Chapter 1

Rational numbers

1.1 Amar, Arun and Aamir use to go for marketing in a team for their household groceries. The benefit of marketing in team is that they can buy their items in whole sale price. After marketing they share their parts of the total expenditure. Once, they went to market and bought three gourds for Rs. 65, three pairs of eggs for Rs 60 and a dozen of banana for Rs. 55. After calculation, they found that each had to share Rs $\frac{65}{3}$ for gourd, Rs. 20 for eggs and Rs. $\frac{55}{3}$ for bananas.

Let us take another example –

The lowest temperatures in Srinagar recorded in a week were as follows –

– 7°C, – 4°C, – 1°C, – 5°C, – 8°C, – 10°C, – 6°C

Therefore the average temperature of the week is

$$\frac{(-7^0C)+(-4^0C)+(-1^0C)+(-5^0C)+(-8^0C)+(-10^0C)+(-6^0C)}{7}$$

$$= \frac{-(7^0C+4^0C+1^0C+5^0C+8^0C+10^0C+6^0C)}{7}$$

$$= \frac{-41^0C}{7}$$

Now, the question that arises in your mind is- what type of numbers are $\frac{65}{3}, \frac{55}{3}, \frac{-41}{7}$?

Are they natural numbers, whole numbers, integers or some other numbers?

Let us discuss numbers of this type. Out of the above numbers 65, 55 ,–41, 3, 7 are integers. If we represent two integers as *p* and *q*, then each of $\frac{65}{3}, \frac{55}{3}, \frac{-41}{7}$ is in the form $\frac{p}{q}$. In $\frac{50}{3}$, $p = 50, q = 3$, In $\frac{-41}{7}$, $p = -41, q = 7$. etc. Here it may be noted that *p* and *q* are integers, but *q* cannot be zero because division by zero is not defined.

The numbers which can be written in the form $\frac{p}{q}$ where *p* and *q* are integers, but *q* $\neq$ 0 are called rational numbers. For example

$$\frac{3}{4}, \frac{11}{-8}, \frac{-19}{6}, \frac{235}{1106}, \frac{-51}{193} \text{ etc.}$$

Activity Each of you write 5 rational numbers and show them to your teachers.

Let us know All integers can be written in the form $\frac{p}{q}$, $(q \neq 0)$ like -

$$6 = \frac{12}{2} \text{ or } \frac{30}{5} \text{ or } \frac{72}{12}$$

$$-8 = \frac{-24}{3} \text{ or } \frac{-48}{6} \text{ or } \frac{-56}{7}$$

$$0 = \frac{0}{17} \text{ or } \frac{0}{21} \text{ etc.}$$

Therefore **all integers are rational numbers.**

Discuss in group : Cite counter examples if needed.

(i) Are all rational numbers integers?
(ii) Are all natural numbers rationals?
(iii) Are all rational numbers natural?
(iv) Are the whole numbers rationals?

You already have come across that decimal numbers can be written as fractions. For example,

$$0.7 = \frac{7}{10}, \quad 0.93 = \frac{93}{100}, \quad -2.367 = \frac{-2367}{1000} \text{ etc.}$$

i.e. the numbers with finite decimal places can be expressed in the form $\frac{p}{q}$. Moreover the numbers, which are expressed in infinite but recurring decimals, can be expressed in the form $\frac{p}{q}$.

For example–

$0.33333............... = \frac{1}{3}$

$0.142857142857.............. = \frac{1}{7}$ etc.

Thus the numbers expressed as infinite but recurring decimals are also rational. Again there are some numbers which can be expressed in infinite but non recurring or non repeating decimals. These numbers cannot be expressed in the form $\frac{p}{q}$. Therefore these are not rational numbers. For example 0.319782714......., 3.14976108....., 0.101100111000.... etc.

Positive and Negative rational numbers :

If p and q are both positive or both negative, then the rational number $\frac{p}{q}$ $(q \neq 0)$ is called **positive rational number**. For example– $\frac{8}{9}, \frac{7}{12}, \frac{-9}{-11}$ etc.

We note here, $\frac{-9}{-11} = \frac{(-1)\times(-9)}{(-1)\times(-11)} = \frac{9}{11}$

Again if any one of p and q is positive and the other is negative, then the rational number $\frac{p}{q}$ $(q \neq 0)$ is called a **negative rational number**. For example – $\frac{-7}{8}, \frac{8}{-9}, \frac{-11}{236}$ etc.

We must note here that the **denominator of a rational number is conventionally taken positively.** for example–

$$\frac{8}{-9} = \frac{(-1)\times 8}{(-1)\times(-9)} = \frac{-8}{9}$$

$$\frac{13}{-219} = \frac{(-1)\times 13}{(-1)\times(-219)} = \frac{-13}{219} \text{ etc.}$$

1.2 Properties of rational numbers

You are already familiar with the fundamental properties of integers. Now we discuss these properties for rational numbers.

1.2.1 Closure Property

(i) Closure Property Under Addition :

Consider two rational numbers $\frac{2}{5}$ and $\frac{-1}{5}$

Adding these two numbers $\frac{2}{5}+(\frac{-1}{5})=\frac{2+(-1)}{5}=\frac{1}{5}$, we get another rational number.

Similarly add 6 and 4. we get 6 + 4 = 10. It is a rational number.

Again adding (– 9) and (– 5) we get (– 9) + (– 5) = – 14, which is also a rational number.

Thus we find that sum of **two rational numbers is again a rational number**.

Hence, *for any two rational numbers a and b, a + b is also a rational number*.

Thus we say that **rational numbers are closed under addition. This property is called the closure property of rational numbers with respect to addition.**

Activity Verify whether the following numbers are closed under addition :

(a) $\frac{2}{3}, 6$ (b) $\frac{1}{8}, \frac{3}{4}$ (c) $-\frac{2}{7}, \frac{3}{4}$ (c) $\frac{-1}{7}, \frac{-1}{3}$

(ii) Closure Property Under Subtraction :

Let us examine through examples if closure property holds for subtraction.

Consider two rational numbers $\frac{3}{7}$ and $\frac{-2}{5}$.

Now subtracting $\frac{-2}{5}$ from $\frac{3}{7}$ we get

$\frac{3}{7}-(\frac{-2}{5})=\frac{15+14}{35}=\frac{29}{35}$ which is a rational number.

Similarly 16 – 14 = 2 is a rational number.

14 – 19 = – 5 is a rational number

Again $(-2)-\frac{5}{6}=\frac{-2\times 6-5}{6}$

$=\frac{-12-5}{6}=\frac{-17}{6}$ which is a rational number.

In each case we get a rational number.

You can try this for other rational numbers also.

∴ **Therefore, we can say that the subtraction of two rational numbers is also a rational number**.

i.e. *if a and b are rational numbers, then, $a - b$ is also a rational number*. Therefore the rational numbers are closed under subtraction.

Activity Are the following numbers closed under subtraction ?

(a) $-6, -9$ (b) $3, \frac{-11}{7}$ (c) $\frac{-1}{4}, \frac{1}{3}$ (d) $\frac{-1}{3}, \frac{-1}{2}$

(iii) Closure Property Under Multiplication :

Let us see whether the closure property holds in case of multiplication.

Consider two rational numbers $\frac{2}{7}$ and $\frac{-11}{5}$

Now $\frac{2}{7} \times \frac{-11}{5} = \frac{2 \times (-11)}{7 \times 5} = \frac{-22}{35}$, which is a rational number.

Similarly,

$(-2) \times 0 = 0$, a rational number

$11 \times 11 = 121$, is also a rational number .etc.

In each case,we see that by multiplying two rational numbers we get a rational number. Take some more pairs of rational numbers and check whether their product is again a rational number or not. So, we can say that *for any two rational numbers a and b, $a \times b$ is also a rational number*. i.e. , the **rational numbers are closed under multiplication**.

Activity Are the following numbers closed under multiplication?

(a) $-5, -11$ (b) $13, \frac{-11}{7}$ (c) $\frac{-1}{4}, \frac{4}{5}$ (d) $\frac{-14}{3}, \frac{-15}{7}$

(iv) Closure Property Under Division

$\frac{2}{6} \div \frac{1}{2} = \frac{2}{6} \times \frac{2}{1} = \frac{4}{6}$, a rational number.

$(-4) \div (-2) = 2$ is also rational number.

$\frac{0}{2} = 0$, is a rational number.

But $6 \div 0$ is meaningless (undefined).

It is seen that division of two rational numbers may not give a rational number again.

i.e *if a and b be two rational numbers, then $a \div b$ may not be a rational numbers*. Therefore **rational numbers are not closed under division**.

However, **excluding zero, rational numbers are closed under division.**

Activity Are the following numbers closed under division?

(a) 4 , 16 (b) – 49, 7 (c) $-\frac{2}{7}, \frac{3}{4}$ (d) $-\frac{1}{8}, -\frac{1}{7}$

1.2.2 Commutative Property

With the help of few examples we shall verify this property also

(i) Commutative Property Under Addition :

Adding two rational numbers $\frac{3}{5}$ and $(\frac{-2}{7})$ we have $\frac{3}{5}+(\frac{-2}{7})=\frac{21-10}{35}=\frac{11}{35}$

In the same way , $\frac{-2}{7}+\frac{3}{5}=\frac{-2\times 5}{7\times 5}+\frac{3\times 7}{5\times 7}=\frac{-10+21}{35}=\frac{11}{35}$

$\therefore \quad \frac{3}{5}+\frac{(-2)}{7}=\frac{(-2)}{7}+\frac{3}{5}$

Similarly, 3 + 2 = 5 = 2 + 3

(– 6) + (– 5) = – 11 = (– 5) + (– 6)

You can take some other rational numbers and verify the property. What do you see? It is seen that **two rational numbers can be added in any order.** i.e, **for any two rational numbers** a **and** b, $a+b=b+a$

So, **The rational numbers are commutative under addition.**

Activity Do the following numbers obey the commutative property of addition? Verify.

(a) 26, 37 (b) – 48, – 17 (c) $\frac{12}{7}$, 0

(ii) Commutative Property Under Subtraction :

Let us take examples to verify if the commutative property holds for subtraction or not.

$$\frac{2}{3}-\frac{7}{4}=\frac{8-21}{12}=\frac{-13}{12}$$

$$\frac{7}{4}-\frac{2}{3}=\frac{21-8}{12}=\frac{13}{12}$$

$$\therefore \quad \frac{2}{3}-\frac{7}{4}\neq\frac{7}{4}-\frac{2}{3}$$

Let us subtract another pair of numbers :

$$4 - 3 = 1$$
$$3 - 4 = -1$$
$$\therefore \quad 4 - 3 \neq 3 - 4$$

Thus, we see that, if we subtract one number from another in different orders we do not get the same result. Therefore, *for any two rational numbers a and b, $a - b \neq b - a$.*

That is **subtraction is not commutative in case of rational numbers**.

Activity Check the commutativity for subtraction of following numbers.

(a) – 3, – 6 (b) – 15, 15 (c) $\frac{1}{4}, \frac{3}{5}$

(iii) Commutative Property Under Multiplication :

Let us see for commutativity of multiplication with the help of the following examples.

$$2 \times 0 = 0 = 0 \times 2$$
$$3 \times (-2) = -6 = (-2) \times 3$$

Similarly, $\frac{2}{3} \times \left(\frac{-5}{7}\right) = \frac{-10}{21}$

$$\left(\frac{-5}{7}\right) \times \frac{2}{3} = \frac{-10}{21}$$

$$\therefore \frac{2}{3} \times \left(\frac{-5}{7}\right) = \left(\frac{-5}{7}\right) \times \frac{2}{3}$$

Thus , we get the same result if two rational numbers are multiplied in any order.

Therefore *for any two rational numbers a and b.* $a \times b = b \times a$

i.e, **the multiplication is commutative in case of rational numbers**.

Activity Check the commutative property of multiplication for following numbers.

(a) 4, – 7 (b) – 5, – 9 (c) $\frac{5}{3}, 1$ (d) $\frac{3}{7}, \frac{-4}{5}$

(iv) Commutative Property Under Division :

Let us verify if the commutativity holds in case of division two rational numbers .

$$\frac{2}{5} \div \frac{4}{15} = \frac{2}{5} \times \frac{15}{4} = \frac{30}{20}$$

$$\frac{4}{15} \div \frac{2}{5} = \frac{4}{15} \times \frac{5}{2} = \frac{20}{30}$$

$$\therefore \frac{2}{5} \div \frac{4}{15} \neq \frac{4}{15} \div \frac{2}{5}$$

Similarly,

$$4 \div 2 = 2, \qquad 2 \div 4 = \frac{2}{4}$$

$$4 \div 2 \neq 2 \div 4$$

In the same way $(-6) \div 2 = -3, \qquad 2 \div (-6) = \frac{2}{-6}$

$$(-6) \div 2 \neq 2 \div (-6)$$

Thus, we see that, if we divide one rational number by another rational number in different orders, we may not get the same result.

i.e, *for any two rational numbers a and b*

$$a \div b \neq b \div a$$

So, **division is not commutative in case of rational numbers.**

Activity Check the commutativity for divison for the following numbers.

(a) 3, 15 (b) – 5, 17 (c) $\frac{7}{4}$, 1

1.2.3 Associative Property

You know that Associative property holds for any three natural numbers . *i.e for any three natural number a, b and c we have, (a + b) + c = a + (b + c)*

Now let us see if associative property holds for rational numbers.

(i) Associative Property Under Addition : Let us take some rational numbers to verify associative property for addition.

$$\left\{\frac{3}{4} + \left(\frac{-2}{3}\right)\right\} + \frac{5}{6}$$

$$= \left\{\frac{9+(-8)}{12}\right\} + \frac{5}{6}$$

$$= \frac{1}{12} + \frac{5}{6}$$

Also, $\frac{3}{4} + \left\{\left(\frac{-2}{3}\right) + \frac{5}{6}\right\}$

$$= \frac{3}{4} + \left\{\left(\frac{-4+5}{6}\right)\right\}$$

$$= \frac{3}{4} + \frac{1}{6}$$

$= \frac{1+10}{12}$ | $= \frac{9+2}{12}$

$= \frac{11}{12}$ | $= \frac{11}{12}$

We have

$$\left\{\frac{3}{4}+\frac{(-2)}{3}\right\}+\frac{5}{6} = \frac{3}{4}+\left\{\frac{(-2)}{3}+\frac{5}{6}\right\}$$

In the same way,

(3 + 4) + 5 = 7 + 5 = 12
3 + (4 + 5) = 3 + 9 = 12
∴ (3 + 4) + 5 = 3 + (4 + 5)

Similarly,

{(– 3) + 2} + (– 5)
= (– 1) + (– 5)
= – 6

(– 3) + {2 + (– 5)}
= (– 3) + (– 3)
= – 6

Hence

{(– 3) + 2} + (– 5) = (– 3) + {2 + (– 5)}

∴ *If a, b, c are three rational numbers, then*

(a + b) + c = a + (b + c)

So, **the rational numbers obey associative property of addition.**

Activity Check the associative property for addition for the following numbers.

(a) 7, – 5, 6 (b) 7, $\frac{1}{5}$, 0 (c) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$

(ii) Associative Property Under Subtraction : With the help of some rational numbers, let us see, if associative property holds for subtraction or not.

$$\left(-\frac{2}{5}-\frac{7}{10}\right)-\frac{1}{2}$$

$$=\left(\frac{-4-7}{10}\right)-\frac{1}{2}$$

$$=\frac{-11}{10}-\frac{1}{2}$$

Also

$$\left(\frac{-2}{5}\right)-\left(\frac{7}{10}-\frac{1}{2}\right)$$

$$=\left(\frac{-2}{5}\right)-\left(\frac{7-5}{10}\right)$$

$$=\left(\frac{-2}{5}\right)-\frac{2}{10}$$

$= \frac{-11-5}{10}$

$= \frac{-16}{10}$

$= \frac{-4-2}{10}$

$= \frac{-6}{10}$

$\therefore \left(-\frac{2}{5}-\frac{7}{10}\right)-\frac{1}{2} \neq \frac{2}{5}-\left(\frac{7}{10}-\frac{1}{2}\right)$

Similarly,

$(6 - 4) - 2 = 2 - 2 = 0$

$6 - (4 - 2) = 6 - 2 = 4$

$\therefore (6 - 4) - 2 \neq 6 - (4 - 2)$

Again, $\{(-2) - (-3)\} - (-1)$

$= 1 - (-1)$

$= 2$

$(-2) - \{(-3) - (-1)\}$

$= (-2) - (-2)$

$= 0$

Thus

$\{(-2) - (-3)\} - (-1) \neq (-2) - \{(-3) - (-1)\}$

i. e, we can say

if a, b, c are three rational numbers

then, $(a - b) - c \neq a - (b - c)$.

Therefore, **the subtraction is not associative in case of rational numbers.**

Activity Check the associative property for subtraction for the following numbers :

(a) 7, – 5, 6 (b) $7, \frac{1}{5}, 0$ (c) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

(iii) Associative Property Under Multiplication :

Like the preceding cases, let us take few examples to verify if the associative property holds with respect to multiplication or not.

$\left\{\left(-\frac{7}{9}\right)\times\frac{2}{5}\right\}\times\frac{7}{3}$

$= \frac{-14}{45}\times\frac{7}{3}$

$= \frac{-98}{135}$

$\left(-\frac{7}{9}\right)\times\left(\frac{2}{5}\times\frac{7}{3}\right)$

$= \left(-\frac{7}{9}\right)\times\frac{14}{15}$

$= \frac{-98}{135}$

We find that

$$\left\{\left(\frac{-7}{9}\right)\times\frac{2}{5}\right\}\times\frac{7}{3}=\left(\frac{-7}{9}\right)\times\left(\frac{2}{5}\times\frac{7}{3}\right)$$

Similarly,

$(3 \times 4) \times 5 = 12 \times 5 = 60.$ and, $3 \times (4 \times 5) = 3 \times 20 = 60$

$\therefore (3 \times 4) \times 5 = 3 \times (4 \times 5)$

Also, $(-9) \times \{(-6) \times 4\} = (-9) \times (-24) = 216$

$\{(-9) \times (-6)\} \times 4 = 54 \times 4 = 216$

$\therefore (-9) \times \{(-6) \times 4\} = \{(-9) \times (-6)\} \times 4$

You may verify associativity for multiplication taking some other rational numbers.

Thus we can say that,

If a, b, c are any three rational numbers then $(a \times b) \times c = a \times (b \times c)$

$\therefore$ **Multiplication is associative with respect to rational numbers.**

Activity Check the associative property of multiplication for the following numbers

(a) 12, –11, 7 (b) – 5, – 4, – 3 (c) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$

(iv) Associative Property Under Division :

Let us see the associativity for division with the help of three rational numbers.

$$\left(\frac{2}{3}\div\frac{1}{5}\right)\div\left(\frac{-2}{5}\right)$$

$$=\left(\frac{2}{3}\times\frac{5}{1}\right)\div\left(\frac{-2}{5}\right)$$

$$=\frac{10}{3}\div\left(\frac{-2}{5}\right)$$

$$=\frac{10}{3}\times\left(\frac{5}{-2}\right)$$

$$=\frac{25}{-3}$$

$$=\frac{-25}{3}$$

$$\frac{2}{3}\div\left\{\frac{1}{5}\div\left(\frac{-2}{5}\right)\right\}$$

$$=\frac{2}{3}\div\frac{1}{5}\times\frac{5}{-2}$$

$$=\frac{2}{3}\div\left(\frac{-5}{10}\right)$$

$$=\frac{2}{3}\times\frac{-10}{5}=\frac{-4}{3}$$

$$\therefore \left(\frac{2}{3}\div\frac{1}{5}\right)\div\left(\frac{-2}{5}\right)\neq\frac{2}{3}\div\left\{\frac{1}{5}\div\left(\frac{-2}{5}\right)\right\}$$

Similarly, $(6 \div 3) \div 2 = 2 \div 2 = 1$ and $6 \div (3 \div 2) = 6 \div \frac{3}{2} = 6 \times \frac{2}{3} = \frac{12}{3} = 4$

$\therefore (6 \div 3) \div 2 \neq 6 \div (3 \div 2)$

Therefore, **rational numbers are not associative under division.**

If a, b, c are three rational numbers, then

$(a \div b) \div c \neq a \div (b \div c)$ always

Activity Check for associativity with respect to division of the following numbers.

(a) $6, -2, -3$ (b) $\frac{2}{7}, \frac{5}{6}, 8$ (c) $-\frac{4}{5}, 8, -\frac{7}{9}$

We have learnt from the above discussion that in case of rational numbers :

Closure property	Closed under addition, subtracton and multiplication but not under division.
Commutative property	Addition and multiplicaton are commutative. Subtraction and division are not commutative.
Associative property	Addition and multiplication are associative Subtraction and division are not associative.

1.2.4 Distributive Law

You already know about distributive property for integers. *If a, b, c are three integers, then, $a \times (b + c) = a \times b + a \times c$.* Which is known as the distributive law of multiplication over addition.

Let us see if the distributive property holds in case of rational numbers as well.

Let $\frac{3}{4}, \frac{-2}{5}, \frac{5}{6}$ be three rational numbers.

Now,

$$\frac{3}{4} \times \left(\frac{-2}{5} + \frac{5}{6}\right)$$

$$= \frac{3}{4} \times \left(\frac{-2 \times 6}{5 \times 6} + \frac{5 \times 5}{6 \times 5}\right)$$

$$= \frac{3}{4} \times \frac{13}{30}$$

$$= \frac{3 \times 13}{4 \times 30} = \frac{39}{120}$$

Also, $\frac{3}{4}\times(\frac{-2}{5})+\frac{3}{4}\times\frac{5}{6}$

$=\frac{-6}{20}+\frac{15}{24}$

$=\frac{-36+75}{120}$

$=\frac{39}{120}$

2	20, 24
2	10, 12
	5, 6

LCM = $2\times2\times5\times6$
= 120

We see that,

$$\frac{3}{4}\times\left(\frac{-2}{5}+\frac{5}{6}\right)=\frac{3}{4}\times\left(\frac{-2}{5}\right)+\frac{3}{4}\times\left(\frac{5}{6}\right)$$

Take some more rational numbers and check distributive property of multiplication over addition.

That is *if a, b , c are any three rational numbers then.*

$$a\times(b+c)=a\times b+a\times c$$

Also,

$$a\times(b-c)=a\times b-a\times c$$

The latter property is known as **distributive property of multiplication over subtraction.**

Activity Check the distributivity of multiplication over addition and subtraction for $-\frac{3}{4}$, $\frac{2}{3}$ and $\frac{5}{6}$

1.3 Additive Identity

$0+2=2=2+0$

$0+(-5)=(-5)=(-5)+0$

$\frac{2}{3}+0=\frac{2}{3}=0+\frac{2}{3}$

$\left(\frac{-5}{7}\right)+0=-\frac{5}{7}=0+\left(\frac{-5}{7}\right)$

Thus, when zero is added to any rational number, the sum is again that rational number

In general, *if 'a' is a rational number, then* $a+0=a=0+a$

Here, zero (0) is called identity for addition. We also call zero the additive identity. Zero is the **additive identity**.

1.4 Multiplicative Identity

Let us look at the operation of multiplication of a rational number by 1.

$5 \times 1 = 5 = 1 \times 5$

$(-3) \times 1 = -3 = 1 \times (-3)$

$$\frac{5}{6} \times 1 = \frac{5}{6} = 1 \times \frac{5}{6}$$

$$\left(\frac{-3}{2}\right) \times 1 = \frac{-3}{2} = 1 \times \left(\frac{-3}{2}\right)$$

If a rational number is multiplied by 1, then the result is the number itself.

$\therefore$ *for any rational number a,*

$a \times 1 = a = 1 \times a$

1 is called the **multiplicative identity** of rational numbers.

1.5 Additive Inverse

If sum of two numbers is zero then one is called additive inverse of the other.

For example $3 + (-3) = 0$

$\therefore$ -3 is the additive inverse of 3. and vice versa.

$(-5) + 5 = 0$

$$\frac{2}{3} + \left(\frac{-2}{3}\right) = 0$$

Therefore 5 is the additive inverse of -5 and $-\frac{2}{3}$ is the additive inverse of $\frac{2}{3}$.

So, *if a is any rational number, then*

$a + (-a) = 0 = (-a) + a$

$-a$ is the additive inverse of a, or, a is the additive inverse of $-a$

Note that 0 is the **additive inverse** of 0.

Activity

Write the additive inverse of $\frac{6}{9}$ [ ]

Write the additive inverse of $\frac{-11}{15}$ [ ]

1.6 Multiplicative Inverse

If multiplication of two numbers is 1, then one is called the **multiplicative inverse** of the other.

For example : $7\times\frac{1}{7}=1$ or $\frac{1}{7}\times 7=1$.

So, $\frac{1}{7}$ is the multiplicative inverse of 7 and 7 is the multiplicative inverse of $\frac{1}{7}$

Similarly,

$(-5)\times\left(\frac{1}{-5}\right)=1$ or $\left(\frac{1}{-5}\right)\times(-5)=1$. Hence, $\frac{1}{-5}$ is the multiplicative inverse of -5, or, -5 is the multiplicative inverse of $\frac{1}{-5}$

Again $\frac{3}{5}\times\frac{5}{3}=1=\frac{5}{3}\times\frac{3}{5}$. Hence, $\frac{5}{3}$ is the multiplicative inverse of $\frac{3}{5}$, or, $\frac{3}{5}$ is the multiplicative inverse of $\frac{5}{3}$.

Similarly,

$\left(-\frac{3}{7}\right)\times\left(-\frac{7}{3}\right)=1=\left(-\frac{7}{3}\right)\times\left(-\frac{3}{7}\right)$ So $-\frac{7}{3}$ is the multiplicative inverse of $-\frac{3}{7}$ and $-\frac{3}{7}$ is the multiplicative inverse of $-\frac{7}{3}$

So, *if a is any non zero rational number, then* $\frac{1}{a}$ *is multiplicative inverse of a* and *vice versa.* Multiplicative inverse is also known as **reciprocal.**

1 is the reciprocal of 1.

Activity Discuss the following questions with your friends and answer:

The multiplicative inverse of 6 is [ ]

The multiplicative inverse of -9 is [ ]

The multiplicative inverse of $\frac{2}{3}$ is [ ]

The multiplicative inverse of $\frac{-5}{7}$ is [ ]

The multiplicative inverse of $\frac{1}{7}$ is [ ]

Example 1 : Using properties of numbers find the value of

$$\frac{2}{5}\times\frac{-2}{7}-\frac{1}{12}-\frac{3}{7}\times\frac{2}{5}$$

Solution :

$$\frac{2}{5}\times\frac{-2}{7}-\frac{1}{12}-\frac{3}{7}\times\frac{2}{5}$$

$$=\left(\frac{2}{5}\times\frac{-2}{7}-\frac{3}{7}\times\frac{2}{5}\right)-\frac{1}{12}$$

$$=\frac{2}{5}\times\left(\frac{-2}{7}-\frac{3}{7}\right)-\frac{1}{12}$$ [distributive property]

$$=\frac{2}{5}\times\left(\frac{-2-3}{7}\right)-\frac{1}{12}$$

$$=\frac{2}{5}\times\left(\frac{-5}{7}\right)-\frac{1}{12}$$

$$=\frac{-2}{7}-\frac{1}{12}$$

$$=\frac{-24-7}{84}$$ [LCM of 7 and 12 is 84]

$$=\frac{-31}{84}$$

Example 2 : Write the additive inverse of $-2\times\frac{5}{7}$

Solution : The additive inverse of $-2\times\frac{5}{7}$ is $-\left(-2\times\frac{5}{7}\right)=2\times\frac{5}{7}=\frac{10}{7}$

Alternatively,

$$-2\times\frac{5}{7}=\frac{-10}{7}$$

Hence, $\frac{10}{7}$ is the additive inverse of $-2\times\frac{5}{7}$

Example 3 : Write the multiplicative inverse of $2\times\frac{-6}{7}$

Solution : $2\times\left(\frac{-6}{7}\right)=\frac{2\times(-6)}{7}=-\frac{12}{7}$

$\therefore\ -\frac{7}{12}$ is the multiplicative inverse of $-\frac{12}{7}$

Example 1.1

1. State whether the following are true or false :
   (i) Rational numbers are commutative under addition and multiplication.
   (ii) Rational numbers are associative under subtraction and division.
   (iii) Rational numbers are not closed under subtraction.
   (iv) Zero does not have any reciprocal.
   (v) Zero is a rational number.
   (vi) $\frac{3}{2}\times\frac{5}{6}\times\frac{4}{7}=\frac{4}{7}\times\frac{5}{6}\times\frac{3}{2}$
   (vii) $\frac{5}{9}\times\left(\frac{1}{3}+\frac{7}{3}\right)=\frac{5}{9}\times\frac{1}{3}+\frac{7}{3}$
   (viii) $\frac{7}{12}-\frac{3}{7}+\frac{11}{12}=\frac{7}{12}-(\frac{3}{7}+\frac{11}{12})$
   (ix) $\frac{-3}{11}\times\frac{2}{9}=\frac{-3}{11}$
   (x) $\frac{4}{9}\left(\frac{11}{12}+\frac{-5}{6}\right)=\frac{4}{9}\times\frac{11}{12}+\frac{4}{9}\times\left(\frac{-5}{6}\right)$

2. Write additive inverse of the following :
   (i) $\frac{-7}{9}$ (ii) 3 (iii) $\frac{-2}{8}$
   (iv) $\frac{19}{-7}$ (v) $\frac{-a}{c}$

3. Write the additive inverse of $-\frac{20}{11}$ and $\frac{5}{6}$

4. Write multiplicative inverse of the following

(i) -13 (ii) $\frac{-4}{9}\times\frac{-2}{7}$ (iii) $-2\times\frac{2}{5}$

(iv) -1 (v) $\frac{2n}{5}$

5. Is $\frac{5}{3}$ a multiplicative inverse of $-1\frac{2}{3}$? Give reason.

6. What are the multiplicative and additive inverses of 1?

7. What are the multiplicative inverses of $\frac{-4}{9}$ and $\frac{11}{16}$?

8. Write the reciprocals of the following two numbers.

(i) $\frac{2}{3}$ (ii) $\frac{-5}{12}$

9. Write the rational numbers whose reciprocals are the numbers themselves.

10. Multiply $\frac{-25}{26}$ with the reciprocal of $\frac{5}{13}$

11. Write the properties which are used in the following

(i) $\frac{-3}{5}\times\frac{-2}{7}=\frac{-2}{7}\times\frac{-3}{5}$ (ii) $\frac{-4}{5}+0=\frac{-4}{5}=0+\frac{-4}{5}$ (iii) $\frac{2}{9}\times 1=1\times\frac{2}{9}=\frac{2}{9}$

12. Find the value of the following using relevant properties

(i) $\frac{6}{7}+\frac{2}{5}+\frac{2}{7}+\frac{1}{5}$

(ii) $670\times\frac{7}{11}+670\times\frac{1}{3}$

(iii) $\frac{7}{5}\times\left(\frac{-3}{8}\right)+\frac{3}{4}\times\frac{7}{5}$

(iv) $-\frac{5}{9}\times\left(\frac{-27}{15}+\frac{36}{25}\right)$

(v) $\frac{2}{3}+\frac{3}{3}-\frac{1}{4}\times\frac{2}{3}+\frac{2}{3}\times\frac{1}{2}$

(vi) $\frac{4}{7}\times\left(\frac{-5}{9}\right)+\frac{4}{7}\times\frac{1}{5}$

(vii) $\frac{2}{3}\times(\frac{-5}{7})-\frac{1}{6}\times\frac{3}{7}+\frac{1}{14}\times\frac{2}{3}$

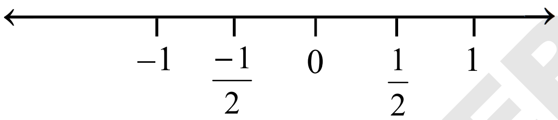
13. Check whether the following numbers are associative under addition and under multiplication : $-\frac{2}{3}$, $\frac{3}{8}$ and $\frac{4}{5}$

14. Give an example to show that the rational numbers are not closed under division.

1.7 Representation of rational numbers on the number line

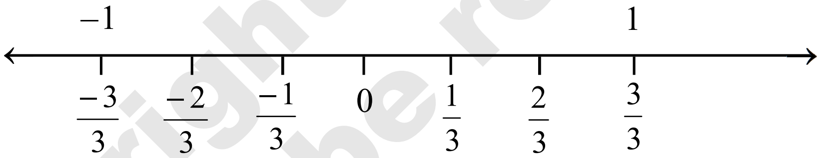
We know that integers can be represented on a number line. In the same way rational numbers can also be represented on a number line.

(i) Observe the following number line



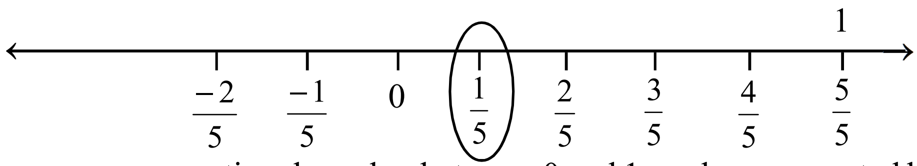
The point on the number line which is half way between 0 and 1 has been labelled $\frac{1}{2}$. That is the line segment between 0 and 1 is equally divided into two equal parts denoted by $\frac{1}{2}$. Similarly by dividing the line segment between –1 and 0 into two equal parts we get the point $-\frac{1}{2}$

(ii)



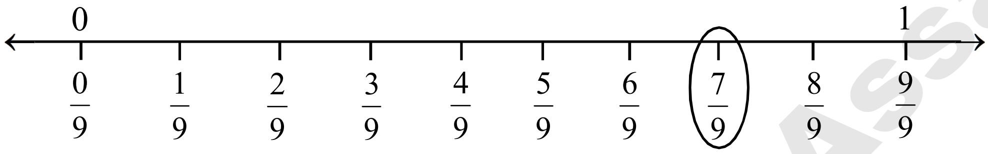
Again, dividing the line segment between 0 and 1 into three equal parts, we get two points which is labelled as $\frac{1}{3}$, $\frac{2}{3}$. Similarly, dividing the line segment between –1 and 0 into three equal parts we get the points $-\frac{1}{3}$ and $-\frac{2}{3}$.

(iii) To show $\frac{1}{5}$ on the number line, the line segment between 0 and 1 must be divided into five equal parts by four points. The four points from 0 are successively $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$ and $\frac{4}{5}$.



In the same way, any rational number between 0 and 1 can be represented by a point on the number line. For such a rational number, the denominator tells the number of equal parts into which the first unit (from 0 to 1) is divided.

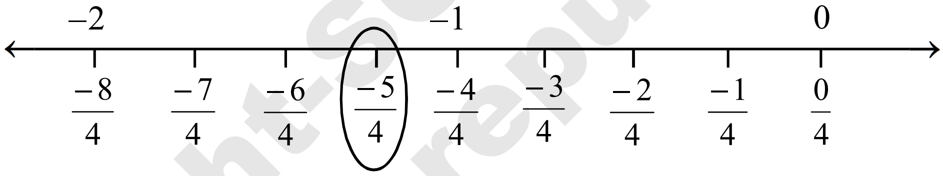
Example 1 : Represent $\frac{7}{9}$ on the number line.



In $\frac{7}{9}$, 9 is the denominator.

The line segment between 0 and 1 is divided into 9 equal parts. Now $\frac{7}{9}$ means a point as shown and it is at a distance equal to 7 of 9 equal parts to the right hand side of 0.

Example 2 : Represent $\frac{-5}{4}$ on the number line.



Activity Represent the following numbers on the number line.

$\frac{2}{3}$, $\frac{1}{5}$, $\frac{-2}{5}$, $\frac{-1}{4}$, $\frac{-7}{9}$, $\frac{5}{7}$, $\frac{-9}{5}$

1.8 Rational numbers between two rational numbers

Note the following points carefully.

3 and 4 are the natural numbers between 2 and 5

– 2, – 1, 0, 1 and 2 are integers between the integers – 3 and 3.

Again, $\frac{6}{11}$, $\frac{7}{11}$, $\frac{8}{11}$ are three rational numbers between $\frac{5}{11}$ and $\frac{9}{11}$.

Also,

$\frac{5}{11}$ can be written as $\frac{5\times10}{11\times10}=\frac{50}{110}$ and

$\frac{9}{11}$ can be written as $\frac{9\times10}{11\times10}=\frac{90}{110}$

Now the rational numbers $\frac{51}{110}, \frac{52}{110}, \frac{53}{110}$ .... $\frac{89}{110}$ lie between $\frac{50}{110}$ and $\frac{90}{110}$

i.e between $\frac{5}{11}$ and $\frac{9}{11}$

Similarly we can write

$$\frac{5}{11}=\frac{50}{110}=\frac{50\times10}{110\times10}=\frac{500}{1100}$$

$$\frac{9}{11}=\frac{90}{110}=\frac{90\times10}{110\times10}=\frac{900}{1100}$$

Here the numbers $\frac{501}{1100}, \frac{502}{1100}, \ldots\ldots \frac{899}{1100}$ all lie between $\frac{500}{1100}$ and $\frac{900}{1100}$

Thus, all the above numbers lie between $\frac{5}{11}$ and $\frac{9}{11}$. In the same way we can find many more numbers between $\frac{5}{11}$ and $\frac{9}{11}$. Therefore, we may conclude the discussion as follows:

There are infinite number of rational numbers between any two rational numbers.

Note There are finite number of natural numbers and integers respectiviely in between two natural numbers and integers.

Example 1 : Find some rational numbers between $\frac{-3}{5}$ and $\frac{2}{7}$.

Solution : Firstly, two numbers must be converted into rational numbers with the same denominators.

$$\frac{-3}{5}=\frac{-3\times7}{5\times7}=\frac{-21}{35}$$

$$\frac{2}{7}=\frac{2\times5}{7\times5}=\frac{10}{35} \qquad \text{[LCM of 5 and 7 = 35]}$$

$\therefore$ Some rational numbers between $\frac{-3}{5}$ and $\frac{2}{7}$ are $\frac{-20}{35}, \frac{-19}{35}, \ldots\ldots, \frac{9}{35}$

Activity Write some rational numbers between $\frac{-3}{5}$ and $\frac{2}{7}$

Example 2 : Let us see how can we find rational numbers between – 3 and – 1

Solution : Here, – 2 is the integer between – 3 and – 1

But apart from integer, we can find any number of rational numbers between – 3 and – 1.

We have, $\frac{-3}{1}=\frac{-3\times 10}{1\times 10}=\frac{-30}{10}$

$$\frac{-1}{1}=\frac{-1\times 10}{1\times 10}=\frac{-10}{10}$$

The numbers $\frac{-29}{10},\frac{-28}{10},........,\frac{-11}{10}$ lie between-3 and -1.

We can also find more rational numbers between -3 and -1. For example :

$$-3=\frac{-30}{10}=\frac{-30\times 10}{10\times 10}=\frac{-300}{100}$$

$$-1=\frac{-10}{10}=\frac{-10\times 10}{10\times 10}=\frac{-100}{100}$$

$\frac{-299}{100},\frac{-298}{100},........,\frac{-102}{100},\frac{-101}{100}$ all lie between –3 and – 1

In this way, *we can find infinite numbers of rational numbers between two given rational numbers.*

In addition, we can use **average or mean method** to find rational numbers between any two rational numbers. It is shown by the following example.

Example 3 : Find two rational numbers between $\frac{1}{3}$ and $\frac{1}{5}$ using mean or average method.

Solution : Mean of $\frac{1}{3}$ and $\frac{1}{5}$

$$=\frac{\left(\frac{1}{3}+\frac{1}{5}\right)}{2}=\frac{4}{15}$$

$\frac{4}{15}$ lies between $\frac{1}{3}$ and $\frac{1}{5}$

Again mean of $\frac{4}{15}$ and $\frac{1}{3}$ $=\left(\frac{1}{3}+\frac{4}{15}\right)\div 2=\frac{3}{10}$

$\frac{3}{10}$ is the mean of $\frac{4}{15}$ and $\frac{1}{3}$.

$\frac{4}{15}$ and $\frac{3}{10}$ are two rational numbers between $\frac{1}{5}$ and $\frac{1}{3}$.

Similarly by finding mean of $\frac{1}{3}$ and $\frac{3}{10}$, we can find other numbers.

Thus if a and b be any two rational numbers then, $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$.

Exercise1.2

1. Represent the following numbers on the number line.

   (i) $\frac{9}{5}$ (ii) $\frac{-2}{13}$ (iii) $\frac{-5}{7}$ (iv) $\frac{-1}{3}$ (v) $\frac{3}{2}$

2. Write 5 rational numbers between the following numbers.

   (i) $\frac{-3}{5}$ and $\frac{1}{3}$ (ii) $\frac{-2}{3}$ and $\frac{5}{3}$

   (iii) -5 and -4 (iv) $\frac{-2}{-3}$ and $\frac{3}{7}$

3. Find 5 rational numbers greater than $\frac{-2}{5}$ and less than $\frac{1}{2}$.

What we have learnt

1. $\frac{p}{q}$ is a rational number, where p and q are integers and $q \neq 0$
2. All integers are rational numbers.
3. Recurring decimals are rational numbers.
4. Rational numbers are closed under addition, subtraction and multiplication.
5. Excluding zero, all other rational numbers are closed under division.
6. Addition and multiplication are commutative for rational numbers.
   But subtraction and division are not commutative for rational numbers.
7. Addition and multiplication are associative for rational numbers.
   But subtraction and division are not associative for rational numbers.
8. Zero is called additive identity and one is called multiplicative identity of rational numbers.
9. If sum of two numbers is zero, then one is called additive inverse of the other.
10. If multiplication of two numbers is 1, then one number is the multiplicative inverse of the other.
11. If a, b and c are three rational numbers, then, $a \times (b + c) = a \times b + a \times c$
    and $a \times (b - c) = a \times b - a \times c$
    These properties are called distributivity of multiplication over addition and subtraction respectively.
12. Rational numbers can be represented on an infinite straight line called the number line.
13. Between any two rational numbers there are infinite number of rational numbers.

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