13. Circle and Tangent

Exercise 13.1

1. Question

Write true or false of each of the following statements. Also give reason for your answer.

(i) A tangent of a circle is that line which intersects the circle at two points.

(ii) A tangent XY touches a circle at a point P and Q is any other point on the tangent. If O is the centre of the circle then, OP = OQ.

(iii) Two tangent LM and XY are drawn respectively at two points P and Q on a circle. If PQ is the diameter than LM || XY.

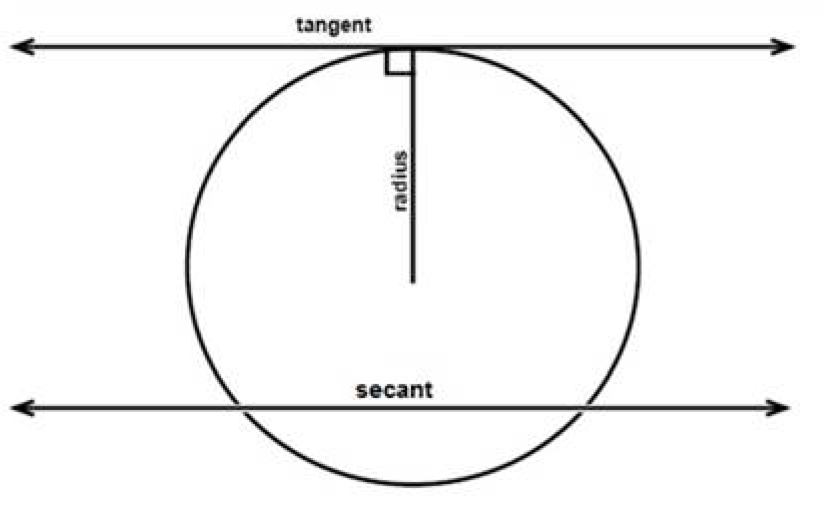
(iv) The centre 'O' of a circle lies on another circle whose centre is 'A'. If the circle with centre O passed through the points A and B such that AOB is a straight line, then the tangents drawn from the point B will pass through the points of intersection of the two points.

Answer

(i) The given statement is false. A tangent is a line which intersect the circle at exactly one point and the point of contact of tangent and circle is called as point of tangency.

The correct statement is as follows:

A secant of a circle is that line which intersects the circle at two points.

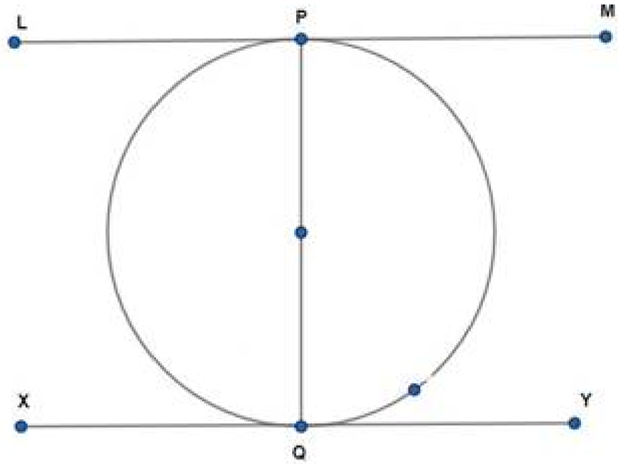


(ii) The given statement is false. Since O is the centre of the circle and hence OP is perpendicular to the tangent and the perpendicular is the smallest of all distances.

Therefore OP ≠ OQ.

A tangent XY touches a circle at a point P and Q is any other point on the tangent. If O is the centre of the circle then, OP < OQ.

(iii) The given statement is true. Since the tangent is perpendicular to the diameter, the two tangents LM and XY has to be parallel.



(iv) The given statement is true. Since AOB is a diameter, a semicircle is formed and the angle in a semicircle is always 90° or right angle.

2. Question

Fill in the blanks:

(i) Through a point on a circle…………………… tangent lines can be drawn.

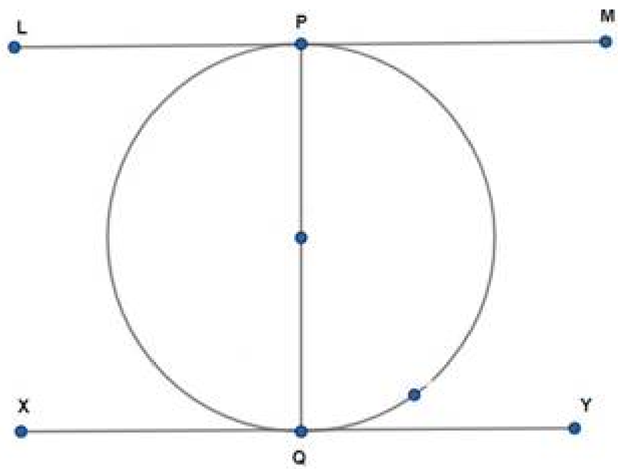
(ii) A line intersecting a circle in two points is called a ……………… .

(iii) A circle can have ………………… Parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called…………….. .

Answer

(i) Through a point on a circle <u>one</u> tangent lines can be drawn.



Let the centre be O.

From this figure it can be interpreted that OP will be the radius of the circle.

For the line LM which contains the point P, the distance OP<OL because OP is the perpendicular distance and perpendicular distance is always the shortest distance.

OP is perpendicular to LP and MP.

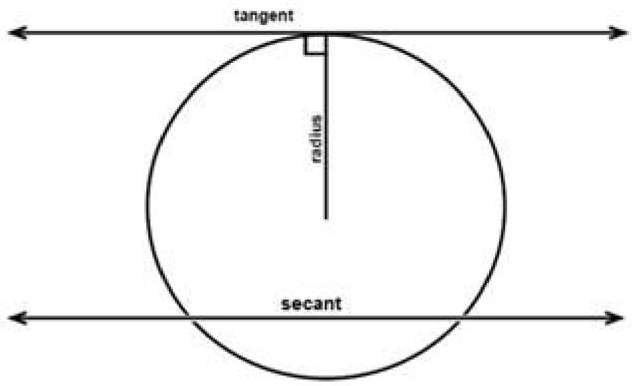
This is possible only when all the L, P and M lie on same line.

Hence LPM is a straight line.

LM is the only tangent which pass through the point P.

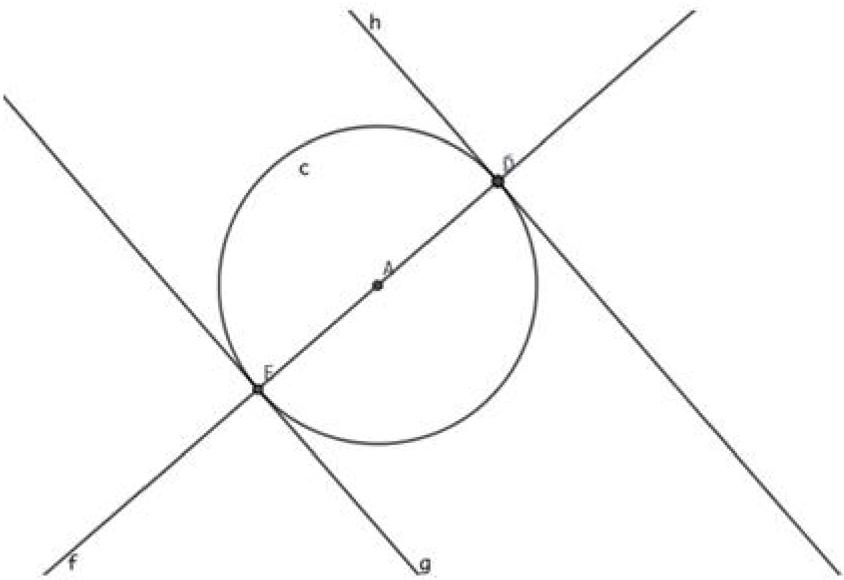
Hence only one tangent can be drawn from a point on a circle.

(ii) A line intersecting a circle in two points is called a <u>secant</u>.

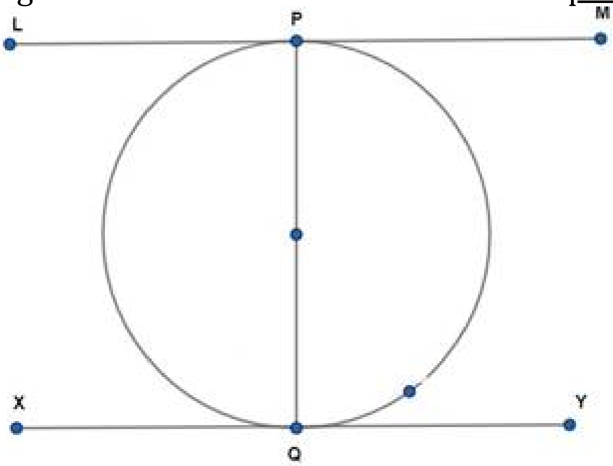


(iii) A circle can have <u>two</u> Parallel tangents at the most.

A tangent line touches a shape at a single point. Two parallel tangents will be on opposite sides of the circle. If you try to add another parallel line, it will pass through two points of the circle, and isn't a tangent.



(iv) The common point of a tangent to a circle and the circle is called point of contact or point of tangency.



In the above figure P is the point of contact for the tangent LM and Q is the point of contact for the tangent XY.

3. Question

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer

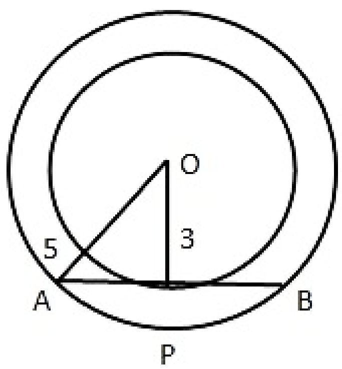
Given:

Radius of larger circle = 5 cm.

Radius of smaller circle = 3 cm.

Solution:

The diagram for the question is as follows:



AB is a tangent for smaller circle, and we know that tangent at any point is perpendicular to the radius through point of contact.

Hence, OP $\perp$ AB

Also, AB is a chord for large circle, and we know perpendicular from center to the chord bisects the chord

Hence,

$AP = BP = \frac{AB}{2}$

Now, In ΔOAP

By Pythagoras theorem,

$OA^2 = AP^2 + OP^2$

$AP^2 = OA^2 - OP^2$

Here, OA = radius of bigger circle = 5 cm

and OB = radius of smaller circle = 3 cm

$AP^2 = 5^2 - 3^2$

$AP^2 = 25 - 9$

$AP^2 = 16$

∴ AP = 4 cm

Hence,

AB = 2 AP

= 2(4) = 8 cm

4. Question

The length of a tangent from a point at a distance of 10 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer

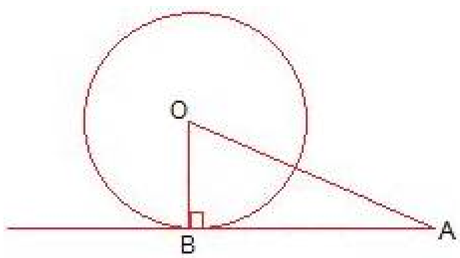
Given:

Distance of point from center, OA = 10 cm

Length of tangent, AB = 4 cm.

Solution:

The diagram for the question is as follows:



Let the centre be O.

From this figure it can be interpreted that OB will be the radius of the circle.

Radius OB is perpendicular to the tangent AB as radius is perpendicular to the tangent.

$OB \perp AB$

Hence Δ OAB is right angled triangle.

$\angle OBA = 90°$

By Pythagoras theorem,

$OA^2 = AB^2 + OB^2$

$OB^2 = OA^2 - AB^2$

$OB^2 = 10^2 - 4^2$

$OB^2 = 100 - 16$

$OB^2 = 84$

$OB^2 = 21 \times 2 \times 2$

$OB = 2\sqrt{21}$ cm

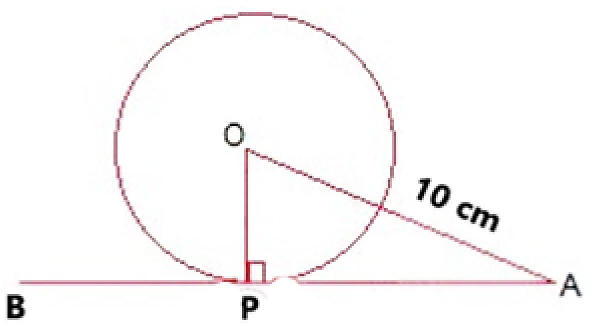
Therefore the radius of circle $OB = 2\sqrt{21}$ cm

5. Question

A circle with centre O touches all the four sides of a quadrilateral ABCD. If the point of contact divides AB in the ratio 3 : 1 and AB = 8 cm, then find the radius of the circle if it is given OA = 10 cm.

Answer

Let the diagram be as follows:



P divides the line AB in the ratio 3:1

$$\Rightarrow \frac{AP}{BP} = \frac{3}{1}$$

Adding 1 both side, we get

$$\frac{AP}{BP} + 1 = \frac{3}{1} + 1 \Rightarrow \frac{AP + BP}{BP} = 4 \Rightarrow \frac{AB}{BP} = 4 \Rightarrow \frac{8}{BP} = 4$$

⇒ BP = 2 cm

And hence,

AP = 3BP = 3(2) = 6 cm

Let the centre be O.

From this figure it can be interpreted that OP will be the radius of the circle.

Radius OP is perpendicular to the tangent AB as radius is perpendicular to the tangent.

If OP is ⊥ to AB which implies that OP is ⊥ AP and OP is ⊥ PB.

Hence Δ OPA is right angled triangle

∠ OPA = 90°

$OA^2 = AP^2 + OP^2$

$OP^2 = OA^2 - AP^2$

$OP^2 = 10^2 - 6^2$

$OP^2 = 100 - 36$

$OP^2 = 64$

∴ OP = 8 cm

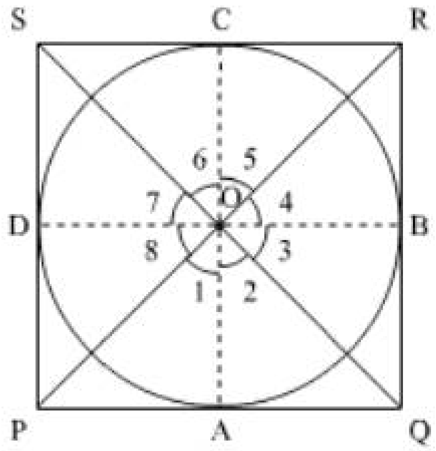
Therefore the radius of circle = 8 cm.

6. Question

A circle touches the sides of a quadrilateral. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

Answer

The diagram is as follows:



Let PQRS be the quadrilateral which has a circle inside it with center O.

Now join AO, BO, CO, DO

From the figure, $\angle 1 = \angle 8$ [Two tangents drawn from an external point to a circle subtend equal angles at the centre.]

We can also state that $\angle 2 = \angle 3$, $\angle 4 = \angle 5$ and $\angle 6 = \angle 7$

Sum of angles in a quadrilateral is 360°

Sum of angles at the center is 360°

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$2(1 + 2 + 5 + 6) = 360°$

$1 + 2 + 5 + 6 = 180°$

or

$2(8 + 3 + 4 + 7) = 360°$

$8 + 3 + 4 + 7 = 180°$

$\angle POQ + \angle ROS = 180°$

Also $\angle QOR + \angle POS = 180°$

[Since $\angle POQ = \angle 1 + \angle 2$

$\angle ROS = \angle 5 + \angle 6$

$\angle QOR = \angle 7 + \angle 8$

$\angle POS = \angle 3 + \angle 4$

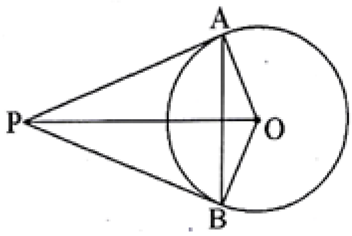
as shown in the figure]

Hence it is proved that PQ and RS subtend supplementary angles at center.

Thus the angles subtended at the centre by a pair of opposite sides are supplementary.

7. Question

In the figure, O is the centre of a circle. Tangents PA and PB drawn from an external point P touch the circle at points A and B respectively. Prove that OP is perpendicular bisector of AB.



Answer

We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\angle OAP = \angle OBP = 90°$

Now, $\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$ [Angle sum property of quadrilaterals]

$\Rightarrow 90° + \angle APB + 90° + \angle AOB = 360°$

$\Rightarrow \angle AOB = 360° - 180° - \angle APB = 180° - \angle APB$.... (1)

Now, in Δ OAB, OA is equal to OB as both are radii.

$\Rightarrow \angle OAB = \angle OBA$ [In a triangle, angles opposite to equal sides are equal]

Now, on applying angle sum property of triangles in Δ AOB,

We obtain $\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow 2\angle OAB + \angle AOB = 180°$

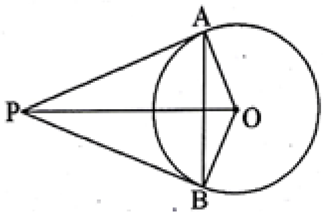
$\Rightarrow 2 \angle OAB + (180° - \angle APB) = 180°$ [Using (1)]

$\Rightarrow 2 \angle OAB = \angle APB$

Thus, the given result is proved

8. Question

In the figure, O is the centre of the circle. Tangents PA and PB drawn from an external point P touch the circle at A and B respectively. Prove that PAOB is a cyclic quadrilateral.



Answer

Since tangents to a circle is perpendicular to the radius.

∴ OA ⊥ AP and OB ⊥ BP.

⟹ ∠OAP = 90° and ∠OBP = 90°

⟹ ∠OAP + ∠OBP = 90° + 90° = 180°........... (1)

In quadrilateral OAPB,

∠OAP + ∠APB + ∠AOB + ∠OBP = 360°

⟹ (∠APB + ∠AOB) + (∠OAP + ∠OBP) = 360°

⟹ ∠APB + ∠AOB + 180° = 360° [From (1)]

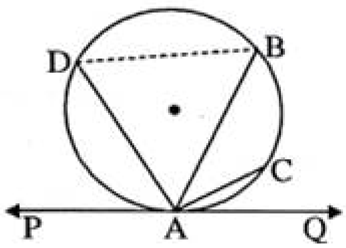
⟹ ∠APB + ∠AOB = 180°................ (2)

Thus from equations (1) and (2) it can be concluded that the quadrilateral PAOB is a cyclic quadrilateral.

Exercise 13.2

1. Question

Answer the following questions from the figure.



(i) Alternate segment of ∠BAQ.

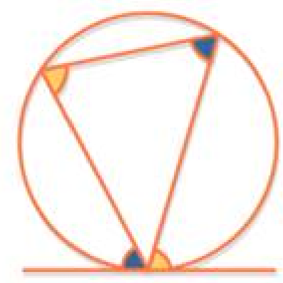
(ii) Alternate segment of ∠DAP.

(iii) If C is jointed to B, then ∠ACB is equal to which angles?

(iv) ∠ABD and ∠ADB are equal to which angles?

Answer

(i) To understand alternate segment following diagram is beneficial:



Angles with same color are equal.

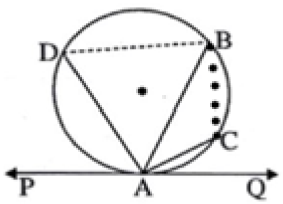
Theorem:

The angle between the tangent and chord at the point of contact is equal to the angle in the alternate segment.

The alternate segment for ∠ BAQ is ∠ ADB.

(ii) The alternate segment for ∠ DAP is ∠ ABD

(iii) The figure is as follows:

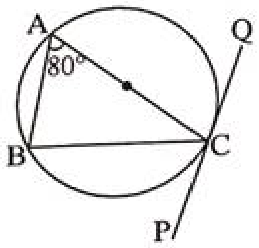


When C will be jointed to B then ∠ ACB will be equal to ∠ BAP.

(iv) ∠ABD and ∠ADB are equal to ∠ DAP and ∠ BAQ because angles in alternate segments are equal.

2. Question

In the figure, if ∠BAC = 80° then find the value of ∠BCP.



Answer

AC is the diameter

∠ ACP = 90° [radius is perpendicular to tangent]

∠ ABC = 90° [angle in a semicircle is 90°]

∠ ABC + ∠ BAC + ∠ ACB = 180° [sum of internal angles in a triangle is 180°]

∠ ACB + 90° + 80° = 180°

∠ ACB = 180 – 170

∠ ACB = 10°

∠ BCP = ∠ ACP - ∠ ACB

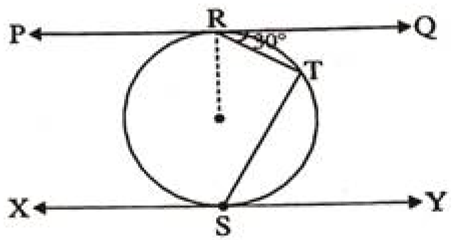
= 90 – 10

= 80°

∴ ∠ BCP = 80°

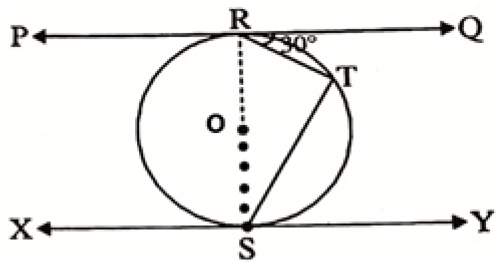
3. Question

In the figure, PQ and XY are parallel tangents. If ∠QRT = 30°, then find the value of ∠TSY.



Answer

In the given figure,



Let O be the centre of the circle

Join OS

Let O be the centre of the circle

So ∠ SRQ = 90° [radius is perpendicular to tangent]

So ∠ SRT = 90 – 30

= 60°

∠ STR = 90° [angle in a semicircle is 90°]

∠ RSY = 90° [radius is perpendicular to tangent]

∠ RST + ∠ STR + ∠ SRT = 180° [sum of internal angles in a triangle is 180°]

$\angle$ RST + 90 + 60 = 180

$\angle$ RST = 180 – 150

$\angle$ RST = 30°

$\angle$ TSY = $\angle$ RSY - $\angle$ RST

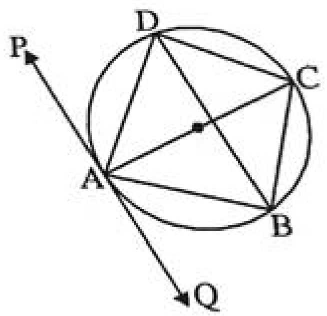
= 90 – 30

= 60°

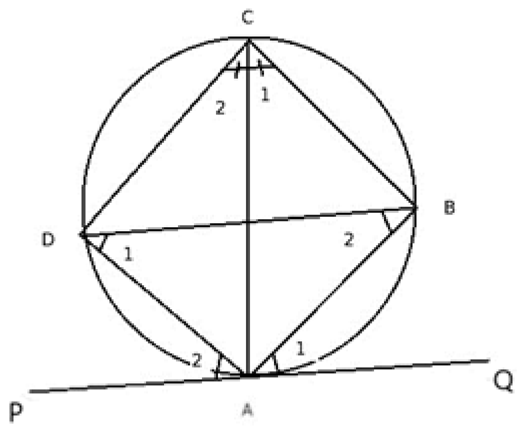
$\therefore$ $\angle$ TSY = 60°

4. Question

In the figure, the diagonal AC or cyclic quadrilateral ABCD bisects ∠C. Prove that the diagonal BD is parallel to the tangent at point A of the circle passing through A, B, C and D.



Answer



$\angle$ ACD = $\angle$ ACB …………. (1) [In the question it is given that AC bisects the angle C which means that the two angles $\angle$ 1 and ∠2 has to be equal]

Here we can see that $\angle$ PAD = $\angle$ ABD [Angle in the alternate segment]

Similarly, $\angle$ QAB = $\angle$ ADB

Also AB is common arc and ADB and ACB are the angle in the same segment, so

$\angle$ ABD = $\angle$ ACD

But by the equation (1) we can say that

$\angle 1 = \angle 2$

So $\angle PAD = \angle ADB$ [Alternate Interior angle]

So $PQ \parallel BD$