CHAPTER 11

Similarity in Geometrical Shapes

Introduction

We see various types of shapes around us, of which some are small in size and some are big. Some of them are circular, some are cubical, some are triangular, and others can be taken as a mix of different shapes.

See figure-1. In the given picture showing a house we are able to see many kinds of geometrical shapes. For example, we can see rectangles and triangles.

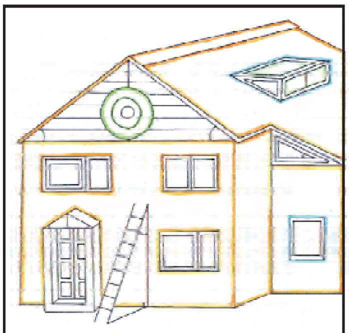


Figure - 1

Can you find some more shapes of different kinds in the figure, which are not triangular or rectangular? What are these shapes? Discuss with your friends.

Same Shapes:- If we carefully observe the figure then we find that there are some shapes where both the shape and the size are same. This means that they are congruent.

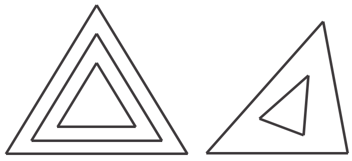


Figure - 2

Now carefully observe figure-2. In figure-2(i), three triangles are drawn. The angles of the triangles appear equal and the sides look bigger or smaller in some fixed proportion. That is why all 3 triangles in figure-2(i) seem similar. But in figure-2(ii), the angles of both triangles are different. Therefore, by looking itself we can say that these two triangles appear different.

Generally, the shapes which look same are called similar but there are certain conditions for similarity in mathematics. Are all 3 triangles similar in 2(i)? When we observe these triangles, they all look similar but how can we confirm? Later in the chapter we will discuss if they are similar or not and why.

Scaling

We often encounter situations where we have to enlarge a picture or draw the map of some farm, house, factory or field on paper. We may be asked to calculate actual distances, size, shape or area from the maps. We use scaling to meet our requirements in such situations.

Meaning of scaling is change in size, that is, increase or decrease (reduce) the size of a given shape. But before we enlarge or reduce a figure we need to keep certain conditions in mind so that the new figure is similar to the earlier one. What do we mean when we say that it should *look the same as earlier*?

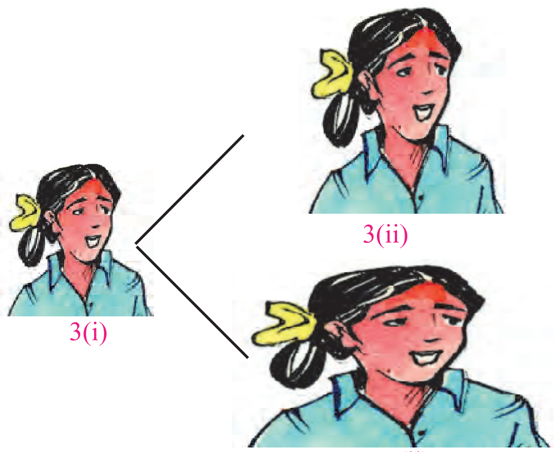


Figure - 3

See the pictures given in figure-3. Our attempts to enlarge figure-3(i), gave us 3(ii) and 3(iii). Which picture will you select from these two? Obviously, the first one. In this, the picture has been enlarged in such a way that it remains close to the original picture, 3(i). To do this, the picture has been enlarged in a fixed ratio. We can say that picture 3(i) and 3(ii) are similar.

Now look at the picture in figure-3(iii). The ratio of the width to height in this picture appears more than that in the original picture. It looks different from the picture in figure-3(i). Hence pictures in 3(i) and 3(ii) are not similar.

Therefore, for scaling, we have to be careful while increasing or decreasing the size so that the shape of the picture is not disturbed and property of similarity remains intact.

Scale Factor

The measurements of two similar shapes are fixed in a particular ratio called the scale factor. By using scale factor size of any figure can be increased or reduced in a fixed proportion, as per our requirements.

For example, to change a 5 cm long line segment to 10 cm long line segment, we have to use scale factor 2, because 5 × 2 = 10. Similarly, to change a 50 cm × 20 cm map into a $10cm \times 4cm$ map, the scale factor is $\frac{1}{5}$ or 0.2 because $50 \times \frac{1}{5} = 10\,cm$, $20 \times \frac{1}{5} = 4\,cm$. So, while scaling the line segment became twice the original length and the map reduces by $\frac{1}{5}$ or 0.2. We can see that in the map, the ratio in which the second side was reduced is the same as the ratio in which the first side was reduced. If the scale factor is more than 1 then new shape is larger and if the scale factor is less than 1 then the new figure is smaller than the original figure.

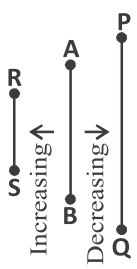


Figure - 4

Now look at figure 4. On increasing the size of $\overline{AB}$ we obtain $\overline{PQ}$ and by reducing the line segment we obtain $\overline{RS}$ Assume that the scale factor is x.

(i) Dilation/Enlargement

PQ $= x$ (AB) (since x is scale factor)

$$\frac{PQ}{AB} = x$$

Because PQ > AB, therefore $x>1$

Hence, to increase the size of any figure the scale factor must be bigger than 1.

(ii) Reduction

RS $= x(\text{AB})$ (Because x is the scale factor)

$$\frac{\text{RS}}{\text{AB}} = x$$

Because RS < AB, therefore $x < 1$

Clearly, the scale factor must be less than 1 to reduce the figure.

Try These

1. What should be the scale factor to change a line segment of length 12 cm into 36 cm and similarly change line segment of 12 cm into 6 cm?

Mapping and Scaling

In drawing the maps of villages, districts, states and countries, we have to depict a large area on paper. Different scales are taken in mapping. If in the map of Chhattisgarh state, the scale is given as l cm : 50,00,000 cm or 1 cm : 50 km, what can we conclude from this information?

According to Rohit, if 1 cm : 50 km is written on the map then it means that 1 cm on the map represents 50 km in reality. Therefore, 2 cm represents, $50 \times 2 = 100$ km and 40 cm represents $50 \times 4 = 200$ km.

Do you agree with Rohit? How much is the scale factor here? Discuss with your friends.

Think & Discuss

1. What size of scale you would take to make the map of your village? Why?
2. (i) You have to draw the map of India (20 cm × 20 cm), which is given in your book onto a wall (3 m × 2 m.). Can you make it by using the scale factor 1 m = 12 cm?
   (ii) If not then why not?
   (iii) What maximum scale factor should be taken to draw the map on a wall of size 6 m × 4 m?
3. To make the map of your district and tehsil, which scale will you take and why?

Exercise - 1

1. In the map of a farm the scale is 1 cm to 1 m. In the map, the area of farm is shown as 3 cm × 4 cm. Find out the actual area of the farm in square meters?
2. You have a square shaped painting of area 3600 square cm. We will take scale factor 0.1 to scale the painting. Find the measure of one side after scaling the painting.
3. In city map, the distance between the Railway station and Airport is given as 3 cm. If the scale of map is 2 cm : 7 km then find out the real distance between the Airport and Railway Station?

Similarity in Squares and Circles

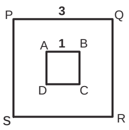


Figure - 5

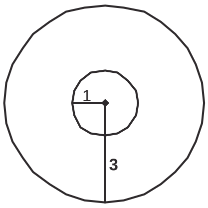


Figure - 6

In this part, we will discuss similarity in squares, circles, parallelograms and triangles. 2 squares of sides 1 cm and 3 cm are shown in figure-4. Are both squares similar?

Because the measure of each angle of every square is 90°, their angles are equal and each side of a square equals the others. Therefore, in both the squares all sides are in the same proportion. Therefore, both the squares are similar. Now, we see that by increasing each side of the square ABCD 3 times, we got the square PQRS. It means that here the scale factor is 3.

Now, we are given 2 circles in figure-5. Radius of one circle is 1 cm and that of the other circle is 3 cm. Are both circles similar?

We can make the circle of 3 cm by expanding the radius of circle of 1 cm. Or, by reducing the radius of circle of 3 cm we can make the circle of radius 1 cm. Hence, we can say that both circles are similar.

Draw circles having different radii in your notebook and check whether they are similar or not.

Think & Discuss

- Are all square similar?
- Are all circles similar?

Similarity in other shapes

Circle and square are two different kinds of shapes and they can be determined by fixing only one component, the radius or the side respectively. During scaling we have to maintain the shape and in these two shapes the properties of similarity are present. But not in other shapes. The parameters to check for similarity are different for different shapes.

Then how can we identify whether two shapes are similar or not. For this, it will be useful to determine and fix some special properties of similar shapes.

Similarity in Polygons

When we say that 2 polygons are similar it means that when we reduce or enlarge by a fixed scale factor, all corresponding sides increase or decrease in the same proportion. Hence, it is necessary **for similarity in polygons that all corresponding angles are equal and all corresponding sides are in same proportion.**

Now, we will study about similarity in triangles and between rectangles and also discuss about the methods to check for similarity.

Test for Similarity in Rectangles

See the figure of a rectangle drawn on graph paper (figure-7). If rectangle ABCD is the original figure, then are rectangles PQRS and LMNO similar to the original figure? They appear to be similar but we have to find out whether these shapes are actually similar or not.

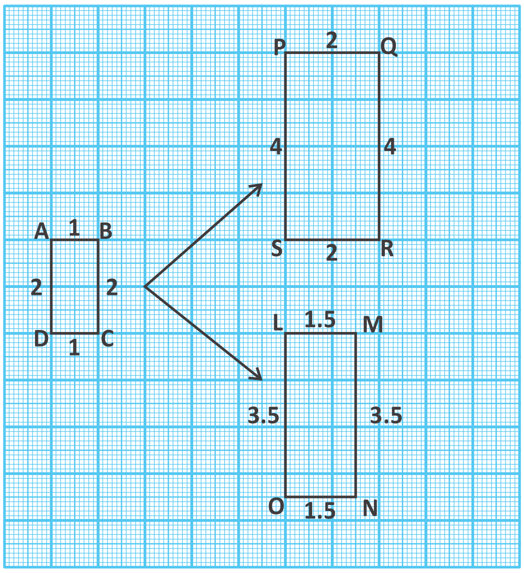


Figure - 7

Each angle of a rectangle is equal to 90°. Hence, the angles of all rectangles are equal. Since the sides opposite to each other in a rectangle are equal therefore to check similarity between rectangles, we need to check only the ratio of two adjacent sides, not all 4 sides. Complete the table given below by looking at figure-7.

Table - 1

Lengths of Sides		**Ratio of Adjacent Sides**
Rectangle ABCD	**Rectangle PQRS**	
AB = 1	PQ = 2	$\frac{PQ}{AB} = \frac{2}{1}$
BC = 2	QR = 4	$\frac{QR}{BC} = \frac{4}{2} = \frac{2}{1}$

Ratio of adjacent sides of rectangles PQRS and ABCD are same. This ratio

$$\frac{PQ}{AB}=\frac{QR}{BC}=2$$

$$\therefore \frac{PQ}{AB}=\frac{QR}{BC}=\frac{RS}{CD}=\frac{SP}{DA}=2$$

Thus, the ratios of all corresponding sides of rectangle PQRS and ABCD are equal. Here, the scale factor is 2. Side PQ is twice of side AB and both rectangles are similar. We can write it as follows: rectangle ABCD ~ rectangle PQRS, where '~' this is the symbol for similarity.

Table - 2

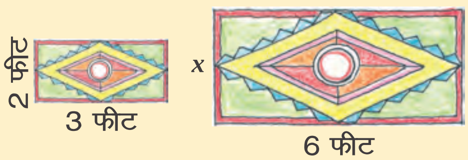
AB= 1	LM = 1.5	$\frac{LM}{AB}=\frac{1.5}{1}$
BC = 2	MN = 3.5	$\frac{MN}{BC}=\frac{3.5}{2}=\frac{1.75}{1}$

Now look at table-2 and compare the rectangles ABCD and LMNO. Is rectangle ABCD similar to rectangle LMNO?

Here, the ratio of one of the corresponding sides of the two rectangles 1.5 is and ratio of other pair of adjacent sides is 1.75. Since corresponding sides are not in the same ratio therefore rectangle ABCD and rectangle LMNO are not similar.

Try These

The adjacent figure depicts two blankets. If both blankets are similar, then:-



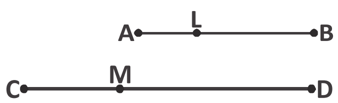
(i) What is the scale factor?

(ii) Find the value of x.

(iii) What is the ratio of the perimeter to the area of the rectangle?

Division of Line Segment in Fixed Proportion

Point L and M are present on line segment $\overline{AB}$ and $\overline{CD}$. If $\frac{AL}{LB}=\frac{CM}{MD}$ then we can say



that, line segments $\overline{AB}$ and $\overline{CD}$ are respectively divided by L and M in the same ratio. We can use this property to check similarity between triangles.

Theorem-1 : If we draw a line which is parallel to any side of a triangle and intersects the other two sides at different points, then this line divides these two lines in the same ratio.

Proof : We are given triangle ABC in which the line DE is parallel to BC, and intersects the two sides AB and AC at points D and E.

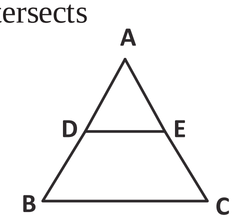


Figure - 8 (i)

We have to prove that : $\dfrac{AD}{DB}=\dfrac{AE}{EC}$

Join B to E and C to D and draw $DM \perp AC$ and $EN \perp AB$.

Because the area of $\Delta ADE = \dfrac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times AD \times EN$$

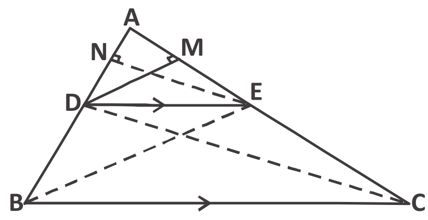


Figure - 8 (ii)

Therefore, we can write the area of ΔADE as ar (ADE)

Therefore $\quad ar\,(ADE) = \dfrac{1}{2} \times AD \times EN$

and $\quad ar\,(BDE) = \dfrac{1}{2} \times DB \times EN$

Similarly, $\quad ar\,(ADE) = \dfrac{1}{2} \times AE \times DM$ and $ar\,(DEC) = \dfrac{1}{2} \times EC \times DM$

Therefore $$\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \text{..............} (1)$$

and $$\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \text{...............} (2)$$

Note that, ΔBDE or ΔDEC are two triangles sharing the same base DE and drawn between the same parallel lines BC and DE.

Therefore, $ar(BDE) = ar(DEC)$ .... (3)

Hence, from (1), (2) and (3)

$\dfrac{AD}{DB}=\dfrac{AE}{EC}$ (this is a basic theorem of proportionality)

Converse of this theorem can also be proved. Let us see:-

Theorem-2: If a line divides any two sides a triangle in the same ratio then this line is parallel to the third side.

Proof: To prove this theorem we can select a line PQ such that:

$\frac{AP}{PB} = \frac{AQ}{QC}$ And then assume the opposite, that is, we assume that PQ is not parallel to BC.

If PQ is not parallel to side BC, then some other line will be parallel to BC.

Let PQ' is one such line which is parallel to BC.

Therefore, $\frac{AP}{PB} = \frac{AQ'}{Q'C}$ (By basic proportionality theorem)

But $\frac{AP}{PB} = \frac{AQ}{QC}$

Thus $\frac{AQ}{QC} = \frac{AQ'}{Q'C}$

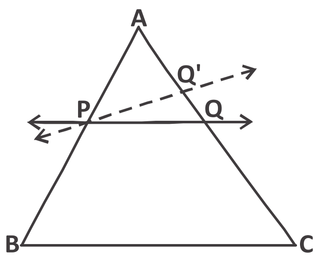


Figure - 9

Adding 1 to both sides

$$\frac{AQ}{QC} + 1 = \frac{AQ'}{Q'C} + 1$$

$$\frac{AQ+QC}{QC} = \frac{AQ'+Q'C}{Q'C}$$

$$\therefore \frac{AC}{QC} = \frac{AC}{Q'C} \qquad \text{Hence, } QC = Q'C$$

But this is possible only if Q and Q' are one and same point.

Therefore, PQ || BC

Let us see some more examples based on this theorem.

Example-1. If line intersects sides AB and AC of ΔABC at points D and E respectively and is parallel to BC then prove that $\frac{AD}{AB} = \frac{AE}{AC}$ 1

Solution : DE || BC (Given)

Therefore, $\dfrac{AD}{DB}=\dfrac{AE}{EC}$

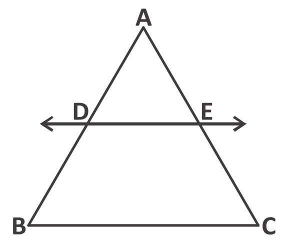


Figure - 10

i.e. $\dfrac{DB}{AD}=\dfrac{EC}{AE}$

Or $\dfrac{DB}{AD}+1=\dfrac{EC}{AE}+1$

Or $\dfrac{DB+AD}{AD}=\dfrac{EC+AE}{AE}$

Or $\dfrac{AB}{AD}=\dfrac{AC}{AE}$

Hence, $\dfrac{AD}{AB}=\dfrac{AE}{AC}$

Example-2. QRST is a trapezium, in which QR || TS Points E and F are located on non-parallel sides QT and RS in such a manner that EF is parallel to side QR. Show that $\dfrac{QE}{ET}=\dfrac{RF}{FS}$.

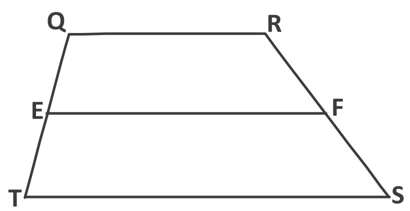


Figure - 11 (i)

Solution : Join Q to S such that QS intersects EF at point G.

QR || TS and EF || QR (Given, see figure-10(ii))

Therefore EF || TS (Two lines parallel to the same line are also mutually parallel)

Now, in ΔQTS, EG || TS (Because EF || TS)

Therefore, $\dfrac{QE}{ET}=\dfrac{QG}{GS}$ ...... (1)

Similarly, in DSQR

$\dfrac{GS}{QG}=\dfrac{FS}{RF}$ which means $\dfrac{QG}{GS}=\dfrac{RF}{FS}$ ...... (2)

By equations (1) and (2) $\dfrac{QE}{ET}=\dfrac{RF}{FS}$

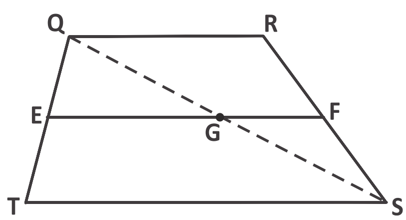


Figure - 11 (ii)

Exercise-2

1. The radius of a circular field is 52 meter. Draw the map of this field on paper in which the scale is 13 m : 1 cm. What is the radius of the field on the map on paper?

2. The measure of 2 corresponding sides of any rectangle are 5 cm and 7.5 cm. Calculate the area and length of sides of new rectangles formed by taking the new scale factors shown below:

   (i) 0.8 (ii) 1.2 (iii) 1.0

   If scale factor of 1 is taken then will the new rectangle formed be congruent to the original figure?

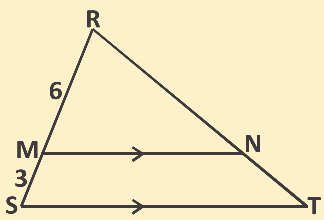


Figure - 12 (i)

3. In figure-12 (i), $MN \parallel ST$ then find the value of following:-

   (i) $\dfrac{TN}{NR}$ (ii) $\dfrac{TR}{NR}$ (iii) $\dfrac{TN}{RT}$

4. By using the basic proportionality theorem prove that the line which is drawn from the midpoint of a side of triangle, parallel to another side, bisects the remaining side?

5. In figure-12 (ii), $DF \parallel AE$ and $DE \parallel AC$

   Prove that $\dfrac{BF}{FE} = \dfrac{BE}{EC}$.

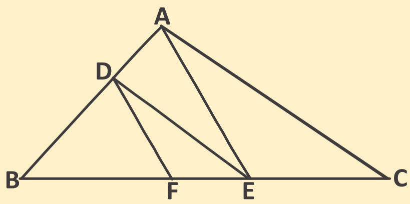


Figure - 12 (ii)

6. In ΔPQR points E and F are located on sides PQ and PR respectively. Then for the given conditions find out whether $EF \parallel QR$:

   (i) PE = 3.9 *cm*, EQ = 3 *cm*, PF = 3.6 *cm* and FR = 2.4 *cm*.

   (ii) PE = 4 *cm*, QE = 4.5 *cm*, PF = 8 *cm* and RF = 9 *cm*.

   (iii) PQ = 1.28 *cm*, PR = 2.56 *cm*, PE = 0.18 *cm* and PF = 0.36 *cm*.

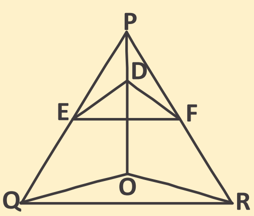


Figure - 12 (iii)

7. In figure-12 (iii), $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.

8. In figure-12 (iv), the points A, B and C are located respectively on OP, OQ or QR in such a way that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.

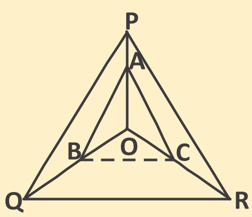


Figure - 12 (iv)

Note : If needed draw a figure to solve the question. It makes it easier to find the solutions.

Similarity in Parallelograms

Are the conditions/properties used to test for similarity in rectangles sufficient to check for similarity in parallelograms? Obviously these are not enough, because in parallelograms all angles are not equal. Therefore, we have to look for one more theorem.

Theorem-3. If in 2 parallelograms corresponding angles are equal, then their corresponding sides are in the same ratio. Hence, such parallelograms are similar.

Proof : According to the statement given in theorem-3, select 2 parallelograms ABCD and PQRS,

Where, $\angle A=\angle P, \angle B=\angle Q$

And, $\angle C=\angle R, \angle D=\angle S$ (Figure-13(i), (ii))

In the parallelogram PQRS join S to Q and take two points A' and C' on PS and SR respectively, such that-

$AD=A'S$, $DC=SC'$

And $\angle DAB=\angle SA'B'$ then join B' to C'

Now, $\triangle A'SB'$ in $\angle SPQ=\angle SA'B'$ (Given)

$\therefore$ $A'B' \parallel PQ$ (Transversal line PS intersects the lines PQ and A'B' and the corresponding angles made are equal).

Now, $A'B' \parallel PQ \parallel SR$ in $\triangle PSQ$, then by basic proportionality theorem-

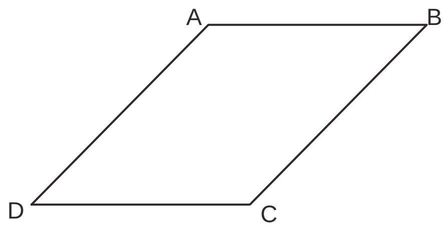


Figure - 13 (i)

$$\frac{PS}{A'S}=\frac{PQ}{A'B'}=\frac{QS}{B'S} \text{ (Why?)}$$

$$\frac{PS}{AD}=\frac{PQ}{AB}=\frac{QS}{B'S} \text{ (Why?)} \quad ....(1)$$

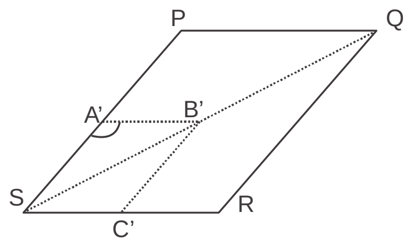


Figure - 13 (ii)

In the same way, in $\triangle SQR$ $\frac{SR}{SC'}=\frac{QR}{B'C'}=\frac{QS}{B'S}$

Or $\frac{SR}{CD}=\frac{QR}{BC}=\frac{QS}{B'S}$ (Why?) ....(2)

From (1) and (2)

$$\frac{PQ}{AB}=\frac{QR}{BC}=\frac{SR}{CD}=\frac{PS}{AD} \text{ (Why?)}$$

Hence, if we assume that the corresponding angles of 2 parallelograms are equal, then we find that their 4 corresponding sides are in the same ratio.

Will the converse also be true?

Theorem-4. If corresponding sides of two parallelograms are in the same ratio and their corresponding angles are equal then these parallelogram are similar.

Proof : Prove the statement given above.

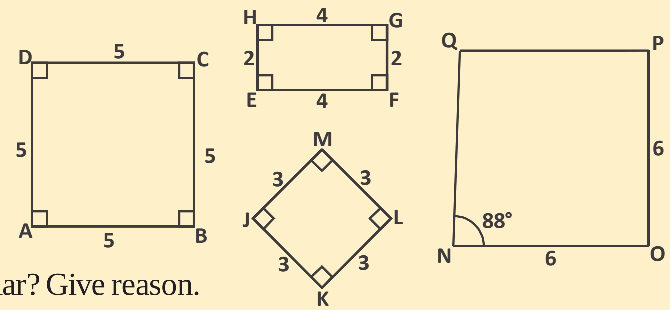
Conclusion :

We can conclude from the two theorems given above that for similarity it is sufficient that any one of the two conditions - that is, (i) corresponding angles are equal (ii) corresponding sides are in same ratio - is satisfied. In similarity between parallelograms it is not required that both the criteria be satisfied because fulfillment of one criteria implies that the other is also satisfied.

Similarly, we can test for similarity in other pairs of polygons (parallelogram, pentagon, etc).

Try These

1. Are given parallelograms similar? Explain your answers –
   (i) ABCD and EFGH
   (ii) ABCD and JKLM
   (iii) ABCD and NOPQ
   (iv) JKLM and NOPQ



2. Are EFGH and JKLM similar? Give reason.

3. Draw a parallelogram similar to EFGH?

Are congruent figures also similar?

Let us understand the relation between similarity and congruency.

2 parallelograms EFGH and WXYZ are congruent which means that EFGH ≅ WXYZ, and therefore their corresponding adjacent sides and corresponding angles are equal.

$$\therefore \text{ EF = WX, FG = XY, GH = YZ and HE = ZW}$$

$$\text{Or } \frac{EF}{WX}=1, \frac{FG}{XY}=1, \frac{GH}{YZ}=1 \text{ and } \frac{HE}{ZW}=1$$

$$\therefore \frac{EF}{WX}=\frac{FG}{XY}=\frac{GH}{YZ}=\frac{HF}{ZW}=1$$

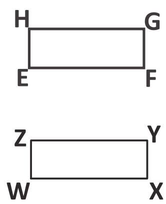


Figure - 14

Clearly, the sides of both the quadrilaterals are in the same proportion, therefore these quadrilaterals are similar. This means that both the conditions of similarity are satisfied in congruency.

Think & Discuss

Are all similar figures, congruent? Explain with reasons.

Relation between Perimeters of Similar Shapes

If two shapes are similar, can we tell the relation between the perimeters of these shapes? Suppose, two similar polygons are given to us in which the scale factor is m. According to the properties of similarity,

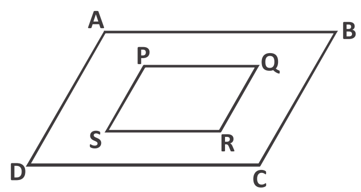
$$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{CD}{RS}=\frac{DA}{SP}=m \text{ (Adjacent sides are in the same proportion).....(1)}$$

AB = mPQ .....(2)

BC = mQR .....(3)

CD = mRS .....(4)

And DA = mSP .....(5)



Let us find their perimeters:

Perimeter of polygon PQRS = AB + BC + CD + DA .....(6)

And, perimeter of polygon PQRS = PQ + QR + RS + SP .....(7)

From (6) and (7)

$$\frac{\text{Perimeter of polygon ABCD}}{\text{Perimeter of polygon PQRS}} = \frac{AB+BC+CD+DA}{PQ+QR+RS+SP}$$

From (2), (3), (4) and (5)

$$\frac{\text{Perimeter of polygon ABCD}}{\text{Perimeter of polygon PQRS}} = \frac{m(PQ+QR+RS+SP)}{(PQ+QR+RS+SP)}$$

$$\frac{\text{Perimeter of polygon ABCD}}{\text{Perimeter of polygon PQRS}} = m = \frac{AB}{PQ}=\frac{BC}{QR}=\frac{CD}{RS}=\frac{DA}{SP} \qquad \text{From (1)}$$

It means that the ratio of the perimeters of 2 similar polygons is equal to the ratio of their corresponding sides and their scale factor.

Example-3. If quadrilateral ABCD ~ quadrilateral PQRS, then:-

(i) What is the scale factor? (that of quadrilateral ABCD with PQRS)

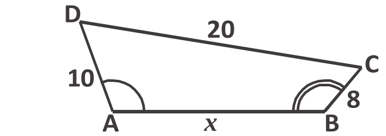
(ii) Find the value of x, y and z.

(iii) What is the perimeter of the quadrilateral ABCD?

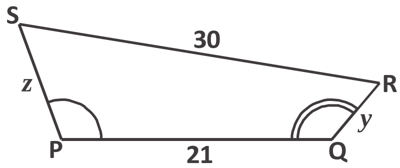
(iv) What is the ratio of the perimeters of the two quadrilaterals?

Solution : (i) Ratio of adjacent sides:-

$$\frac{CD}{RS} = \frac{20}{30} = \frac{2}{3} \text{ (Scale factor)}$$



(ii) Because the given quadrilaterals are similar therefore their adjacent sides are in same proportion:-



$$\frac{CD}{RS} = \frac{AB}{PQ}$$

$$\therefore \frac{2}{3} = \frac{x}{21}$$

$$x = 14$$

$$\text{And } \frac{CD}{RS} = \frac{BC}{QR}$$

$$\frac{2}{3} = \frac{8}{y}$$

$$y = 12$$

$$\frac{CD}{RS} = \frac{AD}{PS}$$

$$\frac{2}{3} = \frac{10}{z}$$

$$z = 15$$

(iii) Perimeters of quadrilaterals ABCD is-

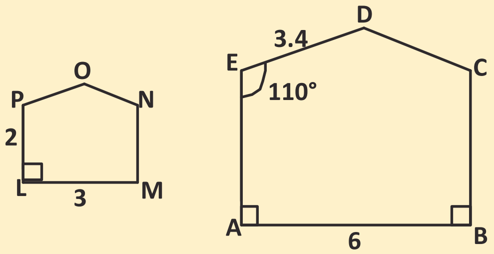
$10 + 20 + 8 + 14 = 52$ units

(iv) Ratio of perimeters of the two quadrilaterals is,

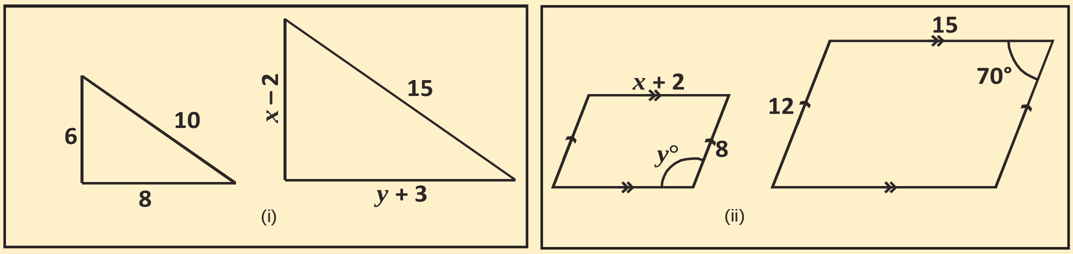
$$\frac{\text{Perimeter of quadrilateral ABCD}}{\text{Perimeter of quadrilateral PQRS}} = \frac{2}{3}$$ which is equal to the scale factor.

Exercise - 3

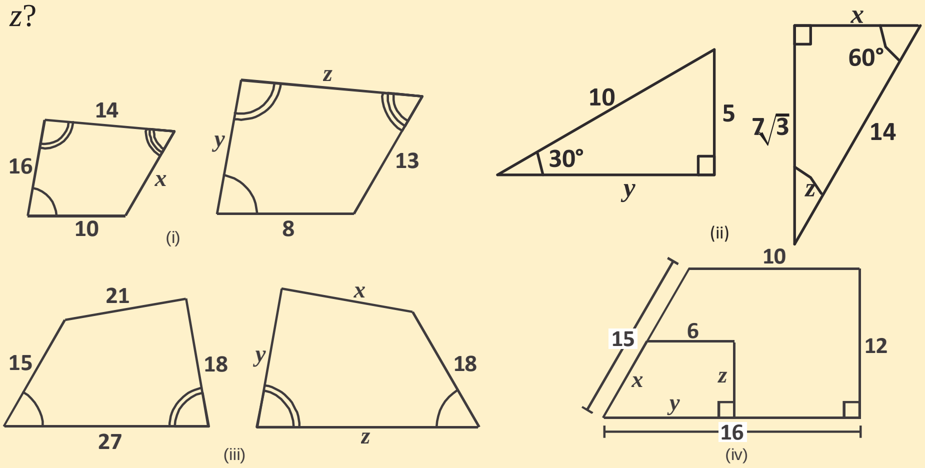
1. If in the given figure polygons are similar then find the value of the following:-
   (i) OP
   (ii) EA
   (iii) m $\angle$OPL
   (iv) m $\angle$LMN
   (v) Polygon DEABC ~ ?
   (vi) The statement polygon BCDEA ~ polygon NOPLM is not correct. Correct it.



2. If in figure-(i) triangle are similar and in figure-(ii) quadrilaterals are similar then find the value of x and y.



3. If in the given figure each pair of polygons is similar, then find the value of x, y and z?



4. The lengths of the sides of a quadrilateral are 4 *cm*, 6 *cm*, 6 *cm* and 8 *cm*. The lengths of the sides of another quadrilateral, which is similar to the first quadrilateral, are 6 *cm*, 9 *cm*, 9 *cm* and 12 *cm*.
   (i) What is the scale factor? (The second quadrilateral from the first quadrilateral)
   (ii) Find the perimeter of both the quadrilaterals.
   (iii) What is the ratio of their perimeters? (The first quadrilateral from the second quadrilateral)

5. Give examples for the following and give reasons:-
   (i) If two polygons are congruent, then they are similar as well.
   (ii) If two polygons are similar, then it is not necessary that they are congruent.
   (iii) Think and write three more statements.

How to check for similarity in triangles?

So far we have seen that two conditions/criteria for proving that any two triangles are similar. They are:-

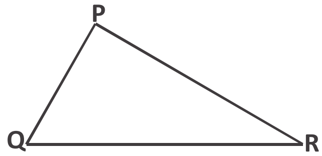


Figure - 15 (i)

(i) Corresponding angles should be equal

$\angle P = \angle X$, $\angle Q = \angle Y$, $\angle R = \angle Z$

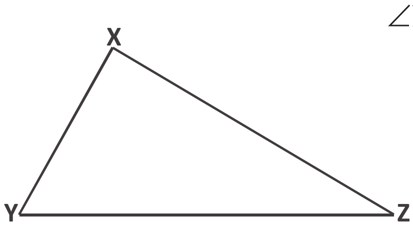


Figure - 15 (ii)

(ii) Ratio of corresponding sides are similar:-

$$\frac{PQ}{XY} = \frac{QR}{YZ} = \frac{RP}{ZX}$$

Then $\Delta PQR \sim \Delta XYZ$

If any one condition is satisfied then we can say that both triangles are similar.

Angle-Angle-Angle (AAA) similarity criterion: If in two triangles, the corresponding angles are equal then their corresponding sides are in the same ratio and thus the two triangles are similar. This is called the angle-angle-angle (AAA) similarity criterion.

Let see what minimum conditions can fulfill these criteria.

Angle-Angle (AA) similarity: If two angles of one triangle are equal to two angles of another triangle respectively then the two triangles are similar.

This is the Angle-Angle criterion for similar triangles.

By using this criterion, we will prove two other criteria, SAS and SSS.

Theorem-5. **Side-Angle-Side (S-A-S) similarity theorem :** If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, then the two triangles are similar.

Proof : To prove this theorem, we take two triangles ABC and DEF in which

$\frac{AB}{DE} = \frac{AC}{DF}$ (<1) which means that DE is greater than AB and $\angle A = \angle D$

We take two points P and Q on DE and DF in such a way that DP = AB and DQ = AC.

Now join P to Q.

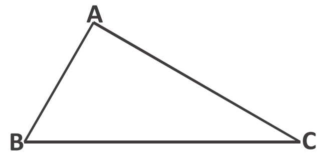


Figure - 16 (i)

$$\frac{DP}{DE}=\frac{DQ}{DF} \qquad \left(\because \frac{AB}{DE}=\frac{AC}{DF}\right)$$

$\therefore$ PQ || EF (By basic proportionality theorem)

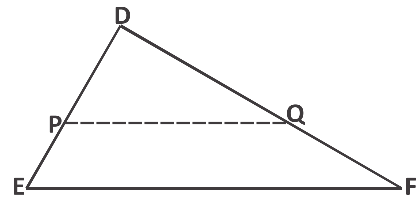


Figure - 16 (ii)

Hence, $\angle P = \angle E$ और $\angle Q = \angle F$ (Why?)

Hence, $\Delta ABC \cong \Delta DPQ$

(AB = DP; AC = DQ and

$\angle BAC = \angle PDQ$)

$\therefore$ $\angle B = \angle E$ और $\angle C = \angle F$

According to Angle-Angle similarity criterion, ΔABC and ΔDEF are similar.

Hence, $\Delta ABC \sim \Delta DEF$

Theorem-6. **Side-Side-Side (S-S-S) Similarity Theorem :** If in 2 triangles sides of one triangle are proportional to the sides of the second triangle then both these triangles are similar.

Proof : To prove this theorem, we take two triangles ABC and DEF in which

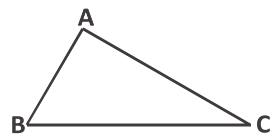


Figure - 17

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$$

In ΔDEF, we select two points P and Q on sides DE and DF such that DP = AB and DQ = AC. Join P to Q.

Here, $\frac{DP}{DE}=\frac{DQ}{DF}$ $\left(\because \frac{AB}{DE}=\frac{CA}{FD}\right.$ and DP = AB, DQ = AC$\left.\right)$

$\therefore$ PQ || EF (Why?) (Theorem-2)

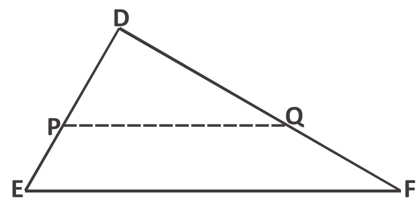


Figure - 18

Thus $\angle P = \angle E$ and $\angle Q = \angle F$

$\therefore$ $\Delta DEF \sim \Delta DPQ$ (Angle-Angle similarity)

We know that $\Delta ABC \cong \Delta DPQ$ (From the given construction)

$\therefore$ $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

Hence, $\Delta ABC \sim \Delta DEF$ (From Angle-Angle similarity)

Example-4. If $PQ \parallel RS$, then prove that $\Delta POQ \sim \Delta SOR$ (Figure-19)

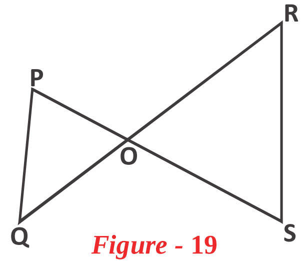


Figure - 19

Solution : $PQ \parallel RS$ (Given)

$\angle P = \angle S$ (Alternet Angles)

And $\angle Q = \angle R$ (Alternet Angles)

Also, $\angle POQ = \angle SOR$ (Vertically opposite angles)

$\therefore$ $\Delta POQ \sim \Delta SOR$ (Angle-Angle-Angle similarity criterion)

Example-5. A girl of height 90 *cm* is walking away from the base of the lamp post at a speed of 1.2 *m/s*. If the lamp bulb is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Solution : Let height of the bulb of lamp-post be AB and height of girl be CD. From the figure, you can see that DE is the shadow of the girl.

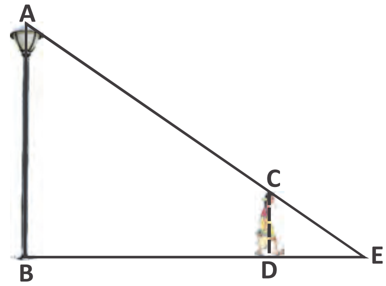


Figure - 20

Let DE = x meter

Because distance = Speed × Time

Therefore, BD = $1.2 \times 4 = 4.8\ m$

In ΔABE and ΔCDE,

$\angle B = \angle D$ (Each is of 90°, because lamp-post as well as the girl are standing vertical on the ground)

$\angle E = \angle E$ (Same angle)

So, $\Delta ABE \sim \Delta CDE$ (AA similarity criterion)

Therefore, $\dfrac{BE}{DE} = \dfrac{AB}{CD}$ (Corresponding sides of similar triangles)

$$\frac{4.8 + x}{x} = \frac{3.6}{0.9} \quad \text{(Because } 1\ m = 100\ cm\text{)}$$

$$4.8 + x = 4x$$

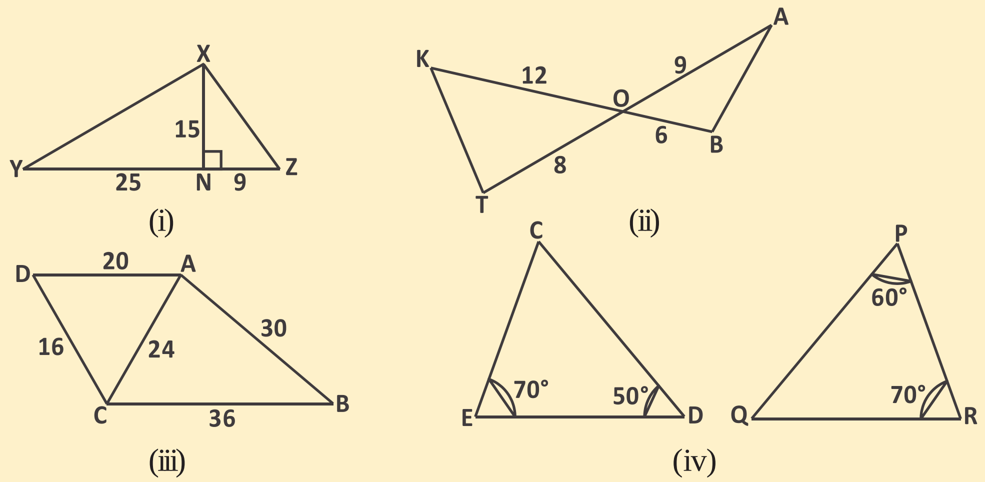
$$3x = 4.8$$

$$x = 1.6$$

So, the shadow of the girl after walking 4 seconds will be 1.6*m* long.

Try These

1. Check for similarity in the triangles given below and explain which criterion is used:-



(i)	ΔYXN and ΔXNZ	(ii)	ΔOAB and ΔOKT
(iii)	ΔADC and ΔACB	(iv)	ΔCED and ΔPRQ

Relations Between the Areas of Similar Triangles

In similar polygons we saw that the ratio of their perimeters is equal to the ratio of their corresponding sides. Then in two triangles ABC and PQR

$$\frac{\text{Perimeter } \Delta ABC}{\text{Perimeter } \Delta PQR} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

Or $$\frac{AB+BC+CA}{PQ+QR+RP} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

Is their any relation between the areas of these triangles and their corresponding sides?

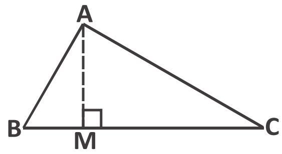
We will see the relation in the next theorem.

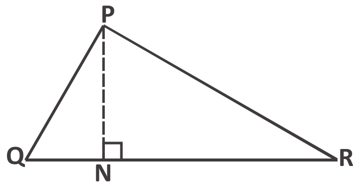
Theorem-7. The ratio of the area of 2 similar triangles is equal to the ratio of their corresponding sides.

Proof : We are given two triangles such that $\Delta ABC \sim \Delta PQR$

We have to prove that:

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$





To calculate the area of both the triangles, we will draw their altitudes AM and PN respectively.

Now, $ar(ABC) = \frac{1}{2} \times BC \times AM$ and

$$ar(PQR) = \frac{1}{2} \times QR \times PN$$

$$\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN} \quad \text{.............} \quad (1)$$

Now, in ΔABM and ΔPQN

$\angle B = \angle Q$ $\quad (\Delta ABC \sim \Delta PQR)$

$\angle M = \angle N$ $\quad$ (Each angle is 90°)

$\therefore \Delta ABM \sim \Delta PQN$ (Angle-Angle similarity)

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ .... (2)

We know that

$\Delta ABC \sim \Delta PQR$ (Given)

Then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ .... (3)

From equation (1) and (3)

$$\frac{ar(ABC)}{ar(PQR)} = \frac{AB}{PQ} \times \frac{AM}{PN}$$

From equation (2)

$$\frac{ar(ABC)}{ar(PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ} = \left(\frac{AB}{PQ}\right)^2$$

Thus, from equation (3) $\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$

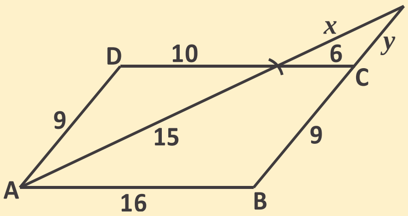
Try These

1. If the ratio of the area of two similar triangles is 25:9 then what will be the ratio of their corresponding sides?
2. If triangle TFR and triangle SPM are similar and their scale factor is 7:4 then what is the ratio of their areas?
3. $\Delta PQR \sim \Delta XYZ$, Where $PQ = 3XY$.

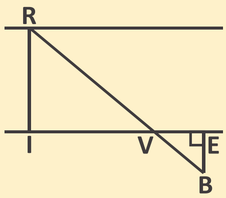
   What is the ratio of their areas?

Exercise-4

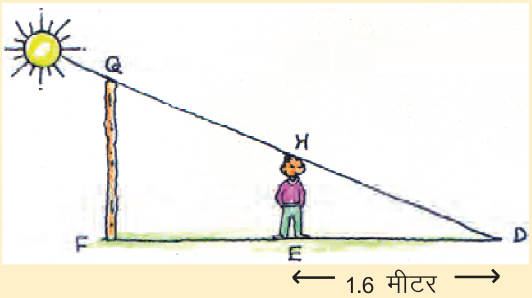
1. ABCD is a parallelogram. Find the values of x and y.



2. We are given a trapezium ABCD in which AB||DC and its diagonals intersect at point O. If AB = 2CD, then find the ratio of the area of triangle AOB and COB.
3. In the given figure if IV=36 meter, VE = 20 meter and EB = 15 meter, then what is the width of the river?

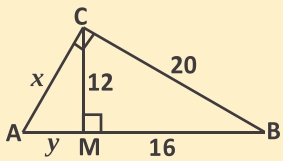


4. If the areas of two similar triangles are equal to each other, then prove that the triangles are congruent.
5. Prove that the ratio of the area of two similar triangles is the square of the ratio of their corresponding medians.
6. Sahid is trying to estimate the height of a pole. He is standing in such a manner that the shadow of his head 'H' and the shadow of the top of the pole 'Q' fall on the same point, D. If DE = 1.6 m and DF = 4.4 m then what is the height of the pole?



7. (i) In the figure, which two triangles are similar to the triangle ABC? Name them.

   (ii) Find the value of x and y.



8. ABC and BDE are two equilateral triangles such that D is the midpoint of side BC. Ratio of area of triangles ABC and BDE is:-

   (i) 2:1 (ii) 1:2 (iii) 4:1 (iv) 1:4

9. If in two similar triangles 9ar (ABC) = 16ar (PQR), then the value of $\frac{AB}{PQ}$ is:-

   (i) 4:3 (ii) 16:3 (iii) 3:4 (iv) 9:4

Pythagoras Theorem

In your earlier classes you solved many problems by using the Pythoagoras Theorem. You also verified this theorem through some activities. Can we prove this theorem by using the concept of similarity of triangles? Let us see –

Theorem-8. If a perpendicular is drawn on the hypotenuse from the vertex of the right angle of a right angle triangle then triangles on both sides of the perpendicular are similar to the original triangle as well as to each other.

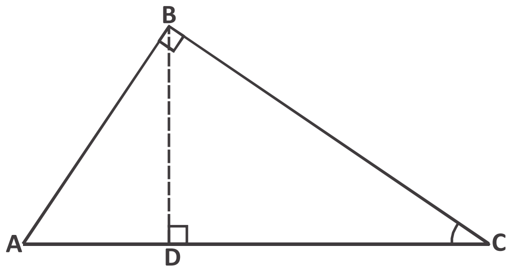
Proof : **Given** : Triangle ΔABC which is right angled at B and BD is perpendicular to the hypotenuse AC.

We have to prove that-

(i) $\triangle ADB \sim \triangle ABC$

(ii) $\triangle BDC \sim \triangle ABC$

(iii) $\triangle ABD \sim \triangle DBC$



We may note that in ΔADB and ΔABC

$\angle A = \angle A$ (Common angle)

$\angle ADB = \angle ABC$ (both angles are 90°)

$\therefore$ $\Delta ADB \sim \Delta ABC$ (How?) .... (1)

Similarly in ΔBDC and ΔABC

$\angle C = \angle C$ and $\angle BDC = \angle ABC$ (Why?)

Therefore, $\Delta BDC \sim \Delta ABC$ .... (2)

From (1) and (2)

$\Delta ADB \sim \Delta BDC$ .... (3)

(If any two triangles are similar to a third triangle then both the triangles are also similar to each other)

Now, we will prove the Pythagoras Theorem by using this theorem.

Theorem-9. In a right angle triangle, the square of the hypotenuse is equal the sum of the squares of the other two sides.

Proof : We are give a right triangle ABC which is right angled at B.

We need to prove that $AC^2 = AB^2 + BC^2$

To prove this theorem, we need to construct or draw a perpendicular BD from the vertex B on side AC.

Now in ΔADB and ΔABC

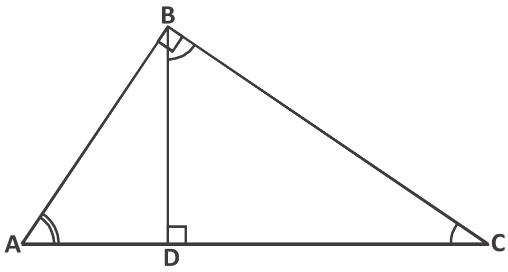
$\angle ADB = \angle ABC = 90°$

$\angle A = \angle A$ (Common angle)

$\therefore \triangle ADB \sim \triangle ABC$ (Angle-Angle similarity)

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ (Proportional sides)

Or $AD.\,AC = AB^2$ .... (1)



Similarly $\Delta BDC \sim \Delta ABC$

$\therefore \frac{CD}{BC} = \frac{BC}{AC}$ (Proportional sides)

Or $CD.\,AC = BC^2$ .... (2)

On adding equations (1) and (2)

$AD.\,AC + CD.\,AC = AB^2 + BC^2$

Or $AC\,(AD + CD) = AB^2 + BC^2$

Or $AC.AC = AB^2 + BC^2$

Or $AC^2 = AB^2 + BC^2$

Can we prove the converse of this theorem?

Theorem-10. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Proof : Prove it on your own.

Let us solve some problems by using these theorems.

Try These

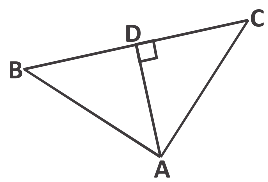
A ladder is placed against a wall such that its foot is at a distance of 2.5 *m* from the wall and its top reaches a window 6 *m* above the ground. Find the length of the ladder.

Example-6. In figure $AD \perp BC$ है।

Prove that $AB^2 + CD^2 = BD^2 + AC^2$ है।

Solution : In ΔADC

$AC^2 = AD^2 + CD^2$ (By Pythagoras Theorem) .... (1)



Now, in ΔADB

$AB^2 = AD^2 + BD^2$ .... (2)

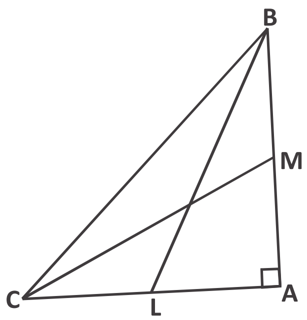
On subtracting (1) from (2)

$AB^2 - AC^2 = BD^2 - CD^2$

Or $AB^2 + CD^2 = BD^2 + AC^2$

Example-7. BL and CM are the medians of a right angle triangle ABC which is right angled at A. Prove that $4(BL^2 + CM^2) = 5\,BC^2$.

Solution : In ΔABC, $\angle A = 90^\circ$ and BL and CM are the medians.



In ΔABC $\quad BC^2 = AB^2 + AC^2$ (Why?)

In ΔABL $\quad BL^2 = AL^2 + AB^2$

$$BL^2 = \left(\frac{AC}{2}\right)^2 + AB^2$$

(L is mid point of AC)

$$BL^2 = \frac{AC^2}{4} + AB^2$$

$$4BL^2 = AC^2 + 4AB^2 \quad \text{............} \quad (2)$$

In ΔCMA $\quad CM^2 = AC^2 + AM^2$

$$CM^2 = AC^2 + \left(\frac{AB}{2}\right)^2$$

(M is the midpoint of AB)

$$4\,CM^2 = 4\,AC^2 + AB^2 \quad \text{............} \quad (3)$$

By adding (2) or (3)

$$4BL^2 + 4CM^2 = AC^2 + 4AB^2 + 4AC^2 + AB^2$$

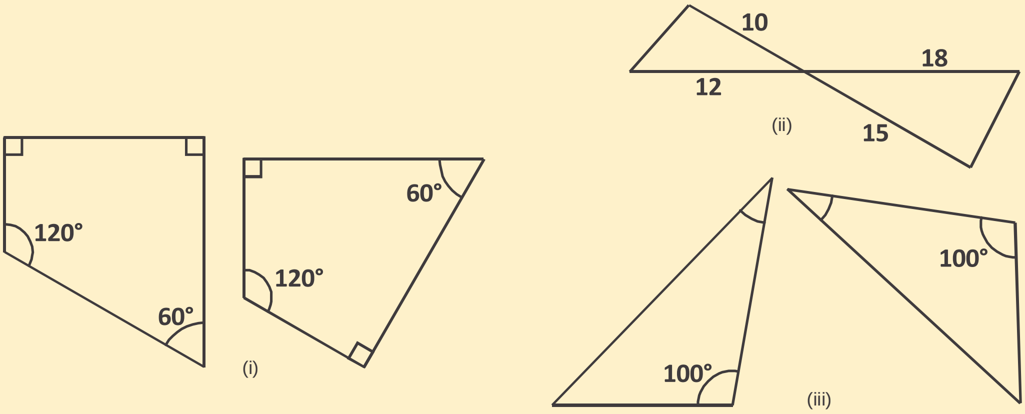
$$4\left(BL^2 + CM^2\right) = 5AC^2 + 5AB^2$$

$$4\,(BL^2 + CM^2) = 5\,(AC^2 + AB^2)$$

$$4\,(BL^2 + CM^2) = 5\,BC^2 \qquad \text{From (1)}$$

Exercise-5

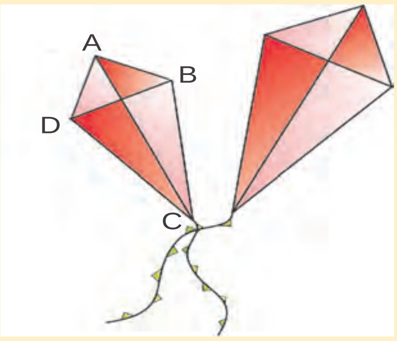
1. Which of the pairs given below is not similar? Why?



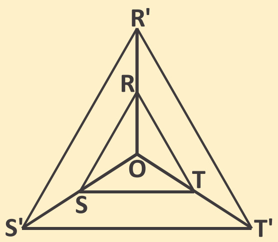
2. Megha made 2 similar kites. The diagonal of the bigger kite is 1.5 times the diagonal of the smaller kite. Then-

   (i) What is the scale factor?

   (ii) Find the length of the diagonal of the bigger kite, given that BD = 40 cm and AC = 68 cm.



3. In a right triangle ΔPQR, P is the right angle and M is the point on QR such that $PM \perp QR$ Show that $PM^2 = QM.MR$.



4. ABC is an equilateral triangle of side $2a$. Find the length of the each altitude.

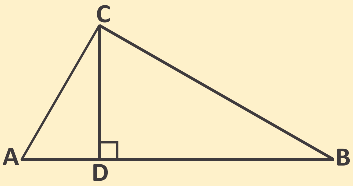
5. ΔABC is an isosceles triangle in which $\angle C = 90^o$. Prove that $AB^2 = 2AC^2$.

6. If in the given figure $OR' = 2.OR$

   $OS' = 2.OS$

   $OT' = 2.OT$

   Then prove taht $\Delta RST \sim \Delta R'S'T'$



7. Triangle ABC is right angled at C. Point 'D' and 'E' are located on sides CA and CB respectively. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

8. In triangle ACB, $\angle ACB = 90^\circ$ and $CD \perp AB$. Prove that $\dfrac{BC^2}{AC^2} = \dfrac{BD}{AD}$

9. The diameter of earth is approximately 8000 miles and that of sun is 864000 miles; the distance between sun and earth is approximately 92 million miles.

   If on paper we take the diameter of earth as 1 inch then what will be the diameter of sun and the distance between the sun and earth on paper? (1 million $=10^6$).

10. If the ratio of the perimeters of two regular hexagon is 5 : 4 then what is the ratio of their sides?

What we have learnt

1. The measurements of 2 similar shapes are in a fixed ratio which is called scale factor.
2. Two polygons having the same number of sides are similar, if

   (i) their corresponding angles are equal, and

   (ii) their corresponding sides are in the same ratio.
3. If a line is drawn parallel to one side of a triangle such that it intersects the other two sides at distinct points then it divides the other two sides in the same ratio.
4. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
5. All congruent polygons are also similar.
6. The ratio of the perimeter of any two similar polygons is same as the ratio of their corresponding sides, or their scale factor.
7. If in two triangles, two angles of one triangle are respectively equal to two angles of other triangle, then two triangles are similar (AA similarity criterion).
8. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio then the two triangles are similar (SAS similarity criterion).
9. If in two triangles corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar.
10. The ratio of the area of two similar triangles is equal to the ratio of their corresponding sides.

11. If in a right triangle a perpendicular is drawn from the vertex of the right angle to its hypotenuse then the triangles on both sides of the perpendicular are similar to the first triangle and also to each other.

12. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

13. If in a triangle, square of one side is equal to the sum of the squares of the other 2 sides, then the angle opposite the first side is a right angle.

ANSWER KEY

Exercise - 1

1. 1200 Square meter 2. $6\ cm$ 3. $10.5\ cm$

Exercise - 2

1. $4\ cm$ 2. (i) $4\ cm$, $6\ cm$, $24\ cm$ (ii) $6\ cm$, $9\ cm$, $54\ cm$

(iii) $5\ cm$, $7.5\ cm$, $37.5\ cm$, Yes 3. (i) $\frac{1}{2}$ (ii) $\frac{3}{2}$ (iii) $\frac{1}{3}$

6. (i) No (ii) Yes (iii) Yes

Exercise - 3

1. (i) 1.7 (ii) 4 (iii) $110°$ (iv) $90°$ (v) Polygon OPLMN

   (vi) Polygon BCDEA is similar to polygon MNOPL

2. (i) $x = 11$, $y = 9$

   (ii) $x = 8$, $y = 110°$

3. (i) $x = 16.25$, $y = 20$, $z = 17.5$

(ii) $x = 7,\ y = 5\sqrt{3},\ z = 30°$

(iii) $x = 25.2, y = 21.6, z = 32.4$ (iv) $x = 9, y = 9.6, z = 7.2$

4. (i) 1.5 (ii) 24 cm, 36 cm (iii) 1.5

Exercise - 4

1. $x = 9, y = \frac{27}{5}$ 2. 4:1 3. 27 meter 6. 3.3 meter

7. (i) Triangle ACM and Triangle CBM (ii) $x = 15, y = 9$

8. (iii) 4:1 9. (i) 4:3

Exercise - 5

1. (i) First is parallelogram, second is not.

   (iii) Since we know the value of only one angle therefore we cannot say anything about similarity.

2. (i) 1.5 (ii) 102 cm and 60 cm

4. $\sqrt{3}a$ 9. 108 $inch$, 11500 $inch$ 10. 5:4