Differentiation

Exercise 28A

Q. 1. Differentiate the following functions:

(i) x^{-3}

(ii) $\sqrt[3]{x}$

Answer : (i) x^{-3}

$$\frac{d}{dx}x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx}x^{-3} = -3x^{-3-1}$$

$$= -3x^{-4}$$

(ii) $\sqrt[3]{x} = x^{\frac{1}{3}}$

Formula:-

$$\frac{d}{dx}x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx}x^{\frac{1}{3}} = \frac{1}{3}x^{\frac{1}{3}-1}$$

$$= \frac{1}{3}x^{-\frac{2}{3}}$$

Q. 2. Differentiate the following functions:

(i) $\frac{1}{x}$ **(ii)** $\frac{1}{\sqrt{x}}$ **(iii)** $\frac{1}{\sqrt[3]{x}}$

Answer : (i) $\frac{1}{x} = x^{-1}$

Formula:-

$$\frac{d}{dx}x^{n} = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx}x^{-1} = -1x^{-1-1}$$

$$= -x^{-2}$$

(ii) $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

Formula:-

$$\frac{d}{dx}x^{n} = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx}x^{\frac{-1}{2}} = \frac{-1}{2}x^{-\frac{1}{2}-1}$$

$$= \frac{-1}{2}x^{-\frac{3}{2}}$$

(iii) $\frac{1}{\sqrt[3]{x}} = x^{\frac{-1}{3}}$

Formula:-

$$\frac{d}{dx}x^{n} = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx}x^{\frac{-1}{3}} = \frac{-1}{3}x^{\frac{-1}{3}-1}$$

$$= -\frac{1}{3}x^{-\frac{4}{3}}$$

Q. 3. Differentiate the following functions:

(i) $3x^{-5}$

(ii) $\frac{1}{5x}$

(iii) $6.\sqrt[3]{x^2}$

Answer : (i) $3x^{-5}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

Differentiating with respect to x,

$\frac{d}{dx}3x^{-5} = 3(-5)x^{-5-1}$

$= -15x^{-6}$

(ii) $1/5x = \frac{1}{5}x^{-1}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

Differentiating with respect to x,

$\frac{1}{5}\frac{d}{dx}x^{-1} = \frac{-1}{5}x^{-1-1}$

$= -\frac{1}{5}x^{-2}$

(iii) $6.\sqrt[3]{x^2} = 6x^{\frac{2}{3}}$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

Differentiating with respect to x,

$\frac{d}{dx}6x^{\frac{2}{3}} = 6 \times \frac{2}{3}x^{\frac{2}{3}-1}$

$= 4x^{-\frac{1}{3}}$

Q. 4 Differentiate the following functions:

(i) 6x5 + 4x3 – 3x2 + 2x – 7
(ii)

$5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x}$

(iii) ax3 + bx2 + cx + d, where a, b, c, d are constants

Answer : (i) $6x^5 + 4x^3 - 3x^2 + 2x - 7$

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

Differentiating with respect to x,

$$\frac{d}{dx}(6x^5 + 4x^3 - 3x^2 + 2x - 7) = 30x^{5-1} + 12x^{3-1} - 6x^{2-1} + 2x^{1-1} + 0$$

$$= 30x^4 + 12x^2 - 6x^1 + 2x$$

(ii) $5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x}$

Formula:-

$$\frac{d}{dx}x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\frac{d}{dx}(5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x})$$

$$= 5 \times -\frac{3}{2}x^{-\frac{3}{2}-1} + 4 \times -\frac{1}{2}x^{-\frac{1}{2}-1} + \frac{1}{2}x^{\frac{1}{2}-1} - 7 \times -1x^{-1-1}$$

$$= -\frac{15}{2}x^{-\frac{5}{2}} - 2x^{-\frac{3}{2}} + \frac{1}{2}x^{\frac{-1}{2}} + 7x^{-2}$$

(iii) $ax^3 + bx^2 + cx + d$, where a, b, c, d are constants

Formula:-

$\frac{d}{dx}x^n = nx^{n-1}$

Differentiating with respect to x,

$\frac{d}{dx}(ax^3 + bx^2 + cx + d) = 3ax^{3-1} + 2bx^{2-1} + cx^{1-1} + d\times 0$

$= 3ax^2 + 2bx + c$

Q. 5. Differentiate the following functions:

(i) $4x^3 + 3.2^x + 6.\sqrt[8]{x^{-4}} + 5\cot x$

(ii) $\frac{x}{3} - \frac{3}{x} + \sqrt{x} - \frac{1}{\sqrt{x}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$

Answer :

(i) $4x^3 + 3.2^x + 6.\sqrt[8]{x^{-4}} + 5\cot x$

$= 4x^3 + 3.2^x + 6x^{-\frac{1}{2}} + 5\cot x$

Formulae:

$\frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$\frac{d}{dx}a^x = \log_n(a)\times a^x$

Differentiating with respect to x,

$$\frac{d}{dx}(4x^3 + 3.2^x + 6x^{-\frac{1}{2}} + 5\cot x)$$

$$= 4.3x^{3-1} + 3.\log_n(2).2^x + 6\times-\frac{1}{2}x^{-\frac{1}{2}-1} + 5\times-\operatorname{cosec}^2 x$$

$$= 12x^2 + 3.\log_n(2).2^x - 3x^{-\frac{3}{2}} - 5\operatorname{cosec}^2 x$$

(ii) $\frac{x}{3} - \frac{3}{x} + \sqrt{x} - \frac{1}{\sqrt{x}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$

$$= \frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$$

$\frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}a^x = \log_n(a)\times a^x$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-\frac{2}{3}} - \frac{2}{3}x^6\right)$$

$= \frac{1}{3} - (-1) \times 3x^{-1-1} + \frac{1}{2}x^{\frac{1}{2}-1} - \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} + 2x^{2-1} - \log(2).2^x + 6\left(-\frac{2}{3}\right)x^{-\frac{2}{3}-1} - \frac{2}{3} \times 6x^{6-1}$

$= \frac{1}{3} + 3x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} + 2x^1 - \log(2).2^x - 4x^{-\frac{5}{3}} - 4x^5$

Q. 6. Differentiate the following functions:

(i) $4\cot x - \frac{1}{2}\cos x + \frac{2}{\cos x} - \frac{3}{\sin x} + \frac{6\cot x}{\operatorname{cosec} x} + 9$

(ii) -5 tan x + 4 tan x cos x – 3 cot x sec x + 2sec x – 13

Answer : Formulae: -

$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}k = 0$, k is constant

(i) $4\cot x - \frac{1}{2}\cos x + \frac{2}{\cos x} - \frac{3}{\sin x} + \frac{6\cot x}{\operatorname{cosec} x} + 9$

$= 4\cot x - \frac{1}{2}\cos x + 2\sec x - 3\operatorname{cosec} x + 6\cos x + 9$

Differentiating with respect to x,

$$\frac{d}{dx}\left(4\cot x - \frac{1}{2}\cos x + 2\sec x - 3\operatorname{cosec} x + 6\cos x + 9\right)$$

$$= 4(-\operatorname{cosec}^2 x) - \frac{1}{2}(-\sin x) + 2\sec x \times \tan x - 3(-\operatorname{cosec} x \times \cot x) + 6(-\sin x) + 0$$

$$= -4\operatorname{cosec}^2 x + \frac{1}{2}\sin x + 2\sec x\tan x + 3\operatorname{cosec} x\cot x - 6\sin x$$

(ii) -5 tan x + 4 tan x cos x – 3 cot x sec x + 2sec x – 13

= -5 tan x + 4 sinx – 3 cosecx+ 2sec x – 13

Differentiating with respect to x,

$\frac{d}{dx}$(-5 tan x + 4 sinx – 3 cosecx+ 2sec x – 13)

= -5 $\sec^2 x$ + 4cosx -3(- cosecx cotx) + 2 secx tanx – 0

= -5 $\sec^2 x$ + 4cosx + 3 cosecx cotx + 2 secx tanx

Q. 7 Differentiate the following functions:

(i) $(2x + 3)(3x - 5)$

(ii) $x(1 + x)^3$

(iii) $\left(\sqrt{x} + \frac{1}{x}\right)\left(x - \frac{1}{\sqrt{x}}\right)$

(iv) $\left(x - \frac{1}{x}\right)^2$

(v) $\left(x^2 - \frac{1}{x^2}\right)^3$

(vi) $(2x^2 + 5x - 1)(x - 3)$

Answer : Formula:

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g)\frac{d}{dx}g$$

Chain rule -

$$\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\frac{d}{dx}u$$

Where u and v are the functions of x.

(i) $(2x + 3)(3x - 5)$

Applying, Chain rule

Here, $u = 2x + 3$

$V = 3x - 5$

$$\frac{d}{dx}(2x + 3)(3x - 5) = (2x + 3)\frac{d}{dx}(3x - 5) + (3x - 5)\frac{d}{dx}(2x + 3)$$

$= (2x + 3)(3x^{1-1}+0) + (3x - 5)(2x^{1-1}+0)$

$= 6x + 9 + 6x - 10$

$= 12x - 1$

(ii) $x(1 + x)^3$

Applying, Chain rule

Here, u = x

V = $(1 + x)^3$

$$\frac{d}{dx}x(1+x)^3 = x\frac{d}{dx}(1 + x)3 + (1 + x)3\frac{d}{dx}(x)$$

= $x \times 3 \times (1 + x)^2 + (1 + x)^3(1)$

= $(1 + x)^2(3x+x+1)$

= $(1 + x)^2(4x+1)$

(iii) $\left(\sqrt{x}+\frac{1}{x}\right)\left(x-\frac{1}{\sqrt{x}}\right)$ = $(x^{1/2} + x^{-1})(x - x^{-1/2})$

Applying, Chain rule

Here, u = $(x^{1/2} + x^{-1})$

V = $(x - x^{-1/2})$

$$\frac{d}{dx}(x^{1/2} + x^{-1})(x - x^{-1/2})$$

$$= (x^{1/2} + x^{-1})\frac{d}{dx}(x - x^{-1/2}) + (x - x^{-1/2})\frac{d}{dx}(x^{1/2} + x^{-1})$$

$$= (x^{1/2} + x^{-1})(1+\frac{1}{2}x^{-3/2}) + (x - x^{-1/2})(\frac{1}{2}x^{-1/2} - x^{-2})$$

$$= x^{1/2} + x^{-1} + \frac{1}{2}x^{-1} + \frac{1}{2}x^{-5/2} + \frac{1}{2}x^{1/2} - x^{-1} - \frac{1}{2}x^{-1} + x^{-5/2}$$

$$= \frac{3}{2}x^{1/2} + \frac{3}{2}x^{-5/2}$$

(iv)

$$\left(x-\frac{1}{x}\right)^2$$

Differentiation of composite function can be done by

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g)\frac{d}{dx}g$$

Here, $f(g) = g^2$, $g(x) = x - \frac{1}{x}$

$$\frac{d}{dx}\left(x-\frac{1}{x}\right)^2 = 2g\times(1 + \frac{1}{x^2})$$

$$= 2(x - \frac{1}{x})\ (1 + \frac{1}{x^2})$$

$$= 2(x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^3})$$

$$= 2(x + \frac{1}{x^3})$$

(v)

$$\left(x^2-\frac{1}{x^2}\right)^3$$

Differentiation of composite function can be done by

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g)\frac{d}{dx}g$$

Here, $f(g) = g^3$, $g(x) = x^2 - \frac{1}{x^2}$

$$\frac{d}{dx}\left(x^2-\frac{1}{x^2}\right)^3 = 3g^2\times(2x - \frac{2}{x^3})$$

$= 3\left(x^2 - \frac{1}{x^2}\right)^2 (2x - \frac{2}{x^3})$

$= 3(2x^3 - \frac{2}{x} - \frac{2}{x} + \frac{2}{x^5})$

$= 3(2x^3 - \frac{4}{x} + \frac{2}{x^5})$

(vi) $(2x^2 + 5x – 1) (x – 3)$

Applying, Chain rule

Here, $u = (2x^2 + 5x – 1)$

$V = (x – 3)$

$\frac{d}{dx}(2x^2 + 5x - 1)\ (x - 3)$

$=(2x^2 + 5x - 1)\frac{d}{dx}(x - 3) + (x - 3)\frac{d}{dx}(2x^2 + 5x - 1)$

$= (2x^2 + 5x - 1)\times 1 + (x - 3)(4x + 5)$

$= 2x^2 + 5x - 1 + 4x^2 - 7x - 15$

$= 6x^2 - 2x - 16$

Q. 8. Differentiate the following functions:

(i) $\frac{3x^2 + 4x - 5}{x}$

(ii) $\frac{(x^3 + 1)(x - 2)}{x^2}$

(iii) $\frac{x-4}{2\sqrt{x}}$

(iv) $\frac{(1+x)\sqrt{x}}{\sqrt[3]{x}}$

(v) $\frac{ax^2+bx+c}{\sqrt{x}}$

(vi) $\frac{a+b\cos x}{\sin x}$

Answer : Formula:

$$\frac{d}{dx}\frac{u}{v}=\frac{v\frac{d}{dx}u-u\frac{d}{dx}v}{u^2}$$

(i) $\frac{3x^2+4x-5}{x}$

Applying, quotient rule

$$\frac{d}{dx}\frac{3x^2+4x-5}{x}=\frac{x\frac{d}{dx}(3x^2+4x-5)-(3x^2+4x-5)\frac{d}{dx}x}{x^2}$$

$=\frac{x(6x+4)-(3x^2+4x-5)1}{x^2}$

$=\frac{6x^2+4x-(3x^2+4x-5)}{x^2}$

$=\frac{3x^2+5}{x^2}$

(ii) $\frac{(x^3+1)(x-2)}{x^2}$

Applying, quotient rule

$$\frac{d}{dx}\frac{(x^3+1)(x-2)}{x^2}=\frac{x^2\frac{d}{dx}(x^3+1)(x-2)-(x^3+1)(x-2)\frac{d}{dx}x^2}{x^4}$$

$=\frac{x^2\{(x^3+1)\frac{d}{dx}(x-2)+(x-2)\frac{d}{dx}(x^3+1)\}-(x^3+1)(x-2)2x}{x^4}$

$= \frac{x^2\{(x^3+1)+(x-2)3x^2\}-(x^3+1)(x-2)2x}{x^4}$

$= \frac{x^2\{x^3+1+3x^3-6x^2\}-2(x^4+x)(x-2)}{x^4}$

$= \frac{4x^5-6x^4+x^2-2(x^5-2x^4+x^2-2x)}{x^4}$

$= \frac{2x^5-2x^4-x^2+4x}{x^4}$

(iii) $\frac{x-4}{2\sqrt{x}}$

Applying, quotient rule

$$\frac{d}{dx}\frac{x-4}{2\sqrt{x}} = \frac{2\sqrt{x}\frac{d}{dx}(x-4)-(x-4)\frac{d}{dx}2\sqrt{x}}{4x}$$

$= \frac{2\sqrt{x}-(x-4)2\frac{1}{2}x^{-\frac{1}{2}}}{4x}$

$= \frac{2\sqrt{x}-(x-4)x^{-\frac{1}{2}}}{4x}$

$= \frac{2\sqrt{x}-x^{\frac{1}{2}}+4x^{-\frac{1}{2}}}{4x}$

$= \frac{\sqrt{x}+4x^{-\frac{1}{2}}}{4x}$

(iv) $\frac{(1+x)\sqrt{x}}{\sqrt[3]{x}}$

Applying, quotient rule

$$\frac{d}{dx}\frac{(1+x)\sqrt{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}\frac{d}{dx}(1+x)\sqrt{x}-(1+x)\sqrt{x}\frac{d}{dx}\sqrt[3]{x}}{x^{\frac{2}{3}}}$$

$= \frac{\sqrt[3]{x}\left\{(1+x)\frac{d}{dx}\sqrt{x}+\sqrt{x}\frac{d}{dx}(1+x)\right\}-(1+x)\sqrt{x}*\frac{1}{3}x^{\frac{-2}{3}}}{x^{\frac{2}{3}}}$

$$= \frac{\sqrt[3]{x}\left\{(1+x)*\frac{1}{2}x^{-\frac{1}{2}}+\sqrt{x}\right\}-(1+x)*\frac{1}{3}x^{\frac{-1}{6}}}{x^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{x}\left\{\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}}+\sqrt{x}\right\}-\frac{1}{3}\left(x^{\frac{-1}{6}}+x^{\frac{5}{6}}\right)}{x^{\frac{2}{3}}}$$

$$= \frac{\sqrt[3]{x}\left\{\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}}+\sqrt{x}\right\}-\frac{1}{3}\left(x^{\frac{-1}{6}}+x^{\frac{5}{6}}\right)}{x^{\frac{2}{3}}}$$

$$= \frac{\frac{1}{2}x^{-\frac{1}{6}}+\frac{1}{2}x^{\frac{5}{6}}+\sqrt{x}-\frac{1}{3}\left(x^{\frac{-1}{6}}+x^{\frac{5}{6}}\right)}{x^{\frac{2}{3}}}$$

$$= \frac{\frac{1}{6}x^{-\frac{1}{6}}+\frac{1}{6}x^{\frac{5}{6}}+\sqrt{x}}{x^{\frac{2}{3}}}$$

(v) $\frac{ax^2+bx+c}{\sqrt{x}}$

Applying, quotient rule

$$\frac{d}{dx}\frac{ax^2+bx+c}{\sqrt{x}} = \frac{\sqrt{x}\frac{d}{dx}(ax^2+bx+c)-(ax^2+bx+c)\frac{d}{dx}\sqrt{x}}{x}$$

$$= \frac{\sqrt{x}(2ax+b)-\frac{1}{2}(ax^2+bx+c)x^{-\frac{1}{2}}}{x}$$

$$= \frac{\frac{3}{2}ax^{\frac{3}{2}}+\frac{1}{2}bx^{\frac{1}{2}}-\frac{1}{2}cx^{-\frac{1}{2}}}{x}$$

(vi) $\frac{a+b\cos x}{\sin x}$

Applying, quotient rule

$$\frac{d}{dx}\frac{a+b\cos x}{\sin x} = \frac{\sin x\frac{d}{dx}(a+b\cos x)-(a+b\cos x)\frac{d}{dx}\sin x}{\sin^2 x}$$

$$= \frac{\sin x(-b\sin x)-(a+b\cos x)\cos x}{\sin^2 x}$$

$= \frac{-b\sin^2 x - a\cos x - b\cos^2 x}{\sin^2 x}$

$= \frac{-b(1) - a\cos x}{\sin^2 x}$

Q. 9. Differentiate the following functions:

(i) If y = 6x^5 – 4x^4 – 2x^2 + 5x – 9, find $\frac{dy}{dx}$ at x = -1.

(ii) If y = (sin x + tan x), find $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$.

(iii) If $y = \frac{(2 - 3\cos x)}{\sin x}$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$.

Answer : Formulae:

$\frac{d}{dx} x^n = nx^{n-1}$

$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \sin x = \cos x$

(i) If y = 6x^5 – 4x^4 – 2x^2 + 5x – 9, find $\frac{dy}{dx}$ at x = -1.

Differentiating with respect to x,

$\frac{d}{dx}(6x^5 - 4x^4 - 2x^2 + 5x - 9)$

$= 30x^4 - 16x^3 - 4x + 5$

substituing $x = -1$

$(\frac{dy}{dx})x = -1 = 30(-1)^4 - 16(-1)^3 - 4(-1) + 5$

= 30+16+4+5

= 55

(ii) If y = (sin x + tan x), find $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$.

Differentiating with respect to x,

$\frac{d}{dx}(\sin x + \tan x) = \cos x + \sec^2 x$

Substituting $x = \frac{\pi}{3}$

$\left(\frac{dy}{dx}\right)x = \pi/3 = \cos\frac{\pi}{3} + \sec^2\frac{\pi}{3}$

$= \frac{1}{2} + 4$

$= \frac{5}{2}$

(iii) If $y = \frac{(2-3\cos x)}{\sin x}$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$.

Differentiating with respect to x,

$\frac{d}{dx}$(2cosec x-3cot x) = 2(- cosecx cotx) – 3(- cosec^{2}x)

Substituting $x = \frac{\pi}{4}$

$\left(\frac{dy}{dx}\right)_{x = \pi/4}$ = 2(- cosec$\frac{\pi}{4}$ cot$\frac{\pi}{4}$) – 3(- cosec$^2\frac{\pi}{4}$)

$= - 2\times\sqrt{2} + 3\times2$

$= 6 - 2\times\sqrt{2}$

Q. 10.

If $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$**, show that** $2x.\frac{dy}{dx} + y = 2\sqrt{x}$ **.**

Answer : To show:

$2x.\frac{dy}{dx} + y = 2\sqrt{x}$

Differentiating with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}$$

Now,

LHS = $2x.\frac{dy}{dx} + y$

LHS = $2x \times \left(\frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}\right) + \sqrt{x} + \frac{1}{\sqrt{x}}$

LHS = $\sqrt{x} - \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{1}{\sqrt{x}}$

LHS = $2\sqrt{x}$

∴ LHS = RHS

Q. 11

If $y = \left(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}\right)$**, prove that** $(2xy)\left(\frac{dy}{dx}\right) = \left(\frac{x}{a} - \frac{a}{x}\right)$**.**

Answer : To prove:

$(2xy)\left(\frac{dy}{dx}\right) = \left(\frac{x}{a} - \frac{a}{x}\right)$.

Differentiating y with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}\right) = \frac{1}{2\sqrt{ax}} - \frac{\sqrt{a}}{2x^{\frac{3}{2}}}$$

Now,

LHS = $(2xy)\left(\frac{dy}{dx}\right)$

$$\text{LHS} = 2x\left(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}\right)\left(\frac{1}{2\sqrt{ax}} - \frac{\sqrt{a}}{2x^{\frac{3}{2}}}\right)$$

$$\text{LHS} = (\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}})(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}})$$

$$\text{LHS} = \left(\frac{x}{a} - \frac{a}{x}\right)$$

∴ LHS = RHS

Q. 12 If $y = \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}$ **, find** $\frac{dy}{dx}$ **.**

Answer :

$$y = \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}$$

Formula:

Using double angle formula:

$$\cos 2x = 2\cos^2 x - 1$$

$= 1 - 2\sin^2 x$

$\therefore 1 + \cos 2x = 2\cos^2 x$

$1 - \cos 2x = 2\sin^2 x$

$$\therefore y = \sqrt{\frac{2\cos^2 x}{2\text{sinn}^2 x}}$$

$$= \sqrt{\cot^2 x}$$

= cotx

Differentiating y with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}(\cot x)$$

$$= -\operatorname{cosec}^2 x$$

Q. 13

$$y = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}, \textbf{ find } \frac{dy}{dx}$$

Answer : Formula:

Using Half angle formula,

$$\cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$$

∴y = cos x

Differentiating y with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\cos x$$

$$= -\sin x$$

Exercise 28B

Q. 1. Find the derivation of each of the following from the first principle:

(ax + b)

Answer : Let f(x) = ax + b

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad ...(i)$$

f(x) = ax + b

f(x + h) = a(x + h) + b

= ax + ah + b

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{ax + ah + b - (ax + b)}{h}$$

$$= \lim_{h \to 0} \frac{ax + ah + b - ax - b}{h}$$

$$= \lim_{h \to 0} \frac{ah}{h}$$

$$= \lim_{h \to 0} a$$

f'(x) = a

Hence, f'(x) = a

Q. 2. Find the derivation of each of the following from the first principle:

$$\left(ax^2 + \frac{b}{x}\right)$$

Answer :

$$\text{Let } f(x) = ax^2 + \frac{b}{x}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad ...(i)$$

$$f(x) = ax^2 + \frac{b}{x}$$

$$f(x+h) = a(x+h)^2 + \frac{b}{(x+h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\left[a(x+h)^2 + \frac{b}{(x+h)}\right] - \left[ax^2 + \frac{b}{x}\right]}{h}$$

$$= \lim_{h \to 0} \frac{a(x+h)^2 + \frac{b}{(x+h)} - ax^2 - \frac{b}{x}}{h}$$

$$= \lim_{h \to 0} \frac{a[(x+h)^2 - x^2] + b\left[\frac{1}{x+h} - \frac{1}{x}\right]}{h}$$

$$= \lim_{h \to 0} \frac{a[x^2 + h^2 + 2xh - x^2] + h\left[\frac{x - (x+h)}{x(x+h)}\right]}{h}$$

$$= \lim_{h \to 0} \frac{a[h^2 + 2xh] + b\left[\frac{x - x - h}{x(x+h)}\right]}{h}$$

$$= \lim_{h \to 0} \frac{a[h^2 + 2xh] + b\left[\frac{-h}{x(x+h)}\right]}{h}$$

$$= \lim_{h \to 0} \left[\frac{ah(h+2x)}{h} + \frac{b(-h)}{hx(x+h)}\right]$$

Taking ‘h’ common from both the numerator and denominator, we get

$$= \lim_{h \to 0} \left[a(h+2x) - \frac{b}{x(x+h)}\right]$$

Putting h = 0, we get

$$= a[(0) + 2x] - \frac{b}{x(x+0)}$$

$$= 2ax - \frac{b}{x^2}$$

Hence,

$$f'(x) = 2ax - \frac{b}{x^2}$$

Q. 3. Find the derivation of each of the following from the first principle:

$3x^2 + 2x – 5$

Answer : Let $f(x) = 3x^2 + 2x – 5$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad ...(i)$$

$f(x) = 3x^2 + 2x – 5$

$f(x + h) = 3(x + h)^2 + 2(x + h) – 5$

$= 3(x^2 + h^2 + 2xh) + 2x + 2h – 5$

$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$= 3x^2 + 3h^2 + 6xh + 2x + 2h – 5$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - (3x^2 + 2x - 5)}{h}$$

$$= \lim_{h\to 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - 3x^2 - 2x + 5}{h}$$

$$= \lim_{h \to 0} \frac{3h^2 + 6xh + 2h}{h}$$

$$= \lim_{h \to 0} 3h + 6x + 2$$

Putting h = 0, we get

f'(x) = 3(0) + 6x + 2

= 6x + 2

Hence, f'(x) = 6x + 2

Q. 4 Find the derivation of each of the following from the first principle:

$x^3 - 2x^2 + x + 3$

Answer : Let $f(x) = x^3 - 2x^2 + x + 3$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

$f(x) = x^3 - 2x^2 + x + 3$

$f(x + h) = (x + h)^3 - 2(x + h)^2 + (x + h) + 3$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - 2(x+h)^2 + (x+h) + 3 - [x^3 - 2x^2 + x + 3]}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - 2(x+h)^2 + (x+h) + 3 - x^3 + 2x^2 - x - 3}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^3 - x^3] - 2[(x+h)^2 - x^2] + [x + h - x]}{h}$$

Using the identities:

$(a + b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$

$(a + b)^2 = a^2 + b^2 + 2ab$

$$= \lim_{h\to 0} \frac{[x^3 + h^3 + 3xh^2 + 3x^2h - x^3] - 2[x^2 + h^2 + 2xh - x^2] + h}{h}$$

$$= \lim_{h\to 0} \frac{[h^3 + 3xh^2 + 3x^2h] - 2[h^2 + 2xh] + h}{h}$$

$$= \lim_{h\to 0} \frac{h[h^2 + 3xh + 3x^2] - 2h[h + 2x] + h}{h}$$

$$= \lim_{h\to 0} h^2 + 3xh + 3x^2 - 2h - 4x + 1$$

Putting h = 0, we get

$f'(x) = (0)^2 + 2x(0) + 3x^2 - 2(0) - 4x + 1$

$= 3x^2 - 4x + 1$

Hence, $f'(x) = 3x^2 - 4x + 1$

Q. 5. Find the derivation of each of the following from the first principle:

x^8

Answer : Let $f(x) = x^8$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \quad \text{…(i)}$$

$f(x) = x^8$

$f(x + h) = (x + h)^8$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^8 - x^8}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^8 - x^8}{(x+h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \to x} \frac{z^8 - x^8}{z - x} \text{ where } z = x + h \text{ and } z \to x \text{ as } h \to 0$$

$$= 8x^{8-1}\left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$= 8x^7$

Hence, $f'(x) = 8x^7$

Q. 6 Find the derivation of each of the following from the first principle:

$$\frac{1}{x^3}$$

Answer :

$$\text{Let } f(x) = \frac{1}{x^3}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

$$f(x) = \frac{1}{x^3}$$

$$f(x+h) = \frac{1}{(x+h)^3}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^{-3} - x^{-3}}{(x+h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z\to x} \frac{z^{-3} - x^{-3}}{z - x} \text{ where } z = x + h \text{ and } z \to x \text{ as } h \to 0$$

$$= (-3)x^{-3-1}\left[\because \lim_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

= $-3x^{-4}$

$$= -\frac{3}{x^4}$$

Hence,

$$f'(x) = -\frac{3}{x^4}$$

Q. 7. Find the derivation of each of the following from the first principle:

$$\frac{1}{x^5}$$

Answer : Let ,

$$f(x) = \frac{1}{x^5}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

$$f(x) = \frac{1}{x^5}$$

$$f(x+h) = \frac{1}{(x+h)^5}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^5} - \frac{1}{x^5}}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^{-5} - x^{-5}}{(x+h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \to x} \frac{z^{-5} - x^{-5}}{z - x} \text{ where } z = x + h \text{ and } z \to x \text{ as } h \to 0$$

$$= (-5)x^{-5-1}\left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$= -5x^{-6}$

$$= -\frac{5}{x^6}$$

Hence,

$$f'(x) = -\frac{5}{x^6}$$

Q. 8. Find the derivation of each of the following from the first principle:

$$\sqrt{ax + b}$$

Answer : Let

$$f(x) = \sqrt{ax + b}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \ldots(i)$$

$$f(x) = \sqrt{ax + b}$$

$$f(x+h) = \sqrt{a(x+h) + b}$$

$$= \sqrt{ax + ah + b}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0}\frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$$\sqrt{ax+ah+b}-\sqrt{ax+b}$$

$$= \lim_{h\to 0}\frac{\sqrt{ax+ah+b}-\sqrt{ax+b}}{h}\times\frac{\sqrt{ax+ah+b}+\sqrt{ax+b}}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$$

Using the formula:

$(a + b)(a – b) = (a^2 – b^2)$

$$= \lim_{h\to 0}\frac{\left(\sqrt{ax+ah+b}\right)^2-\left(\sqrt{ax+b}\right)^2}{h\left(\sqrt{ax+ah+b}+\sqrt{ax+b}\right)}$$

$$= \lim_{h\to 0}\frac{ax+ah+b-ax-b}{h(\sqrt{ax+ah+b}+\sqrt{ax+b})}$$

$$= \lim_{h\to 0}\frac{ah}{h(\sqrt{ax+ah+b}+\sqrt{ax+b})}$$

$$= \lim_{h\to 0}\frac{a}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$$

Putting h = 0, we get

$$= \frac{a}{\sqrt{ax + a(0) + b} + \sqrt{ax + b}}$$

$$= \frac{a}{\sqrt{ax + b} + \sqrt{ax + b}}$$

$$= \frac{a}{2\sqrt{ax + b}}$$

Hence,

$$f'(x) = \frac{a}{2\sqrt{ax+b}}$$

Q. 9. Find the derivation of each of the following from the first principle:

$$\sqrt{5x - 4}$$

Answer : Let

$$f(x) = \sqrt{5x - 4}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

$$f(x) = \sqrt{5x - 4}$$

$$f(x + h) = \sqrt{5(x + h) - 4}$$

$$= \sqrt{5x + 5h - 4}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{5x + 5h - 4} - \sqrt{5x - 4}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$\sqrt{5x+5h-4}-\sqrt{5x-4}$

$$=\lim_{h\to 0}\frac{\sqrt{5x+5h-4}-\sqrt{5x-4}}{h}\times\frac{\sqrt{5x+5h-4}+\sqrt{5x-4}}{\sqrt{5x+5h-4}+\sqrt{5x-4}}$$

Using the formula:

$(a + b)(a – b) = (a^2 – b^2)$

$$=\lim_{h\to 0}\frac{\left(\sqrt{5x+5h-4}\right)^2-\left(\sqrt{5x-4}\right)^2}{h(\sqrt{5x+5h-4}+\sqrt{5x-4})}$$

$$=\lim_{h\to 0}\frac{5x+5h-4-5x+4}{h(\sqrt{5x+5h-4}+\sqrt{5x-4})}$$

$$=\lim_{h\to 0}\frac{5h}{h(\sqrt{5x+5h-4}+\sqrt{5x-4})}$$

$$=\lim_{h\to 0}\frac{5}{\sqrt{5x+5h-4}+\sqrt{5x-4}}$$

Putting h = 0, we get

$$=\frac{5}{\sqrt{5x+5(0)-4}+\sqrt{5x-4}}$$

$$=\frac{5}{\sqrt{5x-4}+\sqrt{5x-4}}$$

$$=\frac{5}{2\sqrt{5x-4}}$$

Hence,

$$f'(x)=\frac{5}{2\sqrt{5x-4}}$$

Q. 10. Find the derivation of each of the following from the first principle:

$$\frac{1}{\sqrt{x+2}}$$

Answer : Let

$f(x) = \frac{1}{\sqrt{x+2}}$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ …(i)

$$f(x) = \frac{1}{\sqrt{x+2}}$$

$$f(x+h) = \frac{1}{\sqrt{x+h+2}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h+2}} - \frac{1}{\sqrt{x+2}}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sqrt{x+2}-\sqrt{x+h+2}}{(\sqrt{x+h+2})(\sqrt{x+2})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$\sqrt{x+2}-\sqrt{x+h+2}$

$$= \lim_{h\to 0} \frac{\sqrt{x+2}-\sqrt{x+h+2}}{h(\sqrt{x+h+2})(\sqrt{x+2})} \times \frac{\sqrt{x+2}+\sqrt{x+h+2}}{\sqrt{x+2}+\sqrt{x+h+2}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h\to 0} \frac{\left(\sqrt{x+2}\right)^2 - \left(\sqrt{x+h+2}\right)^2}{h(\sqrt{x+h\ +\ 2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

$$= \lim_{h\to 0} \frac{x+2-x-h-2}{h(\sqrt{x+h\ +\ 2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

$$= \lim_{h\to 0} \frac{-h}{h(\sqrt{x+h\ +\ 2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

$$= \lim_{h\to 0} \frac{-1}{(\sqrt{x+h\ +\ 2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})}$$

Putting h = 0, we get

$$= \frac{-1}{(\sqrt{x+0\ +\ 2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+0+2})}$$

$$= \frac{-1}{\left(\sqrt{x+2}\right)^2 (2\sqrt{x+2})}$$

$$= \frac{-1}{2\left(\sqrt{x+2}\right)^3}$$

Hence,

$$f'(x) = \frac{-1}{2\left(\sqrt{x+2}\right)^3}$$

Q. 11. Find the derivation of each of the following from the first principle:

$$\frac{1}{\sqrt{2x+3}}$$

Answer : Let

$$f(x) = \frac{1}{\sqrt{2x+3}}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

$$f(x) = \frac{1}{\sqrt{2x + 3}}$$

$$f(x+h) = \frac{1}{\sqrt{2x + 2h + 3}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{2x + 2h + 3}} - \frac{1}{\sqrt{2x + 3}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{2x+3} - \sqrt{2x+2h+3}}{(\sqrt{2x+2h+3})(\sqrt{2x+3})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$$\sqrt{2x+3} - \sqrt{2x+2h+3}$$

$$= \lim_{h \to 0} \frac{\sqrt{2x+3} - \sqrt{2x+2h+3}}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})} \times \frac{\sqrt{2x+3} + \sqrt{2x+2h+3}}{\sqrt{2x+3} + \sqrt{2x+2h+3}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h \to 0} \frac{(\sqrt{2x+3})^2 - (\sqrt{2x+2h+3})^2}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h \to 0} \frac{2x + 3 - 2x - 2h - 3}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h \to 0} \frac{-2h}{h(\sqrt{2x + 2h + 3})(\sqrt{2x + 3})(\sqrt{2x + 3} + \sqrt{2x + 2h + 3})}$$

$$= \lim_{h \to 0} \frac{-2}{(\sqrt{2x + 2h + 3})(\sqrt{2x + 3})(\sqrt{2x + 3} + \sqrt{2x + 2h + 3})}$$

Putting h = 0, we get

$$= \frac{-2}{(\sqrt{2x + 0 + 3})(\sqrt{2x + 3})(\sqrt{2x + 3} + \sqrt{2x + 0 + 3})}$$

$$= \frac{-2}{(\sqrt{2x + 3})^2 (2\sqrt{2x + 3})}$$

$$= \frac{-2}{2(\sqrt{2x + 3})^3}$$

$$= \frac{-1}{(\sqrt{2x + 3})^3}$$

Hence,

$$f'(x) = \frac{-1}{(\sqrt{2x+3})^3}$$

Q. 12. Find the derivation of each of the following from the first principle:

$$\frac{1}{\sqrt{6x - 5}}$$

Answer : Let

$$f(x) = \frac{1}{\sqrt{6x - 5}}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

$$f(x) = \frac{1}{\sqrt{6x - 5}}$$

$$f(x+h) = \frac{1}{\sqrt{6x + 6h - 5}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{6x + 6h - 5}} - \frac{1}{\sqrt{6x - 5}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{6x - 5} - \sqrt{6x + 6h - 5}}{(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{6x - 5} - \sqrt{6x + 6h - 5}$

$$= \lim_{h \to 0} \frac{\sqrt{6x - 5} - \sqrt{6x + 6h - 5}}{h(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})} \times \frac{\sqrt{6x - 5} + \sqrt{6x + 6h - 5}}{\sqrt{6x - 5} + \sqrt{6x + 6h - 5}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h \to 0} \frac{\left(\sqrt{6x - 5}\right)^2 - \left(\sqrt{6x + 6h - 5}\right)^2}{h(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})(\sqrt{6x - 5} + \sqrt{6x + 6h - 5})}$$

$$= \lim_{h \to 0} \frac{6x - 5 - 6x - 6h + 5}{h(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})(\sqrt{6x - 5} + \sqrt{6x + 6h - 5})}$$

$$= \lim_{h \to 0} \frac{-6h}{h(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})(\sqrt{6x - 5} + \sqrt{6x + 6h - 5})}$$

$$= \lim_{h \to 0} \frac{-6}{(\sqrt{6x + 6h - 5})(\sqrt{6x - 5})(\sqrt{6x - 5} + \sqrt{6x + 6h - 5})}$$

Putting h = 0, we get

$$= \frac{-6}{(\sqrt{6x + 6(0) - 5})(\sqrt{6x - 5})(\sqrt{6x - 5} + \sqrt{6x + 6(0) - 5})}$$

$$= \frac{-6}{\left(\sqrt{6x - 5}\right)^2 (2\sqrt{6x - 5})}$$

$$= \frac{-6}{2\left(\sqrt{6x - 5}\right)^3}$$

$$= \frac{-3}{\left(\sqrt{6x - 5}\right)^3}$$

Hence,

$$f'(x) = \frac{-3}{\left(\sqrt{6x-5}\right)^3}$$

Q. 13. Find the derivation of each of the following from the first principle:

$$\frac{1}{\sqrt{2 - 3x}}$$

Answer : Let

$$f(x) = \frac{1}{\sqrt{2-3x}}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

$$f(x) = \frac{1}{\sqrt{2-3x}}$$

$$f(x+h) = \frac{1}{\sqrt{2-3(x+h)}} = \frac{1}{\sqrt{2-3x-3h}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{2-3x-3h}} - \frac{1}{\sqrt{2-3x}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{2-3x} - \sqrt{2-3x-3h}}{\sqrt{2-3x-3h}(\sqrt{2-3x})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{2-3x} - \sqrt{2-3x-3h}$

$$= \lim_{h \to 0} \frac{\sqrt{2-3x} - \sqrt{2-3x-3h}}{h\sqrt{2-3x-3h}(\sqrt{2-3x})} \times \frac{\sqrt{2-3x} + \sqrt{2-3x-3h}}{\sqrt{2-3x} + \sqrt{2-3x-3h}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h \to 0} \frac{\left(\sqrt{2-3x}\right)^2 - \left(\sqrt{2-3x-3h}\right)^2}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h \to 0} \frac{2-3x-2+3x+3h}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h \to 0} \frac{3h}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h \to 0} \frac{3}{(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

Putting h = 0, we get

$$= \frac{3}{(\sqrt{2-3x-3(0)})(\sqrt{2-3x})(\sqrt{2-3x}+\sqrt{2-3x-3(0)})}$$

$$= \frac{3}{(\sqrt{2-3x})^2(2\sqrt{2-3x})}$$

$$= \frac{3}{2(\sqrt{2-3x})^3}$$

Hence,

$$f'(x) = \frac{3}{2(\sqrt{2-3x})^3}$$

Q. 14. Find the derivation of each of the following from the first principle:

$$\frac{2x+3}{3x+2}$$

Answer : Let,

$$f(x) = \frac{2x+3}{3x+2}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad \ldots(i)$$

$$f(x) = \frac{2x+3}{3x+2}$$

$$f(x+h) = \frac{2(x+h)+3}{3(x+h)+2} = \frac{2x+2h+3}{3x+3h+2}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\frac{2x + 2h + 3}{3x + 3h + 2} - \frac{2x + 3}{3x + 2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(2x + 2h + 3)(3x + 2) - (2x + 3)(3x + 3h + 2)}{(3x + 3h + 2)(3x + 2)}}{h}$$

$$= \lim_{h \to 0} \frac{6x^2 + 4x + 6xh + 4h + 9x + 6 - [6x^2 + 6xh + 4x + 9x + 9h + 6]}{h((3x + 3h + 2)(3x + 2))}$$

$$= \lim_{h \to 0} \frac{6x^2 + 4x + 6xh + 4h + 9x + 6 - 6x^2 - 6xh - 4x - 9x - 9h - 6}{h((3x + 3h + 2)(3x + 2))}$$

$$= \lim_{h \to 0} \frac{-5h}{h((3x + 3h + 2)(3x + 2))}$$

$$= \lim_{h \to 0} \frac{-5}{((3x + 3h + 2)(3x + 2))}$$

Putting h = 0, we get

$$= \frac{-5}{((3x + 3(0) + 2)(3x + 2))}$$

$$= \frac{-5}{(3x + 2)(3x + 2)}$$

$$= \frac{-5}{(3x + 2)^2}$$

Hence,

$$f'(x) = \frac{-5}{(3x+2)^2}$$

Q. 15. Find the derivation of each of the following from the first principle:

$$\frac{5 - x}{5 + x}$$

Answer : Let

$$f(x) = \frac{5-x}{5+x}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \ldots(i)$$

$$f(x) = \frac{5-x}{5+x}$$

$$f(x+h) = \frac{5-(x+h)}{5+(x+h)} = \frac{5-x-h}{5+x+h}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0}\frac{\frac{5-x-h}{5+x+h}-\frac{5-x}{5+x}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{(5-x-h)(5+x)-(5-x)(5+x+h)}{(5+x+h)(5+x)}}{h}$$

$$= \lim_{h\to 0}\frac{25+5x-5x-x^2-5h-xh-[25+5x+5h-5x-x^2-xh]}{h(5+x+h)(5+x)}$$

$$= \lim_{h\to 0}\frac{25-x^2-5h-xh-25-5h+x^2+xh}{h(5+x+h)(5+x)}$$

$$= \lim_{h\to 0}\frac{-10h}{h(5+x+h)(5+x)}$$

$$= \lim_{h\to 0}\frac{-10}{(5+x+h)(5+x)}$$

Putting h = 0, we get

$$= \frac{-10}{(5 + x + 0)(5 + x)}$$

$$= \frac{-10}{(5 + x)(5 + x)}$$

$$= \frac{-10}{(5 + x)^2}$$

Hence,

$$f'(x) = \frac{-10}{(5+x)^2}$$

Q. 16. Find the derivation of each of the following from the first principle:

$$\frac{x^2 + 1}{x}, x \neq 0$$

Answer : Let

$$f(x) = \frac{x^2+1}{x}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

$$f(x) = \frac{x^2 + 1}{x}$$

$$f(x + h) = \frac{(x + h)^2 + 1}{x + h} = \frac{x^2 + h^2 + 2xh + 1}{x + h}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\frac{x^2 + h^2 + 2xh + 1}{x + h} - \frac{x^2 + 1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x^2 + h^2 + 2xh + 1)(x) - (x^2 + 1)(x + h)}{(x + h)(x)}}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + xh^2 + 2x^2h + x - [x^3 + x^2h + x + h]}{h(x + h)(x)}$$

$$= \lim_{h \to 0} \frac{x^3 + xh^2 + 2x^2h + x - x^3 - x^2h - x - h}{h(x + h)(x)}$$

$$= \lim_{h \to 0} \frac{xh^2 + x^2h - h}{h(x + h)(x)}$$

$$= \lim_{h \to 0} \frac{xh + x^2 - 1}{(x + h)(x)}$$

Putting h = 0, we get

$$= \frac{x(0) + x^2 - 1}{(x + 0)(x)}$$

$$= \frac{x^2 - 1}{(x)^2}$$

Hence,

$$f'(x) = \frac{x^2 - 1}{x^2}$$

Q. 17. Find the derivation of each of the following from the first principle:

$$\sqrt{\cos 3x}$$

Answer : Let

$$f(x) = \sqrt{\cos 3x}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad \ldots(i)$$

$$f(x) = \sqrt{\cos 3x}$$

$$f(x+h) = \sqrt{\cos 3(x+h)}$$

$$= \sqrt{\cos(3x+3h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}$

$$= \lim_{h\to 0} \frac{\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}}{h} \times \frac{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h\to 0} \frac{\left(\sqrt{\cos(3x+3h)}\right)^2 - \left(\sqrt{\cos 3x}\right)^2}{h\left(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}\right)}$$

$$= \lim_{h\to 0} \frac{\cos(3x+3h) - \cos 3x}{h\left(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}\right)}$$

Using the formula:

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$= \lim_{h\to 0} \frac{-2\sin\frac{3x+3h+3x}{2}\sin\frac{3x+3h-3x}{2}}{h(\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x})}$$

$$= \lim_{h\to 0} \frac{-2\sin\frac{6x+3h}{2}\sin\frac{3h}{2}}{h\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x}}$$

$$= -2\lim_{h\to 0} \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} \times \frac{3}{2}\lim_{h\to 0} \sin(\frac{6x+3h}{2}) \times \lim_{h\to 0} \frac{1}{\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x}}$$

$[\text{Here, we multiply and divide by} \frac{3}{2}]$

$$= -2 \times \frac{3}{2}\lim_{h\to 0} \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} \times \lim_{h\to 0} \sin(\frac{6x+3h}{2}) \times \lim_{h\to 0} \frac{1}{\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x}}$$

$$= -3 \times (1) \times \lim_{h\to 0} \sin(\frac{6x+3h}{2}) \times \lim_{h\to 0} \frac{1}{\sqrt{\cos(3x+3h)}+\sqrt{\cos 3x}}$$

$$\left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

Putting h = 0, we get

$$= -3 \times \sin[\frac{6x+3(0)}{2}] \times \frac{1}{\sqrt{\cos(3x+3(0))}+\sqrt{\cos 3x}}$$

$$= -3\sin 3x \times \frac{1}{2\sqrt{\cos 3x}}$$

$$= -\frac{3\sin 3x}{2(\cos 3x)^{\frac{1}{2}}}$$

Hence,

$$f'(x) = -\frac{3\sin 3x}{2(\cos 3x)^{\frac{1}{2}}}$$

Q. 18. Find the derivation of each of the following from the first principle:

$$\sqrt{\sec x}$$

Answer : Let

$$f(x) = \sqrt{\sec x}$$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \ldots(i)$$

$$f(x) = \sqrt{\sec x}$$

$$f(x+h) = \sqrt{\sec(x+h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h\to 0}\frac{\sqrt{\sec(x+h)} - \sqrt{\sec x}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of $\sqrt{\sec(x+h)} - \sqrt{\sec x}$

$$= \lim_{h\to 0}\frac{\sqrt{\sec(x+h)} - \sqrt{\sec x}}{h} \times \frac{\sqrt{\sec(x+h)} + \sqrt{\sec x}}{\sqrt{\sec(x+h)} + \sqrt{\sec x}}$$

Using the formula:

$(a + b)(a - b) = (a^2 - b^2)$

$$= \lim_{h\to 0}\frac{\left(\sqrt{\sec(x+h)}\right)^2 - \left(\sqrt{\sec x}\right)^2}{h\left(\sqrt{\sec(x+h)} + \sqrt{\sec x}\right)}$$

$$= \lim_{h \to 0} \frac{\sec(x+h) - \sec(x)}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h \to 0} \frac{\frac{\cos x - \cos(x+h)}{\cos(x+h)\cos x}}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h \to 0} \frac{\cos x - \cos(x+h)}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

Using the formula:

$$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\sin\frac{x+(x+h)}{2}\sin\frac{(x+h)-x}{2}}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h \to 0} \frac{2\sin\frac{2x+h}{2}\sin\frac{h}{2}}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= 2\lim_{h \to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2}\lim_{h \to 0} \sin\left(\frac{2x+h}{2}\right) \times \lim_{h \to 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

[Here, we multiply and divide by $\frac{1}{2}$]

$$= 2 \times \frac{1}{2} \lim_{h \to 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}}$$

$$\times \lim_{h \to 0} \sin(x + \frac{h}{2}) \times \lim_{h \to 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= (1) \times \lim_{h \to 0} \sin(x + \frac{h}{2}) \times \lim_{h \to 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$\left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

Putting h = 0, we get

$$= \sin[x + \frac{0}{2}] \times \frac{1}{\cos(x+0)\cos x\left(\sqrt{\sec(x+0)} + \sqrt{\sec x}\right)}$$

$$= \sin x \times \frac{1}{\cos x \cos x\left(\sqrt{\sec x} + \sqrt{\sec x}\right)}$$

$$= \frac{\sin x}{\cos^2 x(2\sqrt{\sec x})}$$

$$= \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \times \frac{1}{2\sqrt{\sec x}}$$

$$= \tan x \times \sec x \times \frac{1}{2\sqrt{\sec x}} \left[\because \frac{\sin x}{\cos x} = \tan x\right] \& \left[\frac{1}{\cos x} = \sec x\right]$$

$$= \frac{1}{2} \tan x \sqrt{\sec x}$$

Hence,

$$f'(x) = \frac{1}{2} \tan x \sqrt{\sec x}$$

Q. 19. Find the derivation of each of the following from the first principle:

$\tan^2 x$

Answer : Let f(x) = $\tan^2 x$

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

f(x) = $\tan^2 x$

f(x + h) = $\tan^2(x + h)$

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\tan^2(x+h) - \tan^2 x}{h}$$

$$= \lim_{h \to 0} \frac{[\tan(x+h) - \tan x][\tan(x+h) + \tan x]}{h}$$

Using:

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \lim_{h \to 0} \frac{\left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}\right]\left[\frac{\sin(x+h)}{\cos(x+h)} + \frac{\sin x}{\cos x}\right]}{h}$$

$$= \lim_{h \to 0} \frac{\left[\frac{\sin(x+h)\cos x - \sin x \cos(x+h)}{\cos(x+h)\cos x}\right]\left[\frac{\sin(x+h)\cos x + \sin x \cos(x+h)}{\cos(x+h)\cos x}\right]}{h}$$

$$= \lim_{h \to 0} \frac{\{\sin[(x+h) - x]\}\{\sin[(x+h) + x]\}}{h[\cos^2(x+h)\cos^2 x]}$$

[∵ sin A cos B – sin B cos A = sin(A – B)

& sin A cos B + sin B cos A = sin(A + B)]

$$= \lim_{h\to 0} \frac{[\sin h][\sin(2x+h)]}{h[\cos^2(x+h)\cos^2 x]}$$

$$= \frac{1}{\cos^2 x} \lim_{h\to 0} \frac{\sin h}{h} \times \lim_{h\to 0} \sin(2x+h) \times \lim_{h\to 0} \frac{1}{\cos^2(x+h)}$$

$$= \frac{1}{\cos^2 x} \times (1) \times \lim_{h\to 0} \sin(2x+h) \times \lim_{h\to 0} \frac{1}{\cos^2(x+h)}$$

$$\left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

Putting h = 0, we get

$$= \frac{1}{\cos^2 x} \times \sin(2x+0) \times \frac{1}{\cos^2(x+0)}$$

$$= \frac{1}{\cos^2 x} \times \sin 2x \times \frac{1}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \times 2 \sin x \cos x \times \sec^2 x$$

[∵ sin2x = 2sinxcosx]

$$= 2\frac{\sin x}{\cos x} \times \sec^2 x \left[\because \frac{1}{\cos x} = \sec x\right]$$

= 2tanx $\sec^2$x

$$\left[\because \frac{\sin x}{\cos x} = \tan x\right]$$

Hence, f'(x) = 2tanx $\sec^2$x

Q. 20. Find the derivation of each of the following from the first principle:

sin (2x + 3)

Answer : Let f(x) = sin (2x + 3)

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

f(x) = sin (2x + 3)

f(x + h) = sin [2(x + h) + 3]

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\sin[2(x+h)+3] - \sin(2x+3)}{h}$$

Using the formula:

$$\sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}$$

$$= \lim_{h \to 0} \frac{2\sin\frac{2(x+h)+3-(2x+3)}{2}\cos\frac{2(x+h)+3+2x+3}{2}}{h}$$

$$= \lim_{h \to 0} \frac{2\sin\frac{2x+2h+3-2x-3}{2}\cos\frac{2x+2h+6+2x}{2}}{h}$$

$$= \lim_{h \to 0} \frac{2\sin\frac{2h}{2}\cos\frac{4x+2h+6}{2}}{h}$$

$$= \lim_{h \to 0} \frac{2\sin(h)\cos(2x+h+3)}{h}$$

$$= 2\lim_{h \to 0} \frac{\sin h}{h} \times \lim_{h \to 0} \cos(2x+h+3)$$

$$= 2(1) \times \lim_{h \to 0} \cos(2x+h+3)$$

$$\left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

Putting h = 0, we get

= 2cos(2x + 0 + 3)

= 2cos(2x + 3)

Hence, f'(x) = 2cos (2x + 3)

Q. 21. Find the derivation of each of the following from the first principle:

tan (3x + 1)

Answer : Let f(x) = tan (3x + 1)

We need to find the derivative of f(x) i.e. f'(x)

We know that,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \ldots(i)$$

f(x) = tan (3x + 1)

f(x + h) = tan [3(x + h) + 1]

Putting values in (i), we get

$$f'(x) = \lim_{h \to 0} \frac{\tan[3(x+h)+1] - \tan[3x+1]}{h}$$

Using the formula:

$$\tan A - \tan B = \frac{\sin(A - B)}{\cos A \cos B}$$

$$= \lim_{h \to 0} \frac{\frac{\sin[3(x+h)+1-(3x+1)]}{\cos[3(x+h)+1]\cos[3x+1]}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin[3x+3h+1-3x-1]}{\cos[3(x+h)+1]\cos[3x+1]}}{h}$$

$$= \lim_{h \to 0} \frac{\sin 3h}{h[\cos[3(x+h)+1]\cos[3x+1]]}$$

$$= \lim_{h \to 0} \frac{\sin 3h}{h} \times \lim_{h \to 0} \frac{1}{\cos[3(x+h)+1]\cos[3x+1]}$$

$$= \lim_{h \to 0} \frac{\sin 3h}{3h} \times 3 \times \lim_{h \to 0} \frac{1}{\cos[3(x+h)+1]\cos[3x+1]}$$

$$= 3(1) \times \lim_{h \to 0} \frac{1}{\cos[3(x+h)+1]\cos[3x+1]}$$

$$\left[\because \lim_{x \to 0} \frac{\sin 3x}{3x} = 1\right]$$

Putting h = 0, we get

$$= 3 \times \frac{1}{\cos[3(x+0)+1]\cos[3x+1]}$$

$$= \frac{3}{\cos[3x+1]\cos[3x+1]}$$

$$= \frac{3}{\cos^2(3x+1)}$$

$$= 3\sec^2(3x+1) \left[\because \frac{1}{\cos x} = \sec x\right]$$

Hence, $f'(x) = 3\sec^2(3x+1)$

Exercise 28C

Q. 1. Differentiate:

$X^2 \sin x$

Answer : To find: Differentiation of $x^2 \sin x$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\sin x}{dx} = \cos x$$

Let us take u = x^2 and v = sinx

$$u' = \frac{du}{dx} = \frac{d(x^2)}{dx} = 2x$$

$$v' = \frac{dv}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

Putting the above obtained values in the formula:-

(uv)′ = u′v + uv′

$(x^2 \sin x)' = 2x \times \sin x + x^2 \times \cos x$

$= 2x\sin x + x^2\cos x$

Ans) $2x\sin x + x^2\cos x$

Q. 2. Differentiate:

e^x cos x

Answer : To find: Differentiation of e^x cos x

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii)

$$\frac{de^x}{dx} = e^x$$

(iii)

$$\frac{d\cos x}{dx} = -\sin x$$

Let us take u = e^x and v = cosx

$$u' = \frac{du}{dx} = \frac{de^x}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{dcosx}{dx} = -sinx$$

Putting the above obtained values in the formula:-

(uv)′ = u′v + uv′

(e^x cosx)’ = e^x× cosx + e^x× -sinx

= e^xcosx - e^xsinx

= e^x (cosx - sinx)

Ans) e^x (cosx - sinx)

Q. 3. Differentiate:

e^x cot x

Answer : To find: Differentiation of e^x cot x

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii)

$$\frac{de^x}{dx} = e^x$$

(iii)

$$\frac{dcotx}{dx} = -cosec^2x$$

Let us take u = e^x and v = cotx

$$u' = \frac{du}{dx} = \frac{de^x}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{dcotx}{dx} = -cosec^2x$$

Putting the above obtained values in the formula:-

$(uv)' = u'v + uv'$

$(e^x \cot x)' = e^x \times \cot x + e^x \times -\operatorname{cosec}^2 x$

$= e^x \cot x - e^x \operatorname{cosec}^2 x$

$= e^x (\cot x - \operatorname{cosec}^2 x)$

Ans) $e^x (\cot x - \operatorname{cosec}^2 x)$

Q. 4. Differentiate:

$x^n \cot x$

Answer : To find: Differentiation of $x^n \cot x$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\cot x}{dx} = -\operatorname{cosec}^2 x$$

Let us take $u = x^n$ and $v = \cot x$

$$u' = \frac{du}{dx} = \frac{dx^n}{dx} = nx^{n-1}$$

$$v' = \frac{dv}{dx} = \frac{d\cot x}{dx} = -\operatorname{cosec}^2 x$$

Putting the above obtained values in the formula :-

$(uv)' = u'v + uv'$

$(x^n \cot x)' = nx^{n-1} \times \cot x + x^n \times -\operatorname{cosec}^2 x$

$= nx^{n-1} \cot x - x^n \operatorname{cosec}^2 x$

$= x^n (nx^{-1}\cot x - \operatorname{cosec}^2 x)$

Ans) $x^n (nx^{-1}\cot x - \operatorname{cosec}^2 x)$

Q. 5. Differentiate:

x^3 sec x

Answer : To find: Differentiation of x^3 sec x

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\sec x}{dx} = \sec x \tan x$$

Let us take u = x^3 and v = sec x

$$u' = \frac{du}{dx} = \frac{dx^3}{dx} = 3x^2$$

$$v' = \frac{dv}{dx} = \frac{d\sec x}{dx} = \sec x \tan x$$

Putting the above obtained values in the formula :-

(uv)′ = u′v + uv′

$(x^3 \sec x)' = 3x^2 \times \sec x + x^3 \times \sec x \tan x$

$= 3x^2 \sec x + x^3 \sec x \tan x$

$= x^2 \sec x(3 + x \tan x)$

Ans) $x^2 \sec x(3 + x \tan x)$

Q. 6. Differentiate:

$(x^2 + 3x + 1)$ sin x

Answer : To find: Differentiation of $(x^2 + 3x + 1) \sin x$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\sin x}{dx} = \cos x$$

Let us take $u = x^2 + 3x + 1$ and $v = \sin x$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 3x + 1)}{dx} = 2x + 3$$

$$v' = \frac{dv}{dx} = \frac{d\sin x}{dx} = \cos x$$

Putting the above obtained values in the formula :-

$(uv)' = u'v + uv'$

$[(x^2 + 3x + 1) \sin x]' = (2x + 3) \times \sin x + (x^2 + 3x + 1) \times \cos x$

$= \sin x\,(2x + 3) + \cos x\,(x^2 + 3x + 1)$

Ans) $(2x + 3) \sin x + (x^2 + 3x + 1) \cos x$

Q. 7. Differentiate:

$x^4 \tan x$

Answer : To find: Differentiation of $x^4 \tan x$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\tan x}{dx} = \sec^2 x$$

Let us take u = x^4 and v = tan x

$$u' = \frac{du}{dx} = \frac{dx^4}{dx} = 4x^3$$

$$v' = \frac{dv}{dx} = \frac{d\tan x}{dx} = \sec^2 x$$

Putting the above obtained values in the formula:-

(uv)′ = u′v + uv′

$(x^4 \tan x)' = 4x^3 \times \tan x + x^4 \times \sec^2 x$

$= 4x^3 \tan x + x^4 \sec^2 x$

$= x^3 (4\tan x + x\sec^2 x)$

Ans) $x^3 (4\tan x + x\sec^2 x)$

Q. 8. Differentiate:

$(3x – 5) (4x^2 – 3 + e^x)$

Answer : To find: Differentiation of $(3x – 5) (4x^2 – 3 + e^x)$

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{de^x}{dx} = e^x$$

Let us take u = (3x – 5) and v = $(4x^2 – 3 + e^x)$

$$u' = \frac{du}{dx} = \frac{d(3x - 5)}{dx} = 3$$

$$v' = \frac{dv}{dx} = \frac{d(4x^2 - 3 + e^x)}{dx} = (8x + e^x)$$

Putting the above obtained values in the formula :-

$(uv)' = u'v + uv'$

$[(3x - 5)(4x^2 - 3 + e^x)]' = 3\times(4x^2 - 3 + e^x) + (3x - 5)\times(8x + e^x)$

$= 12x^2 - 9 + 3e^x + 24x^2 + 3xe^x - 40x - 5e^x$

$= 36x^2 + x(3e^x - 40) - 9 - 2e^x$

Ans) $36x^2 + x(3e^x - 40) - 9 - 2e^x$

Q. 9. Differentiate:

$(x^2 - 4x + 5)(x^3 - 2)$

Answer : To find: Differentiation of $(x^2 - 4x + 5)(x^3 - 2)$

Formula used: (i) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

Let us take $u = (x^2 - 4x + 5)$ and $v = (x^3 - 2)$

$$u' = \frac{du}{dx} = \frac{d(x^2 - 4x + 5)}{dx} = 2x - 4$$

$$v' = \frac{dv}{dx} = \frac{d(x^3 - 2)}{dx} = 3x^2$$

Putting the above obtained values in the formula:-

$(uv)' = u'v + uv'$

$(x^2 - 4x + 5)(x^3 - 2)]' = (2x - 4)\times(x^3 - 2) + (x^2 - 4x + 5)\times(3x^2)$

$= 2x^4 – 4x - 4x^3 + 8 + 3x^4 – 12x^3 + 15x^2$

$= 5x^4 - 16x^3 + 15x^2 – 4x + 8$

Ans) $5x^4 - 16x^3 + 15x^2 – 4x + 8$

Q. 10. Differentiate:

$(x^2 + 2x – 3)(x^2 + 7x + 5)$

Answer : To find: Differentiation of $(x^2 + 2x – 3)(x^2 + 7x + 5)$

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

Let us take $u = (x^2 + 2x – 3)$ and $v = (x^2 + 7x + 5)$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 2x - 3)}{dx} = 2x + 2$$

$$v' = \frac{dv}{dx} = \frac{d(x^2 + 7x + 5)}{dx} = 2x + 7$$

Putting the above obtained values in the formula :-

(uv)′ = u′v + uv′

$[(x^2 + 2x – 3)(x^2 + 7x + 5)]'$

$= (2x + 2) \times (x^2 + 7x + 5) + (x^2 + 2x – 3) \times (2x + 7)$

$= 2x^3 + 14x^2 + 10x + 2x^2 + 14x + 10 + 2x^3 + 7x^2 + 4x^2 + 14x – 6x – 21$

$= 4x^3 + 27x^2 + 32x – 11$

Ans) $4x^3 + 27x^2 + 32x – 11$

Q. 11. Differentiate:

(tan x + sec x) (cot x + cosec x)

Answer : To find: Differentiation of (tan x + sec x) (cot x + cosec x)

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii)

$$\frac{d\tan x}{dx} = \sec^2 x$$

(iii)

$$\frac{d\sec x}{dx} = \sec x \tan x$$

(iv)

$$\frac{d\cot x}{dx} = -\mathrm{cosec}^2 x$$

(v)

$$\frac{d\,\mathrm{cosec}\,x}{dx} = -\mathrm{cosec}\,x \cot x$$

Let us take u = (tan x + sec x) and v = (cot x + cosec x)

$$u' = \frac{du}{dx} = \frac{d(\tan x + \sec x)}{dx} = \sec^2 x + \sec x \tan x = \sec x\,(\sec x + \tan x)$$

$$v' = \frac{dv}{dx} = \frac{d(\cot x + \mathrm{cosec}\,x)}{dx}$$

$$= -\mathrm{cosec}^2 x + (-\mathrm{cosec}\,x \cot x) = -\mathrm{cosec}\,x\,(\mathrm{cosec}\,x + \cot x)$$

Putting the above obtained values in the formula:-

(uv)′ = u′v + uv′

[(tan x + sec x) (cot x + cosec x)]’

= [secx (secx + tanx)] × [(cot x + cosec x)] + [(tan x + sec x)] × [-cosecx (cosecx + cotx)]

= (secx +tanx) [secx(cotx + cosecx) - cosecx(cosecx + cotx)]

= (secx + tanx) (secx – cosecx) (cotx + cosecx)

Ans) (secx + tanx) (secx – cosecx) (cotx + cosecx)

Q. 12. Differentiate:

(x^3 cos x – 2^x tan x)

Answer : To find: Differentiation of (x^3 cos x – 2^x tan x)

Formula used: (i) (uv)′ = u′v + uv′ (Leibnitz or product rule)

(ii)

$$\frac{dx^n}{dx} = nx^{n-1}$$

(iii)

$$\frac{d\cos x}{dx} = -\sin x$$

(iv)

$$\frac{da^x}{dx} = a^x \log a$$

(v)

$$\frac{d\tan x}{dx} = \sec^2 x$$

Here we have two function (x^3 cos x) and (2^x tan x)

We have two differentiate them separately

Let us assume g(x) = (x^3 cos x)

And h(x) = (2^x tan x)

Therefore, f(x) = g(x) – h(x)

⇒ f’(x) = g’(x) – h’(x) … (i)

Applying product rule on g(x)

Let us take u = x^3 and v = cos x

$$u' = \frac{du}{dx} = \frac{d(x^3)}{dx} = 3x^2$$

$$v' = \frac{dv}{dx} = \frac{d(\cos x)}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$(uv)' = u'v + uv'$

$[x^3 \cos x]' = 3x^2 \times \cos x + x^3 \times -\sin x$

$= 3x^2\cos x - x^3\sin x$

$= x^2 (3\cos x – x \sin x)$

$g'(x) = x^2 (3\cos x – x \sin x)$

Applying product rule on h(x)

Let us take $u = 2^x$ and $v = \tan x$

$$u' = \frac{du}{dx} = \frac{d(2^x)}{dx} = 2^x \log 2$$

$$v' = \frac{dv}{dx} = \frac{d(\tan x)}{dx} = \sec^2 x$$

Putting the above obtained values in the formula:-

$(uv)' = u'v + uv'$

$[2^x \tan x]' = 2^x \log 2 \times \tan x + 2^x \times \sec^2 x$

$= 2^x (\log 2 \tan x + \sec^2 x)$

$h'(x) = 2^x (\log 2 \tan x + \sec^2 x)$

Putting the above obtained values in eqn. (i)

$f'(x) = x^2 (3\cos x – x \sin x) - 2^x (\log 2 \tan x + \sec^2 x)$

Ans) $x^2 (3\cos x – x \sin x) - 2^x (\log 2 \tan x + \sec^2 x)$

Exercise 28D

Q. 1. Differentiate

$$\frac{2^x}{x}$$

Answer :

To find: Differentiation of $\frac{2^x}{x}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{da^x}{dx} = a^x \log a$

Let us take $u = 2^x$ and $v = x$

$$u' = \frac{du}{dx} = \frac{d(2^x)}{dx} = 2^x \log 2$$

$$v' = \frac{dv}{dx} = \frac{d(x)}{dx} = 1$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left(\frac{2^x}{x}\right)' = \frac{2^x \log 2 \times x - 2^x \times 1}{(x)^2}$$

$$= \frac{2^x(x\log 2 - 1)}{x^2}$$

Ans) $= \frac{2^x(x\log 2 - 1)}{x^2}$

Q. 2. Differentiate

$$\frac{\log x}{x}$$

Answer :

To find: Differentiation of $\frac{\log x}{x}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\log x}{dx} = \frac{1}{x}$

Let us take u = logx and v = x

$$u' = \frac{du}{dx} = \frac{d(\log x)}{dx} = \frac{1}{x}$$

$$v' = \frac{dv}{dx} = \frac{d(x)}{dx} = 1$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left(\frac{\log x}{x}\right)' = \frac{\frac{1}{x} \times x - \log x \times 1}{(x)^2}$$

$$= \frac{1 - \log x}{x^2}$$

Ans) $= \frac{1 - \log x}{x^2}$

Q. 3. Differentiate

$$\frac{e^x}{(1+x)}$$

Answer :

To find: Differentiation of $\frac{e^x}{(1+x)}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

(ii) $\frac{de^x}{dx} = e^x$

Let us take u = e^x and v = (1+x)

$$u' = \frac{du}{dx} = \frac{d(e^x)}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{d(1+x)}{dx} = 1$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

$$\left(\frac{e^x}{(1+x)}\right)' = \frac{e^x \times (1+x) - e^x \times 1}{(1+x)^2}$$

$$= \frac{xe^x}{(1+x)^2}$$

Ans) = $\frac{xe^x}{(1+x)^2}$

Q. 4. Differentiate

$$\frac{e^x}{(1+x^2)}$$

Answer :

To find: Differentiation of $\frac{e^x}{(1+x^2)}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

(ii) $\frac{de^x}{dx} = e^x$

(iii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take u = e^x and v = $(1+x^2)$

$$u' = \frac{du}{dx} = \frac{d(e^x)}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{d(1+x^2)}{dx} = 2x$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

$$\left(\frac{e^x}{(1+x^2)}\right)' = \frac{e^x \times (1+x^2) - e^x \times 2x}{(1+x^2)^2}$$

$$= \frac{e^x(x^2-2x+1)}{(1+x^2)^2}$$

$$= \frac{e^x(x-1)^2}{(1+x^2)^2}$$

Ans) = $\frac{e^x(x-1)^2}{(1+x^2)^2}$

Q. 5. Differentiate

$$\left(\frac{2x^2-4}{3x^2+7}\right)$$

Answer :

To find: Differentiation of $\frac{(2x^2-4)}{(3x^2+7)}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take u = $(2x^2 – 4)$ and v = $(3x^2 + 7)$

$$u' = \frac{du}{dx} = \frac{d(2x^2-4)}{dx} = 4x$$

$$v' = \frac{dv}{dx} = \frac{d(3x^2+7)}{dx} = 6x$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

$$\left[\frac{(2x^2-4)}{(3x^2+7)}\right]' = \frac{4x \times (3x^2+7) - (2x^2-4) \times 6x}{(3x^2+7)^2}$$

$$= \frac{12x^3 + 28x - 12x^3 + 24x}{(3x^2+7)^2}$$

$$= \frac{52x}{(3x^2+7)^2}$$

Ans) $= \frac{52x}{(3x^2+7)^2}$

Q. 6. Differentiate

$$\left(\frac{x^2+3x-1}{x+2}\right)$$

Answer :

To find: Differentiation of $\left(\frac{x^2 + 3x - 1}{x + 2}\right)$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take u = $(x^2 + 3x – 1)$ and v = $(x + 2)$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 3x - 1)}{dx} = 2x + 3$$

$$v' = \frac{dv}{dx} = \frac{d(x + 2)}{dx} = 1$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v ≠0 (Quotient rule)

$$\left(\frac{x^2 + 3x - 1}{x + 2}\right)' = \frac{(2x + 3) \times (x + 2) - (x^2 + 3x - 1) \times 1}{(x + 2)^2}$$

$$= \frac{2x^2+7x+6-x^2-3x+1}{(x+2)^2}$$

$$= \frac{x^2+4x+7}{(x+2)^2}$$

$$\text{Ans)} = \frac{x^2+4x+7}{(x+2)^2}$$

Q. 7. Differentiate

$$\frac{(x^2-1)}{(x^2+7x+1)}$$

Answer :

To find: Differentiation of $\frac{(x^2-1)}{(x^2+7x+1)}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $u = (x^2 – 1)$ and $v = (x^2 + 7x + 1)$

$$u' = \frac{du}{dx} = \frac{d(x^2-1)}{dx} = 2x$$

$$v' = \frac{dv}{dx} = \frac{d(x^2+7x+1)}{dx} = 2x + 7$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left[\frac{(x^2-1)}{(x^2+7x+1)}\right]' = \frac{2x \times (x^2+7x+1) - (x^2-1) \times (2x+7)}{(x^2+7x+1)^2}$$

$$= \frac{2x^3 + 14x^2 + 2x - 2x^3 - 7x^2 + 2x + 7}{(x^2+7x+1)^2}$$

$$= \frac{7x^2 + 4x + 7}{(x^2+7x+1)^2}$$

Ans) $= \dfrac{7x^2 + 4x + 7}{(x^2+7x+1)^2}$

Q. 8. Differentiate

$$\left(\frac{5x^2 + 6x + 7}{2x^2 + 3x + 4}\right)$$

Answer :

To find: Differentiation of $\left(\dfrac{5x^2 + 6x + 7}{2x^2 + 3x + 4}\right)$

Formula used: (i) $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\dfrac{dx^n}{dx} = nx^{n-1}$

Let us take $u = (5x^2 + 6x + 7)$ and $v = (2x^2 + 3x + 4)$

$$u' = \frac{du}{dx} = \frac{d(5x^2 + 6x + 7)}{dx} = 10x + 6$$

$$v' = \frac{dv}{dx} = \frac{d(2x^2 + 3x + 4)}{dx} = 4x + 3$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{5x^2+6x+7}{2x^2+3x+4}\right)' = \frac{(10x+6)\times(2x^2+3x+4) - \left(5x^2+6x+7\right)\times(4x+3)}{(2x^2+3x+4)^2}$$

$$= \frac{20x^3+30x^2+40x+12x^2+18x+24 -20x^3-15x^2-24x^2-18x-28x-21}{(2x^2+3x+4)^2}$$

$$= \frac{3x^2+12x+3}{(2x^2+3x+4)^2}$$

$$= \frac{3(x^2+4x+1)}{(2x^2+3x+4)^2}$$

$$\text{Ans)} = \frac{3(x^2+4x+1)}{(2x^2+3x+4)^2}$$

Q. 9. Differentiate

$$\frac{x}{(a^2+x^2)}$$

Answer :

To find: Differentiation of $\frac{x}{(a^2+x^2)}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take u = (x) and v = $(a^2 + x^2)$

$$u' = \frac{du}{dx} = \frac{d(x)}{dx} = 1$$

$$v' = \frac{dv}{dx} = \frac{d(a^2+x^2)}{dx} = 2x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{x}{(a^2+x^2)}\right]' = \frac{1 \times (a^2+x^2) - (x) \times (2x)}{(a^2+x^2)^2}$$

$$= \frac{a^2+x^2 - 2x^2}{(a^2+x^2)^2}$$

$$= \frac{a^2 - x^2}{(a^2+x^2)^2}$$

$$\text{Ans)} = \frac{a^2 - x^2}{(a^2+x^2)^2}$$

Q. 10. Differentiate

$$\frac{x^4}{\sin x}$$

Answer :

To find: Differentiation of $\frac{x^4}{\sin x}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

(iii) $\frac{d\sin x}{dx} = \cos x$

Let us take u = (x^4) and v = (sinx)

$$u' = \frac{du}{dx} = \frac{d(x^4)}{dx} = 4x^3$$

$$v' = \frac{dv}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{x^4}{\sin x}\right]' = \frac{4x^3 \times (\sin x) - (x^4) \times (\cos x)}{(\sin x)^2}$$

$$= \frac{x^3[4(\sin x) - x(\cos x)]}{(\sin x)^2}$$

$$\text{Ans)} = \frac{x^3[4(\sin x) - x(\cos x)]}{(\sin x)^2}$$

Q. 11. Differentiate

$$\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}$$

Answer :

To find: Differentiation of $\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}-\sqrt{x}}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $u = (\sqrt{a}+\sqrt{x})$ and $v = (\sqrt{a}-\sqrt{x})$

$$u' = \frac{du}{dx} = \frac{d(\sqrt{a}+\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{a}-\sqrt{x})}{dx} = -\frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}-\sqrt{x}}\right]' = \frac{\frac{1}{2\sqrt{x}} \times (\sqrt{a}-\sqrt{x}) - (\sqrt{a}+\sqrt{x}) \times -\frac{1}{2\sqrt{x}}}{(\sqrt{a}-\sqrt{x})^2}$$

$$= \frac{\frac{\sqrt{a}}{2\sqrt{x}} - \frac{1}{2} + \frac{\sqrt{a}}{2\sqrt{x}} + \frac{1}{2}}{(\sqrt{a}-\sqrt{x})^2}$$

$$= \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a}-\sqrt{x})^2}$$

$$\text{Ans)} = \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a}-\sqrt{x})^2}$$

Q. 12. Differentiate

$$\frac{\cos x}{\log x}$$

Answer :

To find: Differentiation of $\frac{\cos x}{\log x}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where v ≠0 (Quotient rule)

(ii) $\frac{d\cos x}{dx} = -\sin x$

(iii) $\frac{d\log x}{dx} = \frac{1}{x}$

Let us take u = (cosx) and v = (logx)

$$u' = \frac{du}{dx} = \frac{d(\cos x)}{dx} = -\sin x$$

$$v' = \frac{dv}{dx} = \frac{d(\log x)}{dx} = \frac{1}{x}$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where v ≠0 (Quotient rule)

$$\left[\frac{\cos x}{\log x}\right]' = \frac{-\sin x \times (\log x) - (\cos x) \times \left(\frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{-x\sin x(\log x) - (\cos x)}{x(\log x)^2}$$

Ans) $= \frac{-x\sin x(\log x) - (\cos x)}{x(\log x)^2}$

Q. 13. Differentiate

$$\frac{2\cot x}{\sqrt{x}}$$

Answer :

To find: Differentiation of $\frac{2\cot x}{\sqrt{x}}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\cot x}{dx} = -\mathrm{cosec}^2 x$

(iii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take u = (2cotx) and v =

$(\sqrt{x})$

$u' = \frac{du}{dx} = \frac{d(2\cot x)}{dx} = -2\mathrm{cosec}^2 x$

$v' = \frac{dv}{dx} = \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left[\frac{2\cot x}{\sqrt{x}}\right]' = \frac{-2\mathrm{cosec}^2 x \times (\sqrt{x}) - (2\cot x) \times \left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2}$$

$$= \frac{-2x\mathrm{cosec}^2 x - (\cot x)}{\sqrt{x}(\sqrt{x})^2}$$

Ans) $= \frac{-2x\mathrm{cosec}^2 x - \cot x}{x^{3/2}}$

Q. 14. Differentiate

$$\frac{\sin x}{(1+\cos x)}$$

Answer :

To find: Differentiation of $\frac{\sin x}{(1+\cos x)}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\cos x}{dx} = -\sin x$

(iii) $\frac{d\sin x}{dx} = \cos x$

Let us take u = (sinx) and v = (1 + cosx)

$$u' = \frac{du}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

$$v' = \frac{dv}{dx} = \frac{d(1+\cos x)}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left[\frac{\sin x}{(1+\cos x)}\right]' = \frac{\cos x \times (1+\cos x) - (\sin x) \times (-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$$

$$= \frac{\cos x + 1}{(1+\cos x)^2}$$

$$= \frac{1}{(1+\cos x)}$$

Ans) $= \frac{1}{1+\cos x}$

Q. 15. Differentiate

$$\left(\frac{1+\sin x}{1-\sin x}\right)$$

Answer :

To find: Differentiation of $\left(\frac{1 + \sin x}{1 - \sin x}\right)$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\sin x}{dx} = \cos x$

Let us take u = (1 + sinx) and v = (1 - sinx)

$$u' = \frac{du}{dx} = \frac{d(1 + \sin x)}{dx} = \cos x$$

$$v' = \frac{dv}{dx} = \frac{d(1 - \sin x)}{dx} = -\cos x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{1 + \sin x}{1 - \sin x}\right]' = \frac{\cos x \times (1 - \sin x) - (1 + \sin x) \times (-\cos x)}{(1 - \sin x)^2}$$

$$= \frac{\cos x - \cos x \sin x + \cos x + \cos x \sin x}{(1 - \sin x)^2}$$

$$= \frac{2\cos x}{(1 - \sin x)^2}$$

$$\text{Ans)} = \frac{2\cos x}{(1 - \sin x)^2}$$

Q. 16. Differentiate

$$\left(\frac{1-\cos x}{1+\cos x}\right)$$

Answer :

To find: Differentiation of $\left(\frac{1 - \cos x}{1 + \cos x}\right)$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\cos x}{dx} = -\sin x$

Let us take u = (1 - cosx) and v = (1 + cosx)

$$u' = \frac{du}{dx} = \frac{d(1 - \cos x)}{dx} = \sin x$$

$$v' = \frac{dv}{dx} = \frac{d(1 + \cos x)}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$$\left[\frac{1 - \cos x}{1 + \cos x}\right]' = \frac{\sin x \times (1 + \cos x) - (1 - \cos x) \times (-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\sin x + \sin x\cos x + \sin x - \sin x\cos x}{(1 + \cos x)^2}$$

$$= \frac{2\sin x}{(1 + \cos x)^2}$$

Ans) $= \frac{2\sin x}{(1 + \cos x)^2}$

Q. 17. Differentiate

$$\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$$

Answer :

To find: Differentiation of $\left(\frac{\text{sinx - cosx}}{\text{sinx + cosx}}\right)$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v $\neq$0 (Quotient rule)

(ii) $\frac{\text{dsinx}}{\text{dx}}$ = cosx

(iii) $\frac{\text{dcosx}}{\text{dx}}$ = -sinx

Let us take u = (sinx - cosx) and v = (sinx + cosx)

$$u' = \frac{du}{dx} = \frac{d(\text{sinx - cosx})}{dx} = (\text{cosx + sinx})$$

$$v' = \frac{dv}{dx} = \frac{d(\text{sinx + cosx})}{dx} = (\text{cosx - sinx})$$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where v $\neq$0 (Quotient rule)

$$\left[\frac{\text{sinx-cosx}}{\text{sinx+cosx}}\right]' = \frac{(\text{cosx+sinx})\times(\text{sinx+cosx}) - (\text{sinx-cosx})\times(\text{cosx-sinx})}{(\text{sinx+cosx})^2}$$

$$= \frac{\sin^2 x+ \cos^2 x+2\text{sinxcosx} - (\text{sinx-cosx})\times-(\text{sinx-cosx})}{(\text{sinx+cosx})^2}$$

$$= \frac{\sin^2 x + \cos^2 x + 2\text{sinxcosx} + \sin^2 x + \cos^2 x - 2\text{sinxcosx}}{(\text{sinx+cosx})^2}$$

$$= \frac{2(\sin^2 x + \cos^2 x)}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$$

$$= \frac{2}{1 + \sin 2x}$$

Ans) $= \dfrac{2}{1 + \sin 2x}$

Q. 18. Differentiate

$$\left(\frac{\sec x - \tan x}{\sec x + \tan x}\right)$$

Answer :

To find: Differentiation of $\left(\dfrac{\sec x - \tan x}{\sec x + \tan x}\right)$

Formula used: (i) $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\dfrac{d\sec x}{dx} = \sec x \tan x$

(iii) $\dfrac{d\tan x}{dx} = \sec^2 x$

Let us take u = (secx - tanx) and v = (secx + tanx)

$$u' = \frac{du}{dx} = \frac{d(\sec x - \tan x)}{dx} = (\sec x \tan x - \sec^2 x)$$

$$v' = \frac{dv}{dx} = \frac{d(\sec x + \tan x)}{dx} = (\sec x \tan x + \sec^2 x)$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sec x - \tan x}{\sec x + \tan x}\right]' = \frac{(\sec x\tan x - \sec^2 x)(\sec x + \tan x) - (\sec x - \tan x)(\sec x\tan x + \sec^2 x)}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x\tan x - \sec^2 x)(\sec x + \tan x) - (\sec x - \tan x)(\sec x)(\tan x + \sec x)}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[(\sec x\tan x - \sec^2 x) - (\sec x - \tan x)(\sec x)]}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[(\sec x\tan x - \sec^2 x) - (\sec^2 x - \sec x\tan x)]}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[2\sec x\tan x - 2\sec^2 x]}{(\sec x + \tan x)^2}$$

$$= \frac{2\sec x[\tan x - \sec x]}{(\sec x + \tan x)}$$

Ans) $= \dfrac{2\sec x[\tan x - \sec x]}{(\sec x + \tan x)}$

Q. 19. Differentiate

$$\left(\frac{e^x + \sin x}{1 + \log x}\right)$$

Answer :

To find: Differentiation of $\left(\dfrac{e^x + \sin x}{1 + \log x}\right)$

Formula used: (i) $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\dfrac{d\sin x}{dx} = \cos x$

(iii) $\frac{d\log x}{dx} = \frac{1}{x}$

(iv) $\frac{de^x}{dx} = e^x$

Let us take u = $(e^x + \sin x)$ and v = $(1 + \log x)$

$$u' = \frac{du}{dx} = \frac{d(e^x + \sin x)}{dx} = (e^x + \cos x)$$

$$v' = \frac{dv}{dx} = \frac{d(1 + \log x)}{dx} = \frac{1}{x}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{e^x + \sin x}{1 + \log x}\right]' = \frac{(e^x + \cos x) \times (1 + \log x) - (e^x + \sin x) \times \left(\frac{1}{x}\right)}{(1 + \log x)^2}$$

$$= \frac{x(e^x + \cos x)(1 + \log x) - (e^x + \sin x)}{x(1 + \log x)^2}$$

$$\text{Ans)} = \frac{x(e^x + \cos x)(1 + \log x) - (e^x + \sin x)}{x(1 + \log x)^2}$$

Q. 20. Differentiate

$$\frac{e^x \sin x}{\sec x}$$

Answer :

To find: Differentiation of $\left(\frac{e^x \sin x}{\sec x}\right)$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\sin x}{dx} = \cos x$

(iii) $\frac{d\sec x}{dx} = \sec x \tan x$

(iv) $\frac{de^x}{dx} = e^x$

(v) (uv)′ = u′v + uv′ (Leibnitz or product rule)

Let us take $u = (e^x \sin x)$ and $v = (\sec x)$

$$u' = \frac{du}{dx} = \frac{d(e^x \sin x)}{dx}$$

Applying Product rule

(gh)′ = g′h + gh′

Taking g = e^x and h = sinx

= $e^x\sin x + e^x\cos x$

u’ = $e^x\sin x + e^x\cos x$

$$v' = \frac{dv}{dx} = \frac{d(\sec x)}{dx} = \sec x \tan x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{e^x \sin x}{\sec x}\right]' = \frac{(e^x\sin x + e^x\cos x)\times(\sec x) - (e^x \sin x)\times(\sec x \tan x)}{(\sec x)^2}$$

$$= \frac{(e^x\sin x + e^x\cos x) - (e^x \sin x)\times(\tan x)}{(\sec x)}$$

$= \cos x\,[(e^x \sin x + e^x \cos x) - (e^x \sin x) \times (\tan x)]$

$= [(e^x \sin x \cos x + e^x \cos^2 x) - (e^x \sin x \cos x) \times (\tan x)]$

$= \left[(e^x \sin x \cos x + e^x \cos^2 x) - \left(e^x \sin^2 x\right)\right]$

$= (e^x \sin x \cos x + e^x \cos^2 x - e^x \sin^2 x$

$= (e^x \sin x \cos x + e^x \cos^2 x - e^x \sin^2 x$

$= e^x \sin x \cos x + e^x \cos 2x$

$= e^x (\sin x \cos x + \cos 2x)$

Ans) $= e^x (\sin x \cos x + \cos 2x)$

Q. 21. Differentiate

$$\frac{2^x \cot x}{\sqrt{x}}$$

Answer :

To find: Differentiation of $\left(\frac{2^x \cot x}{\sqrt{x}}\right)$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\cot x}{dx} = -\mathrm{cosec}^2 x$

(iii) $\frac{dx^n}{dx} = nx^{n-1}$

(iv) $\frac{da^x}{dx} = a^x \log a$

(v) (uv)′ = u′v + uv′ (Leibnitz or product rule)

Let us take u = $(2^x \cot x)$ and v = $(\sqrt{x})$

$$u' = \frac{du}{dx} = \frac{d(2^x \cot x)}{dx}$$

Applying Product rule

(gh)′ = g′h + gh′

Taking g = 2^x and h = $\cot x$

$$= (2^x \log 2) \cot x + 2^x (-\operatorname{cosec}^2 x)$$

$$u' = (2^x \log 2) \cot x - 2^x (\operatorname{cosec}^2 x)$$

$$u' = 2^x [\log 2 \cot x - \operatorname{cosec}^2 x]$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{2^x \cot x}{\sqrt{x}}\right]' = \frac{\{2^x [\log 2 \cot x - \operatorname{cosec}^2 x] \times \sqrt{x}\} - \left\{(2^x \cot x) \times \left(\frac{1}{2\sqrt{x}}\right)\right\}}{(\sqrt{x})^2}$$

$$= \frac{\{2^x [\log 2 \cot x - \operatorname{cosec}^2 x] \times \sqrt{x}\} - \left\{(2^x \cot x) \times \left(\frac{1}{2\sqrt{x}}\right)\right\}}{x}$$

$$= \frac{\{2^x[\log 2 \cot x - \operatorname{cosec}^2 x] \times \sqrt{x}\} - \left\{\left(2^{x-1} \cot x\right) \times \left(\frac{1}{\sqrt{x}}\right)\right\}}{x}$$

$$= \frac{\{x2^x[\log 2 \cot x - \operatorname{cosec}^2 x]\} - \left\{\left(2^{x-1} \cot x\right)\right\}}{x\sqrt{x}}$$

$$= \frac{\{2^x[x\log 2 \cot x - x\operatorname{cosec}^2 x]\} - \left\{\left(2^{x-1} \cot x\right)\right\}}{x^{\frac{3}{2}}}$$

$$\text{Ans)} = \frac{\{2^x[x\log 2 \cot x - x\operatorname{cosec}^2 x]\} - \left\{\left(2^{x-1} \cot x\right)\right\}}{x^{\frac{3}{2}}}$$

Q. 22. Differentiate

$$\frac{e^x(x-1)}{(x+1)}$$

Answer :

To find: Differentiation of $\frac{e^x(x-1)}{(x+1)}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{de^x}{dx} = e^x$

(iii) $\frac{dx^n}{dx} = nx^{n-1}$

(iv) $(uv)' = u'v + uv'$ (Leibnitz or product rule)

Let us take $u = e^x(x-1)$ and $v = (x+1)$

$$u' = \frac{du}{dx} = \frac{d[e^x(x-1)]}{dx}$$

Applying Product rule

(gh)′ = g′h + gh′

Taking g = e^x and h = x – 1

$[e^x(x-1)]' = e^x(x-1) + e^x (1)$

$= e^x(x-1) + e^x$

$u' = e^x x$

$$v' = \frac{dv}{dx} = \frac{d(x+1)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{e^x(x-1)}{(x+1)}\right]' = \frac{(e^x x)(x+1) - [e^x(x-1)](1)}{(x+1)^2}$$

$$= \frac{e^x x^2 + e^x x - e^x x + e^x}{(x+1)^2}$$

$$= \frac{e^x x^2 + e^x}{(x+1)^2}$$

$$= \frac{e^x(x^2+1)}{(x+1)^2}$$

$$\text{Ans)} = \frac{e^x(x^2+1)}{(x+1)^2}$$

Q. 23. Differentiate

$$\frac{x \tan x}{(\sec x + \tan x)}$$

Answer :

To find: Differentiation of $\frac{x\tan x}{(\sec x+\tan x)}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\sec x}{dx} = \sec x\tan x$

(iii) $\frac{d\tan x}{dx} = \sec^2 x$

(iii) $\frac{dx^n}{dx} = nx^{n-1}$

(iv) (uv)′ = u′v + uv′ (Leibnitz or product rule)

Let us take u = (x tanx) and v = (secx + tanx)

$u' = \frac{du}{dx} = \frac{d[x\tan x]}{dx}$

Applying Product rule for finding u'

(gh)′ = g′h + gh′

Taking g = xand h = tanx

[xtanx]' = (1) (tanx) + x ($\sec^2 x$)

= tanx + x$\sec^2 x$

u' = tanx + x$\sec^2 x$

$v' = \frac{dv}{dx} = \frac{d(\sec x+\tan x)}{dx} = \sec x\tan x + \sec^2 x$

$v' = \sec x(\tan x + \sec x)$

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left[\frac{x\tan x}{(\sec x+\tan x)}\right]' = \frac{(\tan x + x\sec^2 x)(\sec x + \tan x) - [x\tan x][\sec x(\tan x + \sec x)]}{(\sec x+\tan x)^2}$

$= \frac{(\sec x + \tan x)[(\tan x + x\sec^2 x)-(x\tan x)(\sec x)]}{(\sec x+\tan x)^2}$

$= \frac{[\tan x + x\sec^2 x - x\tan x\sec x]}{(\sec x+\tan x)}$

$= \frac{\tan x + x\sec x(\sec x - \tan x)}{(\sec x+\tan x)}$

Ans) $= \frac{\tan x + x\sec x(\sec x - \tan x)}{(\sec x+\tan x)}$

Q. 24. Differentiate

$$\left(\frac{ax^2 + bx + c}{px^2 + qx + r}\right)$$

Answer :

To find: Differentiation of $\frac{ax^2+bx+c}{px^2+qx+r}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $u = (ax^2+bx+c)$ and $v = (px^2+qx+r)$

$u' = \frac{du}{dx} = \frac{d[ax^2+bx+c]}{dx} = 2ax + b$

$$v' = \frac{dv}{dx} = \frac{d(px^2+qx+r)}{dx} = 2px + q$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{ax^2+bx+c}{px^2+qx+r}\right]' = \frac{(2ax+b)(px^2+qx+r) - [ax^2+bx+c][2px+q]}{(px^2+qx+r)^2}$$

$$= \frac{2apx^3+2aqx^2+2axr+bpx^2+bqx+br-[2apx^3+qax^2+2bpx^2+bqx+2pcx+cq]}{(px^2+qx+r)^2}$$

$$= \frac{(aq-bp)x^2+2(ra-pc)x+br-cp}{(px^2+qx+r)^2}$$

$$\text{Ans)} = \frac{(aq-bp)x^2+2(ra-pc)x+br-cp}{(px^2+qx+r)^2}$$

Q. 25. Differentiate

$$\left(\frac{\sin x - x\cos x}{x\sin x + \cos x}\right)$$

Answer :

To find: Differentiation of $\frac{(\sin x - x\cos x)}{(x\sin x + \cos x)}$

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\sin x}{dx} = \cos x$

(iii) $\frac{d\cos x}{dx} = -\sin x$

(iv) (uv)′ = u′v + uv′ (Leibnitz or product rule)

Let us take u = (sinx-xcosx) and v = (xsinx + cosx)

$u' = \frac{du}{dx} = \frac{d[\sin x - x\cos x]}{dx}$

Applying Product rule for finding the term xcosx in u'

(gh)′ = g′h + gh′

Taking g = xand h = cosx

[x cosx]' = (1) (cosx) + x (-sinx)

[x cosx]' = cosx – x sinx

Applying the above obtained value for finding u'

u' = cosx – (cosx – x sinx)

u' = x sinx

$v' = \frac{dv}{dx} = \frac{d(x\sin x + \cos x)}{dx}$

Applying Product rule for finding the term xsinx in v'

(gh)′ = g′h + gh′

Taking g = xand h = sinx

[x sinx]' = (1) (sinx) + x (cosx)

[x sinx]' = sinx + x cosx

Applying the above obtained value for finding v'

v' = sinx + x cosx - sinx

v' = x cosx

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{(\sin x - x\cos x)}{(x\sin x + \cos x)}\right]' = \frac{(x \sin x)\,(x \sin x + \cos x) - (\sin x - x \cos x)(x \cos x)}{(x\sin x + \cos x)^2}$$

$$= \frac{(x^2 \sin^2 x + x\sin x\cos x) - (x\sin x\cos x - x^2 \cos^2 x)}{(x\sin x + \cos x)^2}$$

$$= \frac{x^2 \sin^2 x + x\sin x\cos x - x\sin x\cos x + x^2 \cos^2 x)}{(x\sin x + \cos x)^2}$$

$$= \frac{x^2(\sin^2 x + \cos^2 x)}{(x\sin x + \cos x)^2}$$

$$= \frac{x^2}{(x\sin x + \cos x)^2}$$

$$\text{Ans}) = \frac{x^2}{(x\sin x + \cos x)^2}$$

Q. 26

(i) cotx
(ii) secx

Answer :

To find: Differentiation of cotx

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\sin x}{dx} = \cos x$

(iii) $\frac{d\cos x}{dx} = -\sin x$

We can write cotx as $\frac{\cos x}{\sin x}$

Let us take u = cosx and v = sinx

u’ = (cosx)’ = -sinx

v’ = (sinx)’ = cosx

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{\cos x}{\sin x}\right)' = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\mathrm{cosec}^2 x$$

Ans).

$-\mathrm{cosec}^2 x$

(ii)

To find: Differentiation of secx

Formula used: (i) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

(ii) $\frac{d\cos x}{dx} = -\sin x$

We can write secx as $\frac{1}{\cos x}$

Let us take u = 1 and v = cosx

u' = (1)' = 0

v' = (cosx)' = -sinx

Putting the above obtained values in the formula:-

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ where $v \neq 0$ (Quotient rule)

$\left(\frac{1}{\cos x}\right)' = \frac{(0)(\cos x) - (1)(-\sin x)}{(\cos x)^2}$

$= \frac{\sin x}{\cos^2 x}$

$= \sec x \tan x$

Ans).

$-\operatorname{cosec}^2 x$

Exercise 28E

Q. 1. Differentiate the following with respect to x:

sin 4x

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\sin nu) = \cos (nu) \frac{d}{dx}(nu)$

Let us take y = sin 4x.

So, by using the above formula, we have

$\frac{d}{dx}(\sin 4x) = \cos (4x) \times \frac{d}{dx}(4x) = 4\cos 4x.$

Differentiation of y = sin 4x is 4cos4x

Q. 2. sDifferentiate the following with respect to x:

cos 5x

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\cos nu) = -\sin (nu) \frac{d}{dx}(nu)$.

Let us take y = cos5x.

So, by using the above formula, we have

$\frac{d}{dx}(\cos 5x) = -\sin(5x) \times \frac{d}{dx}(5x) = -5\sin 5x.$

Differentiation of y = cos 5x is - 5sin5x

Q. 3. Differentiate the following with respect to x:

tan3x

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(\tan nu) = \sec^2(nu).\frac{d}{dx}(nu)$.

Let us take y = tan3x

So, by using the above formula, we have

$$\frac{d}{dx}\tan 3x = \sec^2(3x) \times \frac{d}{dx}(3x) = 3\sec^2(3x)$$

Differentiation of y = tan3x is $3\sec^2(3x)$

Q. 4. Differentiate the following with respect to x:

cos x^3

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(\cos nu) = -\sin nu\frac{d}{dx}(nu)$ and $\frac{d\,x^n}{dx} = nx^{n-1}$

Let us take y = cos x^3

So, by using the above formula, we have

$$\frac{d}{dx}\cos x^3 = -\sin(x^3) \times \frac{d}{dx}(x^3) = -3x^2\sin(x^3)$$

Differentiation of y = cos x^3 is $-3x^2\sin(x^3)$

Q. 5. Differentiate the following with respect to x:

$\cot^2 x$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(\cot^a nu) = a\cot^{a-1}(nu) \times \frac{d}{dx}(\cot nu) \times \frac{d}{dx}(nu)$ and $\frac{dx^n}{dx} = nx^{n-1}$

Let us take $y = \cot^2 x$

So, by using the above formula, we have

$$\frac{d}{dx}\cot^2 x = 2\cot(x) \times \frac{d\cot x}{dx} \times \frac{dx}{dx} = -2\cot x\,(\mathrm{cosec}^2 x).$$

Differentiation of $y = \cot^2 x$ is $-2\cot x\,(\mathrm{cosec}^2 x)$

Q. 6. Differentiate the following with respect to x:

$\tan^3 x$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used:

$$\frac{d}{dx}(\tan^a nu) = a\tan^{a-1}nu \times \frac{d(\tan nu)}{dx} \times \frac{d(nu)}{dx} \text{ and } \frac{dx^n}{dx} = nx^{n-1}$$

Let us take $y = \tan^3 x$

So, by using the above formula, we have

$$\frac{d}{dx}\tan^3 x = 3\tan^2(x) \times \frac{d(\tan x)}{dx} \times \frac{dx}{dx} = 3\tan^2 x \times (\sec^2 x).$$

Differentiation of y = $\tan^3 x$ is $3\tan^2 x \times (\sec^2 x)$

Q. 8. Differentiate the following with respect to x:

e^{x^2}

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(e^{a^t}) = e^{a^t} \times \frac{d}{dx}(a^t)$ and $\frac{d\,x^n}{dx} = nx^{n-1}$

Let us take y = e^{x^2}

So, by using the above formula, we have

$$\frac{d}{dx}e^{x^2} = e^{x^2} \times \frac{d}{dx}(x^2) = 2xe^{x^2}$$

Differentiation of y = e^{x^2} is $2xe^{x^2}$

Q. 9. Differentiate the following with respect to x:

$e^{\cot x}$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(e^a) = e^a \times \frac{da}{dx}$ and $\frac{d\,x^n}{dx} = nx^{n-1}$

Let us take $y = e^{cotx}$

So, by using the above formula, we have

$\frac{d}{dx}e^{cotx} = e^{cotx} \times \frac{dcotx}{dx} = -\, e^{cotx}\, cosec^2x.$

Differentiation of $y = e^{cotx}$ is $-\, e^{cotx}\, cosec^2x$

Q. 10. Differentiate the following with respect to x:

$\sqrt{sinx}$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(\sqrt{sinnu}) = \frac{1}{2\sqrt{sinnu}} \times \frac{d}{dx}(sinnu) \times \frac{d}{dx}(nu)$ and $\frac{d\,x^n}{dx} = nx^{n-1}$

Let us take $y = \sqrt{sinx}$

So, by using the above formula, we have

$\frac{d}{dx}\sqrt{sinx} = \frac{1}{2\sqrt{sinx}} \times \frac{d}{dx}(\sin x)\,\frac{d}{dx}(x) = \frac{1}{2\sqrt{sinx}}\, cosx$

Differentiation of $y = \sqrt{sinx}$ is $\frac{1}{2\sqrt{sinx}}\, cosx$

Q. 11. Differentiate the following with respect to x:

$(5 + 7x)^6$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take $y = (5 + 7x)^6$

So, by using the above formula, we have

$$\frac{d}{dx}(5 + 7x)^6 = 6(5 + 7x)^5 \times \frac{d}{dx}(5 + 7x) = 6(5 + 7x)^5 \times 7 = 42(5 + 7x)^5$$

Differentiation of $y = (5 + 7x)^6$ is $42(5 + 7x)^5$

Q. 12. Differentiate the following with respect to x:

$(3 - 4x)^5$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take $y = (3 - 4x)^5$

So, by using the above formula, we have

$\frac{d}{dx}(3 - 4x)^5 = 4(3 - 4x)^5 \times \frac{d}{dx}(3 - 4x) = 4(3 - 4x)^5 \times (-4) = -16(3 - 4x)^5$

Differentiation of $y = (3 - 4x)^5$ is $-16(3 - 4x)^5$

Q. 13. Differentiate the following with respect to x:

$(3x^2 - x + 1)^4$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take $y = (3x^2 - x + 1)^4$

So, by using the above formula, we have

$\frac{d}{dx}(3x^2 - x + 1)^4 = 4(3x^2 - x + 1)^3 \times \frac{d}{dx}(3x^2 - x + 1) = 4(3x^2 - x + 1)^3 \times (3 \times 6x - 1)$
$= 4(3x^2 - x + 1)^3(6x - 1)$

Differentiation of $y = (3x^2 - x + 1)^4$ is $4(3x^2 - x + 1)^3(6x - 1)$

Q. 14. Differentiate the following with respect to x:

$(ax^2 + bx + c)$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take y= $(ax^2 + bx + c)$

So, by using the above formula, we have

$\frac{d}{dx}(ax^2 + bx + c) = 2ax + b$

Differentiation of y = $(ax^2 + bx + c)$ is 2ax + b

Q. 15. Differentiate the following with respect to x:

$$\frac{1}{(x^2 - x + 3)^3}$$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take y= $\frac{1}{(x^2 - x + 3)^3} = (x^2 - x + 3)^{-3}$

So, by using the above formula, we have

$\frac{d}{dx}(x^2 - x + 3)^{-3} = -3(x^2 - x + 3)^{-4} \times (2x - 1) = -3\frac{1}{(x^2 - x + 3)^{-4}}(2x - 1)$

Differentiation of y = $(x^2 - x + 3)^{-3}$ is $\frac{-3(2x-1)}{(x^2 - x + 3)^{-4}}$

Q. 16. Differentiate the following with respect to x:

$\sin^2(2x + 3)$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}\sin^2(ax + b) = 2\sin(ax + b)\frac{d}{dx}\sin(ax + b)\frac{d}{dx}(ax + b)$

Let us take $y = \sin^2(2x + 3)$

So, by using above formula, we have

$$\frac{d}{dx}\sin^2(2x + 3) = 2\sin(2x + 3)\frac{d}{dx}\sin(2x + 3)\frac{d}{dx}(2x + 3) = 4\sin(2x + 3)\cos(2x + 3)$$

Differentiation of $y = \sin^2(2x + 3)$ is $4\sin(2x + 3)\cos(2x + 3)$

Q. 17. Differentiate the following with respect to x:

$\cos^2(x^3)$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(\cos^a nu) = a\cos^{a-1}nu \frac{d}{dx}(\cos nu) \frac{d}{dx}(nu)$

Let us take $y = \cos^2(x^3)$

So, by using the above formula, we have

$$\frac{d}{dx}\cos^2(x^3) = 2\cos x^3(-\sin(x^3))3x^2 = -6x^2\cos(x^3)\sin x^3$$

Differentiation of $y = \cos^2(x^3)$ is $-6x^2\cos(x^3)\sin x^3$

Q. 18. Differentiate the following with respect to x:

$$\sqrt{\sin x^3}$$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(\sqrt{\sin u^a}) = \frac{1}{2\sqrt{\sin u^a}} \times \frac{d}{dx}(\sin u^a) \times \frac{d}{dx}(u^a)$

Let us take y = $\sqrt{\sin x^3}$

So, by using the above formula, we have

$$\frac{d}{dx}\sqrt{\sin x^3} = \frac{1}{2\sqrt{\sin x^3}} \times \frac{d}{dx}(\sin x^3) \times \frac{d}{dx}(x^3) = \frac{1}{2\sqrt{\sin x^3}} \times (\cos x^3) \times 3x^2 = \frac{3x^2(\cos x^3)}{2\sqrt{\sin x^3}}$$

Differentiation of y = $\sqrt{\sin x^3}$ is $\frac{3x^2(\cos x^3)}{2\sqrt{\sin x^3}}$

Q. 19. Differentiate the following with respect to x:

$\sqrt{x\sin x}$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(\sqrt{usinu}) = \frac{1}{2\sqrt{usinu}} \times \frac{d}{dx}(usinu)$

Let us take y = $\sqrt{x \sin x}$

So, by using the above formula, we have

$$\frac{d}{dx}\sqrt{x \sin x} = \frac{1}{2\sqrt{xsinx}} \times \frac{d}{dx}(xsinx) = \frac{1}{2\sqrt{xsinx}} \times (sinx + xcosx) = \frac{(sinx + xcosx)}{2\sqrt{xsinx}}$$

Differentiation of y = $\sqrt{xsin\,x}$ is $\frac{(sinx + xcosx)}{2\sqrt{xsinx}}$

Q. 20. Differentiate the following with respect to x:

$\sqrt{\cot\sqrt{x}}$

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Formula used: $\frac{d}{dx}(\sqrt{cot\sqrt{x}}) = \frac{1}{2\sqrt{cot\sqrt{x}}} \times \frac{d}{dx}(cot\sqrt{x}).\frac{d}{dx}(\sqrt{x})$

Let us take y = $\sqrt{\cot\sqrt{x}}$

So, by using the above formula, we have

$$\frac{d}{dx}\sqrt{\cot\sqrt{x}} = \frac{1}{2\sqrt{\cot\sqrt{x}}} \times \frac{d}{dx}\cot$$

$$\sqrt{x} \times \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{cot\sqrt{x}}} \times (-sec^2\sqrt{x}) \times \frac{1}{2\sqrt{x}} = \frac{-sec^2\sqrt{x}}{4\sqrt{x}\sqrt{\cot\sqrt{x}}}$$

Differentiation of y = $\sqrt{\cot\sqrt{x}}$ is $\frac{-\sec^2\sqrt{x}}{4\sqrt{x}\sqrt{\cot\sqrt{x}}}$

Q. 21. Differentiate the following with respect to x:

cos 3x sin 5x

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Let us take y = cos 3x sin 5x

So, by using the above formula, we have

$$\frac{d}{dx}(\cos 3x \sin 5x) = \sin 5x\frac{d(\cos 3x)}{dx} + \cos 3x\frac{d(\sin 5x)}{dx} =$$

sin5x (- 3sin 3x) + cos 3x(5cos 5x) = 5cos (3x) cos (5x) - 3 sin (5x) 3sin (3x)

Differentiation of y = cos 3x sin 5x is 5cos (3x) cos (5x) - 3 sin (5x) 3sin (3x)

Q. 22. Differentiate the following with respect to x:

sin x sin 2x

Answer : To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e

$$\frac{d(u.v)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Let us take y = sin x sin 2x

So, by using the above formula, we have

$\frac{d}{dx}(\sin x \sin 2x) = \sin x\frac{d(\sin 2x)}{dx} + \sin 2x\frac{d(\sin x)}{dx} =$ sinx (2cos 2x) + sin 2x(sinx) = 2sin(x)cos (2x) + sin 2x(sinx)

Differentiation of y = sin x sin 2x is 2sin(x) cos (2x) + sin 2x(sinx)

Q. 23. Differentiate w.r.t x:

$$\cos(\sin\sqrt{ax+b})$$

Answer :

Let $y = \cos(\sin\sqrt{ax+b})$, $z = \sin\sqrt{ax+b}$ and $w = \sqrt{ax+b}$

Formula :

$$\frac{d(\cos x)}{dx} = -\sin x \text{ and } \frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\sqrt{ax+b})}{dx} = \frac{1}{2} \times (ax+b)^{\frac{1}{2}-1} \times a$$

According to the chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dw} \times \frac{dw}{dx}$$

$$= -\sin(\sin\sqrt{ax+b}) \times \cos\sqrt{ax+b} \times \frac{1}{2} \times (ax+b)^{-\frac{1}{2}} \times a$$

$$= -\frac{a}{2}\sin(\sin\sqrt{ax+b}) \times \cos\sqrt{ax+b} \times (ax+b)^{-\frac{1}{2}}$$

Q. 24. Differentiate w.r.t x: e^{2x} sin 3x

Answer : Let $y = e^{2x} \sin 3x$, $z = e^{2x}$ and $w = \sin 3x$

Formula :

$$\frac{d(e^x)}{dx} = e^x \text{ and } \frac{d(\sin x)}{dx} = \cos x$$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\sin 3x \times (2 \times e^{2x})] + [e^{2x} \times 3\cos 3x]$$

$= e^{2x} \times [2 \sin 3x + 3 \cos 3x\ ]$

Q. 25. Differentiate w.r.t x: e^{3x} cos 2x

Answer : Let y = e^{3x} cos 2x , z = e^{3x} and w = cos 2x

Formula :

$\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to the product rule of differentiation

$^{dy}/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$

$= [\cos 2x\ \times (3 \times e^{3x})] + [e^{3x} \times (-2 \sin 2x)\ ]$

$= e^{3x} \times [3 \cos 2x - 2 \sin 2x]$

Q. 26. Differentiate w.r.t x: e^{-5x} cot 4x

Answer : Let y = e^{-5x} cot 4x , z = e^{-5x} and w = cot 4x

Formula :

$\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to the product rule of differentiation

$^{dy}/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$

$= [\cot 4x\ \times (-5e^{-5x})] + [e^{-5x} \times (-4 \operatorname{cosec}^2 4x\ )]$

$= -e^{-5x} \times [5 \cot 4x + 4 \operatorname{cosec}^2 4x]$

Q. 27. Differentiate w.r.t x: cos (x^3 . e^x)

Answer : Let y = cos (x^3 . e^x) , z = x^3 . e^x , m = e^x and w = x^3

Formula :

$$\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1} \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the product rule of differentiation

$$dz/_{dx} = w \times \frac{dm}{dx} + m \times \frac{dw}{dx}$$

$$= [x^3 \times (e^x)] + [e^x \times (3x^2)]$$

$$= e^x \times [x^3 + 3x^2]$$

According to the chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= -\sin(x^3 \times e^x) \times \{e^x \times [x^3 + 3x^2]\}$$

Q. 28. Differentiate w.r.t x: $e^{(x\sin x+\cos x)}$

Answer : Let y = $e^{(x\sin x+\cos x)}$, z = x sin x+ cos x, m = x and w = sin x

Formula :

$$\frac{d(e^x)}{dx} = e^x, \frac{d(\sin x)}{dx} = \cos x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the product rule of differentiation

$$dz/_{dx} = w \times \frac{dm}{dx} + m \times \frac{dw}{dx} + \frac{d(\cos x)}{dx}$$

$$= [\sin x \times (1)] + [x \times (\cos x)] - \sin x$$

= x cosx

According to the chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= e^{(x\sin x+\cos x)} \times (x\cos x)$$

Q. 29. Differentiate w.r.t x:

$$\frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

Answer :

Let $y = \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$, $u = e^{x} + e^{-x}$, $v = e^{x} - e^{-x}$

Formula :

$$\frac{d(e^{x})}{dx} = e^{x}$$

According to the quotient rule of differentiation

If $y = \frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(e^{x} - e^{-x}) \times (e^{x} - e^{-x}) - (e^{x} + e^{-x}) \times (e^{x} + e^{-x})}{(e^{x} - e^{-x})^2}$$

$$= \frac{(e^{x} - e^{-x})^2 - (e^{x} + e^{-x})^2}{(e^{x} - e^{-x})^2}$$

$$= \frac{(e^{x} - e^{-x} + e^{x} + e^{-x})(e^{x} - e^{-x} - e^{x} - e^{-x})}{(e^{x} - e^{-x})^2}$$

($a^2 - b^2 = (a - b)(a + b)$)

$$= \frac{(2e^{x})(-2e^{-x})}{(e^{x} - e^{-x})^2}$$

$$= \frac{-4}{(e^x - e^{-x})^2}$$

Q. 30. Differentiate w.r.t x:

$$\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$$

Answer :

Let $y = \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$, $u = e^{2x} + e^{-2x}$, $v = e^{2x} - e^{-2x}$

Formula :

$$\frac{d(e^x)}{dx} = e^x$$

According to the quotient rule of differentiation

If $y = \frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(e^{2x} - e^{-2x}) \times (2e^{2x} - 2e^{-2x}) - (e^{2x} + e^{-2x}) \times (2e^{2x} + 2e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{2(e^{2x} - e^{-2x})^2 - 2(e^{2x} + e^{-2x})^2}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{2\,(e^{2x} - e^{-2x} + e^{2x} + e^{-2x})\,(e^{2x} - e^{-2x} - e^{2x} - e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

($a^2 - b^2 = (a - b)(a + b)$)

$$= \frac{2(2e^{2x})(-2e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{-8}{(e^{2x} - e^{-2x})^2}$$

Q. 31. Differentiate w.r.t x:

$$\sqrt{\frac{1-x^2}{1+x^2}}$$

Answer :

Let $y = \sqrt{\frac{1-x^2}{1+x^2}}$, $u = 1 - x^2$, $v = 1 + x^2$, $z = \frac{1-x^2}{1+x^2}$

Formula :

$$\frac{d(x^2)}{dx} = 2x$$

According to the quotient rule of differentiation

If $z = \frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 + x^2) \times (-2x) - (1 - x^2) \times (2x)}{(1 + x^2)^2}$$

$$= \frac{-2x - 2x^3 - 2x + 2x^3}{(1 + x^2)^2}$$

$$= \frac{-4x}{(1 + x^2)^2}$$

According to chain rule of differentiation

$$ {}^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx} $$

$$ = \left[\frac{1}{2} \times \left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{-4x}{(1+x^2)^2}\right] $$

$$ = \left[\frac{-2x}{1} \times \left(\frac{1-x^2}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1+x^2)^{2-\frac{1}{2}}}\right] $$

$$ = \left[-2x \times (1-x^2)^{-\frac{1}{2}}\right] \times (1+x^2)^{-\frac{3}{2}} $$

Q. 32. Differentiate w.r.t x:

$$ \sqrt{\frac{a^2 - x^2}{a^2 + x^2}} $$

Answer :

Let $y = \sqrt{\frac{a^2-x^2}{a^2+x^2}}$, $u = a^2 - x^2$, $v = a^2 + x^2$, $z = \frac{a^2-x^2}{a^2+x^2}$

Formula :

$$ \frac{d(x^2)}{dx} = 2x $$

According to the quotient rule of differentiation

If $z = \frac{u}{v}$

$$ {}^{dz}/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} $$

$$= \frac{(a^2 + x^2) \times (-2x) - (a^2 - x^2) \times (2x)}{(a^2 + x^2)^2}$$

$$= \frac{-2xa^2 - 2x^3 - 2xa^2 + 2x^3}{(1 + x^2)^2}$$

$$= \frac{-4\,x\,a^2}{(1 + x^2)^2}$$

According to the chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{a^2 - x^2}{a^2 + x^2}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{-4x\,a^2}{(a^2+x^2)^2}\right]$$

$$= \left[\frac{-2xa^2}{1} \times \left(\frac{a^2 - x^2}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(a^2+x^2)^{2-\frac{1}{2}}}\right]$$

$$= \left[-2xa^2 \times (a^2 - x^2)^{-\frac{1}{2}}\right] \times (a^2 + x^2)^{-\frac{3}{2}}$$

Q. 33. Differentiate w.r.t x:

$$\sqrt{\frac{1 + \sin x}{1 - \sin x}}$$

Answer :

Let $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$, $u = 1+\sin x$, $v = 1 - \sin x$, $z = \frac{1+\sin x}{1-\sin x}$

Formula :

$$\frac{d(\sin x)}{dx} = \cos x$$

According to the quotient rule of differentiation

If $z = \frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 - \sin x) \times (\cos x) - (1 + \sin x) \times (-\cos x)}{(1 - \sin x)^2}$$

$$= \frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{(1 - \sin x)^2}$$

$$= \frac{2 \cos x}{(1 - \sin x)^2}$$

According to the chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}-1}\right] \times [\frac{2 \cos x}{(1-\sin x)^2}]$$

$$= \left[\frac{\cos x}{1} \times \left(\frac{1+\sin x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1-\sin x)^{2-\frac{1}{2}}}\right]$$

$$= \left[\cos x \times (1 + \sin x)^{-\frac{1}{2}}\right] \times (1 - \sin x)^{-\frac{3}{2}}$$

Q. 34. Differentiate w.r.t x:

$$\sqrt{\frac{1+e^x}{1-e^x}}$$

Answer :

Let $y = \sqrt{\frac{1+e^x}{1-e^x}}$, $u = 1 + e^x$, $v = 1 - e^x$, $z = \frac{1+e^x}{1-e^x}$

Formula :

$$\frac{d(e^x)}{dx} = e^x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 - e^x) \times (e^x) - (1 + e^x) \times (-e^x)}{(1 - e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^x + e^{2x}}{(1 - e^x)^2}$$

$$= \frac{2e^x}{(1 - e^x)^2}$$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{1+e^x}{1-e^x}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{2e^x}{(1-e^x)^2}\right]$$

$$= \left[\frac{e^x}{1} \times \left(\frac{1+e^x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1-e^x)^{2-\frac{1}{2}}}\right]$$

$$= \left[e^x \times (1 + e^x)^{-\frac{1}{2}}\right] \times (1 - e^x)^{-\frac{3}{2}}$$

Q. 35. Differentiate w.r.t x:

$$\frac{e^{2x} + x^3}{\operatorname{cosec} 2x}$$

Answer :

Formula:

$\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1}$ and $\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x$

According to the quotient rule of differentiation

if $y = \frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\operatorname{cosec} 2x) \times (2e^{2x} + 3x^2) - (e^{2x} + x^3) \times (-2 \operatorname{cosec} 2x \cot 2x)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{2e^{2x} \operatorname{cosec} 2x + 3x^2 \operatorname{cosec} 2x + 2 e^{2x} \operatorname{cosec} 2x \cot 2x + 2x^3 \operatorname{cosec} 2x \cot 2x}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{2e^{2x} \operatorname{cosec} 2x (1 + \cot 2x) + 3x^2 \operatorname{cosec} 2x (1 + \cot 2x)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{(1 + \cot 2x)(2e^x \operatorname{cosec} 2x + 3x^2 \operatorname{cosec} 2x)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{(1 + \cot 2x)(2e^x + 3x^2)(\operatorname{cosec} 2x)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{(1+\cot 2x)(2e^x + 3x^2)}{(\operatorname{cosec} 2x)^1}$$

$= (1 + \cot 2x)(2e^x + 3x^2)(\sin 2x)$

Q. 36

Find $\frac{dy}{dx}$,When $y = \sin\sqrt{\sin x + \cos x}$

Answer :

Let $y = \sin(\sqrt{\sin x + \cos x})$, $z = \sqrt{\sin x + \cos x}$

Formula : $\frac{d(\cos x)}{dx} = -\sin x$ and $\frac{d(\sin x)}{dx} = \cos x$

$\frac{d(\sqrt{\sin x+\cos x})}{dx} = \frac{1}{2} \times (\sin x + \cos x)^{\frac{1}{2}-1} \times (\cos x - \sin x)$

According to the chain rule of differentiation

$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \cos(\sin\sqrt{\sin x + \cos x}) \times \frac{1}{2} \times (\sin x + \cos x)^{\frac{1}{2}-1} \times (\cos x - \sin x)$

$= \cos(\sin\sqrt{\sin x + \cos x}) \times \frac{1}{2} \times (\sin x + \cos x)^{-\frac{1}{2}} \times (\cos x - \sin x)$

Q. 37.

Find $\frac{dy}{dx}$,When = e^x log (sin 2x)

Answer :

Let y = e^x log (sin 2x) , z = e^x and w = log (sin 2x)

Formula :

$\frac{d(e^x)}{dx} = e^x$, $\frac{d(\log x)}{dx} = \frac{1}{x}$ and $\frac{d(\sin x)}{dx} = \cos x$

According to the product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\log(\sin 2x) \times (e^x)] + [e^x \times \frac{1}{\sin 2x} \times 2\cos 2x]$$

$$= e^x \times [\log(\sin 2x) + \frac{2\cos 2x}{\sin 2x}]$$

$$= e^x \times [\log(\sin 2x) + 2\cot 2x]$$

Q. 38.

Find $\frac{dy}{dx}$ **,When** $y = \cos\left(\frac{1-x^2}{1+x^2}\right)$

Answer :

Let y = cos ($\frac{1-x^2}{1+x^2}$) , u = $1 - x^2$, v = $1 + x^2$, z= $\frac{1-x^2}{1+x^2}$

Formula :

$\frac{d(x^2)}{dx} = 2x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to the quotient rule of differentiation

If $z = \frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1+x^2) \times (-2x) - (1-x^2) \times (2x)}{(1+x^2)^2}$$

$$= \frac{-2x - 2x^3 - 2x + 2x^3}{(1 + x^2)^2}$$

$$= \frac{-4x}{(1 + x^2)^2}$$

According to the chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[-\sin\frac{1-x^2}{1+x^2}\right] \times \left[\frac{-4x}{(1+x^2)^2}\right]$$

$$= \left[\sin\frac{1-x^2}{1+x^2}\right] \times \left[\frac{4x}{(1+x^2)^2}\right]$$

Q. 39. Find $\frac{dy}{dx}$ **,When** $y = \sin\left(\frac{1+x^2}{1-x^2}\right)$

Answer :

Let $y = \sin\left(\frac{1+x^2}{1-x^2}\right)$, $u = 1 + x^2$, $v = 1 - x^2$, $z = \frac{1+x^2}{1-x^2}$

Formula : $\frac{d(x^2)}{dx} = 2x$ and $\frac{d(\sin x)}{dx} = \cos x$

According to the quotient rule of differentiation

If $z = \frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1-x^2)\times(2x) \;-\; (1+x^2)\times(-2x)}{(1-x^2)^2}$$

$$= \frac{2x - 2x^3 + 2x \;+\; 2x^3}{(1+x^2)^2}$$

$$= \frac{4x}{(1+x^2)^2}$$

According to the chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz}\times\frac{dz}{dx}$$

$$= \left[\cos\frac{1+x^2}{1-x^2}\right]\times\left[\frac{4x}{(1+x^2)^2}\right]$$

Q. 40. Find $\frac{dy}{dx}$ **,When** $y = \frac{\sin x + x^2}{\cot 2x}$

Answer :

Let $y = \frac{\sin x + x^2}{\cot 2x}$, $u = \sin x + x^2$, $v = \cot 2x$

Formula:

$$\frac{d(\sin x)}{dx} = \cos x, \frac{d(x^n)}{dx} = n \times x^{n-1} \text{ and } \frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

According to the quotient rule of differentiation

$$\text{If } y = \frac{u}{v}$$

$$dy/_{dx} = \frac{v\times\frac{du}{dx} - u\times\frac{dv}{dx}}{v^2}$$

$$= \frac{(\cot 2x) \times (\cos x + 2x) \; - \; (\sin x + x^2) \times (-2 \operatorname{cosec}^2 2x)}{(\cot 2x)^2}$$

$$= \frac{\cot 2x \cos x \; + 2x \cot 2x \; + 2 \operatorname{cosec}^2 2x \; \sin x \; + \; 2x^2 \operatorname{cosec}^2 2x}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{\cot 2x\,(\cos x + 2x) \; + \; 2 \operatorname{cosec}^2 2x\,(\sin x + x^2)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{2 \operatorname{cosec}^2 2x\,(\sin x + x^2)}{(\operatorname{cosec} 2x)^2} + \frac{\cot 2x\,(\cos x + 2x)}{(\operatorname{cosec} 2x)^2}$$

$$= \frac{2\,(\sin x + x^2)}{1} + \frac{\cos 2x\;(\cos x + 2x)}{\sin 2x \frac{1}{\sin^2 2x}}$$

$$= 2(\sin x + x^2) + \cos 2x \sin 2x\,(\cos x + 2x)$$

Q. 41.

If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$**, show that** $\frac{dy}{dx} + y^2 + 1 = 0$

Answer :

$$= -\frac{1}{1} - y^2 \;\left(y = \frac{\cos x - \sin x}{\cos x + \sin x} \right)$$

Formula:

$\frac{d(\sin x)}{dx} = \cos x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to the quotient rule of differentiation

If y = u/v

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\cos x + \sin x) \times (-\sin x - \cos x) \; - \; (\cos x - \sin x) \times (-\sin x + \cos x)}{(\cos x \; + \sin x)^2}$$

$$= \frac{-(\cos x \; + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x \; + \sin x)^2}$$

$$= -\frac{(\cos x \; + \sin x)^2}{(\cos x \; + \sin x)^2} - \frac{(\cos x - \sin x)^2}{(\cos x \; + \sin x)^2}$$

$$= -\frac{1}{1} - y^2 \; (\; y = \frac{\cos x - \sin x}{\cos x + \sin x} \;)$$

$$\frac{dy}{dx} + y^2 + 1 = 0$$

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Q. 42.

If $y = \frac{\cos x + \sin x}{\cos x - \sin x}$**, show that** $\frac{dy}{dx} = \sec^2\left(x + \frac{\pi}{4}\right)$**.**

Answer :

Let $y = \frac{\cos x + \sin x}{\cos x - \sin x}$, , u $= \cos x + \sin x$, v $= \cos x - \sin x$

Formula:

$$\frac{d(\sin x)}{dx} = \cos x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the quotient rule of differentiation

If $y = \frac{u}{v}$

$$dy/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\cos x - \sin x) \times (-\sin x + \cos x) \; - \; (\cos x + \sin x) \times (-\sin x - \cos x)}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos x - \sin x)^2 + (\cos x \; + \sin x)^2}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos^2 x + \sin^2 x - 2\cos x \sin x) + (\cos^2 x + \sin^2 x + 2\cos x \sin x)}{(\cos x - \sin x)^2}$$

$$= \frac{2(\cos^2 x + \sin^2 x)}{(\cos x - \sin x)^2}$$

$$= \frac{(1)}{(\cos x - \sin x)^2/_2} \quad (\cos^2 x + \sin^2 x) = 1$$

$$= \frac{1}{\left(\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}\right)^2}$$

$$= \frac{1}{\left(\frac{\cos x \cos 45°}{1} - \frac{\sin x \sin 45°}{1}\right)^2}$$

$= \frac{1}{\cos^2(x + \frac{\pi}{4})}$ [cos a cos b - sin a sin b = cos (a + b)]

$= \sec^2(x + \frac{\pi}{4})$

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Q. 43.

$y = \sqrt{\frac{1-x}{1+x}}$ **, prove that** $(1-x^2)\frac{dy}{dx} + y = 0$

Answer :

Let $y = \sqrt{\frac{1-x^1}{1+x^1}}$, $u = 1 - x^1$, $v = 1 + x^1$, $z = \frac{1-x^1}{1+x^1}$

Formula :

$\frac{d(x^1)}{dx} = 1$

According to quotient rule of differentiation

$\text{If } z = \frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 + x^1) \times (-1) \; - \; (1 - x^1) \times (1)}{(1 + x^1)^2}$$

$$= \frac{-1 - x^1 - 1 \; + \; x}{(1 + x^1)^2}$$

$$= \frac{-2}{(1 + x)^2}$$

According to the chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$= \left[\frac{1}{2} \times \left(\frac{1-x^1}{1+x^1}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{-2}{(1+x^1)^2}\right]$

$= \left[\frac{-1}{1} \times \left(\frac{1-x^1}{1+x}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1+x^1)^2}\right]$

$= \left[-1 \times \frac{(1-x^1)^{-\frac{1}{2}}}{(1+x^1)^{1-\frac{1}{2}}}\right] \times \left[\frac{1}{(1+x^1)^1}\right] \times \frac{1-x}{1-x}$

(Muliplying and dividing by 1-x)

$= \left[-1 \times \frac{(1-x^1)^{1-\frac{1}{2}}}{(1+x^1)^{\frac{1}{2}}}\right] \times \frac{1}{(1-x)(1+x)}$

$= \left[-1 \times \frac{(1-x^1)^{\frac{1}{2}}}{(1+x^1)^{\frac{1}{2}}}\right] \times \frac{1}{(1-x)(1+x)} = -\frac{y}{1-x^2}$

Therefore

$$(1-x^2)\frac{dy}{dx} = -y$$

$$(1-x^2)\frac{dy}{dx} + y = 0$$

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Q. 44.

$y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$, **show that** $\frac{dy}{dx} = \sec x(\tan x + \sec x)$

Answer :

$$y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$$

$y = \sqrt{\frac{\frac{1}{\cos x}-\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+\frac{\sin x}{\cos x}}} = \sqrt{\frac{1-\sin x}{1+\sin x}}$

u =1-sin x, v = 1 + sin x , z=$\frac{1-\sin x}{1+\sin x}$

Formula : $\frac{d(\sin x)}{dx} = \cos x$

According to quotient rule of differentiation

If z = $\frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1+\sin x) \times (-\cos x) \; - \; (1-\sin x) \times (\cos x)}{(1+\sin x)^2}$$

$$= \frac{-\cos x \; - \sin x \cos x - \cos x + \sin x \cos x}{(1+\sin x)^2}$$

$$= \frac{-2\cos x}{(1+\sin x)^2}$$

According to the chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{1-\sin x}{1+\sin x}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{-2\cos x}{(1+\sin x)^2}\right]$$

$$= \left[-\frac{\cos x}{1} \times \left(\frac{1-\sin x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1+\sin x)^{2-\frac{1}{2}}}\right]$$

$$= \left[\cos x \times (1+\sin x)^{-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{3}{2}} \times \left(\frac{1+\sin x}{1+\sin x}\right)^{\frac{3}{2}}$$

(Multiplying and dividing by $(1+\sin x)^{\frac{3}{2}}$)

$$= \left[\cos x \times (1+\sin x)^{\frac{3}{2}-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{3}{2}} \times \left(\frac{1}{1+\sin x}\right)^{\frac{3}{2}}$$

$$= \left[\cos x \times (1+\sin x)^{\frac{3}{2}-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{3}{2}} \times (1+\sin x)^{-\frac{3}{2}}$$

$$= [\cos x \times (1+\sin x)^1] \times (1-\sin^2 x)^{-\frac{3}{2}}$$

$$= [\cos x \times (1+\sin x)^1] \times (\cos^2 x)^{-\frac{3}{2}}$$

$$= [\cos x \times (1+\sin x)^1] \times (\cos x)^{-3}$$

$$= [(1+\sin x)^1] \times (\cos x)^{-3+1}$$

$$= \frac{1+\sin x}{\cos^2 x}$$

$$=\frac{1}{\cos^1 x} \times \frac{1+\sin x}{\cos^1 x}$$

$$=\sec x \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$$

$$=\sec x \,(\sec x + \tan x)$$

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