PUZZLER

A common scene at a carnival is the Ring-the-Bell attraction, in which the player swings a heavy hammer downward in an attempt to project a mass upward to ring a bell. What is the best strategy to win the game and impress your friends? *(Robert E. Daemmrich/Tony Stone Images)*

chapter 8

Potential Energy and Conservation of Energy

Chapter Outline

In Chapter 7 we introduced the concept of kinetic energy, which is the energy associated with the motion of an object. In this chapter we introduce another form of energy—*potential energy,* which is the energy associated with the arrangement of a system of objects that exert forces on each other. Potential energy can be thought of as stored energy that can either do work or be converted to kinetic energy.

The potential energy concept can be used only when dealing with a special class of forces called *conservative forces.* When only conservative forces act within an isolated system, the kinetic energy gained (or lost) by the system as its members change their relative positions is balanced by an equal loss (or gain) in potential energy. This balancing of the two forms of energy is known as the *principle of conservation of mechanical energy.*

Energy is present in the Universe in various forms, including mechanical, electromagnetic, chemical, and nuclear. Furthermore, one form of energy can be converted to another. For example, when an electric motor is connected to a battery, the chemical energy in the battery is converted to electrical energy in the motor, which in turn is converted to mechanical energy as the motor turns some device. The transformation of energy from one form to another is an essential part of the study of physics, engineering, chemistry, biology, geology, and astronomy.

When energy is changed from one form to another, the total amount present does not change. Conservation of energy means that although the form of energy may change, if an object (or system) loses energy, that same amount of energy appears in another object or in the object's surroundings.

8.1 POTENTIAL ENERGY

5.3 An object that possesses kinetic energy can do work on another object—for example, a moving hammer driving a nail into a wall. Now we shall introduce another form of energy. This energy, called **potential energy *U*,** is the energy associated with a system of objects.

Before we describe specific forms of potential energy, we must first define a *system,* which consists of two or more objects that exert forces on one another. **If the arrangement of the system changes, then the potential energy of the system changes.** If the system consists of only two particle-like objects that exert forces on each other, then the work done by the force acting on one of the objects causes a transformation of energy between the object's kinetic energy and other forms of the system's energy.

Gravitational Potential Energy

As an object falls toward the Earth, the Earth exerts a gravitational force $m\mathbf{g}$ on the object, with the direction of the force being the same as the direction of the object's motion. The gravitational force does work on the object and thereby increases the object's kinetic energy. Imagine that a brick is dropped from rest directly above a nail in a board lying on the ground. When the brick is released, it falls toward the ground, gaining speed and therefore gaining kinetic energy. The brick–Earth system has potential energy when the brick is at any distance above the ground (that is, it has the *potential* to do work), and this potential energy is converted to kinetic energy as the brick falls. The conversion from potential energy to kinetic energy occurs continuously over the entire fall. When the brick reaches the nail and the board lying on the ground, it does work on the nail, driving it into the board. What determines how much work the brick is able to do on the nail? It is easy to see that the heavier the brick, the farther in it drives the nail; also the higher the brick is before it is released, the more work it does when it strikes the nail.

The product of the magnitude of the gravitational force mg acting on an object and the height y of the object is so important in physics that we give it a name: the **gravitational potential energy.** The symbol for gravitational potential energy is U_g, and so the defining equation for gravitational potential energy is

Gravitational potential energy

$$U_g \equiv mgy \tag{8.1}$$

Gravitational potential energy is the potential energy of the object–Earth system. This potential energy is transformed into kinetic energy of the system by the gravitational force. In this type of system, in which one of the members (the Earth) is much more massive than the other (the object), the massive object can be modeled as stationary, and the kinetic energy of the system can be represented entirely by the kinetic energy of the lighter object. Thus, the kinetic energy of the system is represented by that of the object falling toward the Earth. Also note that Equation 8.1 is valid only for objects near the surface of the Earth, where **g** is approximately constant.[1]

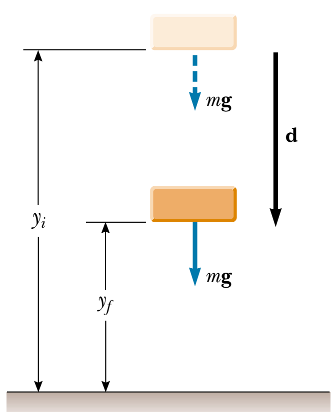

Figure 8.1 The work done on the brick by the gravitational force as the brick falls from a height y_i to a height y_f is equal to $mgy_i - mgy_f$.

Let us now directly relate the work done on an object by the gravitational force to the gravitational potential energy of the object–Earth system. To do this, let us consider a brick of mass m at an initial height y_i above the ground, as shown in Figure 8.1. If we neglect air resistance, then the only force that does work on the brick as it falls is the gravitational force exerted on the brick $m\mathbf{g}$. The work W_g done by the gravitational force as the brick undergoes a downward displacement **d** is

$$W_g = (m\mathbf{g}) \cdot \mathbf{d} = (-mg\mathbf{j}) \cdot (y_f - y_i)\,\mathbf{j} = mgy_i - mgy_f$$

where we have used the fact that $\mathbf{j} \cdot \mathbf{j} = 1$ (Eq. 7.4). If an object undergoes both a horizontal and a vertical displacement, so that $\mathbf{d} = (x_f - x_i)\mathbf{i} + (y_f - y_i)\mathbf{j}$, then the work done by the gravitational force is still $mgy_i - mgy_f$ because $-mg\mathbf{j} \cdot (x_f - x_i)\mathbf{i} = 0$. Thus, the work done by the gravitational force depends only on the change in y and not on any change in the horizontal position x.

We just learned that the quantity mgy is the gravitational potential energy of the system U_g, and so we have

$$W_g = U_i - U_f = -(U_f - U_i) = -\Delta U_g \tag{8.2}$$

From this result, we see that the work done on any object by the gravitational force is equal to the negative of the change in the system's gravitational potential energy. Also, this result demonstrates that it is only the *difference* in the gravitational potential energy at the initial and final locations that matters. This means that we are free to place the origin of coordinates in any convenient location. Finally, the work done by the gravitational force on an object as the object falls to the Earth is the same as the work done were the object to start at the same point and slide down an incline to the Earth. Horizontal motion does not affect the value of W_g.

The unit of gravitational potential energy is the same as that of work—the joule. Potential energy, like work and kinetic energy, is a scalar quantity.

[1] The assumption that the force of gravity is constant is a good one as long as the vertical displacement is small compared with the Earth's radius.

Quick Quiz 8.1

Can the gravitational potential energy of a system ever be negative?

EXAMPLE 8.1 The Bowler and the Sore Toe

A bowling ball held by a careless bowler slips from the bowler's hands and drops on the bowler's toe. Choosing floor level as the $y = 0$ point of your coordinate system, estimate the total work done on the ball by the force of gravity as the ball falls. Repeat the calculation, using the top of the bowler's head as the origin of coordinates.

Solution First, we need to estimate a few values. A bowling ball has a mass of approximately 7 kg, and the top of a person's toe is about 0.03 m above the floor. Also, we shall assume the ball falls from a height of 0.5 m. Holding nonsignificant digits until we finish the problem, we calculate the gravitational potential energy of the ball–Earth system just before the ball is released to be $U_i = mgy_i = (7 \text{ kg})(9.80 \text{ m/s}^2)(0.5 \text{ m}) = 34.3 \text{ J}$. A similar calculation for when the ball reaches his toe gives $U_f = mgy_f = (7 \text{ kg})(9.80 \text{ m/s}^2)(0.03 \text{ m}) = 2.06 \text{ J}$. So, the work done by the gravitational force is $W_g = U_i - U_f = 32.24 \text{ J}$. We should probably keep only one digit because of the roughness of our estimates; thus, we estimate that the gravitational force does 30 J of work on the bowling ball as it falls. The system had 30 J of gravitational potential energy relative to the top of the toe before the ball began its fall.

When we use the bowler's head (which we estimate to be 1.50 m above the floor) as our origin of coordinates, we find that $U_i = mgy_i = (7 \text{ kg})(9.80 \text{ m/s}^2)(-1 \text{ m}) = -68.6 \text{ J}$ and that $U_f = mgy_f = (7 \text{ kg})(9.80 \text{ m/s}^2)(-1.47 \text{ m}) = -100.8 \text{ J}$. The work being done by the gravitational force is still

$W_g = U_i - U_f = 32.24 \text{ J} \approx$ 30 J.

Elastic Potential Energy

Now consider a system consisting of a block plus a spring, as shown in Figure 8.2. The force that the spring exerts on the block is given by $F_s = -kx$. In the previous chapter, we learned that the work done by the spring force on a block connected to the spring is given by Equation 7.11:

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \quad \textbf{(8.3)}$$

In this situation, the initial and final x coordinates of the block are measured from its equilibrium position, $x = 0$. Again we see that W_s depends only on the initial and final x coordinates of the object and is zero for any closed path. The **elastic potential energy** function associated with the system is defined by

$$U_s \equiv \tfrac{1}{2}kx^2 \quad \textbf{(8.4)}$$

Elastic potential energy stored in a spring

The elastic potential energy of the system can be thought of as the energy stored in the deformed spring (one that is either compressed or stretched from its equilibrium position). To visualize this, consider Figure 8.2, which shows a spring on a frictionless, horizontal surface. When a block is pushed against the spring (Fig. 8.2b) and the spring is compressed a distance x, the elastic potential energy stored in the spring is $kx^2/2$. When the block is released from rest, the spring snaps back to its original length and the stored elastic potential energy is transformed into kinetic energy of the block (Fig. 8.2c). The elastic potential energy stored in the spring is zero whenever the spring is undeformed ($x = 0$). Energy is stored in the spring only when the spring is either stretched or compressed. Furthermore, the elastic potential energy is a maximum when the spring has reached its maximum compression or extension (that is, when $|x|$ is a maximum). Finally, because the elastic potential energy is proportional to x^2, we see that U_s is always positive in a deformed spring.

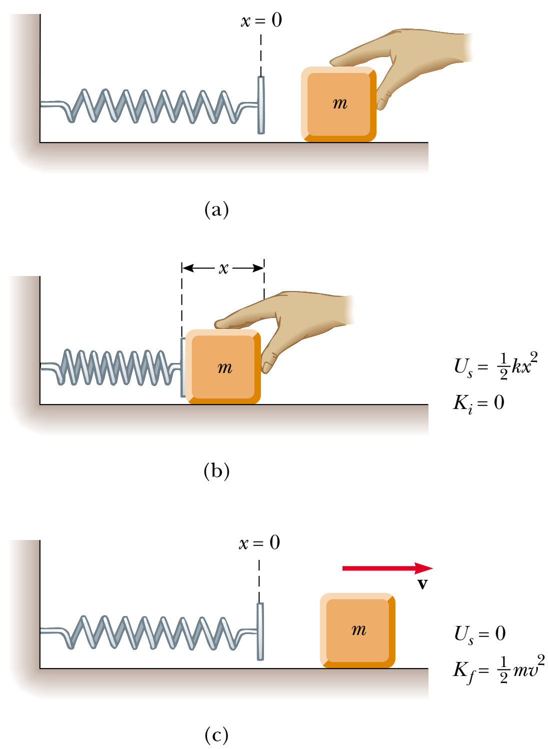

Figure 8.2 (a) An undeformed spring on a frictionless horizontal surface. (b) A block of mass m is pushed against the spring, compressing it a distance x. (c) When the block is released from rest, the elastic potential energy stored in the spring is transferred to the block in the form of kinetic energy.

8.2 CONSERVATIVE AND NONCONSERVATIVE FORCES

The work done by the gravitational force does not depend on whether an object falls vertically or slides down a sloping incline. All that matters is the change in the object's elevation. On the other hand, the energy loss due to friction on that incline depends on the distance the object slides. In other words, the path makes no difference when we consider the work done by the gravitational force, but it does make a difference when we consider the energy loss due to frictional forces. We can use this varying dependence on path to classify forces as either conservative or nonconservative.

Of the two forces just mentioned, the gravitational force is conservative and the frictional force is nonconservative.

Conservative Forces

Properties of a conservative force

Conservative forces have two important properties:

1. A force is conservative if the work it does on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

The gravitational force is one example of a conservative force, and the force that a spring exerts on any object attached to the spring is another. As we learned in the preceding section, the work done by the gravitational force on an object moving between any two points near the Earth's surface is $W_g = mgy_i - mgy_f$. From this equation we see that W_g depends only on the initial and final y coordinates of the object and hence is independent of the path. Furthermore, W_g is zero when the object moves over any closed path (where $y_i = y_f$).

For the case of the object–spring system, the work W_s done by the spring force is given by $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$ (Eq. 8.3). Again, we see that the spring force is conservative because W_s depends only on the initial and final x coordinates of the object and is zero for any closed path.

We can associate a potential energy with any conservative force and can do this *only* for conservative forces. In the previous section, the potential energy associated with the gravitational force was defined as $U_g \equiv mgy$. In general, the work W_c done on an object by a conservative force is equal to the initial value of the potential energy associated with the object minus the final value:

$$W_c = U_i - U_f = -\Delta U \tag{8.5}$$

Work done by a conservative force

This equation should look familiar to you. It is the general form of the equation for work done by the gravitational force (Eq. 8.2) and that for the work done by the spring force (Eq. 8.3).

Nonconservative Forces

5.3 **A force is nonconservative if it causes a change in mechanical energy *E*,** which we define as the sum of kinetic and potential energies. For example, if a book is sent sliding on a horizontal surface that is not frictionless, the force of kinetic friction reduces the book's kinetic energy. As the book slows down, its kinetic energy decreases. As a result of the frictional force, the temperatures of the book and surface increase. The type of energy associated with temperature is *internal energy,* which we will study in detail in Chapter 20. Experience tells us that this internal energy cannot be transferred back to the kinetic energy of the book. In other words, the energy transformation is not reversible. Because the force of kinetic friction changes the mechanical energy of a system, it is a nonconservative force.

Properties of a nonconservative force

From the work–kinetic energy theorem, we see that the work done by a conservative force on an object causes a change in the kinetic energy of the object. The change in kinetic energy depends only on the initial and final positions of the object, and not on the path connecting these points. Let us compare this to the sliding book example, in which the nonconservative force of friction is acting between the book and the surface. According to Equation 7.17a, the change in kinetic energy of the book due to friction is $\Delta K_{\text{friction}} = -f_k d$, where d is the length of the path over which the friction force acts. Imagine that the book slides from A to B over the straight-line path of length d in Figure 8.3. The change in kinetic energy is $-f_k d$. Now, suppose the book slides over the semicircular path from A to B. In this case, the path is longer and, as a result, the change in kinetic energy is greater in magnitude than that in the straight-line case. For this particular path, the change in kinetic energy is $-f_k \pi\, d/2$, since d is the diameter of the semicircle. Thus, we see that for a nonconservative force, the change in kinetic energy depends on the path followed between the initial and final points. If a potential energy is involved, then the change in the total mechanical energy depends on the path followed. We shall return to this point in Section 8.5.

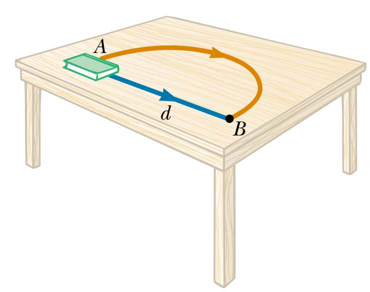

Figure 8.3 The loss in mechanical energy due to the force of kinetic friction depends on the path taken as the book is moved from A to B. The loss in mechanical energy is greater along the red path than along the blue path.

8.3 CONSERVATIVE FORCES AND POTENTIAL ENERGY

In the preceding section we found that the work done on a particle by a conservative force does not depend on the path taken by the particle. The work depends only on the particle's initial and final coordinates. As a consequence, we can define a **potential energy function** U such that the work done by a conservative force equals the decrease in the potential energy of the system. The work done by a conservative force **F** as a particle moves along the x axis is[2]

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\Delta U \tag{8.6}$$

where F_x is the component of **F** in the direction of the displacement. That is, **the work done by a conservative force equals the negative of the change in the potential energy associated with that force,** where the change in the potential energy is defined as $\Delta U = U_f - U_i$.

We can also express Equation 8.6 as

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx \tag{8.7}$$

Therefore, ΔU is negative when F_x and dx are in the same direction, as when an object is lowered in a gravitational field or when a spring pushes an object toward equilibrium.

The term *potential energy* implies that the object has the potential, or capability, of either gaining kinetic energy or doing work when it is released from some point under the influence of a conservative force exerted on the object by some other member of the system. It is often convenient to establish some particular location x_i as a reference point and measure all potential energy differences with respect to it. We can then define the potential energy function as

$$U_f(x) = -\int_{x_i}^{x_f} F_x \, dx + U_i \tag{8.8}$$

The value of U_i is often taken to be zero at the reference point. It really does not matter what value we assign to U_i, because any nonzero value merely shifts $U_f(x)$ by a constant amount, and only the *change* in potential energy is physically meaningful.

If the conservative force is known as a function of position, we can use Equation 8.8 to calculate the change in potential energy of a system as an object within the system moves from x_i to x_f. It is interesting to note that in the case of one-dimensional displacement, a force is always conservative if it is a function of position only. This is not necessarily the case for motion involving two- or three-dimensional displacements.

8.4 CONSERVATION OF MECHANICAL ENERGY

An object held at some height h above the floor has no kinetic energy. However, as
5.9 we learned earlier, the gravitational potential energy of the object–Earth system is equal to mgh. If the object is dropped, it falls to the floor; as it falls, its speed and thus its kinetic energy increase, while the potential energy of the system decreases. If factors such as air resistance are ignored, whatever potential energy the system loses as the object moves downward appears as kinetic energy of the object. In other words, the sum of the kinetic and potential energies—the total mechanical energy E—remains constant. This is an example of the principle of **conservation**

[2] For a general displacement, the work done in two or three dimensions also equals $U_i - U_f$, where $U = U(x, y, z)$. We write this formally as $W = \int_i^f \mathbf{F} \cdot d\mathbf{s} = U_i - U_f$.

of mechanical energy. For the case of an object in free fall, this principle tells us that any increase (or decrease) in potential energy is accompanied by an equal decrease (or increase) in kinetic energy. Note that **the total mechanical energy of a system remains constant in any isolated system of objects that interact only through conservative forces.**

Because the total mechanical energy E of a system is defined as the sum of the kinetic and potential energies, we can write

Total mechanical energy

$$E \equiv K + U \tag{8.9}$$

We can state the principle of conservation of energy as $E_i = E_f$, and so we have

The mechanical energy of an isolated system remains constant

$$K_i + U_i = K_f + U_f \tag{8.10}$$

It is important to note that Equation 8.10 is valid only when no energy is added to or removed from the system. Furthermore, there must be no nonconservative forces doing work within the system.

Consider the carnival Ring-the-Bell event illustrated at the beginning of the chapter. The participant is trying to convert the initial kinetic energy of the hammer into gravitational potential energy associated with a weight that slides on a vertical track. If the hammer has sufficient kinetic energy, the weight is lifted high enough to reach the bell at the top of the track. To maximize the hammer's kinetic energy, the player must swing the heavy hammer as rapidly as possible. The fast-moving hammer does work on the pivoted target, which in turn does work on the weight. Of course, greasing the track (so as to minimize energy loss due to friction) would also help but is probably not allowed!

If more than one conservative force acts on an object within a system, a potential energy function is associated with each force. In such a case, we can apply the principle of conservation of mechanical energy for the system as

$$K_i + \sum U_i = K_f + \sum U_f \tag{8.11}$$

where the number of terms in the sums equals the number of conservative forces present. For example, if an object connected to a spring oscillates vertically, two conservative forces act on the object: the spring force and the gravitational force.

QuickLab

Dangle a shoe from its lace and use it as a pendulum. Hold it to the side, release it, and note how high it swings at the end of its arc. How does this height compare with its initial height? You may want to check Question 8.3 as part of your investigation.

Twin Falls on the Island of Kauai, Hawaii. The gravitational potential energy of the water–Earth system when the water is at the top of the falls is converted to kinetic energy once that water begins falling. How did the water get to the top of the cliff? In other words, what was the original source of the gravitational potential energy when the water was at the top? (*Hint:* This same source powers nearly everything on the planet.)

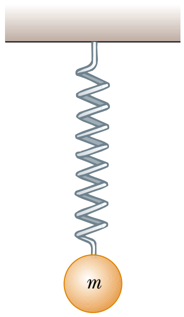

Figure 8.4 A ball connected to a massless spring suspended vertically. What forms of potential energy are associated with the ball–spring–Earth system when the ball is displaced downward?

Quick Quiz 8.2

A ball is connected to a light spring suspended vertically, as shown in Figure 8.4. When displaced downward from its equilibrium position and released, the ball oscillates up and down. If air resistance is neglected, is the total mechanical energy of the system (ball plus spring plus Earth) conserved? How many forms of potential energy are there for this situation?

Quick Quiz 8.3

Three identical balls are thrown from the top of a building, all with the same initial speed. The first is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal, as shown in Figure 8.5. Neglecting air resistance, rank the speeds of the balls at the instant each hits the ground.

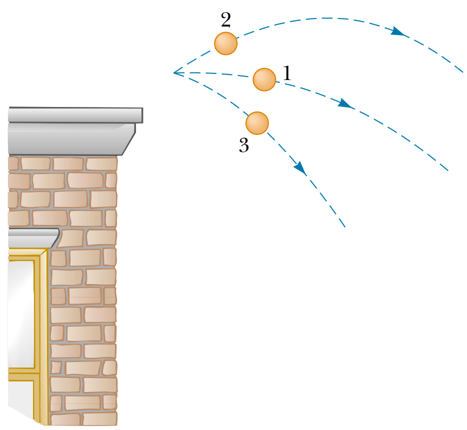

Figure 8.5 Three identical balls are thrown with the same initial speed from the top of a building.

Example 8.2 Ball in Free Fall

A ball of mass m is dropped from a height h above the ground, as shown in Figure 8.6. (a) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.

Solution Because the ball is in free fall, the only force acting on it is the gravitational force. Therefore, we apply the principle of conservation of mechanical energy to the ball–Earth system. Initially, the system has potential energy but no kinetic energy. As the ball falls, the total mechanical energy remains constant and equal to the initial potential energy of the system.

At the instant the ball is released, its kinetic energy is $K_i = 0$ and the potential energy of the system is $U_i = mgh$. When the ball is at a distance y above the ground, its kinetic energy is $K_f = \frac{1}{2}mv_f^2$ and the potential energy relative to the ground is $U_f = mgy$. Applying Equation 8.10, we obtain

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \tfrac{1}{2}mv_f^2 + mgy$$

$$v_f^2 = 2g(h - y)$$

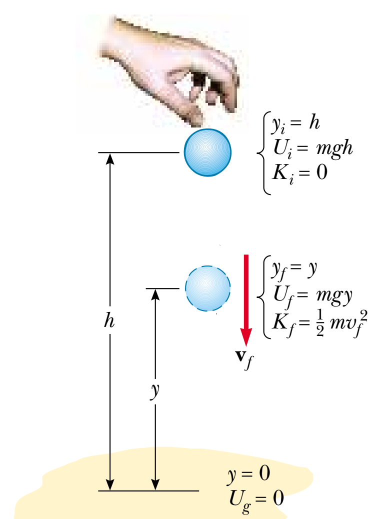

Figure 8.6 A ball is dropped from a height h above the ground. Initially, the total energy of the ball–Earth system is potential energy, equal to mgh relative to the ground. At the elevation y, the total energy is the sum of the kinetic and potential energies.

$$v_f = \sqrt{2g(h - y)}$$

The speed is always positive. If we had been asked to find the ball's velocity, we would use the negative value of the square root as the y component to indicate the downward motion.

(b) Determine the speed of the ball at y if at the instant of release it already has an initial speed v_i at the initial altitude h.

Solution In this case, the initial energy includes kinetic energy equal to $\frac{1}{2}mv_i^2$, and Equation 8.10 gives

$$\tfrac{1}{2}mv_i^2 + mgh = \tfrac{1}{2}mv_f^2 + mgy$$

$$v_f^2 = v_i^2 + 2g(h - y)$$

$$v_f = \sqrt{v_i^2 + 2g(h - y)}$$

This result is consistent with the expression $v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i)$ from kinematics, where $y_i = h$. Furthermore, this result is valid even if the initial velocity is at an angle to the horizontal (the projectile situation) for two reasons: (1) energy is a scalar, and the kinetic energy depends only on the magnitude of the velocity; and (2) the change in the gravitational potential energy depends only on the change in position in the vertical direction.

EXAMPLE 8.3 The Pendulum

A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in Figure 8.7. The sphere is released from rest when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless. (a) Find the speed of the sphere when it is at the lowest point Ⓑ.

Solution The only force that does work on the sphere is the gravitational force. (The force of tension is always perpendicular to each element of the displacement and hence does no work.) Because the gravitational force is conservative, the total mechanical energy of the pendulum–Earth system is constant. (In other words, we can classify this as an "energy conservation" problem.) As the pendulum swings, continuous transformation between potential and kinetic energy occurs. At the instant the pendulum is released, the energy of the system is entirely potential energy. At point Ⓑ the pendulum has kinetic energy, but the system has lost some potential energy. At Ⓒ the system has regained its initial potential energy, and the kinetic energy of the pendulum is again zero.

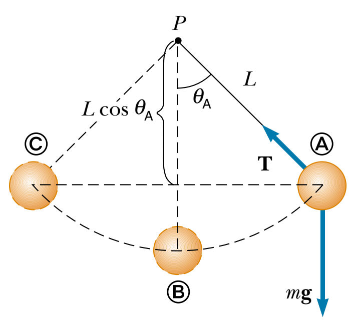

Figure 8.7 If the sphere is released from rest at the angle θ_A it will never swing above this position during its motion. At the start of the motion, position Ⓐ, the energy is entirely potential. This initial potential energy is all transformed into kinetic energy at the lowest elevation Ⓑ. As the sphere continues to move along the arc, the energy again becomes entirely potential energy at Ⓒ.

If we measure the y coordinates of the sphere from the center of rotation, then $y_A = -L\cos\theta_A$ and $y_B = -L$. Therefore, $U_A = -mgL\cos\theta_A$ and $U_B = -mgL$. Applying the principle of conservation of mechanical energy to the system gives

$$K_A + U_A = K_B + U_B$$

$$0 - mgL\cos\theta_A = \tfrac{1}{2}mv_B^2 - mgL$$

$$(1) \qquad v_B = \sqrt{2gL(1 - \cos\theta_A)}$$

(b) What is the tension T_B in the cord at Ⓑ?

Solution Because the force of tension does no work, we cannot determine the tension using the energy method. To find T_B, we can apply Newton's second law to the radial direction. First, recall that the centripetal acceleration of a particle moving in a circle is equal to v^2/r directed toward the center of rotation. Because $r = L$ in this example, we obtain

$$(2) \qquad \sum F_r = T_B - mg = ma_r = m\frac{v_B^2}{L}$$

Substituting (1) into (2) gives the tension at point Ⓑ:

$$(3) \qquad T_B = mg + 2\,mg(1 - \cos\theta_A)$$

$$= mg(3 - 2\cos\theta_A)$$

From (2) we see that the tension at Ⓑ is greater than the weight of the sphere. Furthermore, (3) gives the expected result that $T_B = mg$ when the initial angle $\theta_A = 0$.

Exercise A pendulum of length 2.00 m and mass 0.500 kg is released from rest when the cord makes an angle of 30.0° with the vertical. Find the speed of the sphere and the tension in the cord when the sphere is at its lowest point.

Answer 2.29 m/s; 6.21 N.

8.5 WORK DONE BY NONCONSERVATIVE FORCES

As we have seen, if the forces acting on objects within a system are conservative, then the mechanical energy of the system remains constant. However, if some of the forces acting on objects within the system are not conservative, then the mechanical energy of the system does not remain constant. Let us examine two types of nonconservative forces: an applied force and the force of kinetic friction.

Work Done by an Applied Force

When you lift a book through some distance by applying a force to it, the force you apply does work W_{app} on the book, while the gravitational force does work W_g on the book. If we treat the book as a particle, then the net work done on the book is related to the change in its kinetic energy as described by the work–kinetic energy theorem given by Equation 7.15:

$$W_{\text{app}} + W_g = \Delta K \tag{8.12}$$

Because the gravitational force is conservative, we can use Equation 8.2 to express the work done by the gravitational force in terms of the change in potential energy, or $W_g = -\Delta U$. Substituting this into Equation 8.12 gives

$$W_{\text{app}} = \Delta K + \Delta U \tag{8.13}$$

Note that the right side of this equation represents the change in the mechanical energy of the book–Earth system. This result indicates that your applied force transfers energy to the system in the form of kinetic energy of the book and gravitational potential energy of the book–Earth system. Thus, we conclude that if an object is part of a system, then **an applied force can transfer energy into or out of the system.**

Situations Involving Kinetic Friction

Kinetic friction is an example of a nonconservative force. If a book is given some initial velocity on a horizontal surface that is not frictionless, then the force of kinetic friction acting on the book opposes its motion and the book slows down and eventually stops. The force of kinetic friction reduces the kinetic energy of the book by transforming kinetic energy to internal energy of the book and part of the horizontal surface. Only part of the book's kinetic energy is transformed to internal energy in the book. The rest appears as internal energy in the surface. (When you trip and fall while running across a gymnasium floor, not only does the skin on your knees warm up but so does the floor!)

As the book moves through a distance d, the only force that does work is the force of kinetic friction. This force causes a decrease in the kinetic energy of the book. This decrease was calculated in Chapter 7, leading to Equation 7.17a, which we repeat here:

$$\Delta K_{\text{friction}} = -f_k d \tag{8.14}$$

If the book moves on an incline that is not frictionless, a change in the gravitational potential energy of the book–Earth system also occurs, and $-f_k d$ is the amount by which the mechanical energy of the system changes because of the force of kinetic friction. In such cases,

$$\Delta E = \Delta K + \Delta U = -f_k d \tag{8.15}$$

where $E_i + \Delta E = E_f$.

QuickLab

Find a friend and play a game of racquetball. After a long volley, feel the ball and note that it is warm. Why is that?

Quick Quiz 8.4

Write down the work–kinetic energy theorem for the general case of two objects that are connected by a spring and acted upon by gravity and some other external applied force. Include the effects of friction as $\Delta E_{\text{friction}}$.

Problem-Solving Hints

Conservation of Energy

We can solve many problems in physics using the principle of conservation of energy. You should incorporate the following procedure when you apply this principle:

- Define your system, which may include two or more interacting particles, as well as springs or other systems in which elastic potential energy can be stored. Choose the initial and final points.
- Identify zero points for potential energy (both gravitational and spring). If there is more than one conservative force, write an expression for the potential energy associated with each force.
- Determine whether any nonconservative forces are present. Remember that if friction or air resistance is present, mechanical energy *is not conserved.*
- If mechanical energy is *conserved,* you can write the total initial energy $E_i = K_i + U_i$ at some point. Then, write an expression for the total final energy $E_f = K_f + U_f$ at the final point that is of interest. Because mechanical energy is *conserved,* you can equate the two total energies and solve for the quantity that is unknown.
- If frictional forces are present (and thus mechanical energy is *not conserved*), first write expressions for the total initial and total final energies. In this case, the difference between the total final mechanical energy and the total initial mechanical energy equals the change in mechanical energy in the system due to friction.

EXAMPLE 8.4 Crate Sliding Down a Ramp

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0°, as shown in Figure 8.8. The crate starts from rest at the top, experiences a constant frictional force of magnitude 5.00 N, and continues to move a short distance on the flat floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp.

Solution Because $v_i = 0$, the initial kinetic energy at the top of the ramp is zero. If the y coordinate is measured from the bottom of the ramp (the final position where the potential energy is zero) with the upward direction being positive, then $y_i = 0.500$ m. Therefore, the total mechanical energy of the crate–Earth system at the top is all potential energy:

$$E_i = K_i + U_i = 0 + U_i = mgy_i$$
$$= (3.00\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m}) = 14.7\text{ J}$$

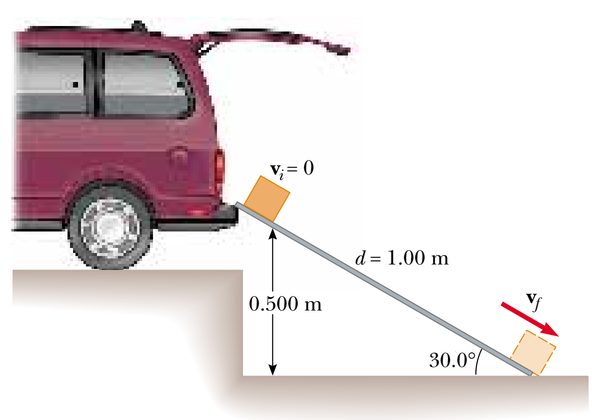

Figure 8.8 A crate slides down a ramp under the influence of gravity. The potential energy decreases while the kinetic energy increases.

When the crate reaches the bottom of the ramp, the potential energy of the system is *zero* because the elevation of the crate is $y_f = 0$. Therefore, the total mechanical energy of the system when the crate reaches the bottom is all kinetic energy:

$$E_f = K_f + U_f = \tfrac{1}{2}mv_f^2 + 0$$

We cannot say that $E_i = E_f$ because a nonconservative force reduces the mechanical energy of the system: the force of kinetic friction acting on the crate. In this case, Equation 8.15 gives $\Delta E = -f_k d$, where d is the displacement along the ramp. (Remember that the forces normal to the ramp do no work on the crate because they are perpendicular to the displacement.) With $f_k = 5.00$ N and $d = 1.00$ m, we have

$$\Delta E = -f_k d = -(5.00\ \text{N})(1.00\ \text{m}) = -5.00\ \text{J}$$

This result indicates that the system loses some mechanical energy because of the presence of the nonconservative frictional force. Applying Equation 8.15 gives

$$E_f - E_i = \tfrac{1}{2}mv_f^2 - mgy_i = -f_k d$$

$$\tfrac{1}{2}mv_f^2 = 14.7\ \text{J} - 5.00\ \text{J} = 9.70\ \text{J}$$

$$v_f^2 = \frac{19.4\ \text{J}}{3.00\ \text{kg}} = 6.47\ \text{m}^2/\text{s}^2$$

$$v_f = \boxed{2.54\ \text{m/s}}$$

Exercise Use Newton's second law to find the acceleration of the crate along the ramp, and use the equations of kinematics to determine the final speed of the crate.

Answer $3.23\ \text{m/s}^2$; 2.54 m/s.

Exercise Assuming the ramp to be frictionless, find the final speed of the crate and its acceleration along the ramp.

Answer 3.13 m/s; $4.90\ \text{m/s}^2$.

EXAMPLE 8.5 Motion on a Curved Track

A child of mass m rides on an irregularly curved slide of height $h = 2.00$ m, as shown in Figure 8.9. The child starts from rest at the top. (a) Determine his speed at the bottom, assuming no friction is present.

Solution The normal force $\mathbf{n}$ does no work on the child because this force is always perpendicular to each element of the displacement. Because there is no friction, the mechanical energy of the child–Earth system is conserved. If we measure the y coordinate in the upward direction from the bottom of the slide, then $y_i = h$, $y_f = 0$, and we obtain

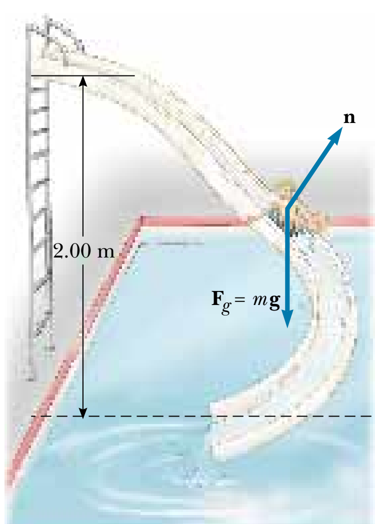

Figure 8.9 If the slide is frictionless, the speed of the child at the bottom depends only on the height of the slide.

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \tfrac{1}{2}mv_f^2 + 0$$

$$v_f = \sqrt{2gh}$$

Note that the result is the same as it would be had the child fallen vertically through a distance h! In this example, $h = 2.00$ m, giving

$$v_f = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(2.00\ \text{m})} = \boxed{6.26\ \text{m/s}}$$

(b) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that $v_f = 3.00$ m/s and $m = 20.0$ kg.

Solution In this case, mechanical energy is *not* conserved, and so we must use Equation 8.15 to find the loss of mechanical energy due to friction:

$$\begin{aligned}\Delta E = E_f - E_i &= (K_f + U_f) - (K_i + U_i)\\ &= (\tfrac{1}{2}mv_f^2 + 0) - (0 + mgh) = \tfrac{1}{2}mv_f^2 - mgh\\ &= \tfrac{1}{2}(20.0\ \text{kg})(3.00\ \text{m/s})^2 - (20.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})\\ &= \boxed{-302\ \text{J}}\end{aligned}$$

Again, ΔE is negative because friction is reducing mechanical energy of the system (the final mechanical energy is less than the initial mechanical energy). Because the slide is curved, the normal force changes in magnitude and direction during the motion. Therefore, the frictional force, which is proportional to n, also changes during the motion. Given this changing frictional force, do you think it is possible to determine μ_k from these data?

EXAMPLE 8.6 Let's Go Skiing!

A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown in Figure 8.10. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210. How far does she travel on the horizontal surface before coming to rest?

Solution First, let us calculate her speed at the bottom of the incline, which we choose as our zero point of potential energy. Because the incline is frictionless, the mechanical energy of the skier–Earth system remains constant, and we find, as we did in the previous example, that

$$v_B = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 19.8 \text{ m/s}$$

Now we apply Equation 8.15 as the skier moves along the rough horizontal surface from Ⓑ to Ⓒ. The change in mechanical energy along the horizontal is $\Delta E = -f_k d$, where d is the horizontal displacement.

To find the distance the skier travels before coming to rest, we take $K_C = 0$. With $v_B = 19.8$ m/s and the frictional force given by $f_k = \mu_k n = \mu_k mg$, we obtain

$$\Delta E = E_C - E_B = -\mu_k mgd$$

$$(K_C + U_C) - (K_B + U_B) = (0 + 0) - (\tfrac{1}{2}mv_B{}^2 + 0) = -\mu_k mgd$$

$$d = \frac{v_B{}^2}{2\mu_k g} = \frac{(19.8 \text{ m/s})^2}{2(0.210)(9.80 \text{ m/s}^2)} = 95.2 \text{ m}$$

Exercise Find the horizontal distance the skier travels before coming to rest if the incline also has a coefficient of kinetic friction equal to 0.210.

Answer 40.3 m.

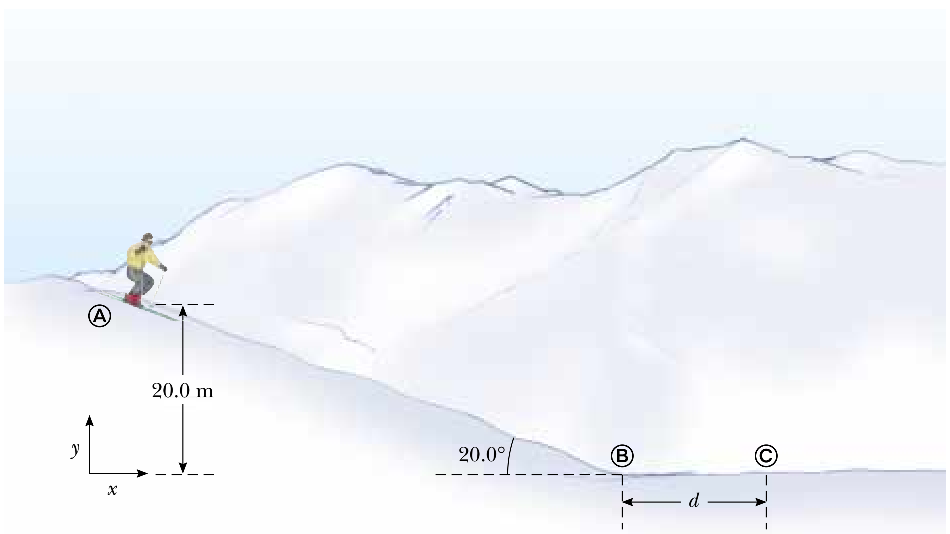

Figure 8.10 The skier slides down the slope and onto a level surface, stopping after a distance d from the bottom of the hill.

EXAMPLE 8.7 The Spring-Loaded Popgun

The launching mechanism of a toy gun consists of a spring of unknown spring constant (Fig. 8.11a). When the spring is compressed 0.120 m, the gun, when fired vertically, is able to launch a 35.0-g projectile to a maximum height of 20.0 m above the position of the projectile before firing. (a) Neglecting all resistive forces, determine the spring constant.

Solution Because the projectile starts from rest, the initial kinetic energy is zero. If we take the zero point for the gravitational potential energy of the projectile–Earth system to be at the lowest position of the projectile x_A, then the initial gravitational potential energy also is zero. The mechanical energy of this system is constant because no nonconservative forces are present.

Initially, the only mechanical energy in the system is the elastic potential energy stored in the spring of the gun, $U_{sA} = kx^2/2$, where the compression of the spring is $x = 0.120$ m. The projectile rises to a maximum height

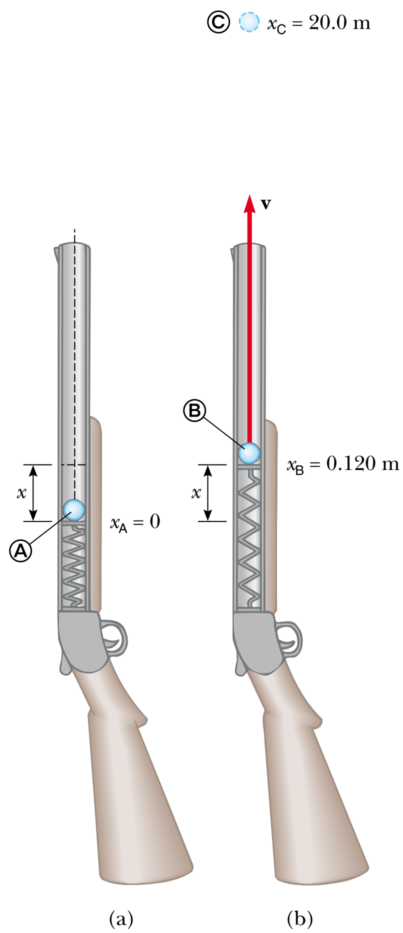

Figure 8.11 A spring-loaded popgun.

$x_C = h = 20.0$ m, and so the final gravitational potential energy when the projectile reaches its peak is mgh. The final kinetic energy of the projectile is zero, and the final elastic potential energy stored in the spring is zero. Because the mechanical energy of the system is constant, we find that

$$E_A = E_C$$
$$K_A + U_{gA} + U_{sA} = K_C + U_{gC} + U_{sC}$$
$$0 + 0 + \tfrac{1}{2}kx^2 = 0 + mgh + 0$$
$$\tfrac{1}{2}k(0.120\ \text{m})^2 = (0.0350\ \text{kg})(9.80\ \text{m/s}^2)(20.0\ \text{m})$$
$$k = \boxed{953\ \text{N/m}}$$

(b) Find the speed of the projectile as it moves through the equilibrium position of the spring (where $x_B = 0.120$ m) as shown in Figure 8.11b.

Solution As already noted, the only mechanical energy in the system at Ⓐ is the elastic potential energy $kx^2/2$. The total energy of the system as the projectile moves through the equilibrium position of the spring comprises the kinetic energy of the projectile $mv_B{}^2/2$, and the gravitational potential energy mgx_B. Hence, the principle of the conservation of mechanical energy in this case gives

$$E_A = E_B$$
$$K_A + U_{gA} + U_{sA} = K_B + U_{gB} + U_{sB}$$
$$0 + 0 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv_B{}^2 + mgx_B + 0$$

Solving for v_B gives

$$v_B = \sqrt{\frac{kx^2}{m} - 2gx_B}$$
$$= \sqrt{\frac{(953\ \text{N/m})(0.120\ \text{m})^2}{0.0350\ \text{kg}} - 2(9.80\ \text{m/s}^2)(0.120\ \text{m})}$$
$$= \boxed{19.7\ \text{m/s}}$$

You should compare the different examples we have presented so far in this chapter. Note how breaking the problem into a sequence of labeled events helps in the analysis.

Exercise What is the speed of the projectile when it is at a height of 10.0 m?

Answer 14.0 m/s.

EXAMPLE 8.8 Block–Spring Collision

A block having a mass of 0.80 kg is given an initial velocity $v_A = 1.2$ m/s to the right and collides with a spring of negligible mass and force constant $k = 50$ N/m, as shown in Figure 8.12. (a) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

Solution Our system in this example consists of the block and spring. Before the collision, at Ⓐ, the block has kinetic energy and the spring is uncompressed, so that the elastic potential energy stored in the spring is zero. Thus, the total mechanical energy of the system before the collision is just $\frac{1}{2}mv_A{}^2$. After the collision, at Ⓒ, the spring is fully compressed; now the block is at rest and so has zero kinetic energy, while the energy stored in the spring has its maximum value $\frac{1}{2}kx^2 = \frac{1}{2}kx_m{}^2$, where the origin of coordinates $x = 0$ is chosen to be the equilibrium position of the spring and x_m is

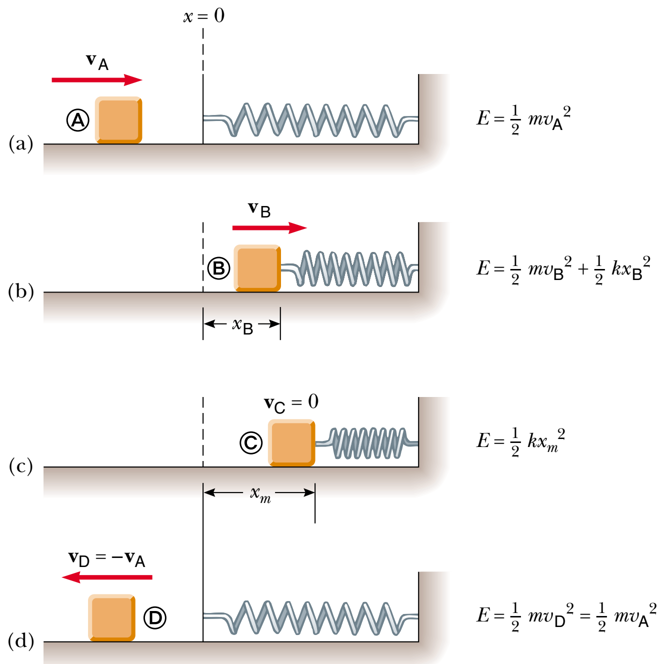

Figure 8.12 A block sliding on a smooth, horizontal surface collides with a light spring. (a) Initially the mechanical energy is all kinetic energy. (b) The mechanical energy is the sum of the kinetic energy of the block and the elastic potential energy in the spring. (c) The energy is entirely potential energy. (d) The energy is transformed back to the kinetic energy of the block. The total energy remains constant throughout the motion.

Multiflash photograph of a pole vault event. How many forms of energy can you identify in this picture?

the maximum compression of the spring, which in this case happens to be x_C. The total mechanical energy of the system is conserved because no nonconservative forces act on objects within the system.

Because mechanical energy is conserved, the kinetic energy of the block before the collision must equal the maximum potential energy stored in the fully compressed spring:

$$E_A = E_C$$

$$K_A + U_{sA} = K_C + U_{sC}$$

$$\tfrac{1}{2}mv_A{}^2 + 0 = 0 + \tfrac{1}{2}kx_m{}^2$$

$$x_m = \sqrt{\frac{m}{k}}\,v_A = \sqrt{\frac{0.80 \text{ kg}}{50 \text{ N/m}}}\,(1.2 \text{ m/s})$$

$$= \boxed{0.15 \text{ m}}$$

Note that we have not included U_g terms because no change in vertical position occurred.

(b) Suppose a constant force of kinetic friction acts between the block and the surface, with $\mu_k = 0.50$. If the speed of the block at the moment it collides with the spring is $v_A = 1.2$ m/s, what is the maximum compression in the spring?

Solution In this case, mechanical energy is *not* conserved because a frictional force acts on the block. The magnitude of the frictional force is

$$f_k = \mu_k n = \mu_k mg = 0.50(0.80 \text{ kg})(9.80 \text{ m/s}^2) = 3.92 \text{ N}$$

Therefore, the change in the block's mechanical energy due to friction as the block is displaced from the equilibrium position of the spring (where we have set our origin) to x_B is

$$\Delta E = -f_k x_B = -3.92x_B$$

Substituting this into Equation 8.15 gives

$$\Delta E = E_f - E_i = (0 + \tfrac{1}{2}kx_B{}^2) - (\tfrac{1}{2}mv_A{}^2 + 0) = -f_k x_B$$

$$\tfrac{1}{2}(50)x_B{}^2 - \tfrac{1}{2}(0.80)(1.2)^2 = -3.92x_B$$

$$25x_B{}^2 + 3.92x_B - 0.576 = 0$$

Solving the quadratic equation for x_B gives $x_B = 0.092$ m and $x_B = -0.25$ m. The physically meaningful root is $x_B = \boxed{0.092 \text{ m.}}$ The negative root does not apply to this situation because the block must be to the right of the origin (positive value of x) when it comes to rest. Note that 0.092 m is less than the distance obtained in the frictionless case of part (a). This result is what we expect because friction retards the motion of the system.

EXAMPLE 8.9 Connected Blocks in Motion

Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 8.13. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

Solution The key word *rest* appears twice in the problem statement, telling us that the initial and final velocities and kinetic energies are zero. (Also note that because we are concerned only with the beginning and ending points of the motion, we do not need to label events with circled letters as we did in the previous two examples. Simply using i and f is sufficient to keep track of the situation.) In this situation, the system consists of the two blocks, the spring, and the Earth. We need to consider two forms of potential energy: gravitational and elastic. Because the initial and final kinetic energies of the system are zero, $\Delta K = 0$, and we can write

$$(1) \qquad \Delta E = \Delta U_g + \Delta U_s$$

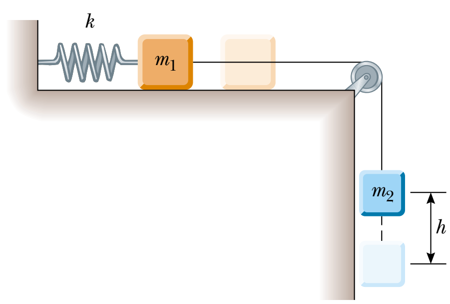

Figure 8.13 As the hanging block moves from its highest elevation to its lowest, the system loses gravitational potential energy but gains elastic potential energy in the spring. Some mechanical energy is lost because of friction between the sliding block and the surface.

where $\Delta U_g = U_{gf} - U_{gi}$ is the change in the system's gravitational potential energy and $\Delta U_s = U_{sf} - U_{si}$ is the change in the system's elastic potential energy. As the hanging block falls a distance h, the horizontally moving block moves the same distance h to the right. Therefore, using Equation 8.15, we find that the loss in energy due to friction between the horizontally sliding block and the surface is

$$(2) \qquad \Delta E = -f_k h = -\mu_k m_1 g h$$

The change in the gravitational potential energy of the system is associated with only the falling block because the vertical coordinate of the horizontally sliding block does not change. Therefore, we obtain

$$(3) \qquad \Delta U_g = U_{gf} - U_{gi} = 0 - m_2 g h$$

where the coordinates have been measured from the lowest position of the falling block.

The change in the elastic potential energy stored in the spring is

$$(4) \qquad \Delta U_s = U_{sf} - U_{si} = \tfrac{1}{2}kh^2 - 0$$

Substituting Equations (2), (3), and (4) into Equation (1) gives

$$-\mu_k m_1 g h = -m_2 g h + \tfrac{1}{2}kh^2$$

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

This setup represents a way of measuring the coefficient of kinetic friction between an object and some surface. As you can see from the problem, sometimes it is easier to work with the changes in the various types of energy rather than the actual values. For example, if we wanted to calculate the numerical value of the gravitational potential energy associated with the horizontally sliding block, we would need to specify the height of the horizontal surface relative to the lowest position of the falling block. Fortunately, this is not necessary because the gravitational potential energy associated with the first block does not change.

EXAMPLE 8.10 A Grand Entrance

You are designing apparatus to support an actor of mass 65 kg who is to "fly" down to the stage during the performance of a play. You decide to attach the actor's harness to a 130-kg sandbag by means of a lightweight steel cable running smoothly over two frictionless pulleys, as shown in Figure 8.14a. You need 3.0 m of cable between the harness and the nearest pulley so that the pulley can be hidden behind a curtain. For the apparatus to work successfully, the sandbag must never lift above the floor as the actor swings from above the stage to the floor. Let us call the angle that the actor's cable makes with the vertical θ. What is the maximum value θ can have before the sandbag lifts off the floor?

Solution We need to draw on several concepts to solve this problem. First, we use the principle of the conservation of mechanical energy to find the actor's speed as he hits the floor as a function of θ and the radius R of the circular path through which he swings. Next, we apply Newton's second law to the actor at the bottom of his path to find the cable tension as a function of the given parameters. Finally, we note that the sandbag lifts off the floor when the upward force exerted on it by the cable exceeds the gravitational force acting on it; the normal force is zero when this happens.

Applying conservation of energy to the actor–Earth system gives

$$K_i + U_i = K_f + U_f$$

$$(1) \qquad 0 + m_{\text{actor}}\, g y_i = \tfrac{1}{2} m_{\text{actor}} v_f^{\,2} + 0$$

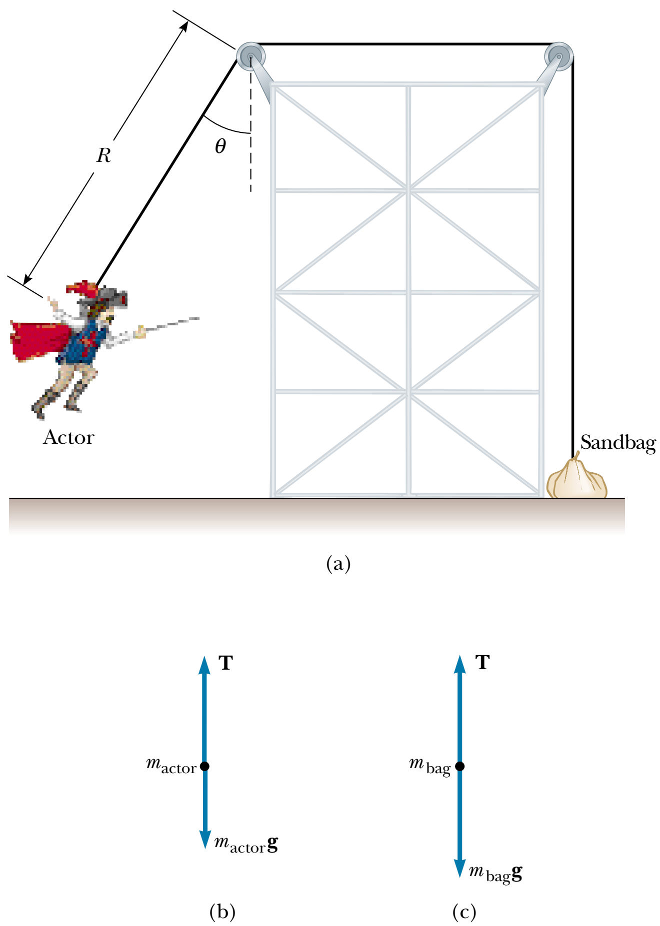

Figure 8.14 (a) An actor uses some clever staging to make his entrance. (b) Free-body diagram for actor at the bottom of the circular path. (c) Free-body diagram for sandbag.

where y_i is the initial height of the actor above the floor and v_f is the speed of the actor at the instant before he lands. (Note that $K_i = 0$ because he starts from rest and that $U_f = 0$ because we set the level of the actor's harness when he is standing on the floor as the zero level of potential energy.) From the geometry in Figure 8.14a, we see that $y_i = R - R\cos\theta = R(1 - \cos\theta)$. Using this relationship in Equation (1), we obtain

$$(2) \qquad v_f^{\,2} = 2gR(1 - \cos\theta)$$

Now we apply Newton's second law to the actor when he is at the bottom of the circular path, using the free-body diagram in Figure 8.14b as a guide:

$$\sum F_y = T - m_{\text{actor}} g = m_{\text{actor}} \frac{v_f^{\,2}}{R}$$

$$(3) \qquad T = m_{\text{actor}} g + m_{\text{actor}} \frac{v_f^{\,2}}{R}$$

A force of the same magnitude as T is transmitted to the sandbag. If it is to be just lifted off the floor, the normal force on it becomes zero, and we require that $T = m_{\text{bag}} g$, as shown in Figure 8.14c. Using this condition together with Equations (2) and (3), we find that

$$m_{\text{bag}} g = m_{\text{actor}} g + m_{\text{actor}} \frac{2gR(1 - \cos\theta)}{R}$$

Solving for θ and substituting in the given parameters, we obtain

$$\cos\theta = \frac{3m_{\text{actor}} - m_{\text{bag}}}{2m_{\text{actor}}} = \frac{3(65\text{ kg}) - 130\text{ kg}}{2(65\text{ kg})} = \frac{1}{2}$$

$$\theta = 60°$$

Notice that we did not need to be concerned with the length R of the cable from the actor's harness to the leftmost pulley. The important point to be made from this problem is that it is sometimes necessary to combine energy considerations with Newton's laws of motion.

Exercise If the initial angle $\theta = 40°$, find the speed of the actor and the tension in the cable just before he reaches the floor. (*Hint:* You cannot ignore the length $R = 3.0$ m in this calculation.)

Answer 3.7 m/s; 940 N.

8.6 RELATIONSHIP BETWEEN CONSERVATIVE FORCES AND POTENTIAL ENERGY

Once again let us consider a particle that is part of a system. Suppose that the particle moves along the x axis, and assume that a conservative force with an x component F_x acts on the particle. Earlier in this chapter, we showed how to determine the change in potential energy of a system when we are given the conservative force. We now show how to find F_x if the potential energy of the system is known.

In Section 8.2 we learned that the work done by the conservative force as its point of application undergoes a displacement Δx equals the negative of the change in the potential energy associated with that force; that is, $W = F_x \Delta x = -\Delta U$. If the point of application of the force undergoes an infinitesimal displacement dx, we can express the infinitesimal change in the potential energy of the system dU as

$$dU = -F_x\, dx$$

Therefore, the conservative force is related to the potential energy function through the relationship[3]

Relationship between force and potential energy

$$F_x = -\frac{dU}{dx} \qquad \textbf{(8.16)}$$

That is, **any conservative force acting on an object within a system equals the negative derivative of the potential energy of the system with respect to *x*.**

We can easily check this relationship for the two examples already discussed. In the case of the deformed spring, $U_s = \frac{1}{2}kx^2$, and therefore

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\tfrac{1}{2}kx^2\right) = -kx$$

which corresponds to the restoring force in the spring. Because the gravitational potential energy function is $U_g = mgy$, it follows from Equation 8.16 that $F_g = -mg$ when we differentiate U_g with respect to y instead of x.

We now see that U is an important function because a conservative force can be derived from it. Furthermore, Equation 8.16 should clarify the fact that adding a constant to the potential energy is unimportant because the derivative of a constant is zero.

Quick Quiz 8.5

What does the slope of a graph of $U(x)$ versus x represent?

Optional Section

8.7 ENERGY DIAGRAMS AND THE EQUILIBRIUM OF A SYSTEM

The motion of a system can often be understood qualitatively through a graph of its potential energy versus the separation distance between the objects in the system. Consider the potential energy function for a block–spring system, given by $U_s = \frac{1}{2}kx^2$. This function is plotted versus x in Figure 8.15a. (A common mistake is to think that potential energy on the graph represents height. This is clearly not

[3] In three dimensions, the expression is $\mathbf{F} = -\mathbf{i}\frac{\partial U}{\partial x} - \mathbf{j}\frac{\partial U}{\partial y} - \mathbf{k}\frac{\partial U}{\partial z}$, where $\frac{\partial U}{\partial x}$, and so forth, are partial derivatives. In the language of vector calculus, **F** equals the negative of the gradient of the scalar quantity $U(x, y, z)$.

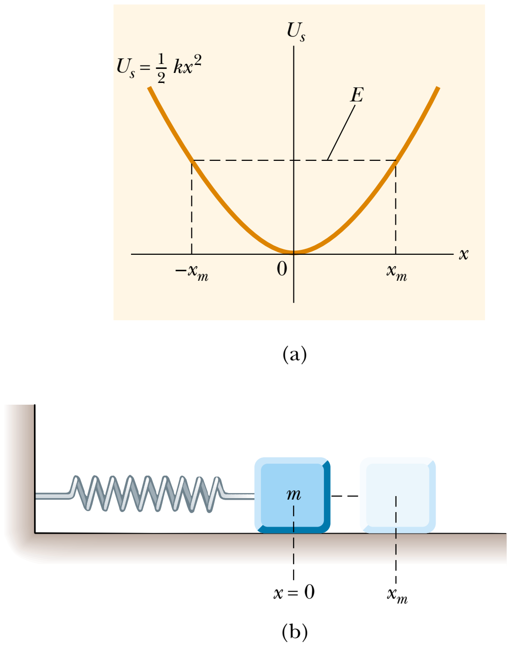

Figure 8.15 (a) Potential energy as a function of x for the block–spring system shown in (b). The block oscillates between the turning points, which have the coordinates $x = \pm x_m$. Note that the restoring force exerted by the spring always acts toward $x = 0$, the position of stable equilibrium.

the case here, where the block is only moving horizontally.) The force F_s exerted by the spring on the block is related to U_s through Equation 8.16:

$$F_s = -\frac{dU_s}{dx} = -kx$$

As we saw in Quick Quiz 8.5, the force is equal to the negative of the slope of the U versus x curve. When the block is placed at rest at the equilibrium position of the spring ($x = 0$), where $F_s = 0$, it will remain there unless some external force F_{ext} acts on it. If this external force stretches the spring from equilibrium, x is positive and the slope dU/dx is positive; therefore, the force F_s exerted by the spring is negative, and the block accelerates back toward $x = 0$ when released. If the external force compresses the spring, then x is negative and the slope is negative; therefore, F_s is positive, and again the mass accelerates toward $x = 0$ upon release.

From this analysis, we conclude that the $x = 0$ position for a block–spring system is one of **stable equilibrium.** That is, any movement away from this position results in a force directed back toward $x = 0$. In general, **positions of stable equilibrium correspond to points for which $U(x)$ is a minimum.**

From Figure 8.15 we see that if the block is given an initial displacement x_m and is released from rest, its total energy initially is the potential energy stored in the spring $\frac{1}{2}kx_m^2$. As the block starts to move, the system acquires kinetic energy and loses an equal amount of potential energy. Because the total energy must remain constant, the block oscillates (moves back and forth) between the two points $x = -x_m$ and $x = +x_m$, called the *turning points.* In fact, because no energy is lost (no friction), the block will oscillate between $-x_m$ and $+x_m$ forever. (We discuss these oscillations further in Chapter 13.) From an energy viewpoint, the energy of the system cannot exceed $\frac{1}{2}kx_m^2$; therefore, the block must stop at these points and, because of the spring force, must accelerate toward $x = 0$.

Another simple mechanical system that has a position of stable equilibrium is a ball rolling about in the bottom of a bowl. Anytime the ball is displaced from its lowest position, it tends to return to that position when released.

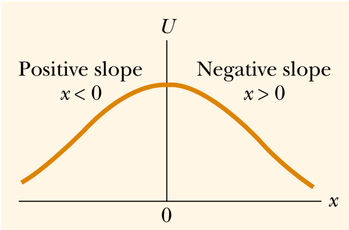

Figure 8.16 A plot of U versus x for a particle that has a position of unstable equilibrium located at $x = 0$. For any finite displacement of the particle, the force on the particle is directed away from $x = 0$.

Now consider a particle moving along the x axis under the influence of a conservative force F_x, where the U versus x curve is as shown in Figure 8.16. Once again, $F_x = 0$ at $x = 0$, and so the particle is in equilibrium at this point. However, this is a position of **unstable equilibrium** for the following reason: Suppose that the particle is displaced to the right $(x > 0)$. Because the slope is negative for $x > 0$, $F_x = -dU/dx$ is positive and the particle accelerates *away from* $x = 0$. If instead the particle is at $x = 0$ and is displaced to the left $(x < 0)$, the force is negative because the slope is positive for $x < 0$, and the particle again accelerates *away from* the equilibrium position. The position $x = 0$ in this situation is one of unstable equilibrium because for any displacement from this point, the force pushes the particle *farther away from* equilibrium. The force pushes the particle toward a position of lower potential energy. A pencil balanced on its point is in a position of unstable equilibrium. If the pencil is displaced slightly from its absolutely vertical position and is then released, it will surely fall over. In general, **positions of unstable equilibrium correspond to points for which $U(x)$ is a maximum.**

Finally, a situation may arise where U is constant over some region and hence $F_x = 0$. This is called a position of **neutral equilibrium.** Small displacements from this position produce neither restoring nor disrupting forces. A ball lying on a flat horizontal surface is an example of an object in neutral equilibrium.

EXAMPLE 8.11 Force and Energy on an Atomic Scale

The potential energy associated with the force between two neutral atoms in a molecule can be modeled by the Lennard–Jones potential energy function:

$$U(x) = 4\epsilon\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

where x is the separation of the atoms. The function $U(x)$ contains two parameters σ and ϵ that are determined from experiments. Sample values for the interaction between two atoms in a molecule are $\sigma = 0.263$ nm and $\epsilon = 1.51 \times 10^{-22}$ J. (a) Using a spreadsheet or similar tool, graph this function and find the most likely distance between the two atoms.

Solution We expect to find stable equilibrium when the two atoms are separated by some equilibrium distance and the potential energy of the system of two atoms (the molecule) is a minimum. One can minimize the function $U(x)$ by taking its derivative and setting it equal to zero:

$$\frac{dU(x)}{dx} = 4\epsilon\frac{d}{dx}\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right] = 0$$

$$= 4\epsilon\left[\frac{-12\sigma^{12}}{x^{13}} - \frac{-6\sigma^{6}}{x^{7}}\right] = 0$$

Solving for x—the equilibrium separation of the two atoms in the molecule—and inserting the given information yield

$$x = 2.95 \times 10^{-10}\ \text{m}.$$

We graph the Lennard–Jones function on both sides of this critical value to create our energy diagram, as shown in Figure 8.17a. Notice how $U(x)$ is extremely large when the atoms are very close together, is a minimum when the atoms are at their critical separation, and then increases again as the atoms move apart. When $U(x)$ is a minimum, the atoms are in stable equilbrium; this indicates that this is the most likely separation between them.

(b) Determine $F_x(x)$—the force that one atom exerts on the other in the molecule as a function of separation—and argue that the way this force behaves is physically plausible when the atoms are close together and far apart.

Solution Because the atoms combine to form a molecule, we reason that the force must be attractive when the atoms are far apart. On the other hand, the force must be repulsive when the two atoms get very close together. Otherwise, the molecule would collapse in on itself. Thus, the force must change sign at the critical separation, similar to the way spring forces switch sign in the change from extension to compression. Applying Equation 8.16 to the Lennard–Jones potential energy function gives

$$F_x = -\frac{dU(x)}{dx} = -4\epsilon\frac{d}{dx}\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

$$= 4\epsilon\left[\frac{12\sigma^{12}}{x^{13}} - \frac{6\sigma^{6}}{x^{7}}\right]$$

This result is graphed in Figure 8.17b. As expected, the force is positive (repulsive) at small atomic separations, zero when the atoms are at the position of stable equilibrium [recall how we found the minimum of $U(x)$], and negative (attractive) at greater separations. Note that the force approaches zero as the separation between the atoms becomes very great.

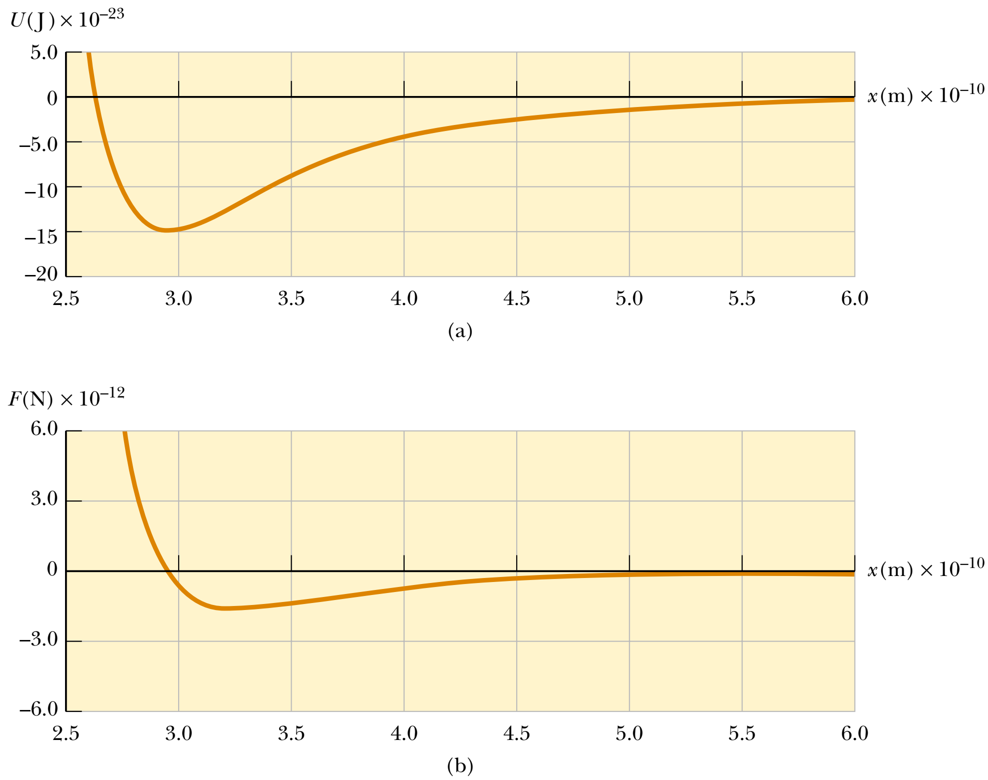

Figure 8.17 (a) Potential energy curve associated with a molecule. The distance x is the separation between the two atoms making up the molecule. (b) Force exerted on one atom by the other.

8.8 CONSERVATION OF ENERGY IN GENERAL

We have seen that the total mechanical energy of a system is constant when only conservative forces act within the system. Furthermore, we can associate a potential energy function with each conservative force. On the other hand, as we saw in Section 8.5, mechanical energy is lost when nonconservative forces such as friction are present.

In our study of thermodynamics later in this course, we shall find that mechanical energy can be transformed into energy stored *inside* the various objects that make up the system. This form of energy is called *internal energy*. For example, when a block slides over a rough surface, the mechanical energy lost because of friction is transformed into internal energy that is stored temporarily inside the block and inside the surface, as evidenced by a measurable increase in the temperature of both block and surface. We shall see that on a submicroscopic scale, this internal energy is associated with the vibration of atoms about their equilibrium positions. Such internal atomic motion involves both kinetic and potential energy. Therefore, if we include in our energy expression this increase in the internal energy of the objects that make up the system, then energy is conserved.

This is just one example of how you can analyze an isolated system and always find that the total amount of energy it contains does not change, as long as you account for all forms of energy. That is, **energy can never be created or destroyed. Energy may be transformed from one form to another, but the**

Total energy is always conserved

total energy of an isolated system is always constant. From a universal point of view, we can say that the **total energy of the Universe is constant.** If one part of the Universe gains energy in some form, then another part must lose an equal amount of energy. No violation of this principle has ever been found.

Optional Section

8.9 MASS–ENERGY EQUIVALENCE

This chapter has been concerned with the important principle of energy conservation and its application to various physical phenomena. Another important principle, **conservation of mass,** states that **in any physical or chemical process, mass is neither created nor destroyed.** That is, the mass before the process equals the mass after the process.

For centuries, scientists believed that energy and mass were two quantities that were separately conserved. However, in 1905 Einstein made the brilliant discovery that the mass of any system is a measure of the energy of that system. Hence, energy and mass are related concepts. The relationship between the two is given by Einstein's most famous formula:

$$E_R = mc^2 \tag{8.17}$$

where c is the speed of light and E_R is the energy equivalent of a mass m. The subscript R on the energy refers to the **rest energy** of an object of mass m—that is, the energy of the object when its speed is $v = 0$.

The rest energy associated with even a small amount of matter is enormous. For example, the rest energy of 1 kg of any substance is

$$E_R = mc^2 = (1\text{ kg})(3 \times 10^8\text{ m/s})^2 = 9 \times 10^{16}\text{ J}$$

This is equivalent to the energy content of about 15 million barrels of crude oil—about one day's consumption in the United States! If this energy could easily be released as useful work, our energy resources would be unlimited.

In reality, only a small fraction of the energy contained in a material sample can be released through chemical or nuclear processes. The effects are greatest in nuclear reactions, in which fractional changes in energy, and hence mass, of approximately 10^{-3} are routinely observed. A good example is the enormous amount of energy released when the uranium-235 nucleus splits into two smaller nuclei. This happens because the sum of the masses of the product nuclei is slightly less than the mass of the original ^{235}U nucleus. The awesome nature of the energy released in such reactions is vividly demonstrated in the explosion of a nuclear weapon.

Equation 8.17 indicates that *energy has mass.* Whenever the energy of an object changes in any way, its mass changes as well. If ΔE is the change in energy of an object, then its change in mass is

$$\Delta m = \frac{\Delta E}{c^2} \tag{8.18}$$

Anytime energy ΔE in any form is supplied to an object, the change in the mass of the object is $\Delta m = \Delta E/c^2$. However, because c^2 is so large, the changes in mass in any ordinary mechanical experiment or chemical reaction are too small to be detected.

EXAMPLE 8.12 Here Comes the Sun

The Sun converts an enormous amount of matter to energy. Each second, 4.19×10^9 kg—approximately the capacity of 400 average-sized cargo ships—is changed to energy. What is the power output of the Sun?

Solution We find the energy liberated per second by means of a straightforward conversion:

$$E_R = (4.19 \times 10^9 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 3.77 \times 10^{26} \text{ J}$$

We then apply the definition of power:

$$\mathcal{P} = \frac{3.77 \times 10^{26} \text{ J}}{1.00 \text{ s}} = \boxed{3.77 \times 10^{26} \text{ W}}$$

The Sun radiates uniformly in all directions, and so only a very tiny fraction of its total output is collected by the Earth. Nonetheless this amount is sufficient to supply energy to nearly everything on the Earth. (Nuclear and geothermal energy are the only alternatives.) Plants absorb solar energy and convert it to chemical potential energy (energy stored in the plant's molecules). When an animal eats the plant, this chemical potential energy can be turned into kinetic and other forms of energy. You are reading this book with solar-powered eyes!

Optional Section

8.10 QUANTIZATION OF ENERGY

Certain physical quantities such as electric charge are *quantized;* that is, the quantities have discrete values rather than continuous values. The quantized nature of energy is especially important in the atomic and subatomic world. As an example, let us consider the energy levels of the hydrogen atom (which consists of an electron orbiting around a proton). The atom can occupy only certain energy levels, called *quantum states,* as shown in Figure 8.18a. The atom cannot have any energy values lying between these quantum states. The lowest energy level E_1 is called the

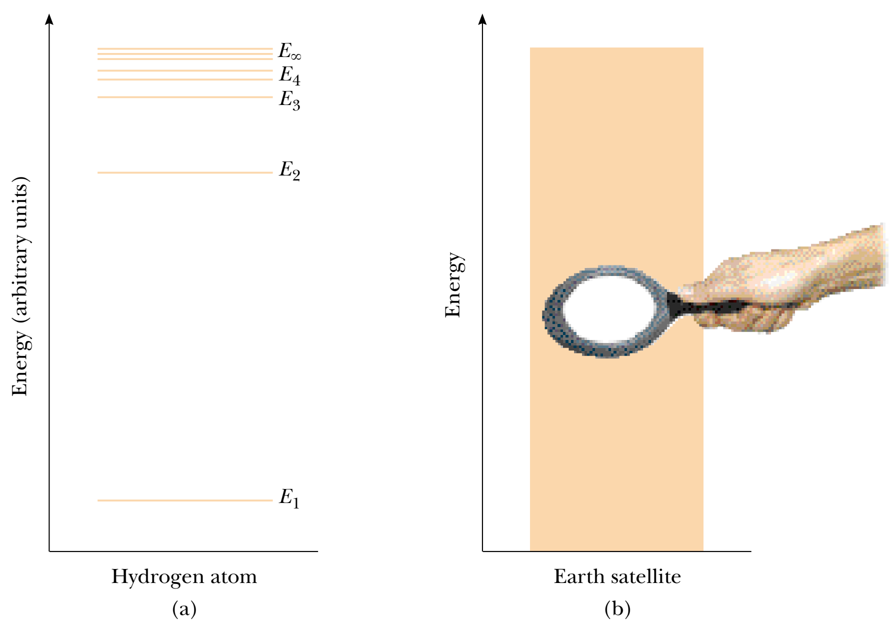

Figure 8.18 Energy-level diagrams: (a) Quantum states of the hydrogen atom. The lowest state E_1 is the ground state. (b) The energy levels of an Earth satellite are also quantized but are so close together that they cannot be distinguished from one another.

ground state of the atom. The ground state corresponds to the state that an isolated atom usually occupies. The atom can move to higher energy states by absorbing energy from some external source or by colliding with other atoms. The highest energy on the scale shown in Figure 8.18a, E_∞, corresponds to the energy of the atom when the electron is completely removed from the proton. The energy difference $E_\infty - E_1$ is called the **ionization energy.** Note that the energy levels get closer together at the high end of the scale.

Next, consider a satellite in orbit about the Earth. If you were asked to describe the possible energies that the satellite could have, it would be reasonable (but incorrect) to say that it could have any arbitrary energy value. Just like that of the hydrogen atom, however, **the energy of the satellite is quantized.** If you were to construct an energy level diagram for the satellite showing its allowed energies, the levels would be so close to one another, as shown in Figure 8.18b, that it would be difficult to discern that they were not continuous. In other words, we have no way of experiencing quantization of energy in the macroscopic world; hence, we can ignore it in describing everyday experiences.

SUMMARY

If a particle of mass m is at a distance y above the Earth's surface, the **gravitational potential energy** of the particle–Earth system is

$$U_g = mgy \qquad \textbf{(8.1)}$$

The **elastic potential energy** stored in a spring of force constant k is

$$U_s \equiv \tfrac{1}{2}kx^2 \qquad \textbf{(8.4)}$$

You should be able to apply these two equations in a variety of situations to determine the potential an object has to perform work.

A force is **conservative** if the work it does on a particle moving between two points is independent of the path the particle takes between the two points. Furthermore, a force is conservative if the work it does on a particle is zero when the particle moves through an arbitrary closed path and returns to its initial position. A force that does not meet these criteria is said to be **nonconservative.**

A **potential energy** function U can be associated only with a conservative force. If a conservative force $\mathbf{F}$ acts on a particle that moves along the x axis from x_i to x_f, then the change in the potential energy of the system equals the negative of the work done by that force:

$$U_f - U_i = -\int_{x_i}^{x_f} F_x\,dx \qquad \textbf{(8.7)}$$

You should be able to use calculus to find the potential energy associated with a conservative force and vice versa.

The **total mechanical energy of a system** is defined as the sum of the kinetic energy and the potential energy:

$$E \equiv K + U \qquad \textbf{(8.9)}$$

If no external forces do work on a system and if no nonconservative forces are acting on objects inside the system, then the total mechanical energy of the system is constant:

$$K_i + U_i = K_f + U_f \qquad \textbf{(8.10)}$$

If nonconservative forces (such as friction) act on objects inside a system, then mechanical energy is not conserved. In these situations, the difference between the total final mechanical energy and the total initial mechanical energy of the system equals the energy transferred to or from the system by the nonconservative forces.

QUESTIONS

1. Many mountain roads are constructed so that they spiral around a mountain rather than go straight up the slope. Discuss this design from the viewpoint of energy and power.
2. A ball is thrown straight up into the air. At what position is its kinetic energy a maximum? At what position is the gravitational potential energy a maximum?
3. A bowling ball is suspended from the ceiling of a lecture hall by a strong cord. The bowling ball is drawn away from its equilibrium position and released from rest at the tip of the student's nose as in Figure Q8.3. If the student remains stationary, explain why she will not be struck by the ball on its return swing. Would the student be safe if she pushed the ball as she released it?

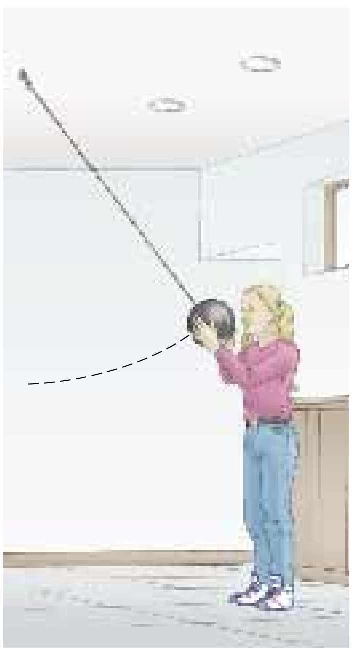

Figure Q8.3

4. One person drops a ball from the top of a building, while another person at the bottom observes its motion. Will these two people agree on the value of the potential energy of the ball–Earth system? on its change in potential energy? on the kinetic energy of the ball?
5. When a person runs in a track event at constant velocity, is any work done? (*Note:* Although the runner moves with constant velocity, the legs and arms accelerate.) How does air resistance enter into the picture? Does the center of mass of the runner move horizontally?
6. Our body muscles exert forces when we lift, push, run, jump, and so forth. Are these forces conservative?
7. If three conservative forces and one nonconservative force act on a system, how many potential energy terms appear in the equation that describes this system?
8. Consider a ball fixed to one end of a rigid rod whose other end pivots on a horizontal axis so that the rod can rotate in a vertical plane. What are the positions of stable and unstable equilibrium?
9. Is it physically possible to have a situation where $E - U < 0$?
10. What would the curve of U versus x look like if a particle were in a region of neutral equilibrium?
11. Explain the energy transformations that occur during (a) the pole vault, (b) the shot put, (c) the high jump. What is the source of energy in each case?
12. Discuss some of the energy transformations that occur during the operation of an automobile.
13. If only one external force acts on a particle, does it necessarily change the particle's (a) kinetic energy? (b) velocity?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
☐ = paired numerical/symbolic problems

Section 8.1 Potential Energy
Section 8.2 Conservative and Nonconservative Forces

1. A 1 000-kg roller coaster is initially at the top of a rise, at point A. It then moves 135 ft, at an angle of 40.0° below the horizontal, to a lower point B. (a) Choose point B to be the zero level for gravitational potential energy. Find the potential energy of the roller coaster–Earth system at points A and B and the change in its potential energy as the coaster moves. (b) Repeat part (a), setting the zero reference level at point A.

2. A 40.0-N child is in a swing that is attached to ropes 2.00 m long. Find the gravitational potential energy of the child–Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a 30.0° angle with the vertical, and (c) the child is at the bottom of the circular arc.

3. A 4.00-kg particle moves from the origin to position *C*, which has coordinates $x = 5.00$ m and $y = 5.00$ m (Fig. P8.3). One force on it is the force of gravity acting in the negative *y* direction. Using Equation 7.2, calculate the work done by gravity as the particle moves from *O* to *C* along (a) *OAC*, (b) *OBC*, and (c) *OC*. Your results should all be identical. Why?

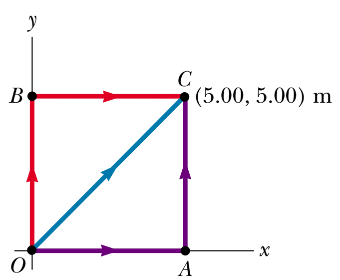

Figure P8.3 Problems 3, 4, and 5.

4. (a) Suppose that a constant force acts on an object. The force does not vary with time, nor with the position or velocity of the object. Start with the general definition for work done by a force

$$W = \int_i^f \mathbf{F} \cdot d\mathbf{s}$$

and show that the force is conservative. (b) As a special case, suppose that the force $\mathbf{F} = (3\mathbf{i} + 4\mathbf{j})$ N acts on a particle that moves from *O* to *C* in Figure P8.3. Calculate the work done by **F** if the particle moves along each one of the three paths *OAC*, *OBC*, and *OC*. (Your three answers should be identical.)

5. A force acting on a particle moving in the *xy* plane is given by $\mathbf{F} = (2y\mathbf{i} + x^2\mathbf{j})$ N, where *x* and *y* are in meters. The particle moves from the origin to a final position having coordinates $x = 5.00$ m and $y = 5.00$ m, as in Figure P8.3. Calculate the work done by **F** along (a) *OAC*, (b) *OBC*, (c) *OC*. (d) Is **F** conservative or nonconservative? Explain.

Section 8.3 Conservative Forces and Potential Energy
Section 8.4 Conservation of Mechanical Energy

6. At time t_i, the kinetic energy of a particle in a system is 30.0 J and the potential energy of the system is 10.0 J. At some later time t_f, the kinetic energy of the particle is 18.0 J. (a) If only conservative forces act on the particle, what are the potential energy and the total energy at time t_f? (b) If the potential energy of the system at time t_f is 5.00 J, are any nonconservative forces acting on the particle? Explain.

WEB **7.** A single conservative force acts on a 5.00-kg particle. The equation $F_x = (2x + 4)$ N, where *x* is in meters, describes this force. As the particle moves along the *x* axis from $x = 1.00$ m to $x = 5.00$ m, calculate (a) the work done by this force, (b) the change in the potential energy of the system, and (c) the kinetic energy of the particle at $x = 5.00$ m if its speed at $x = 1.00$ m is 3.00 m/s.

8. A single constant force $\mathbf{F} = (3\mathbf{i} + 5\mathbf{j})$ N acts on a 4.00-kg particle. (a) Calculate the work done by this force if the particle moves from the origin to the point having the vector position $\mathbf{r} = (2\mathbf{i} - 3\mathbf{j})$ m. Does this result depend on the path? Explain. (b) What is the speed of the particle at **r** if its speed at the origin is 4.00 m/s? (c) What is the change in the potential energy of the system?

9. A single conservative force acting on a particle varies as $\mathbf{F} = (-Ax + Bx^2)\mathbf{i}$ N, where *A* and *B* are constants and *x* is in meters. (a) Calculate the potential energy function $U(x)$ associated with this force, taking $U = 0$ at $x = 0$. (b) Find the change in potential energy and change in kinetic energy as the particle moves from $x = 2.00$ m to $x = 3.00$ m.

10. A particle of mass 0.500 kg is shot from *P* as shown in Figure P8.10. The particle has an initial velocity $\mathbf{v}_i$ with a horizontal component of 30.0 m/s. The particle rises to a maximum height of 20.0 m above *P*. Using the law of conservation of energy, determine (a) the vertical component of $\mathbf{v}_i$, (b) the work done by the gravitational force on the particle during its motion from *P* to *B*, and (c) the horizontal and the vertical components of the velocity vector when the particle reaches *B*.

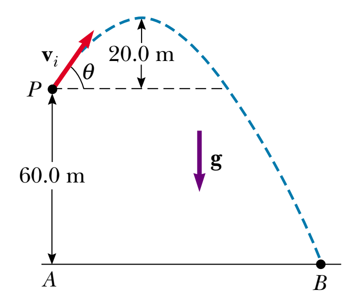

Figure P8.10

11. A 3.00-kg mass starts from rest and slides a distance *d* down a frictionless 30.0° incline. While sliding, it comes into contact with an unstressed spring of negligible mass, as shown in Figure P8.11. The mass slides an additional 0.200 m as it is brought momentarily to rest by compression of the spring ($k = 400$ N/m). Find the initial separation *d* between the mass and the spring.

12. A mass m starts from rest and slides a distance d down a frictionless incline of angle θ. While sliding, it contacts an unstressed spring of negligible mass, as shown in Figure P8.11. The mass slides an additional distance x as it is brought momentarily to rest by compression of the spring (of force constant k). Find the initial separation d between the mass and the spring.

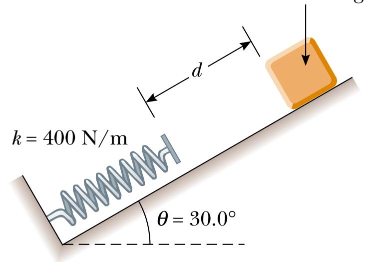

Figure P8.11 Problems 11 and 12.

13. A particle of mass $m = 5.00$ kg is released from point Ⓐ and slides on the frictionless track shown in Figure P8.13. Determine (a) the particle's speed at points Ⓑ and Ⓒ and (b) the net work done by the force of gravity in moving the particle from Ⓐ to Ⓒ.

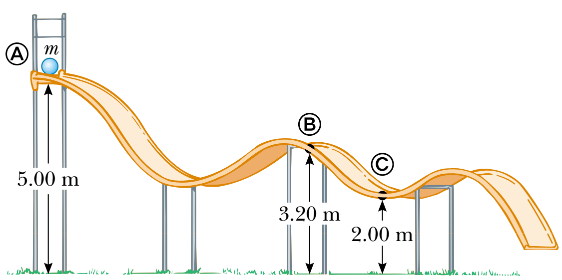

Figure P8.13

14. A simple, 2.00-m-long pendulum is released from rest when the support string is at an angle of 25.0° from the vertical. What is the speed of the suspended mass at the bottom of the swing?

15. A bead slides without friction around a loop-the-loop (Fig. P8.15). If the bead is released from a height $h = 3.50R$, what is its speed at point A? How great is the normal force on it if its mass is 5.00 g?

16. A 120-g mass is attached to the bottom end of an unstressed spring. The spring is hanging vertically and has a spring constant of 40.0 N/m. The mass is dropped. (a) What is its maximum speed? (b) How far does it drop before coming to rest momentarily?

17. A block of mass 0.250 kg is placed on top of a light vertical spring of constant $k = 5\,000$ N/m and is pushed downward so that the spring is compressed 0.100 m. After the block is released, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise?

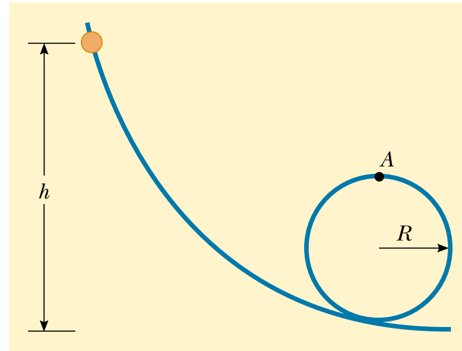

Figure P8.15

18. Dave Johnson, the bronze medalist at the 1992 Olympic decathlon in Barcelona, leaves the ground for his high jump with a vertical velocity component of 6.00 m/s. How far up does his center of gravity move as he makes the jump?

19. A 0.400-kg ball is thrown straight up into the air and reaches a maximum altitude of 20.0 m. Taking its initial position as the point of zero potential energy and using energy methods, find (a) its initial speed, (b) its total mechanical energy, and (c) the ratio of its kinetic energy to the potential energy of the ball–Earth system when the ball is at an altitude of 10.0 m.

20. In the dangerous "sport" of bungee-jumping, a daring student jumps from a balloon with a specially designed

Figure P8.20 Bungee-jumping. *(Gamma)* elastic cord attached to his ankles, as shown in Figure P8.20. The unstretched length of the cord is 25.0 m, the student weighs 700 N, and the balloon is 36.0 m above the surface of a river below. Assuming that Hooke's law describes the cord, calculate the required force constant if the student is to stop safely 4.00 m above the river.

21. Two masses are connected by a light string passing over a light frictionless pulley, as shown in Figure P8.21. The 5.00-kg mass is released from rest. Using the law of conservation of energy, (a) determine the speed of the 3.00-kg mass just as the 5.00-kg mass hits the ground and (b) find the maximum height to which the 3.00-kg mass rises.

22. Two masses are connected by a light string passing over a light frictionless pulley, as shown in Figure P8.21. The mass m_1 (which is greater than m_2) is released from rest. Using the law of conservation of energy, (a) determine the speed of m_2 just as m_1 hits the ground in terms of m_1, m_2, and h, and (b) find the maximum height to which m_2 rises.

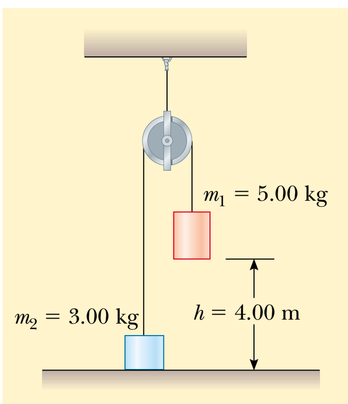

Figure P8.21 Problems 21 and 22.

23. A 20.0-kg cannon ball is fired from a cannon with a muzzle speed of 1 000 m/s at an angle of 37.0° with the horizontal. A second ball is fired at an angle of 90.0°. Use the law of conservation of mechanical energy to find (a) the maximum height reached by each ball and (b) the total mechanical energy at the maximum height for each ball. Let $y = 0$ at the cannon.

24. A 2.00-kg ball is attached to the bottom end of a length of 10-lb (44.5-N) fishing line. The top end of the fishing line is held stationary. The ball is released from rest while the line is taut and horizontal ($\theta = 90.0°$). At what angle θ (measured from the vertical) will the fishing line break?

25. The circus apparatus known as the *trapeze* consists of a bar suspended by two parallel ropes, each of length ℓ. The trapeze allows circus performers to swing in a vertical circular arc (Fig. P8.25). Suppose a performer with mass m and holding the bar steps off an elevated platform, starting from rest with the ropes at an angle of θ_i with respect to the vertical. Suppose the size of the performer's body is small compared with the length ℓ, that she does not pump the trapeze to swing higher, and that air resistance is negligible. (a) Show that when the ropes make an angle of θ with respect to the vertical, the performer must exert a force

$$F = mg\,(3\cos\theta - 2\cos\theta_i)$$

in order to hang on. (b) Determine the angle θ_i at which the force required to hang on at the bottom of the swing is twice the performer's weight.

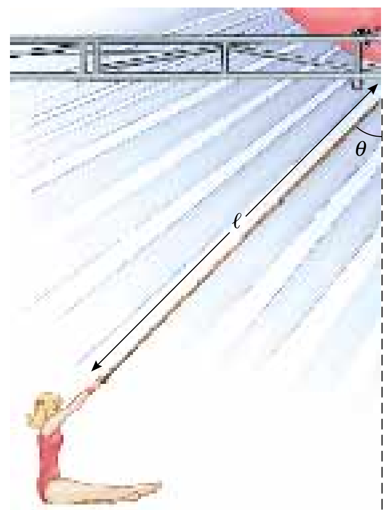

Figure P8.25

26. After its release at the top of the first rise, a roller-coaster car moves freely with negligible friction. The roller coaster shown in Figure P8.26 has a circular loop of radius 20.0 m. The car barely makes it around the loop: At the top of the loop, the riders are upside down and feel weightless. (a) Find the speed of the roller coaster car at the top of the loop (position 3). Find the speed of the roller coaster car (b) at position 1 and (c) at position 2. (d) Find the difference in height between positions 1 and 4 if the speed at position 4 is 10.0 m/s.

27. A light rigid rod is 77.0 cm long. Its top end is pivoted on a low-friction horizontal axle. The rod hangs straight down at rest, with a small massive ball attached to its bottom end. You strike the ball, suddenly giving it a horizontal velocity so that it swings around in a full circle. What minimum speed at the bottom is required to make the ball go over the top of the circle?

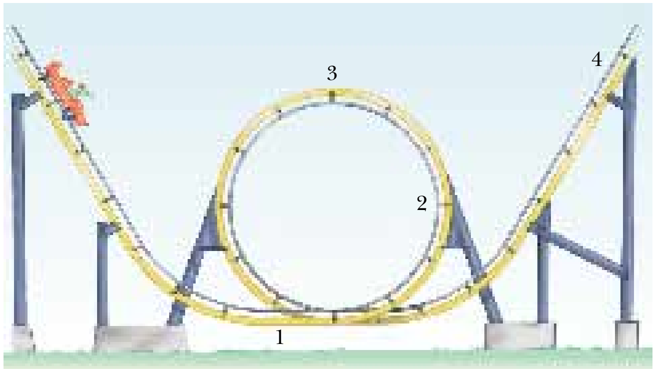

Figure P8.26

Section 8.5 Work Done by Nonconservative Forces

28. A 70.0-kg diver steps off a 10.0-m tower and drops straight down into the water. If he comes to rest 5.00 m beneath the surface of the water, determine the average resistance force that the water exerts on the diver.

29. A force F_x, shown as a function of distance in Figure P8.29, acts on a 5.00-kg mass. If the particle starts from rest at $x = 0$ m, determine the speed of the particle at $x = 2.00$, 4.00, and 6.00 m.

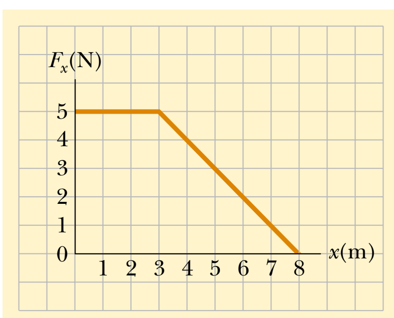

Figure P8.29

30. A softball pitcher swings a ball of mass 0.250 kg around a vertical circular path of radius 60.0 cm before releasing it from her hand. The pitcher maintains a component of force on the ball of constant magnitude 30.0 N in the direction of motion around the complete path. The speed of the ball at the top of the circle is 15.0 m/s. If the ball is released at the bottom of the circle, what is its speed upon release?

WEB **31.** The coefficient of friction between the 3.00-kg block and the surface in Figure P8.31 is 0.400. The system starts from rest. What is the speed of the 5.00-kg ball when it has fallen 1.50 m?

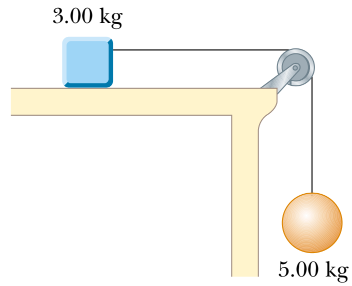

Figure P8.31

32. A 2 000-kg car starts from rest and coasts down from the top of a 5.00-m-long driveway that is sloped at an angle of 20.0° with the horizontal. If an average friction force of 4 000 N impedes the motion of the car, find the speed of the car at the bottom of the driveway.

33. A 5.00-kg block is set into motion up an inclined plane with an initial speed of 8.00 m/s (Fig. P8.33). The block comes to rest after traveling 3.00 m along the plane, which is inclined at an angle of 30.0° to the horizontal. For this motion determine (a) the change in the block's kinetic energy, (b) the change in the potential energy, and (c) the frictional force exerted on it (assumed to be constant). (d) What is the coefficient of kinetic friction?

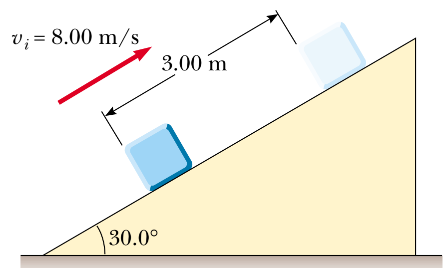

Figure P8.33

34. A boy in a wheelchair (total mass, 47.0 kg) wins a race with a skateboarder. He has a speed of 1.40 m/s at the crest of a slope 2.60 m high and 12.4 m long. At the bottom of the slope, his speed is 6.20 m/s. If air resistance and rolling resistance can be modeled as a constant frictional force of 41.0 N, find the work he did in pushing forward on his wheels during the downhill ride.

35. A parachutist of mass 50.0 kg jumps out of a balloon at a height of 1 000 m and lands on the ground with a speed of 5.00 m/s. How much energy was lost to air friction during this jump?

36. An 80.0-kg sky diver jumps out of a balloon at an altitude of 1 000 m and opens the parachute at an altitude of 200.0 m. (a) Assuming that the total retarding force on the diver is constant at 50.0 N with the parachute closed and constant at 3 600 N with the parachute open, what is the speed of the diver when he lands on the ground? (b) Do you think the sky diver will get hurt? Explain. (c) At what height should the parachute be opened so that the final speed of the sky diver when he hits the ground is 5.00 m/s? (d) How realistic is the assumption that the total retarding force is constant? Explain.

37. A toy cannon uses a spring to project a 5.30-g soft rubber ball. The spring is originally compressed by 5.00 cm and has a stiffness constant of 8.00 N/m. When it is fired, the ball moves 15.0 cm through the barrel of the cannon, and there is a constant frictional force of 0.032 0 N between the barrel and the ball. (a) With what speed does the projectile leave the barrel of the cannon? (b) At what point does the ball have maximum speed? (c) What is this maximum speed?

38. A 1.50-kg mass is held 1.20 m above a relaxed, massless vertical spring with a spring constant of 320 N/m. The mass is dropped onto the spring. (a) How far does it compress the spring? (b) How far would it compress the spring if the same experiment were performed on the Moon, where $g = 1.63\ \text{m/s}^2$? (c) Repeat part (a), but this time assume that a constant air-resistance force of 0.700 N acts on the mass during its motion.

39. A 3.00-kg block starts at a height $h = 60.0$ cm on a plane that has an inclination angle of 30.0°, as shown in Figure P8.39. Upon reaching the bottom, the block slides along a horizontal surface. If the coefficient of friction on both surfaces is $\mu_k = 0.200$, how far does the block slide on the horizontal surface before coming to rest? (*Hint:* Divide the path into two straight-line parts.)

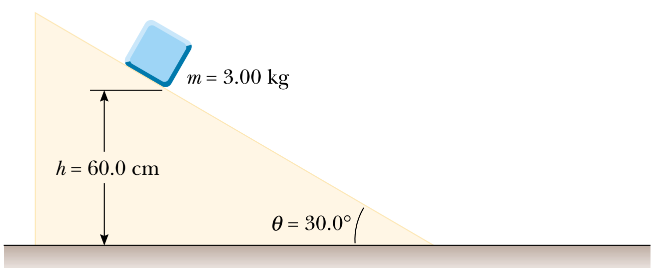

Figure P8.39

40. A 75.0-kg sky diver is falling with a terminal speed of 60.0 m/s. Determine the rate at which he is losing mechanical energy.

Section 8.6 Relationship Between Conservative Forces and Potential Energy

WEB **41.** The potential energy of a two-particle system separated by a distance r is given by $U(r) = A/r$, where A is a constant. Find the radial force $\mathbf{F}_r$ that each particle exerts on the other.

42. A potential energy function for a two-dimensional force is of the form $U = 3x^3y - 7x$. Find the force that acts at the point (x, y).

(Optional)
Section 8.7 Energy Diagrams and the Equilibrium of a System

43. A particle moves along a line where the potential energy depends on its position r, as graphed in Figure P8.43. In the limit as r increases without bound, $U(r)$ approaches $+1$ J. (a) Identify each equilibrium position for this particle. Indicate whether each is a point of stable, unstable, or neutral equilibrium. (b) The particle will be bound if its total energy is in what range? Now suppose the particle has energy -3 J. Determine (c) the range of positions where it can be found, (d) its maximum kinetic energy, (e) the location at which it has maximum kinetic energy, and (f) its *binding energy*—that is, the additional energy that it would have to be given in order for it to move out to $r \to \infty$.

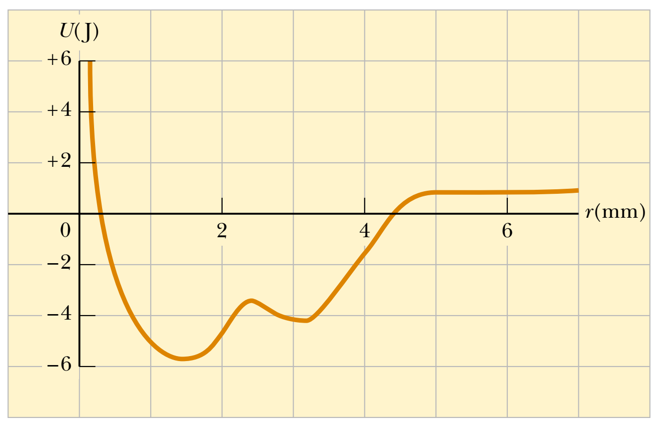

Figure P8.43

44. A right circular cone can be balanced on a horizontal surface in three different ways. Sketch these three equilibrium configurations and identify them as positions of stable, unstable, or neutral equilibrium.

45. For the potential energy curve shown in Figure P8.45, (a) determine whether the force F_x is positive, negative, or zero at the five points indicated. (b) Indicate points of stable, unstable, and neutral equilibrium. (c) Sketch the curve for F_x versus x from $x = 0$ to $x = 9.5$ m.

46. A hollow pipe has one or two weights attached to its inner surface, as shown in Figure P8.46. Characterize each configuration as being stable, unstable, or neutral equilibrium and explain each of your choices ("CM" indicates center of mass).

47. A particle of mass m is attached between two identical springs on a horizontal frictionless tabletop. The

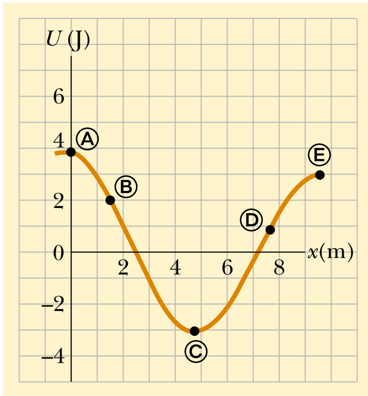

Figure P8.45

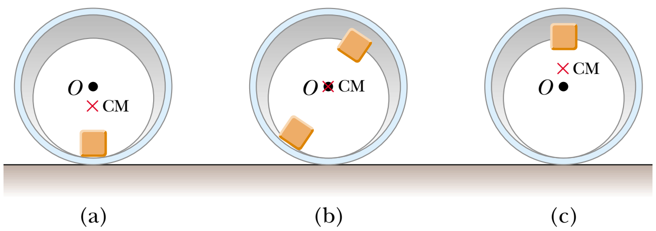

Figure P8.46

springs have spring constant k, and each is initially unstressed. (a) If the mass is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as in Figure P8.47, show that the potential energy of the system is

$$U(x) = kx^2 + 2kL(L - \sqrt{x^2 + L^2})$$

(*Hint:* See Problem 66 in Chapter 7.) (b) Make a plot of $U(x)$ versus x and identify all equilibrium points. Assume that $L = 1.20$ m and $k = 40.0$ N/m. (c) If the mass is pulled 0.500 m to the right and then released, what is its speed when it reaches the equilibrium point $x = 0$?

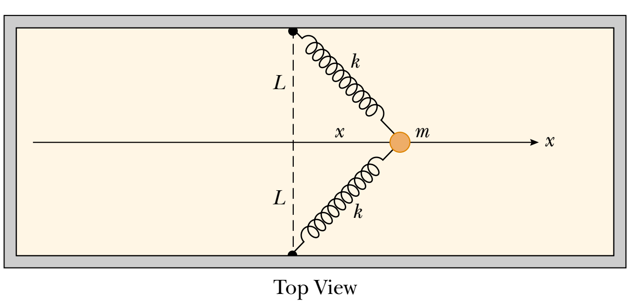

Figure P8.47

(Optional)

Section 8.9 Mass–Energy Equivalence

48. Find the energy equivalents of (a) an electron of mass 9.11×10^{-31} kg, (b) a uranium atom with a mass of 4.00×10^{-25} kg, (c) a paper clip of mass 2.00 g, and (d) the Earth (of mass 5.99×10^{24} kg).

49. The expression for the kinetic energy of a particle moving with speed v is given by Equation 7.19, which can be written as $K = \gamma mc^2 - mc^2$, where $\gamma = [1 - (v/c)^2]^{-1/2}$. The term γmc^2 is the total energy of the particle, and the term mc^2 is its rest energy. A proton moves with a speed of $0.990c$, where c is the speed of light. Find (a) its rest energy, (b) its total energy, and (c) its kinetic energy.

ADDITIONAL PROBLEMS

50. A block slides down a curved frictionless track and then up an inclined plane as in Figure P8.50. The coefficient of kinetic friction between the block and the incline is μ_k. Use energy methods to show that the maximum height reached by the block is

$$y_{\max} = \frac{h}{1 + \mu_k \cot \theta}$$

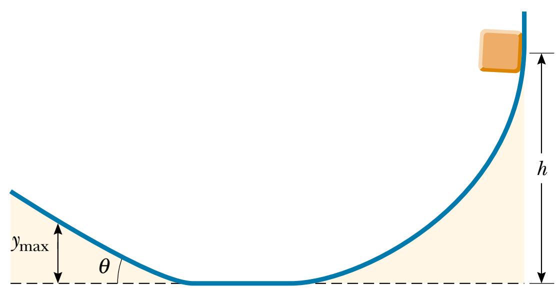

Figure P8.50

51. Close to the center of a campus is a tall silo topped with a hemispherical cap. The cap is frictionless when wet. Someone has somehow balanced a pumpkin at the highest point. The line from the center of curvature of the cap to the pumpkin makes an angle $\theta_i = 0°$ with the vertical. On a rainy night, a breath of wind makes the pumpkin start sliding downward from rest. It loses contact with the cap when the line from the center of the hemisphere to the pumpkin makes a certain angle with the vertical; what is this angle?

52. A 200-g particle is released from rest at point Ⓐ along the horizontal diameter on the inside of a frictionless, hemispherical bowl of radius $R = 30.0$ cm (Fig. P8.52). Calculate (a) the gravitational potential energy when the particle is at point Ⓐ relative to point Ⓑ, (b) the kinetic energy of the particle at point Ⓑ, (c) its speed at point Ⓑ, and (d) its kinetic energy and the potential energy at point Ⓒ.

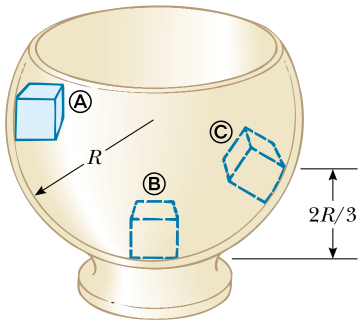

Figure P8.52 Problems 52 and 53.

WEB **53.** The particle described in Problem 52 (Fig. P8.52) is released from rest at Ⓐ, and the surface of the bowl is rough. The speed of the particle at Ⓑ is 1.50 m/s. (a) What is its kinetic energy at Ⓑ? (b) How much energy is lost owing to friction as the particle moves from Ⓐ to Ⓑ? (c) Is it possible to determine μ from these results in any simple manner? Explain.

54. Review Problem. The mass of a car is 1 500 kg. The shape of the body is such that its aerodynamic drag coefficient is $D = 0.330$ and the frontal area is 2.50 m^2. Assuming that the drag force is proportional to v^2 and neglecting other sources of friction, calculate the power the car requires to maintain a speed of 100 km/h as it climbs a long hill sloping at 3.20°.

55. Make an order-of-magnitude estimate of your power output as you climb stairs. In your solution, state the physical quantities you take as data and the values you measure or estimate for them. Do you consider your peak power or your sustainable power?

56. A child's pogo stick (Fig. P8.56) stores energy in a spring ($k = 2.50 \times 10^4$ N/m). At position Ⓐ ($x_A = -0.100$ m), the spring compression is a maximum and the child is momentarily at rest. At position Ⓑ ($x_B = 0$), the spring is relaxed and the child is moving upward. At position Ⓒ, the child is again momentarily at rest at the top of the jump. Assuming that the combined mass of the child and the pogo stick is 25.0 kg, (a) calculate the total energy of the system if both potential energies are zero at $x = 0$, (b) determine x_C, (c) calculate the speed of the child at $x = 0$, (d) determine the value of x for which the kinetic energy of the system is a maximum, and (e) calculate the child's maximum upward speed.

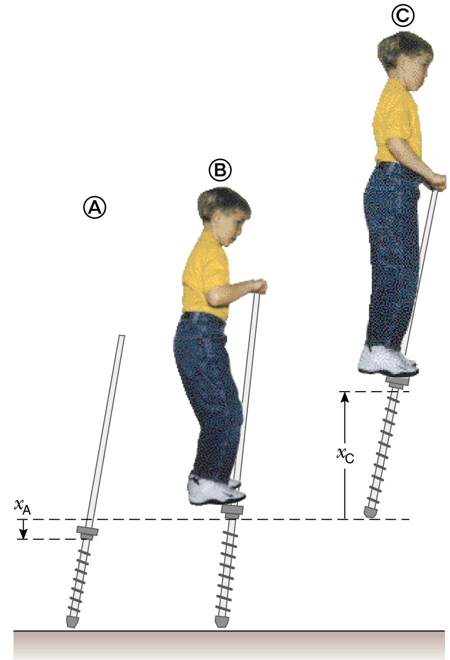

Figure P8.56

57. A 10.0-kg block is released from point Ⓐ in Figure P8.57. The track is frictionless except for the portion between Ⓑ and Ⓒ, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant $k = 2\,250$ N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between Ⓑ and Ⓒ.

58. A 2.00-kg block situated on a rough incline is connected to a spring of negligible mass having a spring constant of 100 N/m (Fig. P8.58). The pulley is frictionless. The block is released from rest when the spring is unstretched. The block moves 20.0 cm down the incline before coming to rest. Find the coefficient of kinetic friction between block and incline.

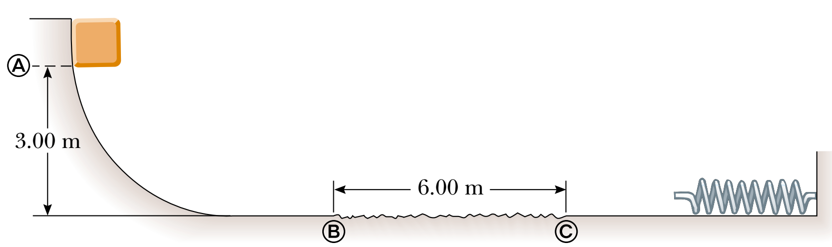

Figure P8.57

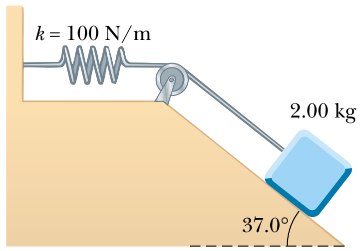

Figure P8.58 Problems 58 and 59.

59. **Review Problem.** Suppose the incline is frictionless for the system described in Problem 58 (see Fig. P8.58). The block is released from rest with the spring initially unstretched. (a) How far does it move down the incline before coming to rest? (b) What is its acceleration at its lowest point? Is the acceleration constant? (c) Describe the energy transformations that occur during the descent.

60. The potential energy function for a system is given by $U(x) = -x^3 + 2x^2 + 3x$. (a) Determine the force F_x as a function of x. (b) For what values of x is the force equal to zero? (c) Plot $U(x)$ versus x and F_x versus x, and indicate points of stable and unstable equilibrium.

61. A 20.0-kg block is connected to a 30.0-kg block by a string that passes over a frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of 250 N/m, as shown in Figure P8.61. The spring is unstretched when the system is as shown in the figure, and the incline is frictionless. The 20.0-kg block is pulled 20.0 cm down the incline (so that the 30.0-kg block is 40.0 cm above the floor) and is released from rest. Find the speed of each block when the 30.0-kg block is 20.0 cm above the floor (that is, when the spring is unstretched).

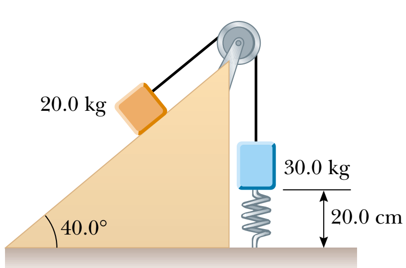

Figure P8.61

62. A 1.00-kg mass slides to the right on a surface having a coefficient of friction $\mu = 0.250$ (Fig. P8.62). The mass has a speed of $v_i = 3.00$ m/s when it makes contact with a light spring that has a spring constant $k = 50.0$ N/m. The mass comes to rest after the spring has been compressed a distance d. The mass is then forced toward the left by the spring and continues to move in that direction beyond the spring's unstretched position. Finally, the mass comes to rest at a distance D to the left of the unstretched spring. Find (a) the distance of compression d, (b) the speed v of the mass at the unstretched position when the mass is moving to the left, and (c) the distance D between the unstretched spring and the point at which the mass comes to rest.

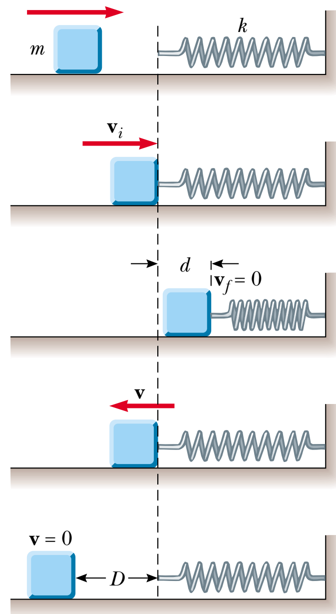

Figure P8.62

WEB **63.** A block of mass 0.500 kg is pushed against a horizontal spring of negligible mass until the spring is compressed a distance Δx (Fig. P8.63). The spring constant is 450 N/m. When it is released, the block travels along a frictionless, horizontal surface to point B, at the bottom of a vertical circular track of radius $R = 1.00$ m, and continues to move up the track. The speed of the block at the bottom of the track is $v_B = 12.0$ m/s, and the block experiences an average frictional force of 7.00 N while sliding up the track. (a) What is Δx? (b) What speed do you predict for the block at the top of the track? (c) Does the block actually reach the top of the track, or does it fall off before reaching the top?

64. A uniform chain of length 8.00 m initially lies stretched out on a horizontal table. (a) If the coefficient of static friction between the chain and the table is 0.600, show that the chain will begin to slide off the table if at least 3.00 m of it hangs over the edge of the table. (b) Determine the speed of the chain as all of it leaves the table, given that the coefficient of kinetic friction between the chain and the table is 0.400.

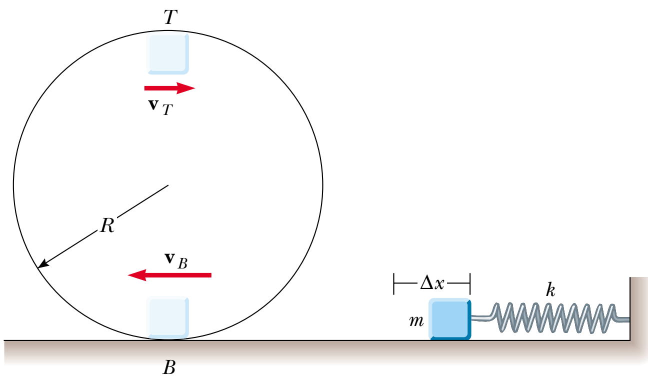

Figure P8.63

65. An object of mass m is suspended from a post on top of a cart by a string of length L as in Figure P8.65a. The cart and object are initially moving to the right at constant speed v_i. The cart comes to rest after colliding and sticking to a bumper as in Figure P8.65b, and the suspended object swings through an angle θ. (a) Show that the speed is $v_i = \sqrt{2gL(1 - \cos\theta)}$. (b) If $L = 1.20$ m and $\theta = 35.0°$, find the initial speed of the cart. (*Hint:* The force exerted by the string on the object does no work on the object.)

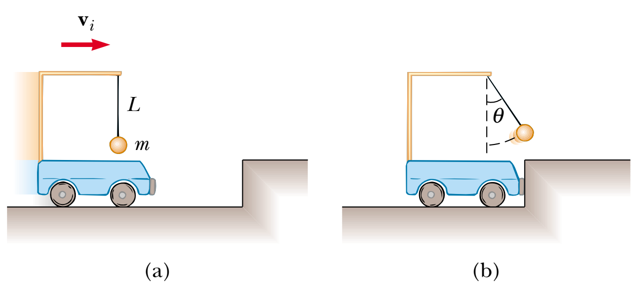

Figure P8.65

66. A child slides without friction from a height h along a curved water slide (Fig. P8.66). She is launched from a height $h/5$ into the pool. Determine her maximum airborne height y in terms of h and θ.

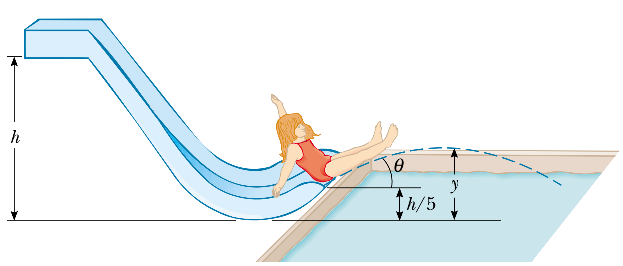

Figure P8.66

67. A ball having mass m is connected by a strong string of length L to a pivot point and held in place in a vertical position. A wind exerting constant force of magnitude F is blowing from left to right as in Figure P8.67a. (a) If the ball is released from rest, show that the maximum height H it reaches, as measured from its initial height, is

$$H = \frac{2L}{1 + (mg/F)^2}$$

Check that the above formula is valid both when $0 \le H \le L$ and when $L \le H \le 2L$. (*Hint:* First determine the potential energy associated with the *constant* wind force.) (b) Compute the value of H using the values $m = 2.00$ kg, $L = 2.00$ m, and $F = 14.7$ N. (c) Using these same values, determine the *equilibrium* height of the ball. (d) Could the equilibrium height ever be greater than L? Explain.

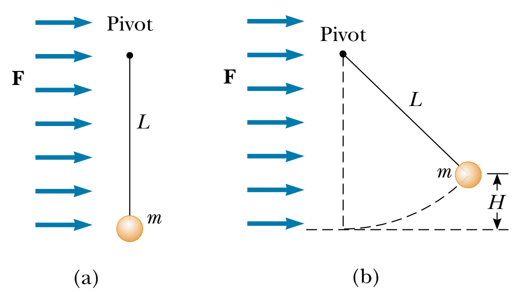

Figure P8.67

68. A ball is tied to one end of a string. The other end of the string is fixed. The ball is set in motion around a vertical circle without friction. At the top of the circle, the ball has a speed of $v_i = \sqrt{Rg}$, as shown in Figure P8.68. At what angle θ should the string be cut so that the ball will travel through the center of the circle?

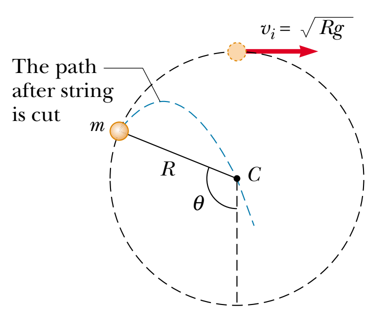

Figure P8.68

69. A ball at the end of a string whirls around in a vertical circle. If the ball's total energy remains constant, show that the tension in the string at the bottom is greater than the tension at the top by a value six times the weight of the ball.

70. A pendulum comprising a string of length L and a sphere swings in the vertical plane. The string hits a peg located a distance d below the point of suspension (Fig. P8.70). (a) Show that if the sphere is released from a height below that of the peg, it will return to this height after striking the peg. (b) Show that if the pendulum is released from the horizontal position ($\theta = 90°$) and is to swing in a complete circle centered on the peg, then the minimum value of d must be $3L/5$.

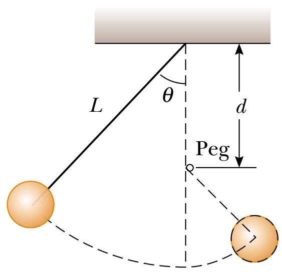

Figure P8.70

71. Jane, whose mass is 50.0 kg, needs to swing across a river (having width D) filled with man-eating crocodiles to save Tarzan from danger. However, she must swing into a wind exerting constant horizontal force **F** on a vine having length L and initially making an angle θ with the vertical (Fig. P8.71). Taking $D = 50.0$ m, $F = 110$ N, $L = 40.0$ m, and $\theta = 50.0°$, (a) with what minimum speed must Jane begin her swing to just make it to the other side? (*Hint:* First determine the potential energy associated with the wind force.) (b) Once the rescue is complete, Tarzan and Jane must swing back across the river. With what minimum speed must they begin their swing? Assume that Tarzan has a mass of 80.0 kg.

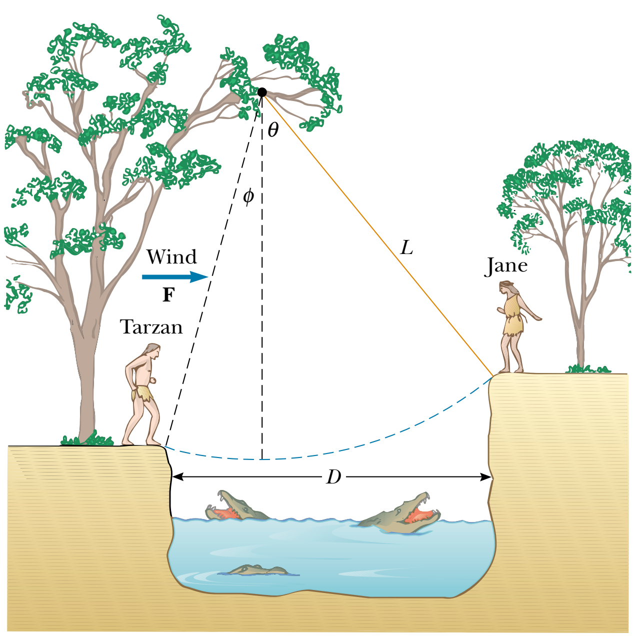

Figure P8.71

72. A child starts from rest and slides down the frictionless slide shown in Figure P8.72. In terms of R and H, at what height h will he lose contact with the section of radius R?

Figure P8.72

73. A 5.00-kg block free to move on a horizontal, frictionless surface is attached to one end of a light horizontal spring. The other end of the spring is fixed. The spring is compressed 0.100 m from equilibrium and is then released. The speed of the block is 1.20 m/s when it passes the equilibrium position of the spring. The same experiment is now repeated with the frictionless surface replaced by a surface for which $\mu_k = 0.300$. Determine the speed of the block at the equilibrium position of the spring.

74. A 50.0-kg block and a 100-kg block are connected by a string as in Figure P8.74. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50.0-kg block and the incline is $\mu_k = 0.250$. Determine the change in the kinetic energy of the 50.0-kg block as it moves from Ⓐ to Ⓑ, a distance of 20.0 m.

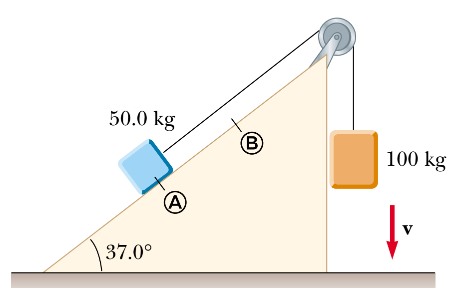

Figure P8.74

Answers to Quick Quizzes

8.1 Yes, because we are free to choose any point whatsoever as our origin of coordinates, which is the $U_g = 0$ point. If the object is below the origin of coordinates that we choose, then $U_g < 0$ for the object–Earth system.

8.2 Yes, the total mechanical energy of the system is conserved because the only forces acting are conservative: the force of gravity and the spring force. There are two forms of potential energy: (1) gravitational potential energy and (2) elastic potential energy stored in the spring.

8.3 The first and third balls speed up after they are thrown, while the second ball initially slows down but then speeds up after reaching its peak. The paths of all three balls are parabolas, and the balls take different times to reach the ground because they have different initial velocities. However, all three balls have the same speed at the moment they hit the ground because all start with the same kinetic energy and undergo the same change in gravitational potential energy. In other words, $E_{\text{total}} = \frac{1}{2}mv^2 + mgh$ is the same for all three balls at the start of the motion.

8.4 Designate one object as No. 1 and the other as No. 2. The external force does work W_{app} on the system. If $W_{\text{app}} > 0$, then the system energy increases. If $W_{\text{app}} < 0$, then the system energy decreases. The effect of friction is to decrease the total system energy. Equation 8.15 then becomes

$$\begin{aligned}\Delta E &= W_{\text{app}} - \Delta E_{\text{friction}} \\ &= \Delta K + \Delta U \\ &= [K_{1f} + K_{2f}) - (K_{1i} + K_{2i})] \\ &\quad + [(U_{g1f} + U_{g2f} + U_{sf}) - (U_{g1i} + U_{g2i} + U_{si})]\end{aligned}$$

You may find it easier to think of this equation with its terms in a different order, saying

total initial energy + net change = total final energy

$$K_{1i} + K_{2i} + U_{g1i} + U_{g2i} + U_{si} + W_{\text{app}} - f_k d = K_{1f} + K_{2f} + U_{g1f} + U_{g2f} + U_{sf}$$

8.5 The slope of a $U(x)$-versus-x graph is by definition $dU(x)/dx$. From Equation 8.16, we see that this expression is equal to the negative of the x component of the conservative force acting on an object that is part of the system.