Statistics

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in Seconds)	0–20	20–40	40–60	60–80	80–100
No. of students	8	10	13	6	3

(A) The mean time taken by a student to finish the race is:

(a) 54

(b) 63

(c) 43

(d) 50

(B) The construction of cumulative frequency table is useful in determining:

(a) mean

(b) median

(c) mode

(d) all of the above

(C) The lower limit of the median class is:

(a) 20

(b) 40

(c) 60

(d) 80

(D) The algebraic sum of absolute values of deviations from the mean is:

(a) 0

(b) 788

(c) 1720

(d) none of these

(E) The value of mean deviation from mean is:

(a) 19.7

(b) 20.7

(c) 18.7

(d) 17.7

Ans. Calculation of Mean, Mean deviation and cumulative

C.I.	Mid-Valcue x_i	Fre-quency fi	$f_i x_i$	$\lvert x_i - \bar{x} \rvert =$	$\Sigma f_i \lvert x_i - \bar{x} \rvert$
0–20	10	8	80	33	264
20–40	30	10	300	13	130
40–60	50	13	650	7	91
60–80	70	6	420	27	162
80–100	90	3	270	47	141
		N = 40	$\Sigma f_i x_i =$ 1720		$\Sigma f_i \lvert x_i - \underline{x} \rvert =$ 788

$\therefore$ Mean is given by $\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{1720}{40} = 43$

And Mean deviation from mean

$$= \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{N} = \frac{788}{40} = 19.7$$

And $\frac{N}{2} = \frac{40}{2} = 20,$ which lies in the class 40-60

$\therefore$ Median class is 40 – 60.

(A) (c) 43

Explanation: mean time = 43

(B) (b) Median

Explanation: The construction of the cumulative frequency table is useful in determining the median.

(C) (b) 40

Explanation: The median class is 40-60. So, the lower limit = 40.

(D) (b) 788

Explanation: The absolute value of algebraic sum of deviation from the mean is 788.

(E) (a) 19.7

Explanation: From the calculation above, mean deviation from mean is 19.7.

2. A seminar on prime numbers was organised in a college to motivate students.

(A) Find the mean deviation of the first three primes from their median.

(B) Find the median of first three primes and find the algebraic sum of deviations of first three prime numbers from the mean.

(C) Find the mean deviation from the mean for the following data

6.5, 5, 5.25, 5.5, 4.75, 4.5, 6.25, 7.75, 8.5

Ans. (A) The first three prime numbers in ascending order are 2, 3 and 5.

Median of these three prime numbers

= Value of 2nd item = 3.

The sum of absolute values of deviations from the median

= 2-3+3-3+5-3=1+0+2=3

Mean deviations of first three prime

numbers from median = $\frac{3}{3} = 1$

(B) Median is 3.

He algebraic sum of deviations from the mean is always zero.

(C) Given observations are

6.5, 5, 5.25, 5.5, 4.75, 4.5, 6.25, 7.75, 8.5

Here number of observation n = 9

Let x be the mean of given data.

Then,

$$\bar{x} = \frac{6.5+5+5.25+5.5+4.75+4.5+6.25+7.75+8.5}{9}$$

$$= \frac{54}{9} = 6$$

Let us make the table for deviation and absolute deviation.

x_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$
6.5	0.5	0.50
5.0	−1	1.00
5.25	−0.75	0.75
5.5	−0.5	0.50
4.75	−1.25	1.25
4.5	−1.50	1.50
6.25	0.25	0.25
7.75	1.75	1.75
8.5	2.5	2.50
Total		$\sum_{i=1}^{9} \lvert x_i - \bar{x} \rvert = 10.00$

∴ Mean deviation about mean,

$$MD(\bar{x}) = \frac{\sum_{i=1}^{6} \lvert x_i - \bar{x} \rvert}{9} = \frac{10}{9} = 1.1$$

Hence, the mean deviation about mean is 1.1.

3. For a group of 200 candidates, the mean and the standard deviation of scores were found to be 40 and 15, respectively. Later on, it was discovered that the scores of 43 and 35 were misread as 34 and 53, respectively:

(A) The sum of correct scores is:

(a) 7991

(b) 8000

(c) 8550

(d) 6572

(B) The correct mean is:

(a) 42.924

(b) 39.955

(c) 38.423

(d) 41.621

(C) The correct variance is:

(a) 280.3

(b) 235.6

(c) 224.143

(d) 226.521

(D) The correct standard deviation is:

(a) 14.971

(b) 11.321

(c) 16.441

(d) 12.824

(E) Assertion (A): The variance of first n even natural numbers is $\frac{n^2-1}{4}$.

Reason (R): The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural number is $\frac{n(n+1)(2n+1)}{6}$.

Ans. (A) (a) 7991

Explanation: We have, n = 200, incorrect mean = 40 and incorrect deviation = 15

Now, incorrect mean = 40

$$\Rightarrow \quad \frac{\text{Incorrect} \sum x_i}{200} = 40$$

Incorrect $\sum x_i = 8000$

$$\Rightarrow \text{correct} \sum x_i = 8000 - (34 + 53) + (43 + 35)$$
$$= 8000 - 87 + 78 = 7991$$

(B) (b) 39.955

Explanation: Correct mean = $\frac{7991}{200}$

$= 39.955$

(C) (c) 224.143

Explanation: Incorrect SD = 15

$\Rightarrow$ incorrect variance = $(15)^2 = 225$

$$\Rightarrow \quad \frac{\text{Incorrect} \sum x_i^2}{200} - (\text{incorrect mean})^2$$

$$= 225$$

$$\Rightarrow \quad \frac{\text{Incorrect} \sum x_i^2}{200} - (40)^2 = 225$$

$$\Rightarrow \quad \text{Incorrect} \sum x_i^2 = 200(1600 + 225)$$

$$= 200 \times 1825 = 365000$$

$$\Rightarrow \quad \text{correct} \sum x_i^2 = \text{Incorrect } x_i^2 - (34^2 + 53^2)$$

$$+ (43^2 + 35^2)$$

$$= 365000 - 3965 + 3074$$

$$= 364109$$

So, correct variance

$$= \frac{1}{200}(\text{correct} \sum x_i^2) - (\text{correct mean})^2$$

$$= \frac{1}{200}(364109) - \left(\frac{7991}{200}\right)^2$$

$$= 1820.545 - 1596.402$$

$$= 224.143$$

(D) (a) 14.971

Explanation: Correct standard deviation

$$= \sqrt{\textit{correct variance}}$$

$$= \sqrt{224.143} \qquad \text{(using part (C))}$$

$$= 14.971$$

(E) Assertion: Sum of n even natural numbers

$= n(n+1)$

$$\text{Mean } (\bar{x}) = \frac{n(n+1)}{n} = n+1$$

$$\text{Variance} = \left[\frac{1}{n}\Sigma(x_i)^2\right] - (\bar{x})^2$$

$$= \frac{1}{n}[2^2 + 4^2 + ... + (2n)^2] - (n+1)^2$$

$$= \frac{1}{n}2^2[1^2 + 2^2 + ... + n^2] - (n+1)^2$$

$$= \frac{4}{n}\frac{n(n+1)(2n+1)}{6} - (n+1)^2$$

$$= \frac{(n+1)[2(2n+1) - 3(n+1)]}{3}$$

$$= \frac{(n+1)(n-1)}{3}$$

$$= \frac{n^2 - 1}{3}$$

4. The following values are calculated in respect of length and mass of the students of class XII

	Length	Mass
Mean	164cm	64kg
Variance	125cm^2	25kg^2

(A) Find the coefficient of variation of length.

(B) Find the coefficient of variation of mass and standard deviation of length.

(C) Find the variance and standard deviation
for the following data 6, 7, 10, 12, 13, 4, 8,12.

Ans. (A) S.D. of length = √25 = 11.18

The coefficient of variation of length

$$= \frac{S.D.}{Mean} \times 100$$

$$= \frac{11.18}{164} \times 100$$

$= 6.81$

(B) S.D. of mass = √25 = 5

$$C.V. \text{ of mass} = \frac{S.D}{Mean} \times 100$$

$$= \frac{5}{64} \times 100$$

$$= 7.81$$

$$S.D. = \sqrt{\text{variance}} = \sqrt{125} = 11.18$$

(C) Given observations are 6, 7, 10, 12, 13, 4,8,12

Number of observations = 8

$$\therefore \text{Mean}(\bar{x}) = \frac{6+7+10+12+13+4+8+12}{8}$$

$$= \frac{72}{8} = 9$$

Now, let us make the following table for deviation.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	−3	9	13	4	16
7	−2	4	4	−5	25
10	1	1	8	−1	1
12	3	9	12	3	9
	Total	74		Total	74

$\therefore$ Sum of squares of deviations

$$= \sum_{i=1}^{8}(x_i - \bar{x})^2 = 74$$

Hence, variance, $\sigma^2 = \dfrac{\sum_{i=1}^{8}(x_i - \bar{x})^2}{n} = \dfrac{74}{8} = 9.25$

and standard deviation $= \sqrt{\sigma} = \sqrt{9.25} = 3.04$