Units And Measurements

Systems of Units

Measurement

It is the process of assigning a number to an attribute (or phenomenon) according to a rule or set of rules.

Units

- A unit is the chosen standard of measurement of quantity, which has the same nature as the quantity.
- To express any physical quantity completely, we need the numerical value and the unit (*u*).

Physical quantity = *nu*

Fundamental Units: Units for fundamental or base quantities (length, mass and time)

Derived Units: Units obtained from fundamental units

Example:

Unit of speed (ms^{-1})

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\therefore \text{Unit of speed} = \frac{\text{Unit of Distance}}{\text{Unit of Time}}$$

$$\Rightarrow \frac{\text{m}}{\text{s}} = \text{ms}^{-1}$$

Systems of Units

- CGS System: Base units for length, mass and time in this system are centimeter, gram and second respectively.
- FPS System: Base units for length, mass and time in this system are the foot, pound and second respectively.
- MKS System: Base units in this system are metre, kilogram and second.
- International System (SI) of Units: Based on seven base units; at present the internationally accepted system

SI Base Quantities and Units

- Length − metre (m)
- Mass − kilogram (kg)
- Time − second (s)
- Electric current − ampere (A)
- Thermodynamic temperature − kelvin (K)
- Amount of substance − mole (mol)
- Luminous intensity − candela (cd)

Derived Units

- The units of different physical quantities can be derived from the seven basic fundamental units. These are called derived units.
- Some common derived units are mentioned in the given table.

S.No.	Physical Quantity	Relationship with Fundamental Unit	Symbol
1.	Volume	Length cube	m^3
2.	Density	Mass per unit volume	$kg\ m^{-3}$
3.	Velocity	Distance covered in unit time	ms^{-1}
4.	Acceleration	Velocity changes per unit time	ms^{-2}
5.	Force	Mass × Acceleration	$kg\ ms^{-2}$
6.	Work	Force × Distance traveled	$kg\ m^2s^{-2}$
7.	Pressure	Force per unit area	$kg\ m^{-1}s^{-2}$

We have seen various objects as large as a mountain to as small as a speck. Therefore, to measure such large and small quantities, we have to use a simple method.

Example:

Diameter of the sun = 1,391,000,000 m

Diameter of a hydrogen atom = 0.000,000,000,106 m

Thus, when we are using metre, we find that the content is either quiet bulky or very small. At the same time, it is very inconvenient. Therefore, to counter this, we use a standard form of expression as:

Diameter of the sun = 1,391,000,000 m = 1.39×10^9 m

Diameter of a hydrogen atom = 0.000,000,000,106 m = 1.06×10^{-10} m

The exponential part of a particular measurement is called the order of magnitude of a quantity.

The prefixes and symbols for such order of magnitude are listed in the given table.

Multiple	**Prefix**	**Symbol**
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k

10^6	mega	M
10^9	giga	G

Bigger Units:

To know the distance between two or more heavenly bodies, for measuring heavy materials used in daily life and to count the large span of time we require bigger units.

- For length, the bigger units used are:

  (i) Astronomical unit (A.U.): It is the mean distance between Earth and Sun. 1 A.U. = 1.496×10^{11} m
  (ii) Light year (ly): It is the distance travelled by light in vacuum, in one year. 1 ly =9.46×10^{12} km
  (iii) Parsec: 1 Parsec = 3.26 ly

- For mass, the bigger units use are:

  (i) quintal: 1 quintal = 100 kg
  (ii) metric tonne: 1 metric tonne = 1000 kg = 10 quintal

- For time:

  (i) lunar month: 1 lunar month =29.5 days
  (ii) Leap year
  (iii) Decade
  (iv) Century
  (v) Millennium

Directions for writing units:

The following rules have been observed while writing the physical quantity:

- Symbols of units which are not named after scientists are written in the small letter. For example, m, kg, q and so on.
- The unit which is named after a scientist is written with the first letter of his/her name in the capital. For example, N for Newton, J for joule, W for watt and so on.
- While writing the full name of a unit we do not consider if the unit is named after scientist or not, it is always written with lower initial letter. For example, the unit of length

as meter, mass as kilogram, force as Newton and so on.

- Compound units, formed by the product of two or more units are written after placing a dot, cross or leaving a space between the two symbols. For example, the unit of electric dipole C.m or C×m or C m.

- The compound unit uses negative powers when one unit is divided by another. For example, unit of power = joule/second = J s^{-1}.

- Shorter forms of units are never written in the plural. For example, 10 kilograms cannot be written as 10 kgs.

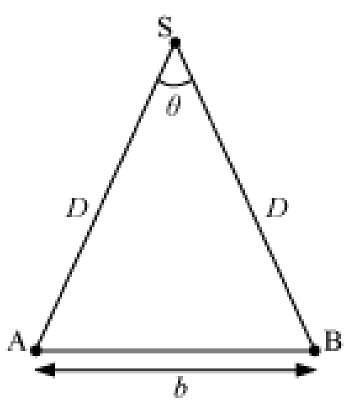
- Units cannot be written with more than one prefix. In spite of writing kMW, we must write GW.

- The prefix and symbol combined together become a new symbol for the unit. For example, km^3 means $(10^3\ m)^3 = 10^9\ m^3$. It does not mean $10^3\ m^3$

Measurement of Length

- **Measurement of Large Distances**

- Method used − Parallax method
- Parallax − Name given to the apparent change in position of an object with respect to the background, when the object is seen from two different positions



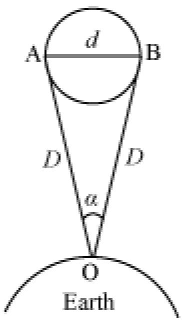
Far-away object (S) is observed from two different positions (A and B).

Distance between the two observation points is called basis (*b*).

The angle θ is called parallax angle. When θ is very small, we can consider the line segment AB as an arc of length *b* having radius *D*. By geometry,

$$\boxed{D = \frac{b}{\theta}} \qquad \left(\because \text{angle} = \frac{\text{arc}}{\text{radius}}\right)$$

- **Size of an Astronomical Object**



Size of a far-away planet can also be determined by using the same method.

$$\boxed{\alpha = \frac{d}{D}}$$

- **Estimation of Small Distances**
- The size of small microscopic particles cannot be estimated by optical microscopes.
- Optical microscopes use visible light. The wavelength range of visible light is 4×10^{-7} m to 7×10^{-7} m. Hence, the distances smaller than this cannot be measured by using visible light.
- Instead of visible light, electron beams can be used to measure very small distance. An electron microscope uses such electron beams.
- Electron microscope with a resolution of $0.6\ \overset{0}{\text{A}}$ can be used to measure the size of subatomic particles.

Measurement of Mass

- SI unit of mass − Kilogram
- While dealing with atoms and molecules, we use unified atomic mass unit (u or amu) as standard unit.

$1\ \text{u} = \frac{1}{12}$ of the mass of C^{12} atom

Or, 1 u = 1.66×10^{-27} kg

- Masses of common objects can be measured by balance.
- Large masses can be measured by using gravitational method.
- Masses of sub-atomic particles can be measured by using mass spectrograph.
- Range of variation of mass − from 10^{-30} kg to 10^{55} kg

Measurement of Time

Before going into the subject, let us look at this animation.

So, you have learned the importance of measuring time. You also have learned about some time-measuring instruments, some from ancient and medieval ages and some from modern times.

Let us learn about the units of time in detail.

Units of Time

In ancient times, people used large units for measuring time. For example, the time interval between two consecutive sunrises was considered **a day**. However, they were not able to accurately measure the time taken by relatively shorter events such as lightning, rain, or time taken to cover a distance.

Period	Method of measuring time in ancient times
Day	Time between two consecutive sunrises
Month	Time between two consecutive new or full moons
Year	Time taken by the Earth to complete one revolution around the sun

Take a rubber ball and drop it from a height on to a hard surface. The ball will bounce repeatedly and stop after some time. Which device would you require to measure the time taken by the ball to stop?

We require some convenient units that can be used for measuring short as well as long intervals of time.

- The internationally accepted unit of time is **second**. Its symbolic representation is **s**. However, this unit is used for representing shorter durations of time only. Minutes (min) and hours (h) are used for representing longer durations of time.

- We use different units of time depending on the requirement and convenience. For example, it is easy to express the pulse rate using minutes as units of time. However, while expressing the time taken to travel from Delhi to Jaipur, we use hours as the unit of time.

Modern Technology

Scientists use modern technology for scientific research to determine shorter intervals of time such as millisecond (ms), microsecond (μs), and nanosecond (ns); and larger intervals of time such as million years (MY), billion years (BY), etc.

Discuss with your teacher how the larger units of time are used in astronomy.

<table>
<tr><td rowspan="4">The given table shows the respective amount of time taken by all the participants to cross the finish line in a cycling race. The time displayed at the start of the race was 10:45:00

Who took the least amount of time to finish the race?</td><th>Participant</th><th>Display on digital clock</th></tr>
<tr><td>Ram</td><td>10:46:45</td></tr>
<tr><td>Shyam</td><td>10:46:35</td></tr>
<tr><td>Ajit</td><td>10:46:57</td></tr>
</table>

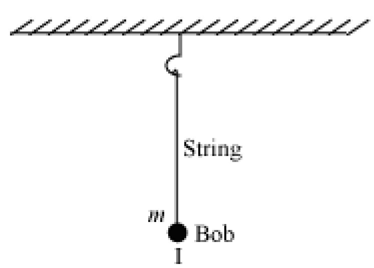
	Aditya	10:46:13
	Jay	10:47:01
	Ramesh	10:46:41

Till now, we were discussing the different types of clocks that are used by people today to measure time. **Now, the question that arises is – how is a clock able to measure time?**

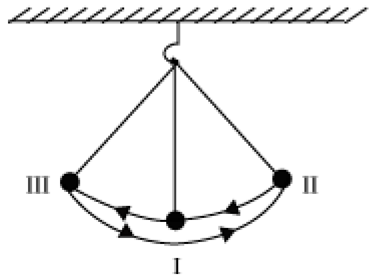
The answer lies in the fact that a clock uses some kind of periodic motion in its working. One such object that exhibits periodic motions is a **simple pendulum.** In this section, we will discuss some common characteristics of a simple pendulum.

Characteristics of a simple pendulum

- A simple pendulum consists of a mass *m* that is suspended by a piece of string. The mass is known as the **bob** of the pendulum. A simple pendulum is shown in the given figure. Here, point **I** represents the **mean position** of the bob.



- The bob begins to move to and from when it is released from the extreme position **II** i.e., it will go up to another extreme position **III** via mean position **I**, and then go up to position **II** again via mean position **I**. It will continue to move in the same way. This back and forth motion of the bob via mean position **I** is called the **periodic or the oscillatory motion of the bob.**



- The bob completes one **oscillation** when it moves from extreme position **II** to extreme position **III** via mean position **I** and then returns to extreme position **II**, following the same path.

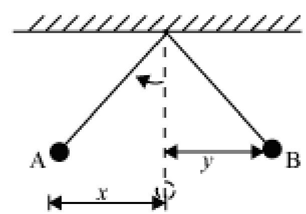
- The time taken to complete one oscillation is termed as the **time period** of the pendulum. If a simple pendulum completes 15 oscillations in 5 seconds, then the time period of the pendulum is 1/3 seconds to complete one oscillation.

- Consider *T* is the time period of a simple pendulum which means that in *T* seconds the pendulum completes 1 oscillation. This implies that in 1 second, the number of oscillations will be 1/*T* which is equal to the frequency of the pendulum i.e. *f*.

$$f = \frac{1}{T} \text{ or } T = \frac{1}{f}$$

Take a stone and attach it to the ceiling by a thread. Now, move it to one side up to a distance *x* (point A) and release it.

It will go up to point B on the other side. Measure the horizontal distance *y* with the help of a ruler. Is *y* = *x*?



You will find that the distance *y* < *x*. This is because air particles present in the medium oppose the motion of the pendulum. Hence, its distance decreases continuously until the bob comes to rest.

What will happen to the motion of the pendulum if it is allowed to oscillate in space?

Suspend a bob in a vertical stand and allow it to oscillate. Measure the time required by it to complete 10 oscillations. Wait until the bob comes to rest at the mean position. Now, move the bob to a distance greater than the first distance and release it. Measure the time taken by the bob to complete 10 oscillations again. Repeat the same steps for different distances and record the time period in the following table. **Is the time period dependent on displacement *x*?** Discuss this with your teacher.

S. No	Value of x (in cm)	Time taken to complete 10 oscillations	Time period
1.	-	-	-
2.	-	-	-
3.	-	-	-
4.	-	-	-

After performing this activity, you will find that the time periods for all distances are nearly equal to each other. This activity shows that the time period of a pendulum does not depend on the distance through which it is displaced.

Repeat the same activity by changing the mass of the bob and find out whether the **time period of a pendulum depends on the mass of the bob**.

Modern quartz clocks and watches lose or gain one second once in 2 to 10 years!

Galileo and the swinging lamp

Galileo was one of the greatest scientists of his time. He was a physicist, astronomer, and mathematician.

Once when he was sitting in a cathedral, he noticed that the lamp, suspended by a long chain from the ceiling, swung back and forth because of wind currents. He measured the time taken by the lamp to complete one oscillation using his pulse rate. He was surprised to find that each oscillation took an equal amount of time. He set up his own pendulum and obtained the same result as that obtained with the

swinging lamp. He concluded from his findings that a pendulum takes equal time to complete each oscillation.	

Measurement of Time

- Atomic standard of time → Periodic vibrations produced in a cesium atom
- 1 second → time required for 9, 192, 631, 770 vibrations of the radiation of cesium 133 atom
- In our country, the National Physical Laboratory (NPL) has the responsibility of maintaining Indian standard time.

Errors in Measurement

Errors

Error = Actual value − Measured value

Types of Errors

	Type	Source	Minimisation
a	Systematic errors	Instrumental errors Imperfection in experimental technique or procedure Personal errors (carelessness in taking observations, improper setting of the apparatus, etc.)	By improving experimental techniques Selecting better instruments Removing personal bias

b	Random errors	Arise due to random and unpredictable fluctuations in experimental conditions (fluctuations in temperature, voltage, etc.)	By repeating the observation a large number of times Taking the arithmetic mean of all the observations
c	Least-count errors	Errors associated with the resolution of the instrument	By using instruments of higher resolution
d	Personal errors	Personal errors arise due to fault of an observer in taking reading, lack of proper setting of the apparatus etc.	Repeated observations by different observers

Absolute Error

It is the difference between the arithmetic mean values of the different measurements and the individual measured value of the quantity.

$$|\Delta X| = \overline{X} - X_0$$

Here,

ΔX = Absolute error

$\overline{X}$ = Mean value

X_0 = Measured value

Relative Error and Percentage (%) Error

It is the ratio of the mean absolute error to the mean value of the measured quantity.

$\frac{\Delta X}{\overline{X}}$ is the relative error in X.

% Error → Relative error multiplied by 100.

that is, $\frac{\Delta X}{\overline{X}} \times 100$

Errors in Calculations

- **Error of a Sum or a Difference**

A, B = Two physical quantities

A ± ΔA = Measured value of A

B ± ΔB = Measured value of B

We have to find the error ΔZ in the sum,

Z = A + B

Or Z ± ΔZ = (A ± ΔA) + (B ± ΔB)

The maximum possible error in Z,

ΔZ = ± (ΔA + ΔB)

For the difference, Z = A − B, we have

Z ± ΔZ = (A ± ΔA) − (B ± ΔB)

= (A − B) ± ΔA ± ΔB

± ΔZ = ± ΔA ∓ ΔB

The maximum possible value of error in Z,

ΔZ = ± (ΔA + ΔB)

Rule: When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

- **Error of a Product or a Quotient**

Suppose Z = AB, and the measured values of A and B are A ± ΔA and B ± ΔB respectively. Then, Z ± ΔZ = (A ± ΔA) (B ± ΔB)

$Z \pm \Delta Z = AB \pm B\Delta A \pm A\Delta B \pm \Delta A\Delta B$

Dividing LHS by Z and RHS by AB, we have, $1 \pm (\Delta Z/Z) = 1 \pm (\Delta A/A) \pm (\Delta B/B) \pm \left(\frac{\Delta A}{A}\right)\left(\frac{\Delta B}{B}\right)$

Since ΔA and ΔB are small, we shall ignore their product.

Maximum relative error, $\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)$

This is true for the division as well.

- **Error in the Case of a Measured Quantity Raised to a Power**

Suppose $Z = A^2$

Then, $\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta A}{A}\right) = 2\left(\frac{\Delta A}{A}\right)$

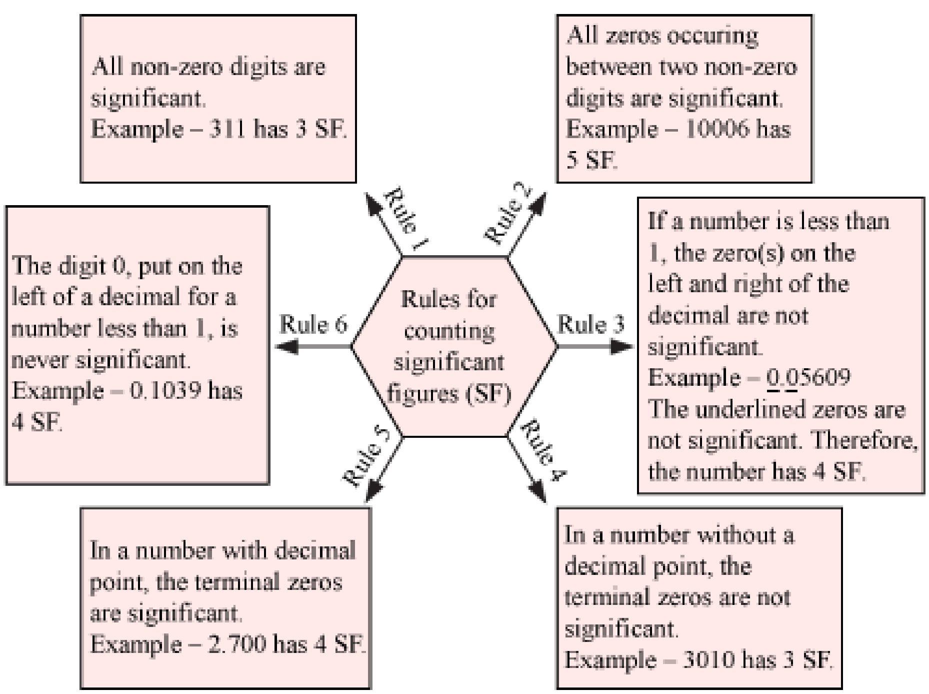
Hence, the relative error in A^2 is two times the error in A.

In general, if $Z = \frac{A^p B^q}{C^r}$

Then, $\frac{\Delta Z}{Z} = p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta A}{A}\right) + r\left(\frac{\Delta C}{C}\right)$

Significant Figures

- Significant figures − Numbers of digits up to which one could express a quantity measured



Rules for Arithmetic Operations with Significant Figures

- **Addition or subtraction −** The final result should retain **as many decimal places** as there are in the number with the least decimal places.

  Example − The sum of three numbers 1.3 m, 2.76 m, and 3.071 m is 7.131 m. It is rounded off to 7.1 m [up to smallest number (1.3 m) of decimal places (2)].

- **Multiplication and division −** The final result should retain **as many significant figures** as there are in the original number with the least significant figures.

  Example − Suppose x = 2.7 and y = 0.326

  Therefore, xy = 0.8802

  As least number of significant figure is 2 (in x = 2.7), therefore, the final result is rounded off to xy = 0.88

  For division, suppose x = 2.367 and y = 1.3, then $\frac{x}{y} = 1.82076,$, but the final result is rounded off to $\frac{x}{y} = 1.8$

Rounding Off the Uncertain Digits and Determining Uncertainty

Rule 1

If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example: 9.83 is rounded off to 9.8

Rule 2

If the digit to be dropped is more than 5, then the preceding digit is raised by one.

Example: 6.38 is rounded off to 6.4

Rule 3

If the digit to be dropped is 5, followed by a non-zero digit, then the preceding digit is raised by one.

Example: 14.252 is rounded off to 14.3

Rule 4

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit (if even) is left unchanged.

Example: 6.250 becomes 6.2 on rounding off

Rule 5

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit (if odd) is raised by one.

Example: 9.750 is rounded off to 9.8

Rules for Determining the Uncertainty in the Results of Arithmetic Calculation

Rule 1

Example:

The length and breadth of a thin rectangular sheet, measured using a metre scale, are 18.2 cm and 14.5 cm respectively. Find the uncertainty or error in the estimation of the area of the rectangular sheet.

Solution:

There are three significant figures in each measurement and a precision upto first place. This means that the length, l may be written as

$l = 18.2 \pm 0.1$ cm

$= 18.2$ cm $\pm 0.5\%$

Similarly, the breadth, b may be written as

$b = 14.5 \pm 0.1$ cm

$= 14.5$ cm $\pm 0.7\%$

Then, the error of the product, using the combination of errors rule

$lb = 263.9\ cm^2 \pm 1.2\%$

The final result will be

$lb = 264 \pm 1\%$

(∵ As per the rule, the final value of area can contain only 3 significant figures and the error can contain only 1 significant figure.)

Rule 2

If a set of experimental data is specified to *n* significant figures, the result obtained by combining the data will be valid to *n* significant figures.

However, if the data are subtracted, the number of significant figures would reduce.

Example: (13.9 g − 6.06 g) both specified to three significant figures gives 7.84 g.

But in addition and subtraction, the final result should retain as many decimal places as there are in the number with the least number of decimal places. So, the answer would be 7.8 g.

Rule 3

The relative error of a value of number specified to significant figures depends not only on ‘*n*’ but also on the number itself.

Example:

Suppose

$m_1 = (1.02 \pm 0.1 \text{ kg})$, and

$m_2 = (9.32 \pm 0.01 \text{ kg})$

Both the measured masses have an error or uncertainty of 0.01 kg.

The relative error in 1.02 kg is

$$= \pm\left(\frac{0.01}{1.02}\right)\times 100 = \pm 1\%$$

Similarly, the relative error in 9.32 kg is

$$= \pm\left(\frac{0.01}{9.32}\right)\times 100 = \pm 0.1\%$$

Thus, the relative error depends on the number itself.

Rule 4

In a multi-step computation, the intermediate results in each measurement should be calculated to a significant figure that is one more than the number of digit in the least precise measurement.

Dimensions of Physical Quantities and Its Applications

- **Dimensions**

They are the powers (or exponents) to which the units of base quantities are raised for representing a derived unit of that quantity.

Examples: Dimensional formula of the volume is $[M^0L^3T^0]$.

Dimensional formula of velocity is $[M^0LT^{-1}]$.

Dimensional formula of acceleration is $[M^0LT^{-2}]$.

- **Applications of Dimensional Analysis**

- Checking the dimensional consistency of equations

- Deducing the relations among physical quantities
- Converting one system of units to another

- **Checking the Dimensional Consistency of Equations**

Based on the principle of homogeneity of dimensions:

- According to this principle, only that formula is correct in which the dimensions of the various terms on one side of the relation are equal to the respective dimensions of these terms on the other side of the relation.

- Example:

Check the correctness of the relation, $t = 2\pi\sqrt{\frac{l}{g}}$, where *l* is length and *t* is time period of a simple pendulum; *g* is acceleration due to gravity.

Solution:

$t = 2\pi\sqrt{\frac{l}{g}}$

Dimension of L.H.S, $t = [\mathrm{T}]$

Dimension of R.H.S, $2\pi\sqrt{\frac{l}{g}} = \sqrt{\frac{\mathrm{L}}{\mathrm{LT^{-2}}}}$

(∵ 2π is a constant)

$= \sqrt{\mathrm{T}^2} = [\mathrm{T}]$

Dimensionally, L.H.S = R.H.S; therefore, the given relation is correct.

- **Deducing the Relations Among Various Physical Quantities**

    Based on the principle of homogeneity of dimensions:

- Example:

The centripetal force, *F,* acting on a particle moving uniformly in a circle may depend upon the mass (*m*), velocity (*v*) and radius (*r*) of the circle. Derive the formula for *F* using the method of dimensions.

Solution:

Let $F = km^a v^b r^c \ldots (i)$

Here, *k* is the dimensionless constant of proportionality, and *a*, *b*, *c* are the powers of *m*, *v*, *r,* respectively.

On writing the dimensions of various quantities in (*i*), we get:

$[M^1L^1T^{-2}] = M^a [LT^{-1}]^b L^c$

$= M^aL^bT^{-b}L^c$

$M^1L^1T^{-2} = M^aL^{b+c}T^{-b}$

On applying the principle of homogeneity of dimensions, we get:

a = 1,

b = 2,

b + *c* = 1 …(*ii*)

From (*ii*), *c* = 1 − *b* = 1 − 2 = −1

On putting these values in (*i*), we get:

$$F = km^1 v^2 r^{-1}$$

Or

$$F = k\frac{mv^2}{r}$$

This is the required relation for centripetal force.

- **Converting one system of units to another**

  Example:

The dimension of force is $\mathrm{F} = [\mathrm{MLT}^{-2}]$
Let the conversion factor from SI to C.G.S be *x*.

Let $1\ kgms^{-2} = x \times 1\ gcms^{-2}$
Dimensional formula for force is $[\mathrm{MLT}^{-2}]$

$$[\mathrm{M_1}^1\mathrm{L_1}^1\mathrm{T_1}^{-2}] = x\,[\mathrm{M_2}^1\mathrm{L_2}^1\mathrm{T_2}^{-2}]$$

$$\therefore\ x = \left[\frac{\mathrm{M_1}}{\mathrm{M_2}}\right]^1 \times \left[\frac{\mathrm{L_1}}{\mathrm{L_2}}\right]^1 \times \left[\frac{\mathrm{T_1}}{\mathrm{T_2}}\right]^{-2}$$

$$\therefore\ x = \left[\frac{1\ \mathrm{kg}}{1\ \mathrm{g}}\right]^1 \times \left[\frac{1\ \mathrm{m}}{1\ \mathrm{cm}}\right] \times \left[\frac{1\ \mathrm{s}}{1\ \mathrm{s}}\right]^{-2}$$

$$\therefore\ x = \left[\frac{10^3\ \mathrm{g}}{1\ \mathrm{g}}\right] \times \left[\frac{10^2\ \mathrm{cm}}{1\ \mathrm{cm}}\right] \times 1$$

$$\therefore\ x = 10^3 \times 10^2 = 10^5$$

Hence, $1\ N = 10^5\ dyne$