymbol{E_{net} = \frac{Q}{2\epsilon_0 L^2}}$

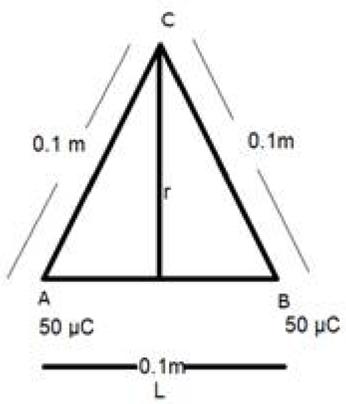
As L=π R.Hence electric field at the centre of the curvature is $\boldsymbol{\frac{Q}{2\epsilon_0 L^2}}$

41. Question

A 10 cm long rod carries a charge of +50 μC distributed uniformly along its length. Find the magnitude of the electric field at a point 10 cm from both the ends of the rod.

Answer

Given: Length of the rod : L = 10cm = 0.1mCharge on the rod; q = +50 μC= 50× 10^{-6} C



Here, C is 10 cm = 0.1 m away from both ends of the rod AB.Distance between C and centre of AB : r**Formula used:**From Pythagoras Theorem,$r^2+(L/2)^2=(BC)^2$

$\therefore\ r^2= [0.1]^2- [0.05]^2$

$\therefore\ r^2= 7.5\times10^{-3}$

$\therefore r=\sqrt{(7.5\times10^{-3})}$

∴r=0.0866 mNow we know that, Electric field at a point on the perpendicular bisector of a uniformly charged rod is:$\boldsymbol{E = \frac{2kQ}{r\left(\sqrt{L^2+4r^2}\right)}}$Here, k is a constant and $k=\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} . q is the point charge, L is the length of the rod and Q is the magnitude of the charge.Substituting we get,

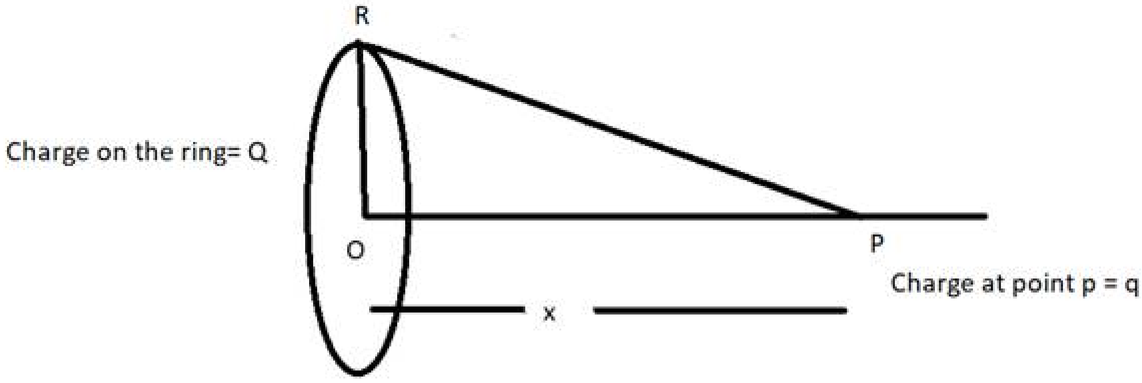
$E = \frac{2 \times 9 \times 10^9 \times 50 \times 10^{-6}}{0.0866\left(\sqrt{0.1^2 + 4(0.0866)^2}\right)} \therefore \boldsymbol{E = 5.2 \times 10^7\ NC^{-1}}$Hence, electric field at a point 10 cm away from the ends of the rod is 5.2 × 10^7 NC^{-1}.

42. Question

Consider a uniformly charged ring of radius R. Find the point on the axis where the electric filed is maximum.

Answer

Given: Charge of the ring: QRadius of the ring :RLet P be the point where electric field is found.Distance between center of the ring and P is x.



Formula used: We know that electric field at any point on the axis at a distance x from the center is:$E = \frac{kQx}{(R^2 + x^2)^{\frac{3}{2}}}$Where k is a constant and k=$\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2}. Q is the charge of the ring, x is the distance between center of the ring and point P and R is the radius of the ring.Now for the electric field to be maximum, we use the maxima property:$\frac{dE}{dx} = 0$Taking derivative of E w.r.t x,

$$\frac{dE}{dx} = kQ\left[(R^2 + x^2)^{-\frac{3}{2}} - \left(\frac{3}{2}x\right)(R^2 + x^2)^{-\frac{5}{2}} \times 2x\right] = 0$$

$$\therefore\ (R^2 + x^2)^{-\frac{3}{2}} - \left(\frac{3}{2}x\right)(R^2 + x^2)^{-\frac{5}{2}} \times 2x = 0$$

$$\therefore\ (R^2 + x^2)^{-\frac{3}{2}} = \ (3x^2)(R^2 + x^2)^{-\frac{5}{2}} \therefore\ (R^2 + x^2)^{-\frac{3}{2}} \times \frac{1}{(R^2 + x^2)^{-\frac{5}{2}}} = 3x^2$$

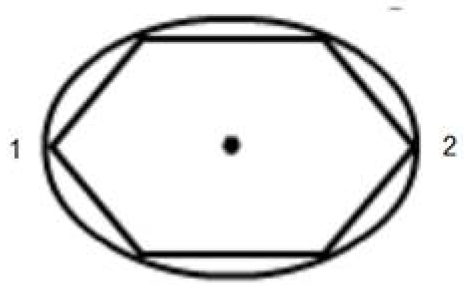
$\therefore R^2 + x^2 = 3x^2 \therefore R^2 = 2x^2 \therefore \boldsymbol{x = \frac{R}{\sqrt{2}}}$Hence Electric field is maximum at $x = \frac{R}{\sqrt{2}}$ on the axis.

43. Question

A wire is bent in the form of a regular hexagon and a total charge q is distributed uniformly on it. What is the electric field at the center? You many answer this part without making any numerical calculations.

Answer

Given: Uniformly distributed Charge on the regular hexagon : q



When a wire is bent in the form of a regular hexagon having charge q uniformly distributed, each point on the hexagon will contribute same magnitude of electric field.Say, 6 vertices of the hexagon have same charge q.Thus they will produce same electric field E at the center.This electric field gets nullified as same magnitude is acting from both sides.**Formula used:**Electric field at a point due to a point charge is given as:$E = \frac{kq}{r^2}$Where k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2}. q is the charge and r is the distance between the point and the charge.Mathematically:E_1=E_2From the figure E_1 is the electric field due to vertex 1 at the center and E_2 is the electric field due to vertex 2 at the center diametrically opposite to vertex 1.Thus net electric field at center due to 1 and 2 isE_{net} = E_1-E_2 ∴ $E_{net} = \frac{kq}{r^2} - \frac{kq}{r^2}$∴ E_{net} = 0 Eventually electric field due to all 6 vertices cancel out each other at the center.Hence, electric field at the center of the regular hexagon is zero.

44. Question

A circular wire-loop of radius a carries a total charge Q distributed uniformly over its length. A small length dL of the wire is cut off. Find the electric field at the center due to the remaining wire.

Answer

Given: Radius of the circular loop : aTotal charge on the wire : QLength of the cut off wire: dL**Formula used:**Electric field is given as:$\boldsymbol{E = \frac{kq}{r^2}}$Here, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} . q is the point charge and r is the distance between the charge and the point of influence.Here r=a=radius of the loop. We know that, electric field at the center of the uniformly charged circular wire $E_{cutoff} + E_{remaining} = 0$Which means that sum of electric field due to cut off wire and remaining wire is zero.We know that,$\lambda = \frac{Q}{L}$Where λ is the linear charge density. Q is the total charge and L is the length of the wire.Charge on the element dL be dq:$\lambda = \frac{dq}{dL}$∴ $dq = \lambda dL$ ∴ $dq = \frac{Q}{L} dL$Here L = 2πa=circumference. Here, a is the radius of the loop and L is the Length of the loop.∴ $dq = \frac{Q}{2\pi a} dL$Thus, electric field due to dL(cutoff wire) at the center is:$E_{cutoff} = \frac{kdq}{a^2}$ ∴ $E_{cutoff} = \frac{k\frac{Q}{2\pi a}dL}{a^2}$

$\therefore E_{cutoff} = \frac{kQdL}{2\pi a^3}$Since$E_{cutoff} + E_{remaining} = 0 \therefore \frac{kQdL}{2\pi a^3} + E_{remaining} = 0$

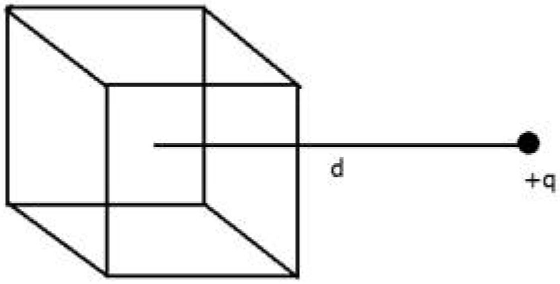
$\therefore E_{remaining} = -\left(\frac{kQdL}{2\pi a^3}\right)$ Thus, magnitude of electric field at the center of the circular wire due to remaining wire is E=$\frac{kQdL}{2\pi a^3}$ but in opposite direction to that of the field due to cut off wire.

45. Question

A positive charge q is placed in front of a conducting solid cube at a distance d from its center. Find the electric field at the center of the cube due to the charges appearing on its surface.

Answer

Given: Charge placed in front of a solid cube: +qDistance between the charge and the center of the cube: d



Formula used: Formula for electric field is:$E = \frac{kq}{r^2}$Here, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} . q is the point charge and r is the distance between the charge and the point of influence.Since we have to find electric field at the center of the cube$r = d$

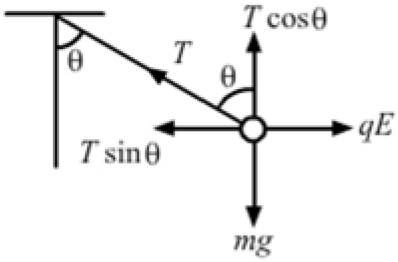
$\therefore E = \frac{kq}{d^2} \therefore E = \frac{q}{4\pi\epsilon_0 d^2}$ Hence, electric field at the center of the cube due to a positive charge at a distance d from the center of cube is $E = \frac{q}{4\pi\epsilon_0 d^2}$.

46. Question

A pendulum bob of mass 80 mg and carrying a charge of 2 × 10^{-8} C is at rest in a uniform, horizontal electric field of 20 kVm^{-1}. Find the tension in the thread.

Answer

Given: Mass of the bob: m = 80 mg= 80× 10^{-6} kgCharge on the bob: q = 2 × 10^{-8} CElectric field : E = 20 kVm^{-1} = 20× 10^3 Vm^{-1}.



Here the electric field is horizontal.The tension in the string is resolved into two components:Horizontal component : Tsinθ Vertical component : Tcosθ As shown in the figure.**Formula Used:**Since electric field acts on the bob having charge q, it experiences an electric field in horizontal direction.Electric force is given as:F=qEF is electric force, q is the charge on the bob and E is the horizontal electric field. This electric force is balanced by the horizontal component of the tension.Tsinθ = qENow the vertical component of tension is balanced by the weight of the bob.Tcosθ=mgDividing we get,$\frac{Tsin\theta}{Tcos\theta} = \frac{qE}{mg}$$\therefore tan\theta = \frac{2 \times 10^{-8} \times 20 \times 10^{3}}{80 \times 10^{-6} \times 9.8}$ $\therefore \tan\theta = 0.510$$\therefore \theta = \tan^{-1}(0.510)$$\therefore \theta = 27.02°$ Substituting the value of θ , we can calculate tension in the string.$\therefore Tcos(27.02) = 80 \times 10^{-6} \times 9.8$ $\therefore T = \frac{7.84 \times 10^{-4}}{0.89}$$\therefore \boldsymbol{T = 8.8 \times 10^{-4} N}$ Hence the tension in the string is 8.8×10^{-4} N.

47. Question

A particle of mass m and charge q is thrown at a speed u against a uniform electric field E. How much distance will it travel before coming to momentary rest?

Answer

Given: Mass of the particle: mCharge of the particle: qinitial velocity of the particle when thrown : uUniform electric field : E**Formula used:**When a charge q moves in an uniform electric field E, it experiences an electric force F.F=qEBut this particle is thrown **against** the electric field, hence the force experience would be negative.F=-qEAccording to Newton's Second Law:F=maWhere a is the acceleration and m is the mass of the body.Thus,$a = \frac{F}{m} = -\left(\frac{qE}{m}\right)$We will be using one of the equations of motion.$\boldsymbol{v^2 = u^2 + 2as}$Here, v is the final velocity of the particle, u is the initial velocity, a is the acceleration and s is the displacement.Since particle comes to rest, final velocity: v=0Thus, $0 = u^2 - \frac{2qE}{m} \times s$$\therefore \frac{2qE}{m} \times s = u^2$$\therefore \boldsymbol{s = \frac{u^2 m}{2qE}\ units}$. Hence, the particle will travel $\frac{u^2 m}{2qE}$ units before coming to rest.

48. Question

A particle of mass 1g and charge 2.5×10^{-4} C is released from rest in an electric field of 1.2×10^{4} N C^{-1}.

(a) Find the electric force and the force of gravity acing on this particle. Can one of these forces be neglected in comparison with the other for approximate analysis?

(b) How long will it take for the particle to travel a distance of 40 cm?

(c) What will be the speed of the particle after travelling this distance?

(d) How much is the work done by the electric force on the particle during this period?

Answer

Given: Mass of the particle : m= 1g = 10^{-3} kgCharge of the particle: q = 2.5×10^{-4} CElectric Field: E = 1.2×10^{4} N C^{-1}Distance travelled : s = 40 cm = 0.4 mInitial velocity: u = 0**Formula used:**(a)When a charge q moves in an uniform electric field E, it experiences an electric force F_e.F_e = qEsubstituting the values we get, $F_e = 2.5 \times 10^{-4} \times 1.2 \times 10^4$ ∴ F_e = 3NForce of gravity would be:$F_g = mg$ Where m is the mass of the particle and g is the acceleration due to gravity. $\therefore F_g = 10^{-3} \times 9.8 \therefore \boldsymbol{F_g = 9.8 \times 10^{-3} N}$Since the mass of the particle is very low, force due to gravity can be neglected.(b)From Newton's Second Law,F=mawhere a is the acceleration of the body and m is the mass.$\therefore a = \frac{F}{m}$acceleration of the particle is:$a = \frac{F_e}{m} = \frac{3}{10^{-3}} = 3 \times 10^3 \frac{m}{s^2}$Using one of the equations of motion: $\boldsymbol{s = ut + \frac{1}{2}at^2}$Here, s is the distance travelled, u is the initial velocity, t is the time required to travel s, and a is the acceleration of the particle.Substituting we get,$0.4 = 0 + 0.5 \times 3 \times 10^3 \times t^2 \therefore t^2 = \frac{0.4}{1500} \therefore t = (2.67 \times 10^{-4})^{\frac{1}{2}}$ $\boldsymbol{\therefore t = 0.0163\ seconds}$ It will take 0.0163 seconds for the particle to travel a distance of 40 cm.(c)Using another equation of motion$\boldsymbol{v^2 = u^2 + 2as}$Here v is the final velocity of the particle,u is the initial velocity of the particle, a is the acceleration and s is the distance travelled by the particle.Substituting we get, $v^2 = 0 + 2 \times 3 \times 10^3 \times 0.4 \therefore v^2 = 2400 \therefore \boldsymbol{v = \sqrt{2400} = 48.9 \frac{m}{s}}$After travelling 40cm the speed of the particle will be 48.9 m/s.(d)We know that,Work=Force×DisplacementThus work done by the electric force:W = F × s ∴ W = 3× 0.4 ∴ W = 1.2 J

Hence , work of 1.2 J is being done by the electric force on the particle.

49. Question

A ball of mass 100g and having a charge of 4.9×10^{-5} C is released from rest in a region where a horizontal electric field of 2.0×10^4 C NC^{-1} exists.

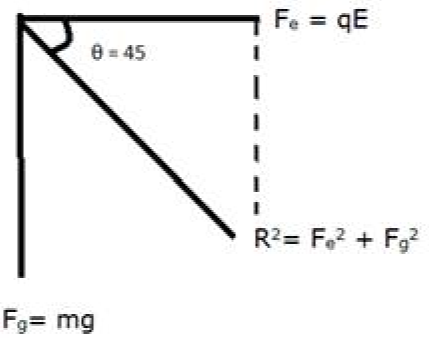
(a) Find the resultant force acing on the ball.

(b) What will be the path of the ball?

(c) Where will the ball be at the end of 2s?

Answer

Given: Mass of the ball : m = 100 g = 100×10^{-3} kg =0.1 kgCharge on the ball : q = 4.9×10^{-5} CHorizontal electric field : E= 2.0×10^4 C NC^{-1}Initial Velocity of the ball: u

= 0 

Here, R is the resultant Force due to gravitational force F_g and electric force F_e.**Formula used:**(a)Electric force F_e due to charge q and electric field E isF_e = qE$\therefore F_e = 4.9 \times 10^{-5} \times 2.0 \times 10^4$ $\therefore \mathbf{F_e = 0.98\ N}$ Gravitational force F_g experience due to mass of the ball m and acceleration due to gravity g is:F_g=mg$\therefore F_g = 0.1 \times 9.8 \therefore \boldsymbol{F_g = 0.98\ N}$Thus we see that $F_e = F_g$The Resultant force R can be calculated by:$R^2=F_e^2+F_g^2$

$\therefore R^2= (0.98)^2+(0.98)^2$

$\therefore R=\sqrt{1.9208}$

$\therefore$R=1.3859 N Hence the resultant force of 1.3859 N is acting on the ball.(b)We take tangent of the angle θ $tan\theta = \frac{Opposite\ side}{Adjacent\ side}$$tan\theta = \frac{F_g}{F_e}$But $F_g= F_e$,$\therefore tan\theta = 1$ $\therefore \theta = 45°$ Hence, the path of the ball is along a straight line and inclined at an angle of 45° with the horizontal electric field.(c)Here, we will be using one of the equations of motion.$\boldsymbol{s = ut + \frac{1}{2}at^2}$Here s is the distance covered by the ball, u is the initial velocity of the ball, a is the acceleration of the ball and t is the time required to cover s.We need to find s at t=2sFirstly ,vertical displacement due to gravitational force is:a=g$\therefore s = 0 + 0.5 \times 9.8 \times 2^2 \therefore s_v = 19.6\ m$. Secondly, $a = \frac{F}{m}$Horizontal displacement due to electric force is:$s_h = 0 + \frac{1}{2} \times \frac{F_e}{m} \times 2^2$ $\therefore s = 0.5 \times \frac{0.98}{0.1} \times 4 \therefore s = 19.6\ m$Thus net displacement = $(s_v^2+ s_h^2)^{1/2}$$\therefore$ Net displacement = $((19.6)^2+(19.6^2))^{1/2}$$\therefore$ Net displacement = 27.71 m

Thus, the ball will be at a distance of 27.71m after 2s

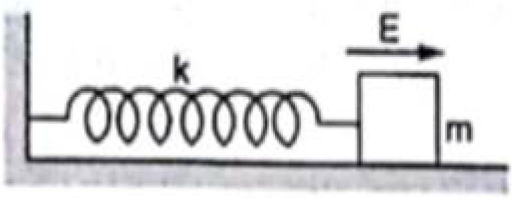
50. Question

The bob of a simple pendulum as a mass of 40g and a positive charge of 4.0×10^{-6} C. It makes 20 oscillations in 45s. A vertical electric field pointing upward and the magnitude 2.5×10^4 N C^{-1} is switched on. How much time will it now take to complete 20 oscillations?

Answer

Mass of the bob: m = 40 g = 0.04 kgCharge of the bob: q = 4.0×10^{-6} COscillations in t=45s : 20Vertical electric field: E = 2.5×10^4 N C^{-1}.**Formula used:**Time period of a simple pendulum is:$T = 2\pi\sqrt{\frac{l}{g}}$here l is the length of the string .Thus, when there is no electric field, the time period is:$T_1 = 2\pi\sqrt{\frac{l}{g}}$When a vertical electric field is applied, the positively charged bob will experience a vertical acceleration due to electric force. Thus net acceleration of the bob = g-aTime period of the simple pendulum when vertical electric field applied is:$T_2 = 2\pi\sqrt{\frac{l}{g-a}}$Now,Since a is due to electric force, by Newton's Second Law and Formula for electric force, we get $a = \frac{F}{m} = \frac{qE}{m} \therefore a = \frac{4.0 \times 10^{-6} \times 2.5 \times 10^{4}}{0.04} \therefore a = 2.5\frac{m}{s^2}$Thus g-a = 9.8-2.5 = 7.3 m/s^2Taking ratio we get:$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{l}{g}}}{2\pi\sqrt{\frac{l}{g-a}}} \therefore \frac{T_1}{T_2} = \sqrt{\frac{g-a}{g}} \therefore \frac{T_1}{T_2} = \sqrt{\frac{7.3}{9.8}}$ $\therefore \frac{T_1}{T_2} = 0.863 \therefore T_2 = \frac{45}{0.863} \therefore T_2 = 52\ seconds.$Hence time required by the bob to complete 20 oscillations in presence of a vertical electric field is 52 seconds.

51. Question

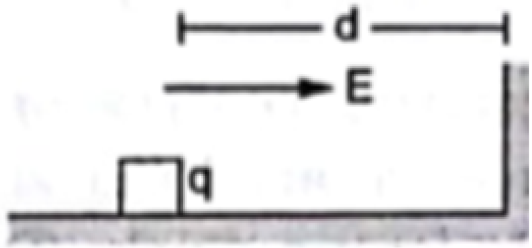
A block a mass m having a charge q is placed on a smooth horizontal table and is connected to a wall through an unstressed spring of spring constant κ as shown in figure. A horizontal electric field E parallel to the spring is switched on. Find the amplitude of the resulting SHM of the block. 

Answer

Given: Mass of the block: mCharge of the block: q**Formula used:**When a charged body of charge q and mass m is brought under an horizontal electric field E, it will experience electric force F_e in the direction of the field.F_e=qEWhen the block is accelerated due to the electric force, the spring will cause a restoring spring force in the opposite direction.Spring force is:F_s = -kxWhere, k is the spring constant and x is the distance the spring is stretched or compressed.Here x is the amplitude.Thus,F_e = F_s ∴ qE = -kx∴ $x = \left|-\frac{qE}{k}\right| \therefore \boldsymbol{x} = \frac{\boldsymbol{qE}}{\boldsymbol{k}}$We used modulus as the amplitude cannot be negative.Hence, $\frac{qE}{k}$ is the amplitude of the resulting SHM of the block.

52. Question

A block of mass m containing a net positive charge q is placed on a smooth horizontal table which terminates in a vertical wall as shown in figure. The distance of the block from the wall is d. A horizontal electric field E towards right is switched on. Assuming elastic collisions (if any) find the time period of the resulting oscillatory motion. Is it a simple harmonic motion?



Answer

Given: Mass of the block : mCharge on the block: qDistance between block and the wall : dHorizontal electric field: EInitial velocity: u=0

Formula used: It is not Simple Harmonic Motion. In SHM the acceleration is directly proportional to the displacement of the body and in opposite direction of the displacement. In this case, acceleration is proportional to the displacement **but** not in the opposite direction.We know that,$F = qE \text{ } and \text{ } F = ma \therefore a = \frac{qE}{m}$Here, F is the horizontal Electric force , q is the charge of the body and E is the electric force (horizontal), a is the acceleration and m is the mass of the bodyTo find time required by the block to collide into the wall can be calculated by laws of motion. $s = ut + \left(\frac{1}{2}\right) at^2$Here s is the displacement; s=d. u is the initial velocity , a is the acceleration of the block and t is the time required to travel the displacement. $\therefore d = 0 + \frac{1}{2} \times \frac{qE}{m} \times t^2 \therefore t = \sqrt{\frac{2dm}{qE}}$This is the time required by the block to travel d and hit the wall.Since it is assumed to be elastic collision , after collision with the wall the block will take same time ‘t’ to come back till it’s velocity is zero.Thus Total time take:T$T = 2t \therefore \boldsymbol{T = 2\sqrt{\frac{2dm}{qE}}}$Time period of the resulting oscillatory motion is $T = 2\sqrt{\frac{2dm}{qE}}$

53. Question

A uniform electric field of 10 N C^{-1} exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 50 cm.

Answer

Given: Electric field: E= 10 N C^{-1}Change in Height : dh = 50 cm= 0.5 m**Formula used:**Since the electric field exists in vertically downward direction, E becomes negative as one goes up. E= -10 N C^{-1}Change in potential is :dV = -E.drHere, dV is change in potential, dr is the distance moved and E is the electric field.In this case dr=dh∴ dV = -(-10)× 0.5 ∴ dV = 5 VHence, Increase in potential as one goes up by 50cm is 5V.

54. Question

12 J of work has to be done against an existing electric field to take a charge of 0.01 C from A to B. How much is the potential difference $V_B - V_A$?

Answer

Given: Work done against electric field: W= 12 JCharge : q= 0.01 C**Formula used:**Work done is given by:W=Potential difference×charge

∴W=(V_B-V_A)×qHere, V_B-V_A is the potential difference and q is the charge.

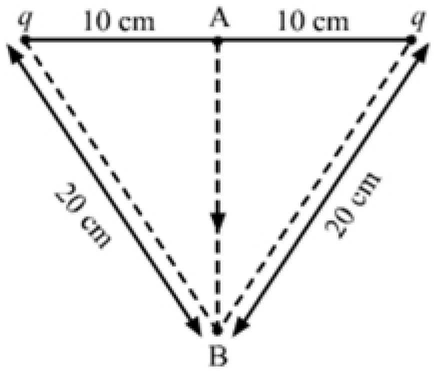
$\therefore\ V_B - V_A = \frac{W}{q}$ $\therefore\ V_B - V_A = \frac{12}{0.01}$ $\therefore\ \boldsymbol{V_B - V_A = 1200\ V}$Hence, potential difference is 1200 V

55. Question

Two equal charges, 2.0 × 10^{-7} C each, are held fixed at a separation of 20 cm. A third charge of equal magnitude is placed midway between the two charges. It is now moved to a point 20 cm from both the charges. How much work is done by the electric field during the process?

Answer

Given: Magnitude of all three charges: q_1=q_2=q_3= q= 2.0 × 10^{-7} CSeparation between first two charges: d = 20 cm = 0.2m



From the figure,Distance between charge at A and both the charges:r =10cm= 0.1 mDistance between charge at b and both the charges:r'= 20 cm= 0.2 m**Formula used:**Potential is given as$\boldsymbol{V = \frac{Kq}{r}}$Here, Here, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} . q is the point charge and r is the distance between the charge and the point of influence.Since A is midway between two point charges, potential at A will be due to both the charges:$V_A = 2 \times \frac{kq}{r}$ $\therefore\ V_A = \frac{2 \times 9 \times 10^9 \times\ 2.0\ \times\ 10^{-7}}{0.1}$

$\therefore\ V_A = 36000\ V$ Now, when charge at A is displaced to B, potential difference is created. Potential at B is due to both the charges at 0.2 m equally from B.Potential at B:$V_B = 2 \times \frac{kq}{r'}$ $\therefore\ V_B = \frac{2 \times 9 \times 10^9 \times\ 2.0\ \times\ 10^{-7}}{0.2}$ $\therefore\ V_B = 18000\ V$Thus, Potential difference is:V_A-V_B =36000-18000 ∴ V_A-V_B = 18000Work done is:W= (V_A-V_B) × q ∴ W = 18000 × 2.0 × 10^{-7} ∴ W = 3.6 × 10^{-3} JHence, 3.6× 10^{-3} J of work is being done by the electric field during whole process.

56. Question

An electric field of 20 N C^{-1} exists along the x-axis in space. Calculate the potential difference $V_B - V_A$ where the points A and B are given by,

(a) A = (0, 0); B = (4m, 2m)

(b) A = (4m, 2m); B = (6m, 5m)

(c) A = (0, 0); B = (6m, 5m)Do you find any relation between the answers of parts (a), (b) and (c)?

Answer

Given: Magnitude of Electric field: E = 20 N C^{-1}E is along x-axis**Formula used:**As Electric field is along x-axis, potential difference will be along x-direction. Which means only x co-ordinates will be considered.We know that,$dV = -E.ds$Here dV is the change in potential : dV= V_B-V_AE is the electric field along positive x axis and ds is the change in displacement.(a)A = (0, 0); B = (4m, 2m) ∴ V_B-V_A= -20×(4-0) = -80 V(b)A = (4m, 2m); B = (6m, 5m) ∴ V_B-V_A= -20×(6-4) = -40 V(c)A = (0, 0); B = (6m, 5m) ∴ V_B-V_A= -20×(6-0) = -120 V(d)From (a),(b) and (c), we conclude that:Potential difference of at points A = (0, 0), B = (6m, 5m)= Potential difference at points A = (0, 0), B = (4m, 2m)+ Potential difference at points A = (4m, 2m), B = (6m, 5m)

57. Question

Consider the situation of the previous problem. A charge of –2.0 × 10^{-4} C is moved from the point A to the point B. Find the change in electrical potential energy $U_B - U_A$ for the cases (a), (b) and (c).

Answer

Given: Magnitude of Electric field: E = 20 N C^{-1}Magnitude of charge moved from A to B = –2.0 × 10^{-4} C**Formula used:**Change in Electrical potential energy is $\boldsymbol{\Delta U = \Delta V \times q}$Here, ΔU is change in Electrical potential energy, Δ V is change in potential and q is the charge displaced.ΔU = U_B-U_A where U_B is electric potential energy at B and U_A is electric potential energy at A(a)A = (0, 0); B = (4m, 2m) ∴ V_B-V_A= -20×(4-0) = -80 VThus Change in Electrical potential energy isU_B-U_A = (V_B – V_A)× q ∴ Δ U = -80 × -2.0× 10^{-4} ∴ Δ U = 0.016 J(b)A = (4m, 2m); B = (6m, 5m) ∴ V_B-V_A= -20×(6-4) = -40 VThus Change in Electrical potential energy isU_B-U_A = (V_B – V_A)× q ∴ Δ U = -40 × -2.0× 10^{-4} ∴ Δ U = 0.008 J(c)A = (0, 0); B = (6m, 5m) ∴ V_B-V_A= -20×(6-0) = -120 VThus Change in Electrical potential energy isU_B-U_A = (V_B – V_A)× q ∴ Δ U = -120 × -2.0× 10^{-4} ∴ Δ U = 0.024Hence, for the cases (a),(b) and (c) the change in electric potential energy when the charge is moved from A to B is 0.016 J,0.008 J , 0.024 J respectively.

58. Question

An electric field $\vec{E} = (\vec{i}\,20 + \vec{j}\,30)\,NC^{-1}$ exists in the space. If the potential at the origin is taken to be zero, find the potential at (2m, 2m).

Answer

Given: Electric field: $\vec{E}$ = 20$\vec{\imath}$+ 30$\vec{\jmath}$ NC^{-1}V(0,0) = 0Final position: $\vec{r} = (2\vec{\imath} + 2\vec{\jmath})$
Formula used: We know that Electric potential is given as:$V = -E.ds$Where ds is the change in displacement.In vector form$\vec{V} = -\vec{E}.\vec{r}$Here, r̄ is the changed position from origin.∴ $\vec{V} = -(20\vec{\imath} + 30\vec{\jmath}).(2\vec{\imath} + 2\vec{\jmath})$
∴ $\vec{V} = -(20 \times 2 + 30 \times 2)$∴ $\boldsymbol{\vec{V} = -100\ V}$

Hence potential at (2m,2m) is -100 V.

59. Question

An electric field exists in the space, where A = 10 Vm^{-2}. Take the potential at (10m, 20m) to be zero. Find the potential at the origin.

Answer

Given: Electric Field : $\vec{E} = Ax\vec{\imath}$A = 10 Vm^{-2}V(10m,20m) = 0**Formula used:**Change in potential is:$dV = -E.dx$Where dV is change in potential, E is the electric field and dx is the change in displacement.$\vec{E} = 10x\vec{\imath}$In vector form:$dV = -\vec{E}.\overrightarrow{dx}$
∴ $dV = -10xdx$Integrating we get:$V = -10\int_{10}^{0} xdx$∴ $V = -10\left[\frac{x^2}{2}\right]_{10}^{0}$
∴ $\boldsymbol{V = 10 \times \frac{100}{2} = 500\ V}$Here limits 10 to 0 is taken as 10 m is the x co-ordinate and origin has (0,0)Hence. potential of 500 V exists at the origin.

60. Question

The electric potential existing in space isV(x, y, z) = A (xy + yz + zx).

(a) Write the dimensional formula of A.

(b) Find the expression for the electric field.

(c) If A is 10 SI units, find the magnitude of the electric filed at (1m, 1m, 1m).

Answer

Given: Electric Potential :V(x,y,z) = A (xy + yz + zx).A = 10 SI units.**Formula used:**
(a)From the given data V(x,y,z) = A (xy + yz + zx), we can say that Voltage is the product of A and length^2(xy+yz+zx)Volt = A× m^2∴ $A = \frac{Volt}{m^2}$We know that Volt = $\left[\frac{ML^2}{QT^2}\right]$, where Q= $[IT^{-1}]$∴ $A = \frac{[ML^2I^{-1}T^{-3}]}{[L^2]}$ ∴ $\boldsymbol{A = [MI^{-1}T^{-3}]}$Thus, dimensions of A are $[\boldsymbol{MI^{-1}T^{-3}}]$(b)Formula for electric field is:$\boldsymbol{\vec{E} = -\nabla.V}$Where
∇= $\frac{\partial}{\partial x}\vec{\imath} + \frac{\partial}{\partial y}\vec{\jmath} + \frac{\partial}{\partial z}\vec{k}$∴ $\boldsymbol{\vec{E}} = -\left(\left(\frac{\partial}{\partial x}\vec{\imath} + \frac{\partial}{\partial y}\vec{\jmath} + \frac{\partial}{\partial z}\vec{k}\right).A\,(xy + yz + zx)\right)$

$$\therefore \vec{E} = -\left(A\left(\frac{\partial(xy + yz + zx)}{\partial x}\right)\vec{i} + A\left(\frac{\partial(xy + yz + zx)}{\partial y}\right)\vec{j} + A\left(\frac{\partial(xy + yz + zx)}{\partial z}\right)\vec{k}\right)$$

$\therefore \boldsymbol{\vec{E} = -A(y+z)\vec{i} - A(x+z)\vec{j} - A(x+y)\vec{k}}$Hence , expression of electric field is $\vec{E} = -A(y+z)\vec{i} - A(x+z)\vec{j} - A(x+y)\vec{k}$(c)A = 10 SI units(x,y,z) = (1m,1m,1m)Substituting in the expression of E we get:
$\vec{E} = -10(1+1)\vec{i} - 10(1+1)\vec{j} - 10(1+1)\vec{k} \therefore \vec{E} = -20\vec{i} - 20\vec{j} - 20\vec{k}$
Magnitude of Electric field is$E = \sqrt{(-20)^2 + (-20)^2 + (-20)^2} \therefore E = \sqrt{1200}$
$\therefore \boldsymbol{E = 34.64\ NC^{-1}}$

Hence, magnitude of Electric field at (1m,1m,1m0 is 34.64 NC^{-1}.

61. Question

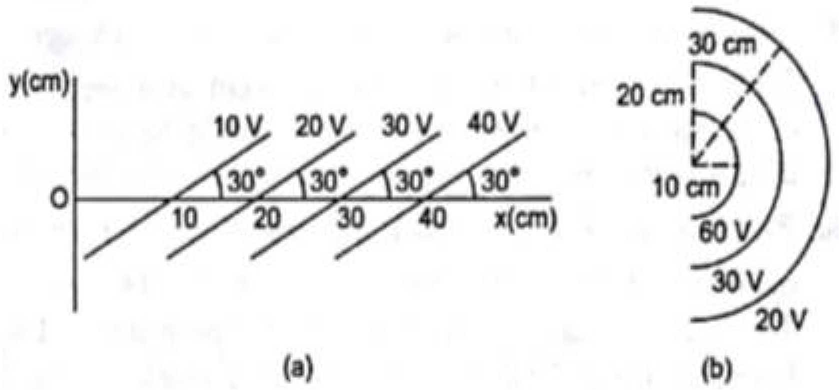
Two charged particles, having equal charges of 2.0 × 10^{-5} C each, are brought from infinity to within a separation of 10 cm. Find the increase in the electric potential energy during the process.

Answer

Given: Charge of two particles: q_1=q_2= 2.0 × 10^{-5} CSeparation between two charges: r = 10 cm = 0.1 m**Formula used:**Electric potential energy is given as:
$U = \frac{kq_1q_2}{r}$Where,k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} . q is the point charge and r is the separation between two charges.When two charges were at infinity, the separation between them was infiniteThus,$U_\infty = \frac{kq_1q_2}{\infty} \therefore U_\infty = 0$Now, when the separation between them was 10 cm;$U_f = \frac{kq_1q_2}{r}$Where U_f is the final electric potential energy.Substituting the values we get,
$U_f = \frac{9 \times 10^9 \times 2.0 \times 10^{-5} \times 2.0 \times 10^{-5}}{0.1} \therefore \mathbf{U_f = 36\ J}$ Now, increase in electric potential energy: Δ U$\Delta U = U_f - U_\infty \therefore \Delta U = 36 - 0 \therefore \boldsymbol{\Delta U = 36\ J}$Hence, Electric potential energy increased by 36 J during the process.

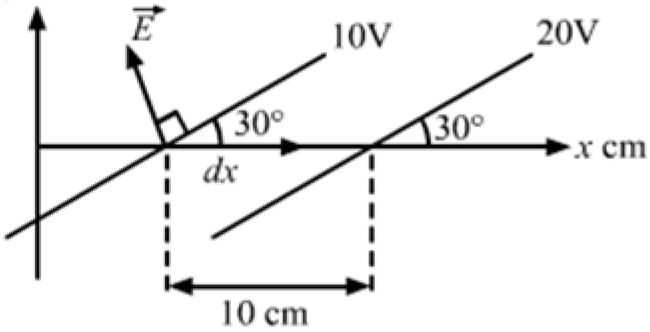
62. Question

Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field?

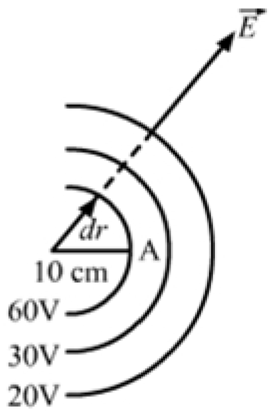


Answer

Given: From figure (a)Angle between equipotential surfaces and the displacement x:θ = 30°Change in potential : dV = 10 VChange in displacement between two consecutive equipotential surfaces: dx = 10 cm = 0.1 mFrom figure (b)Increase in radius from center: dr= 10cm=0.1m**Formula used:**(a)As we know that the electric field $\vec{E}$ is always perpendicular to the equipotential surface



From the above diagram, the angle between Electric field $\vec{E}$ and dx ; θ' = 90° +30° =120°Change in electric potential is given as:$dV = -\vec{E}.dx$∴ $10 = -Edx\cos(\theta')$ ∴ $10 = -E \times 0.1 \times \cos(120°)$∴ $E = \frac{10}{0.1 \times 0.5}$∴ $\boldsymbol{E = 200\frac{V}{m}}$ Hence the magnitude of electric field is 200 V/m making an angle of 120° with the x axis.(b)As we know that the electric field $\vec{E}$ is always perpendicular to the equipotential surface



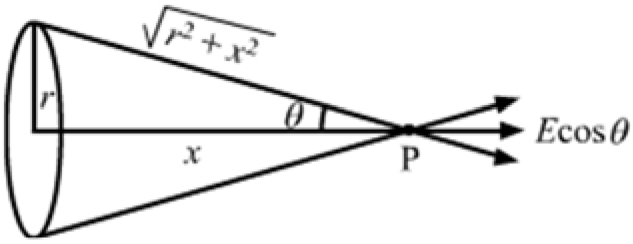
Radius increases by: dr= 10 cm = 0.1 mAs $\vec{E}$ is perpendicular to the equipotential surface, $\vec{E}$ and dr would be along same line as shown in the figure above.Thus angle between $\vec{E}$ and dr : θ = 0We know that potential at a point due to a charge q is given as:$V = \frac{kq}{r}$Where, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} . q is the point charge and r is radius of that surface.Consider potential at point A where r=0.1 m$V_A = 60V$∴ $60 = \frac{kq}{0.1}$ ∴ $kq = 60 \times 0.1 = 6$Electric field is given as:$E = \frac{kq}{r^2}$Substituting value of kq we get,$\boldsymbol{E = \frac{6}{r^2}}$Hence, the magnitude of the electric field is $\frac{6}{r^2}$ and it's direction is radially outward with decreasing with increasing radii.

63. Question

Consider a circular ring of radius r, uniformly charged with linear charge density λ. Find the electric potential at a point on the axis at a distance x from the center of the ring. Using this expression for the potential, find the electric field at this point.

Answer

Given: Radius of the circular ring: rLinear charge density : λDistance of a point from the center of the ring : x



From the diagram we can see that,Point P is at a distance x from the center of the ring.Point P is at a distance of $\sqrt{r^2+x^2}$ from the surface of the ring: r' = $\sqrt{r^2+x^2}$ Circumference of the ring is : L = 2π r**Formula used:**We can see that, Electric field as p is resolved into vertical and horizontal components. As the ring is symmetric, vertical components are cancelled out and horizontal components add.Thus E_{net} =Ecosθ , where θ is the angle between x and $\sqrt{r^2+x^2}$.We know that,$\boldsymbol{\lambda} = \frac{\boldsymbol{Q}}{\boldsymbol{L}}$Where, λ is the linear charge density, Q is the Total charge due to whole ring and L is the circumference of the ring.$\therefore\ Q = 2\pi r\lambda$Potential at a point due to charge Q is: $\boldsymbol{V} = \frac{\boldsymbol{kQ}}{\boldsymbol{r'}}$Here, k is a constant and k=$\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} . q is the point charge. $\therefore\ V = \frac{k2\pi r\lambda}{\sqrt{r^2+x^2}}$If we substitute value of k then:$V = \frac{1}{2\epsilon_0}\frac{r\lambda}{\sqrt{r^2+x^2}}$

Hence, Electric potential at a point x from the center of the ring is $V = \frac{k2\pi r\lambda}{\sqrt{r^2+x^2}}$.Net electric field at P is Ecosθ and $E = \frac{V}{r'}$From the figure:$cos\theta = \frac{x}{\sqrt{r^2+x^2}}$ $\therefore\ E_{net} = \frac{V}{r'}cos\theta \therefore\ E_{net} = \frac{1}{2\epsilon_0}\frac{r\lambda}{\sqrt{r^2+x^2}} \times \frac{1}{\sqrt{r^2+x^2}} \times \frac{x}{\sqrt{r^2+x^2}}$ $\therefore\ \boldsymbol{E_{net}} = \frac{\boldsymbol{1}}{\boldsymbol{2\epsilon_0}}\frac{\boldsymbol{r\lambda}}{(\boldsymbol{r^2+x^2})^{\frac{3}{2}}}$Hence, Electric field at P is $\frac{\boldsymbol{1}}{\boldsymbol{2\epsilon_0}}\frac{\boldsymbol{r\lambda}}{(\boldsymbol{r^2+x^2})^{\frac{3}{2}}}$

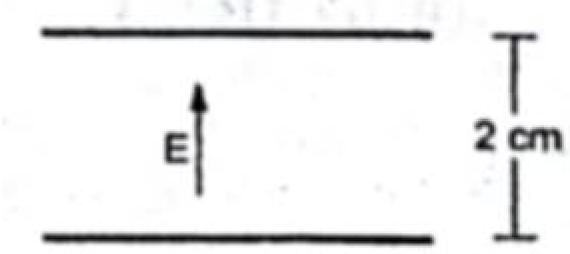
64. Question

An electric field of magnitude 1000 NC^{-1} is produced between two parallel plates having a separation of 2.0 cm as shown in figure.

(a) What is the potential difference between the plate?

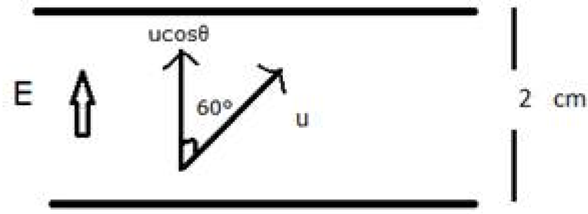
(b) With what minimum speed should an electron be projected from the lower plate in the direction of the field so that it may reach the upper plate?

(c) Suppose the electron is projected from the lower plate with the speed calculated in part (b). The direction of projection makes an angle of 60° with the field. Find the maximum height reached by the electron.



Answer

Given: Magnitude of the electric field : E = 1000 NC^{-1}Separation between the plates: r = 2 cm = 0.02 mAngle made by the projection with the field : θ = 60°



Formula used:(a)The Potential difference is:V = -E.rHere, E is the electric field and r is the separation between the plates.V = -1000 × 0.2 ∴ V = -200 = |-200| = 200 V

Hence, the potential difference between the plates is of 200 V(b)Charge on an electron: e = -1.6× 10^{-19} CWe know thatF=qE=maWhere F is the electric force, E is the electric field, m is the mass of the body and a is the acceleration of the body.Here, q=e of electron. And m = 9.7× 10^{-31} kg mass of the electron. $\therefore a = \frac{eE}{m} = \frac{-1.6\times\ 10^{-19}\times\ 1000}{9.7\times\ 10^{-31}}\therefore \mathrm{a} =\ -1.75\times 10^{14}\frac{m}{s^2}$Using one of the equations of motion we get,$v^2=u^2+2as$Here, v is the final velocity of the electron= 0Here v is zero as it we have to calculate u when it just reaches the upper plate, u is the initial velocity of the electron, s is the distance between the plates: s=r and a is the acceleration of the electrons.$\therefore\ 0 = u^2 - 2\times\ 1.75\times 10^{14}\times 0.02$ $\therefore u^2\ = 7\times 10^{12}\therefore u = \sqrt{7\times 10^{12}}\therefore \boldsymbol{u} = \mathbf{2.64}\times \mathbf{10^6}\,\frac{\boldsymbol{m}}{\boldsymbol{s}}$Hence, with a minimum speed of 2.64× 10^6 m/s^2 should the electron be projected from the lower plate in the direction of the field for it to reach the upper plate.(c)As direction of projection makes an angle 60° with the electric field.Initial velocity is resolved into its cos component which is along the direction of the electric field as shown in the figure above.Hence, u'=ucos(60°), u' is the resolved initial velocity.Using the same equation of motion used in (b) we get,$\therefore\ v^2 = u'^2 - 2\times a\times h$Where h is the maximum height reached by the electron.

$\therefore\ 0 = (\ 2.64\times\ 10^6\times 0.5)^2 -\ 2\times\ 1.75\times 10^{14}\times h\therefore\ h = \frac{1.74\times 10^{12}}{3.5\times 10^{14}}$

$\therefore\ \mathbf{h = 4.97\times 10^{-3}\ m}$ Hence, the maximum height reached by the electron is 4.7× 10^{-3} m.

65. Question

A uniform field of 2.0 NC^{-1} exists in space in x-direction.

(a) Taking the potential at the origin to be zero, write an expression for the potential at a general point (x, y, z).

(b) At which points, the potential is 25 V?

(c) If the potential at the origin is taken to be 100 V, what will be the expression for the potential at ta general point?

(d) What will be the potential at the origin if the potential at infinity is taken to be zero? Is it practical to choose the potential at infinity to be zero?

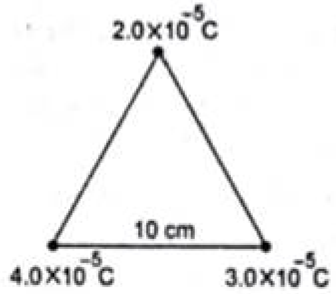
Answer

Given:(a)Magnitude of Electric field: E = 2.0 NC^{-1}V(0,0,0) = 0 V(b) V= 25 V(c) V(0,0,0) = 100 V**Formula used:**(a) Electric field exists in x-direction.We know that, Potential difference is :

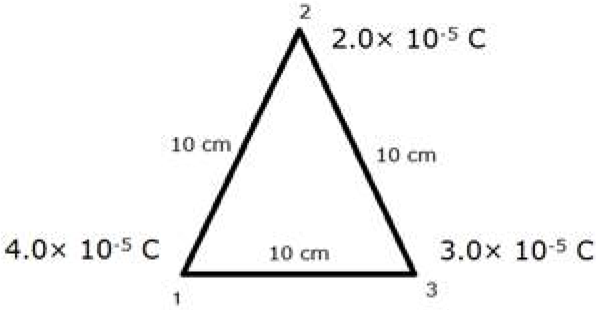
V=V_B-V_A=V_B-0=V_BHere V_B is the potential at general point (x,y,z) and V_A is the potential at origin = 0.Potential is given aV = -E.rWhere E is the electric field and r is the position of the point in space.In vector form: $\boldsymbol{E = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}}$ $\boldsymbol{r = x\hat{\imath} + y\hat{\jmath} + z\hat{k}}$ Substituting ,$\therefore V = -(E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}).(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$ $\therefore V = -E_x x$Here, y and z components are not considered as electric field is along x direction.$\therefore$ $\boldsymbol{V = -2x\ V}$Hence expression of potential at a general point (x,y,z) in space is -2x V.(b)Here, V_B is 25 V.$V_B - V_A = -2x \therefore 25 - 0 = -2x$ $\therefore \boldsymbol{x = \frac{25}{-2} = -12.5\ m}$Hence at x= -12.5 m, potential is 25 V.(c)Now similar to part (a), here potential at origin is given and we need to find potential at general point.V(0,0,0) = 100 VUsing formula for potential derived in (a) we get, $V_B - V_A = -2x \therefore V_B - 100 = -2x \therefore V_B = 100 - 2x \therefore \boldsymbol{V(x,y,z) = 100 - 2x\ V}$ Hence, potential at general point is 100 – 2x V(d)Let potential at infinity be : V_∞ = 0 and x=∞Potential at origin is : V_0Using the formula for potential and result of (a):$V_\infty - V_0 = -2x V_0 = V_\infty + 2x \therefore V_0 = 0 + 2 \times \infty \therefore \boldsymbol{V_0} = \infty$Hence, potential at origin is infinite.It is not practical to choose potential at infinity to be zero as it will make potential at origin to be infinite as we derived which will make the calculations impossible.

66. Question

How much work has to be done in assembling three charged particles at the vertices of an equilateral triangle as shown in figure?



Answer

Given: 

Charge at 1 : $q_1 = 4.0 \times 10^{-5}$ CCharge at 2 : $q_2 = 2.0 \times 10^{-5}$ CCharge at 3 : $q_3 = 3.0 \times 10^{-5}$ CLet side of the equilateral triangle be L = 10 cm = 0.1 m**Formula used:**To assemble the charges at three vertices electric potential energy is required.Thus, the work done in assembling these charges is equal to the total electric potential energy used.W = U_{12} +U_{23} + U_{31}Here, W is the work done.U_{12}: Electric Potential energy due to charges at 1 and 2 having separation 0.1 m.U_{23} : Electric Potential energy due to charges at 2 and 3 having separation 0.1 m.U_{31}: Electric Potential energy due to charges at 3 and 1 having separation 0.1 m.Formula for Electric potential energy is: $\boldsymbol{U = \frac{kq_1q_2}{r}}$Here, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} , q_1 and q_2 are point charges and r is the separation between them and U is the electric potential energy required to moves charges q_1 and q_2 apart.Substituting, $W = \frac{kq_1q_2}{L} + \frac{kq_2q_3}{L} + \frac{kq_3q_1}{L}$$\therefore W = \frac{k}{L} \times 10^{-10}(4 \times 2 + 2 \times 3 + 3 \times 4)$ $\therefore W = \frac{9 \times 10^9 \times 10^{-10} \times 26}{0.1}$$\boldsymbol{\therefore W = 234\, J}$Hence, 234 J of work is required for assembling the charges at the vertices of the equilateral triangle.

67. Question

The kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100V to a point at potential 200V. Find the charge on the particle.

Answer

Given: Change in Kinetic Energy of a charged particle: ΔE = 10 JPotential difference : dV = 200-100 =100 VWe know that, when energy of a body is changed, work is being done.Change in kinetic energy = Work DoneThus, Work done : W = 10 JAlso Work is given as:W = dV× qHere dV is the potential difference and q is the charge of the particle.Substituting the values:$10 = 100 \times q$$\boldsymbol{\therefore q = \frac{10}{100} = 0.1\, C}$Hence the charge on the particle is 0.1 C

68. Question

Two identical particles, each having a charge of 2.0 × 10^{-4} C and mass of 10g, are kept at a separation of 10 cm and then released. What would be the speeds of the particles when the separation becomes large?

Answer

Given: Charge on two identical particles: q_1=q_2= 2.0 × 10^{-4} CMass of the two particles : m_1=m_2= m = 10 g = 0.01 kgSeparation between the charges : r = 10 cm =

0.1 m**Formula used:**When they are released, the force of repulsion will be acting on both the particles making them drift apart thereby increasing distance between them.Potential Energy at the start will be$\boldsymbol{U_{initial} = \frac{kq_1q_2}{r}}$where $U_{initial}$ is the initial potential energy between the charges ,k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} , q_1 and q_2 are point charges and r is the separation between them.Final Potential energy would be zero as the particles will gain Kinetic energy and the separation between the charges would approach infinity : U_{final} = 0Now by conservation of energy:$\boldsymbol{Total\ Potential\ energy = Total\ Kinetic\ energy}$ Total kinetic energy is : $\boldsymbol{KE = \frac{1}{2}m_1V^2 + \frac{1}{2}m_2V^2 = 2 \times \frac{1}{2}mV^2}$Where , m is the mass of the particles and V is the velocity gained by the particles.

$\therefore\ U_{initial} - U_{final} = K.E \therefore \frac{kq_1q_2}{r} = 2 \times \frac{1}{2}mV^2 \therefore V^2 = \frac{kq_1q_2}{mr}$

$\therefore V^2 = \frac{9 \times 10^9 \times 2.0 \times 10^{-4} \times 2.0 \times 10^{-4}}{0.01 \times 0.1} \therefore V = \sqrt{360000} \therefore \boldsymbol{V = 600\ m/s}$

Hence, when the separation becomes large the speed of the particles should be 600 m/s.

69. Question

Two particles have equal masses of 5.0g each and opposite charges of +4.0 × 10^{-5} C and −4.0 × 10^{-5} C. They are released from rest with a separation of 1.0 m between them. Find the speeds of the particles when the separation is reduced to 50 cm.

Answer

Given: Mass of the two particles : m_1=m_2= m = 5.0 g = 0.005 kgCharge on particle 1 : q_1 = +4.0 × 10^{-5} CCharge on particle 2 : q_2 = -4.0 × 10^{-5} CSeparation between the charges : r = 1 mInitial velocity: v = 0**Formula used:**Conservation of Energy is:$U_{initial}$ + $K.E_{initial}$ = U_{final} + $K.E_{final}$where, $U_{initial}$ and $K.E_{initial}$ are the Initial potential energy and initial kinetic energy respectively .U_{final} and $K.E_{final}$ are the final potential energy and final kinetic energy respectively.Now, as initial velocity is zero : $K.E_{initial}$ = 0Final Kinetic Energy of both the particles is: $\boldsymbol{K.E_{final} = \frac{1}{2}m_1V^2 + \frac{1}{2}m_2V^2 = 2 \times \frac{1}{2}mV^2}$Also, Electric potential energy is given as;$\boldsymbol{U = \frac{kq_1q_2}{r}}$Where k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} , q_1 and q_2 are point charges and r is the separation between them.For initial situation : r = 1 mFor final situation : r = 50 cm = 0.5 m = r/2Substituting in the conservation formula we get,$\therefore \frac{kq_1q_2}{r} + 0 = \frac{kq_1q_2}{\frac{r}{2}} + 2 \times \frac{1}{2}mV^2 \therefore \frac{kq_1q_2}{r} - \frac{2kq_1q_2}{r} = mV^2$

$\therefore \frac{kq_1q_2}{r}(1-2) = mV^2$

$-\frac{9 \times 10^9 \times 4.0 \times 10^{-5} \times 4.0 \times 10^{-5}}{1} \times (-1) = 0.005 \times V^2 \therefore V^2 = \frac{14.4}{0.005}$

$\therefore V = \sqrt{2880} \therefore \boldsymbol{V = 53.66\ m/s}$ Hence the velocity of the particles when separation is reduced to 50 cm is 53.66 m/s

70. Question

A sample of HCl gas is placed in an electric field of 2.5×10^4 N C^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} Cm. Find the maximum torque that can act on a molecule.

Answer

Given: Electric field : E = 2.5×10^4 N C^{-1}Dipole moment of each HCl molecule :P = 3.4×10^{-30} cm = 3.4×10^{-30} Cm .**Formula used:** Torque acting on a dipole is given as$\boldsymbol{\tau} = \vec{\boldsymbol{P}} \times \vec{\boldsymbol{E}}$Where, τ is the torque acting on the dipole, P is the dipole moment of the HCl molecules and E is the electric field.τ = PEsinθFor maximum torque θ = 90°Thus sin(90) = 1$\therefore \tau = PE \therefore \tau = 3.4 \times 10^{-30} \times 2.5 \times 10^4$

$\therefore \boldsymbol{\tau} = \mathbf{8.5} \times \mathbf{10^{-26}}\ \boldsymbol{Nm}$ Hence a maximum of 8.5×10^{-26} Nm of torque can act on a HCl molecule.

71. Question

Two particles A and B, having opposite charges 2.0×10^{-6} C and -2.0×10^{-6} C, are placed at a separation of 1.0 cm.

(a) Write down the electric dipole moment of this pair.

(b) Calculate the electric field at a point on the axis of the dipole 1.0 m away from the center.

(c) Calculate the electric field at a point on the perpendicular bisector of the dipole and 1.0 m sway from the center.

Answer

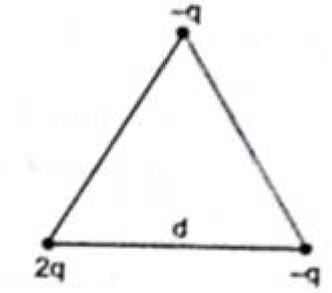
Given:(a)Charge on particle A : $q_1 = 2.0 \times 10^{-6}$ CCharge on particle B : $q_2 = -2.0 \times 10^{-6}$ CMagnitude of both the charges : q = 2.0×10^{-6} CSeparation between A and B : d = 1.0 cm = 0.01 m(b)Distance between center and the point on the axis of dipole: r= 1 cm= 0.01 m(c)Distance between center and the point on the perpendicular bisector of the dipole:r'= 1 m**Formula used:**(a)Electric dipole moment is given as: $\vec{\boldsymbol{p}} = \boldsymbol{q}\vec{\boldsymbol{d}}$Where, $\vec{p}$ is the electric dipole moment, q is the magnitude of the charges at the end of the dipole and $\vec{d}$ is the vector joining the two charges.$\therefore p = qd$

$\therefore p = 2.0 \times 10^{-6} \times 0.01 \therefore \mathbf{p = 2 \times 10^{-8}\ Cm}$ Hence, electric dipole moment between A and B is 2×10^{-8}Cm(b)Electric field at a point on the axis of the dipole is: $\boldsymbol{E} = \frac{\boldsymbol{k2p}}{\boldsymbol{r^3}}$Here k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9×10^9 Nm^2C^{-2}, p is the Electric dipole moment and r is the distance between the point on the axis and it's center.Substituting we get,$E = \frac{9 \times 10^9 \times 2 \times 2 \times 10^{-8}}{(0.01)^3} \therefore \boldsymbol{E} = \mathbf{3.6} \times \mathbf{10^8}\ \boldsymbol{NC^{-1}}$

Hence, electric field at a distance 1 cm away from the center of the dipole to the point on it's axis is 3.6×10^8 NC^{-1}.(c)Electric field at a point on the perpendicular bisector of the dipole is given as:$\boldsymbol{E} = \frac{\boldsymbol{kp}}{\boldsymbol{r'^3}}$Here, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9×10^9 Nm^2C^{-2}, p is the Electric dipole moment and r' is the distance between the point on the perpendicular bisector of the dipole and it's centerSubstituting we get: $E = \frac{9 \times 10^9 \times 2 \times 10^{-8}}{1^3}$ $\therefore \boldsymbol{E = 180\ NC^{-1}}$Hence, electric field at a point on the perpendicular bisector of the dipole 1 m away from it's center is 180 NC^{-1}.

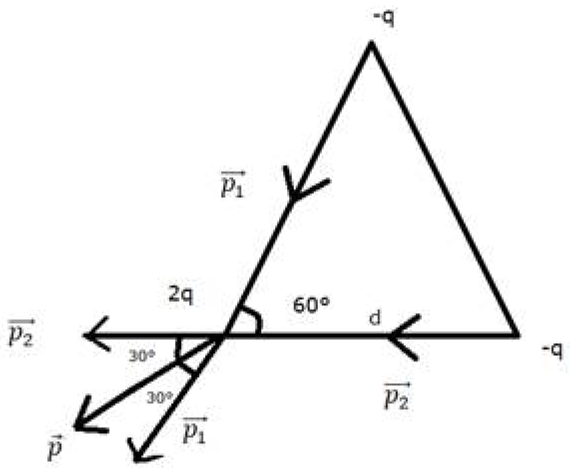
72. Question

Three charges are arranged on the vertices of n equilateral triangle as shown in figure. Find the dipole moment of the combination.



Answer

Given: Distance between two charges :d

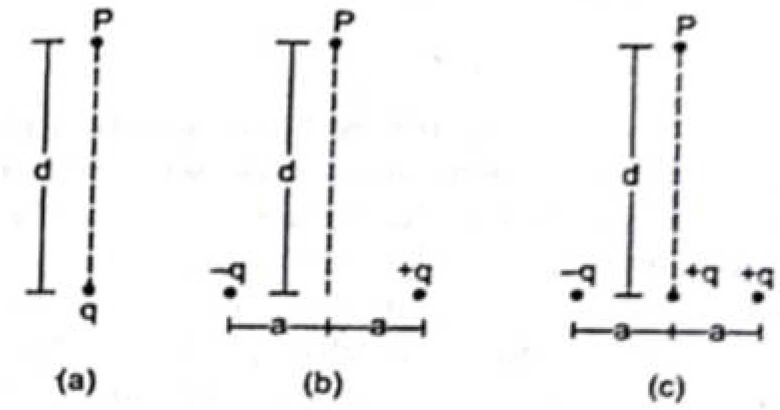


In the diagram, the dipole moments $\overrightarrow{p_1}$ and $\overrightarrow{p_2}$ are extended and resolved into the resultant component $\vec{p}$.**Formula used:**The resultant $\vec{p}$ is given as vector sum of $\overrightarrow{p_1}$ and $\overrightarrow{p_2}$ $\therefore$ $\vec{p} = \overrightarrow{p_1} + \overrightarrow{p_2}$ As angle between $\vec{p}$ and $\vec{p}_1$ is 30° , the value of the resultant:$p = 2p_1 cos30$ As $p_1=p_2$, the resultant due to $\vec{p}_1$ and $\overrightarrow{p_2}$ is : $p = 2p_1 cos(30)$We know that:p=qdWhere p is the dipole moment, q is the magnitude of charges and d is the distance between the charges forming the dipole.Substituting, the resultant dipole moment is:$p = \frac{2qd\sqrt{3}}{2}$ $\therefore \boldsymbol{p = qd\sqrt{3}}$

Hence, dipole moment of the combination is qd√3

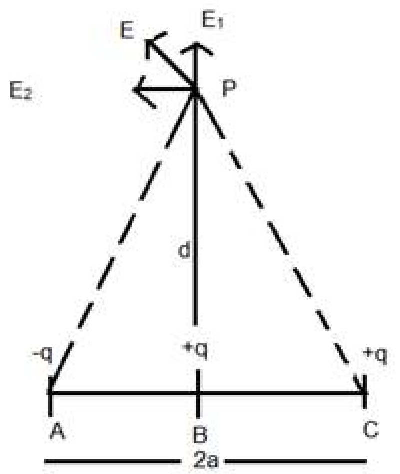
73. Question

Find the magnitude of the electric field at the point P in the configuration shown in figure for d >> a. Take 2qa = p.



Answer

Given:(a)Distance between charge and P : d(b)Distance between two charges +q and -q = 2aDipole moment : p =2qa**Formula used:**(a)It's a direct formula of the electric field due to a point charge:$\boldsymbol{E_1 = \frac{kq}{d^2}}$Here, k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} . q is the point charge and d is the distance between the charge and the point P(b)For the second configuration too we have a direct formula.Opposite charges at both ends separated by a distance forms a dipole and electric field at a point on the perpendicular bisector of the dipole is given as:$\boldsymbol{E_2 = \frac{kp}{d^3}}$Here, p is the dipole moment and is : p= 2qa and k is a constant and k= $\frac{1}{4\pi\epsilon_0}$ =9× 10^9 Nm^2C^{-2} , d is the distance between Point P and the center of the dipole.(c)Figure (c) is made up of combination of system in (a) and (b)



Point P is influenced by two dipoles.1.-q and +q (A and B)2. +q and +q (B and C)1. Is the result of (b) = $E_2$2. Is the result of (a) = E_1Hence net electric field at P in the configuration (c) would be resultant of E_1 and $E_2$$\therefore \vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$\therefore E_{net} = \sqrt{E_1^2 + E_2^2}$

$$\therefore E_{net} = \sqrt{\left(\frac{kq}{d^2}\right)^2 + \left(\frac{kp}{d^3}\right)^2} \therefore E_{net} = k\sqrt{\frac{q^2}{d^4} + \frac{p^2}{d^6}} \therefore E_{net} = k\sqrt{\frac{1}{d^6}(q^2d^2 + p^2)}$$

$$\boldsymbol{\therefore E_{net} = \frac{k}{d^3}\sqrt{(q^2d^2 + p^2)}}$$

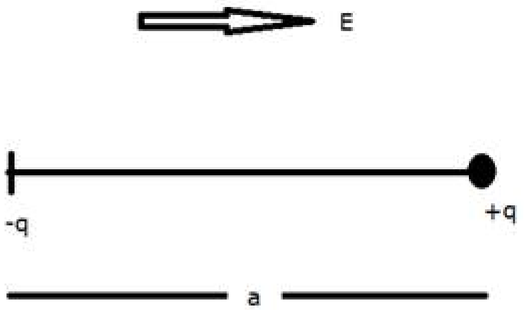
Hence, results for (a), (b) and (c) are $\boldsymbol{\frac{kq}{d^2}}$, $\boldsymbol{\frac{kp}{d^3}}$, and $\boldsymbol{\frac{k}{d^3}\sqrt{(q^2d^2 + p^2)}}$ respectively.

74. Question

Two particles, carrying –q and +q and having equal masses m each, are fixed at the ends of a light rod of length a to form a dipole. The rod is clamped at an end and is placed in a uniform electric field E with the axis of the dipole along the electric field. The rod is slightly tilted and then released. Neglecting gravity find the time period of small oscillations.

Answer

Given: Magnitude of charge on both the particles: qMass of both the particles: mElectric field : EThe axis of the dipole is along the electric field.Length of the dipole: l=a



Formula used: We know that, a charge placed in an electric field E will experience electric force.When a dipole with charge of magnitude q at its ends is suspended under a magnetic field and having one end clamped (-q), the free end (+q) will experience an electric force and it will start oscillating.Electric force is given as:F=qEandF = ma ∴ qE = ma $\therefore a = \frac{qE}{m}$Where a is the acceleration of the particle, q is the charge on the particle, m is the mass of the particle and E is the electric field.Also, time period for Simple pendulum is:$T = 2\pi\sqrt{\frac{l}{g}}$Here, g=a(acceleration) as we are neglecting gravity.Substituting we get,$T = 2\pi\sqrt{\frac{a}{\frac{qE}{m}}}$$\therefore \boldsymbol{T = 2\pi\sqrt{\frac{am}{qE}}}$Hence, time period of the small oscillations is $2\pi\sqrt{\frac{am}{qE}}$.

75. Question

Assume that each atom in a copper wire contributes one free electron. Estimate the number of free electrons in a copper wire having a mass of 6.4 g (take the atomic weight of copper to be 64 g mol^{-1}).

Answer

Given: Atomic weight of copper = 64 g mol^{-1}Mass of copper wire : m = 6.4 g**Formula used:**We know that,Number of moles in 64 g of copper = 1Thus, number of moles in 6.4 g of copper = 0.1Also,Number of atoms in one mole=N_AHere, N_A is the Avogadro Number : N_A = 6.023× 10^{23} atoms∴No.of atoms in 0.1 mole of copper=0.1× 6.023× 10^{23}∴No.of atoms in 0.1 mole of copper=0.1× 6.023× 10^{22}It is given that each atom contributes one free electron,Therefore in 0.1 mole of copper 6.023×10^{22} atoms will contribute 6.023×10^{22} free electrons.Hence, there will be 6.023×10^{22} free electron in 6.4 g of copper.