Chapter 5. Arithmetic Progression

Question-1

Find the sum of the following A.P. 1, 3, 5, 7, ...,199.

Solution:

Given, $a = 1, d = 2, a_n = l = 199,$

$a + (n - 1) d = 199$

$1 + (n - 1) 2 = 199$

$\Rightarrow 1 + 2n - 2 = 199$

$\Rightarrow 2n = 200$

$\therefore \; n = \frac{200}{2}$

$n = 100.$

$S_n = n/2 (a + l)$

$= 50(1 + 199)$

$= 50(200)$

$= 10000$

Question-2

Find the A.P. whose 10^{th} term is 5 and 18^{th} term is 77.

Solution:

given, 10^{th} term of an A.P= 5

Þ $a + (10 - 1) d = 5$

$\Rightarrow a + 9d = 5$ ....(i)

and 18^{th} term = 77

Þ$a + (18 - 1) d = 77$

$\Rightarrow a + 17d = 77$ ....(ii)

(ii) − (i), $8d = 72$

$\therefore d = 9$

Substituting the value of d = 9 in (i),

$a + 81 = 5$

$a = 5 - 81 = -76$

∴ The A.P. is − 76, - 67, ...

Question-3

In a certain A.P the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

Solution:

Given, $a_{24} = 2a_{10}$

$a_{24} = a + 23d$ and $a_{10} = a + 9d$

To prove: $a_{72} = 2a_{34}$

$a_{72} = a + 71d$

$a_{34} = a + 33d$

$a_{24} = 2a_{10}$ (Given)

$a + 23d = 2(a + 9d)$

$a + 23d = 2a + 18d$

$a - 5d = 0$

$a = 5d$..........................(i)

$a_{72} = 2a_{34}$

$a + 71d = 2(a + 33d)$

$a + 71d = 2a + 66d$

$a - 5d = 0$

$a = 5d$..........................(ii)

from, (1) and (2) $a_{72} = 2a_{34}$

Hence proved.

Question-4

a, b and c are in A.P. Prove that b + c, c + a and a + b are in A.P.

Solution:

Given, a, b and c are in A.P.

$\therefore\ b - a = c - b$

To prove : **b + c, c + a and a + b are in A.P.**

$c + a - (b + c) = a + b - (c + a)$

$\Rightarrow\ c + a - b - c = a + b - c - a$

$a - b = b - c$

$\Rightarrow b - a = c - b$

$\therefore$ a, b, c are in A.P.

$\therefore$ b + c, c + a and a + b are in A.P.

Question-5

If 9^{th} term of an A.P. is zero, prove that its 29^{th} term is double the 19^{th} term.

Solution:

9^{th} term = 0

$a_1 + 8d = 0$

$a_{29} = a_1 + 28d = a_1 + 8d + 20d = 0 + 20d = 20d$

$a_{19} = a_1 + 18d = a_1 + 8d + 10d = 0 + 10d = 10d$

$a_{29} = 2a_{19}$.

Question-6

Determine the A.P whose third term is 16 and the difference of 5^{th} from 7^{th} term is 12.

Solution:

Let the A.P. be a, a + d, a + 2d, ...

$\Rightarrow$ The third term = $a_3 = a + 2d = 16$ ....(i)

and seventh term = $a_7 = a + 6d$

Given that $a_7 - a_5 = 12$

$\Rightarrow (a + 6d) - (a + 4d) = 12$

$\Rightarrow a + 6d - a - 4d = 12$

$\Rightarrow 2d = 12$

$\Rightarrow d = 6$

Substituting the value of d = 6 in (i),

$a + 12 = 16$

$a = 4$

$\therefore$ The first term of the A.P. is 4 and the common difference is 6.

$\therefore$ The A.P. is 4, 10, 16, 22, 28, 34, ...

$\therefore$ The fifth term = $a_5 = a + 4d$.

Question-7

The sum of the first six terms of an A.P is zero and the fourth term is 2. Find the sum of its first 30 terms.

Solution:

Let the sum of first 30 terms be S_{30}, first term be a, fourth term be a_4 and the sum of first six terms be S_6.

Given that $S_6 = 0$ and fourth term $a_4 = 2$

$\Rightarrow a + 3d = 2$ ....(i)

$S_6 = 0$

$\frac{n}{2}(2a + 5d) = 0$

$\Rightarrow 2a + 5d = 0$ ....(ii)

(i) × 2,

$2a + 6d = 4$ ....(iii)

(iii) − (ii),

$\therefore d = 4$

Substituting the value of d = 4 in (i),

$a + 3 \times (4) = 2$

$\Rightarrow a = 2 - 12 = -10$

$\therefore a_{30} = a + 29d$

$= -10 + 29 \times (4)$

$= -10 + 116$

$= 106$

$\therefore$ Sum to first 30 terms = $S_{30} = \frac{n}{2}(a + l)$

$= \frac{30}{2}(-10 + 106)$

$= 15 \times 96$

$= 1140.$

Question-8

An A.P consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32^{nd} term.

Solution:

Given, $n = 60$, $a_1 = 7$,

and $a_{60} = 125$

$\Rightarrow a_1 + 59d = 125$

$7 + 59d = 125$

$59d = 118$

$d = 118/59 = 2$

$a_{32} = a_1 + 31d = 7 + 31(2) = 7 + 62$

$\therefore a_{32} = 69.$

Question-9

Find the sum of the series 51 + 50 + 49 + ... + 21.

Solution:

51 + 50 + 49 + ... + 21

$a = 51$, $d = -1$, $a_n = 21$

$\therefore a + (n - 1)d = a_n$

$51 + (n - 1)(-1) = 21$

$(n - 1)(-1) = 21 - 51$

$n - 1 = 30$

$\therefore n = 31$

$\therefore$ Sum of the series = $\frac{31}{2}(51 + 21)$

$= \frac{31}{2} \times 72$

$= 1116$

$\therefore$ The sum of the series 51 + 50 + 49 + ... + 21 = 1116.

Question-10

Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Solution:

Let the three numbers be $a - d$, a, $a + d$.

$a - d + a + a + d = 27$

$3a = 27$

$a = 9$

$(a - d)(a)(a + d) = 648$

$a(a^2 - d^2) = 648$

$9(9^2 - d^2) = 648$

$9^2 - d^2 = 72$

$d^2 = 81 - 72$

$d^2 = 9$

$d = 3$

The numbers are 6, 9, 12.

Question-11

How many terms of A.P -10, -7, -4, -1, ... must be added to get the sum -104?

Solution:

-10, -7, -4, -1, ...

$a = -10$, $d = 3$

$S_n = \frac{1}{2}n\{2a + (n - 1)\, d\}$

$-104 = \frac{1}{2}n\,\{2(-10) + (n - 1)\, 3\}$

$= \frac{1}{2}n\,(-20 + 3n - 3)$

$-208 = n\,(3n - 23)$

$3n^2 - 23n + 208 = 0$

$3n^2 - 39n + 16n + 208 = 0$

$3n\,(n - 13) + 16\,(n - 13) = 0$

$(n - 13)\,(3n + 16) = 0$

$\therefore n = 13$

$\therefore$ 13 terms must be added to get the sum of the A.P -104.

Question-12

If the sum of p terms of an A.P is $3p^2 + 4p$, find its n^{th} term.

Solution:

$S_p = 3p^2 + 4p$

$t_n = S_n - S_{n-1}$

$= (3n^2 + 4n) - [3(n - 1)^2 + 4(n - 1)]$

$= (3n^2 + 4n) - [3(n^2 - 2n + 1) + 4(n - 1)]$

$= (3n^2 + 4n) - [3n^2 - 6n + 3 + 4n - 4]$

$= (3n^2 + 4n) - [3n^2 - 2n - 1]$

$= 3n^2 + 4n - 3n^2 + 2n + 1$

$= 6n + 1$

Therefore the n^{th} term is $6n + 1$.