Transformers

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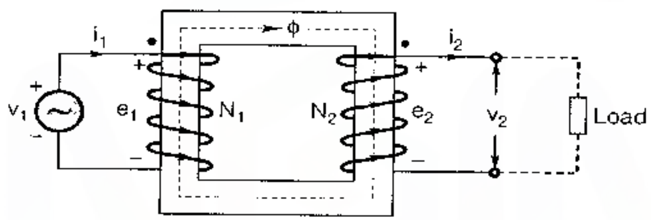
A transformer is a static device which consists of two or more stationary electric circuits interlinked by a common magnetic circuit for the purpose of transferring electrical energy between them while keeping the frequency of operation consant.

Remember:

- Basic principal behind the transformer action is Faraday's law of electromagnetic induction.
- Transformer is a constant frequency and constant power device.

Induced EMF

Direction of induced emf can be found by "Lenz's law" and magnitude of induced emf can be given by "Faraday's law of electromagnetic induction".



❑ RMS value of induced Emf.

In primary winding

$$E_1 = \sqrt{2}\,\pi N_1 f \phi_m \text{ Volts}$$

In secondary winding

$$E_2 = \sqrt{2}\,\pi N_2 f \phi_m \text{ Volts}$$

where, N_1 = Number of turns in primary winding

N_2 = Number of turns in secondary winding

ϕ_m = Maximum value of the magnetic flux, in webers

f = Supply frequency, in Hz

Remember:

- Emf per turn in primary = Emf per turn in secondary
- Compensating primary mmf = Secondary mmf

$$I_1' N_1 = I_2 N_2$$

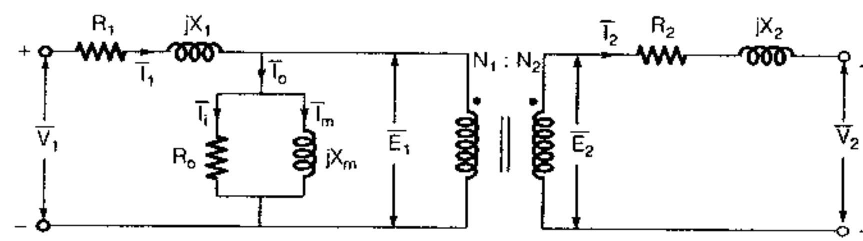
where, I_1' = Load component of primary current I_1

- Primary volt-amperes = Secondary volt-amperes.
- Instantaneous power input into primary is equal to the instantaneous power output from the secondary.
- Step up transformer : $N_1 < N_2$.
- Step down transformer : $N_1 > N_2$.

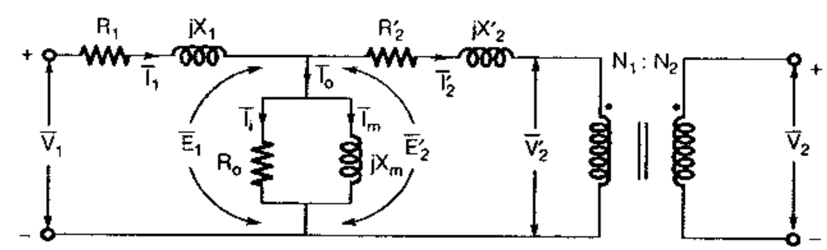
Ideal Transformer

- The transformer core has infinite permeability. So, flux established without any magnetising current.
- The primary and secondary windings have zero resistance. So, there is no ohmic power loss and no resistive voltage drop.
- There is no magnetic leakage flux.
- The core loss considered to be zero.

Equivalent Circuit



Exact equivalent circuit of transformer



Equivalent circuit referred to primary side where, $\bar{V}_1$ = Applied voltage to primary side.

$\bar{V}_2'$ = Secondary terminal voltage referred to primary side.

$\bar{E}_1, \bar{E}_2$ = Induced emf in primary and secondary side.

$\bar{E}_2'$ = Secondary induced emf referred to primary side.

$\bar{I}_o$ = No load current.

$\bar{I}_m, \bar{I}_i$ = Magnetizing and core loss component of exciting current.

R_1, R_2 = Primary and secondary winding resistances.

X_1, X_2 = Primary and secondary winding leakage reactances.

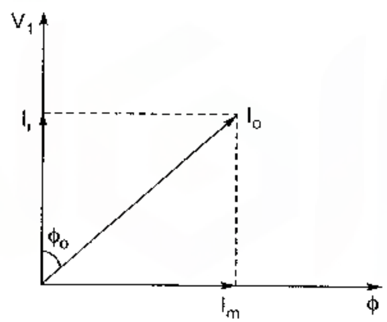
R_2', X_2' = Secondary resistance and leakage reactance referred to primary side.

R_o = Core loss equivalent resistance.

X_m = Magnetising reactance.

Note: ....

At no load, current drawn from the supply is I_o



$\bar{I}_o = \bar{I}_m + \bar{I}_i = (2 \text{ to } 5\% \text{ of } I_{fl})$

$\bar{I}_m = I_o \sin\phi_o$

= Magnetizing current and is responsible for the production of flux.

$\bar{I}_i = I_o \cos\phi_o$

= Core loss current responsible for the active power being drawn from the source to provide hysteresis and eddy current losses.

ϕ_o = No load phase angle (80 to 85°)

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Secondary Side Parameters Referred to Primary Side

- If $\frac{N_1}{N_2} = a$
- Secondary resistance referred to primary side

$$R_2' = a^2 R_2$$

- Secondary leakage reactance referred to primary side

$$X_2' = a^2 X_2$$

- Secondary current referred to primary side

$$\bar{I}_2' = \frac{\bar{I}_2}{a}$$

- Secondary induced emf referred to primary side

$$\bar{E}_2' = a\bar{E}_2$$

- Secondary terminal voltage referred to primary side

$$\bar{V}_2' = a\bar{V}_2$$

Basic Equations for Transformer

$$\bar{E}_1 = a\bar{E}_2$$

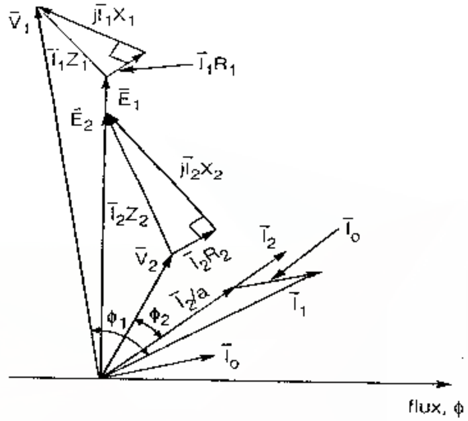
$$\bar{I}_o = \bar{I}_m + \bar{I}_i$$

$$\bar{E}_2 = \bar{V}_2 + \bar{I}_2 R_2 + j\bar{I}_2 X_2$$

$$\bar{V}_1 = \bar{E}_1 + \bar{I}_1 R_1 + j\bar{I}_1 X_1$$

$$\bar{I}_1 = \frac{\bar{I}_2}{a} + \bar{I}_o$$

Phasor Diagram



Step down transformer at lagging power factor

Note:

- $R_{01} = R_1 + R_2'$ = Equivalent resistance referred to primary side.
- $X_{01} = X_1 + X_2'$ = Equivalent reactance referred to primary side.
- $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$ = Equivalent impedance referred to primary side.
- Core of transformer should have low reluctance and high permeability.
- It is made from CRGO (cold rolled grain oriented) material with 4% of silicon.
- It is laminated to reduce eddy current losses.

Types of Transformer Construction

1. Core type 2. Shell type

S.No.	Core type transformer	Shell type transformer
1.	Series magnetic circuit	Parallel magnetic circuit
2.	2 limb and 2 yoke	3 limb and 2 yoke
3.	Suitable for low flux density application	Suitable for high flux density application
4.	Required more amount of copper	Required less amount of copper
5.	Required less amount of insulation so suitable for high voltage and high power applications	Suitable for low voltage and lower power applications

Remember:

- Transformer windings are made up of stranded conductors instead of solid conductor.
- Low voltage windings are placed nearer to the core, for reducing insulation requirement.

Losses in Transformer

1. Copper Loss (P_{cu})

This loss is variable loss.

Total copper loss = Primary copper loss + Secondary copper loss

$$P_{cu} = I_1^2 R_1 + I_2^2 R_2$$

Resistance referred to any one side.

$$P_{cu} = I_1^2 R_{01} = I_2^2 R_{02}$$

Remember:

p.u. resistance = p.u. full load copper loss

2. Iron Loss (P_i)

This loss is constant loss.

Iron loss = Hysteresis loss + Eddy current loss

(a) Hysteresis loss (P_h)

$$P_h = K_h f B_m^x$$

where, K_h = Proportionality constant depends upon volume, quality of the core material and units used

B_m = Maximum flux density in the core

f = Supply frequency

x = Steinmetz exponent (value vary from 1.5 to 2.5)

But, $B_m \propto \dfrac{V}{f}$

(i) If $\frac{V}{f}$ = constant

$$P_h \propto f$$

(ii) If $\frac{V}{f} \neq$ constant

$$P_h = A V^{1.6} f^{-0.6}$$ [if x = 1.6 i.e. for silicon steel]

where, A = constant

(b) Eddy current loss (P_e)

$$P_e = K_e f^2 B_m^2 t^2$$

where, K_e = Proportionality constant depends upon volume and resistivity of the core material, thickness of laminations and units employed

f = Supply frequency

B_m = Maximum flux density

t = Thickness of lamination

But, $B_m \propto \frac{V}{f}$

(i) If $\frac{V}{f}$ = constant

$$P_e \propto f^2$$

(ii) If $\frac{V}{f} \neq$ constant

$$P_e \propto V^2 \text{ or } P_e = BV^2$$

where, B = constant

Remember:

- The iron-loss can be reduced by reducing the core flux density which means that the for same flux, the core cross-sectional area must be increased.
- Size of distribution transformer is larger as compare to similar power transformer as iron to copper ratio of distribution transformer is higher.

(c) Stray load loss

It largely results from leakage flux including eddy currents in the tank walls, conductors etc.

(d) Dielectric loss

The seat of this loss is in the insulating materials particularly in transformer oil and solid insulations of high voltage transformers.

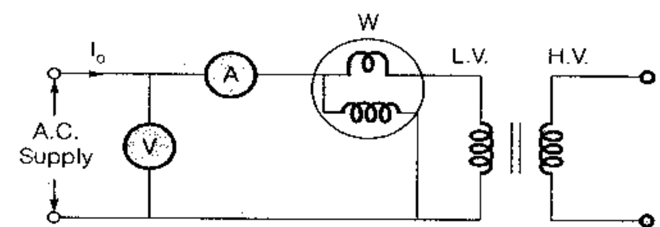
Remember:

Rating of transformer is given in kVA because constant loss is proportional to voltage and variable loss is proportional to current so total loss in transformer is proportional to volt ampere.

Open Circuit Test and Short Circuit Test

- These two tests on transformer helps to determine are
  - (a) The parameters of the equivalent circuit
  - (b) Voltage regulation
  - (c) Efficiency
- They help in predicting performance of transformer without actually loading it.

1. Open Circuit (OC) Test



Main objectives of this test are:

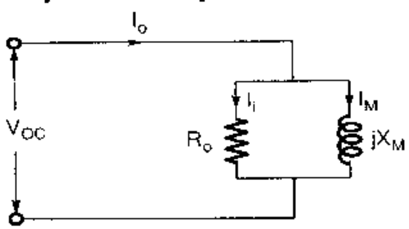
(a) To find out parameters of no load branch i.e. R_o and X_m of equivalent circuit (shunt branch parameter).

(b) To find out constant losses in the transformer (i.e. core loss).

Remember:

- This test must be conducted at rated flux condition i.e. at rated voltage and rated frequency.
- This test is perform on L.V. side while H.V. side is open circuited.

Sequence of equations used to find different parameters



Wattmeter reading = P_i (core loss or iron loss)
Voltmeter reading = V_{oc} (Rated voltage)
Ammeter reading = I_o (No load current)
No load power factor = Cos ϕ_o

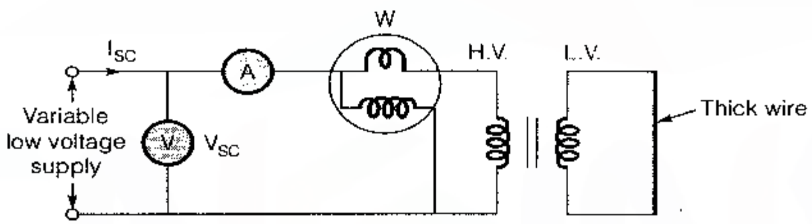
$$P_i = V_{oc} I_o \cos\phi_o$$

$$\cos\phi_0 = \frac{P_i}{V_{oc} I_o}$$

$$I_i = I_o \cos\phi_o \quad \text{and} \quad I_m = I_o \sin\phi_o$$

$$R_o = \frac{V_{oc}}{I_i}, \quad \text{and} \quad X_m = \frac{V_{oc}}{I_m}$$

2. Short-Circuit (SC) Test



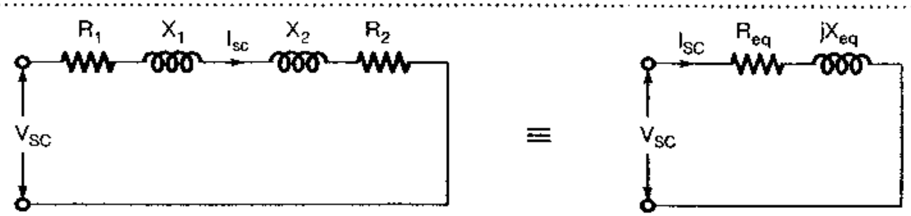
Main objectives of this test are:

(a) To find out total resistance and reactance of transformer, when referred to the winding in which the measuring instruments are placed to conduct this test.

(b) To find out variable losses in the transformer (i e. copper loss).

Remember:

- This test should be conducted at rated current condition.
- This test is conducted at H.V. side while L.V. side is short circuited with a thick conductor.



Wattmeter reading = P_{sc} (rated copper loss)
Voltmeter reading = V_{sc}
Ammeter reading = I_{sc} (rated current)

Sequence of equations used to find different parameters

$$R_{eq} = \frac{P_{SC}}{(I_{SC})^2}$$

$$Z_{eq} = \frac{V_{SC}}{I_{SC}}$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

Note:

- R_{eq}, X_{eq} and Z_{ec} are referred to high voltage side
- Power factor on short circuit $\cos\phi_{sc} = \frac{R_{eq}}{Z_{eq}}$
- Back to back or Sumpner's test is used to determine maximum temperature rise of transformer

Efficiency

$$\text{Efficiency } (\eta) = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output}}{\text{Output + losses}}$$

$$(\eta)_{\text{Full load}} = \frac{(kVA)_{fl} \cos\phi}{(kVA)_{fl} \cos\phi + P_i + (P_{cu})_{fl}}$$

where, P_i = Core loss or iron loss
$(P_{cu})_{fl}$ = Full load copper loss

Note:

Efficiency at any fraction 'x' of full load

$$(\eta)_{x\cdot\text{Full load}} = \frac{x(kVA)_{fl} \cos\phi}{x(kVA)_{fl} \cos\phi + P_i + x^2 (P_{cu})_{fl}}$$

Condition for Maximum Efficiency

$$\text{Copper loss} = \text{Iron loss}$$

$$I_{\eta max} = \sqrt{\frac{P_i}{R_{eq}}}$$

KVA load for Maximum Efficiency

$$S_{\eta max} = S_{(known\ load)} \cdot \sqrt{\frac{P_i}{P_{cu(known\ load)}}}$$

where, $S_{\eta max}$ = KVA rating at maximum efficiency

$S_{(known\ load)}$ = KVA rating at known load

$P_{cu(known\ load)}$ = Copper loss at known load

For full load

$$S_{\eta max} = S_{full\ load} \cdot \sqrt{\frac{P_i}{P_{cu(full\ load)}}}$$

All day efficiency

$$\eta_{all\ day} = \frac{\text{Output energy (kWh) in 24 hrs.}}{\text{Input energy (kWh) in 24 hrs.}}$$

Remember:

- The maximum efficiency for a constant load current occurs at unity power factor.
- Efficiency is independent of type of power factor.

Voltage Regulation

Voltage regulation of transformer is defined as the rise in output (secondary) voltage express as the fraction of full load rated voltage when full load (at specified pf) is reduced to zero keeping the primary impressed terminal voltage constant.

$$\text{Voltage regulation} = \frac{\text{No load voltage} - \text{Full load voltage}}{\text{Full load voltage}}$$

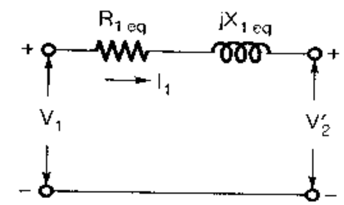
where, full load voltage = Rated voltage

Remember:

- Regulation is always calculated on full load and should be as low as possible.

Approximated Equivalent Circuit Referred to Secondary

- Voltage regulation



$$V.R = \frac{I_1 R_{1eq}}{V_{rated}}\cos\phi + \frac{I_1 X_{1eq}}{V_{rated}}\sin\phi$$

$$V.R = R_{p.u.}\cos\phi + X_{p.u.}\sin\phi$$

where, ϕ = angle between V_2' and I_1

$R_{p.u.}$ = p.u. equivalent resistance

$X_{p.u.}$ = p.u. equivalent reactance

- ϕ is positive for lagging power factor and negative for leading power factor.
- Condition for maximum voltage regulation

$$\cos\phi = \frac{|R_{p.u.}|}{|Z_{p.u.}|} = \frac{R_{1eq}}{Z_{1eq}} \quad \text{lagging}$$

- Maximum voltage regulation

$$(V.R)_{max} = Z_{p.u.}$$

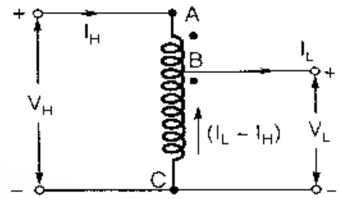
- Condition for zero voltage regulation

$$\cos\phi = \frac{X_{p.u.}}{Z_{p.u.}} = \frac{X_{1eq}}{Z_{1eq}} \quad \text{Leading}$$

Remember:

- For leading p.f., greater than $\frac{X_{p.u.}}{Z_{p.u.}}$, the voltage regulation will be negative
- Regulation is always positive at all lagging p.f. loads and also at unity p.f. load.
- Regulation may be positive at large leading power factor, near to unity and regulation is negative at low leading power factor, away from unity.

Auto Transformer



AB = Series winding
BC = Common winding
N_H = Number of turns in winding AC
N_L = Number of turns in winding BC

- Ratio of transformation

$$a_{auto} = \frac{V_H}{V_L} = \frac{N_H}{N_L}$$

Note:

- Always try to keep a_{auto} closer to 1 i.e. a ≈ 1 in worst case we try to keep $a_{auto} = 3:1$
- Since there is no electrical isolation between primary winding and secondary winding, so when there is common winding is open circuit then full high voltage is appears across low voltage winding at the time of step down mode.

Comparison Between two Winding Transformer & Auto Transformer

- Copper weight

$$(\text{Cu weight})_{auto} = \left(1 - \frac{1}{a_{auto}}\right) \times (\text{Cu weight})_{2\text{-wdg}}$$

- Copper saving

$$\%\text{Copper saving} = \frac{1}{a_{auto}} \times 100$$

- kVA rating of autotransformer

$$kVA_{auto} = \left(\frac{a_{auto}}{a_{auto} - 1}\right) kVA_{2\text{-wdg}}$$

where, kVA_{auto} = kVA rating of auto transformer
$kVA_{2\text{-wdg}}$ = kVA rating of two winding-transformer

Inductive and Conductive Transfer

$$V_L I_L = \underbrace{V_L(I_L - I_H)}_{\text{inductive transfer}} + \underbrace{V_L I_H}_{\text{Conductive transfer}}$$

$$\frac{\text{Inductive transfer}}{\text{total transfer}} = \left(1 - \frac{1}{a_{(auto)}}\right)$$

$$\frac{\text{Conductive transfer}}{\text{Total transfer}} = \left(\frac{1}{a_{(auto)}}\right)$$

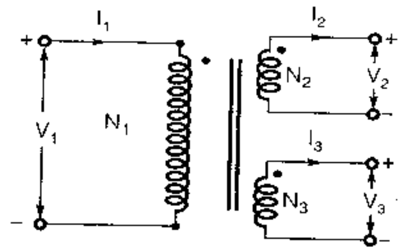
Remember:

Auto transformer has higher efficiency, lower p.u. impedance and lower voltage regulation as compare to 2-wdg transformer.

Three Winding Tra©1nsformer

- Voltage per turn

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} = \frac{V_3}{N_3}$$



- MMF balance equation

$$N_1 I_1 = N_2 I_2 + N_3 I_3 \quad \text{(neglect magnetizing current)}$$

$$I_1 = \frac{N_2}{N_1} \cdot I_2 + \frac{N_3}{N_1} I_3$$

- kVA balance equation

$$V_1 I_1 = V_2 I_2 + V_3 I_3$$

$$S_1 = S_2 + S_3$$

Remember:

- Three winding transformer can be used to interconnect 3 power systems at different voltage levels.
- Tertiary winding is used as a supporting winding in YY transformer to stabilize neutral and to avoid third harmonics.

Application of Different Connection

1. Δ/Δ → This connection is used where we have simple 3ϕ load, specially at low voltage level. No mix loading possible.
2. Y/Δ → This connection is used for step down application.
3. Δ/Y → This connection is used for step up application except in distribution system where, Δ/Y is used for step down application for mix loading.
4. Y/Y → This connection is quite attractive for HV applications. It is not generally used without a tertiary Δ.

Condition to be Satisfied for Parallel Operation

Must Condition

- Same polarity.
- Same voltage ratio and voltage rating.
- Same phase sequence.
- Zero phase difference, which means transformer belongs to same phasor group can only operate in parallel.

Desirable Conditions

- Same X/R ratio for same power factor operation.
- Same p.u. impedance on their own kVA base for proportional load sharing.