Short Answer Type Questions – II

[3 marks]

Que 1. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

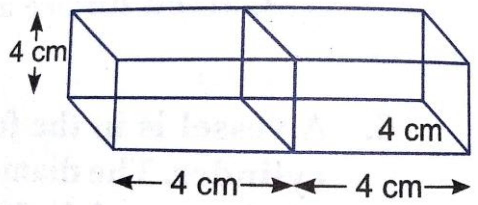

Fig. 13.9

Sol. Let the length of each edge of the cube of volume 64 cm^3 be x cm.

Then, Volume = 64 cm^3

$\Rightarrow \quad x^3 = 64$

$\Rightarrow \quad x^3 = 4^3 \quad\quad \Rightarrow \quad x = 4\ cm$

The dimensions of cuboid so formed are

l = Length = (4 + 4) cm = 8 cm

B = Breadth = 4 cm and h = Height = 4 cm

$\therefore$ Surface area of the cuboid = 2 (lb + bh + lh)

= 2 (8 × 4 + 4 × 4 + 8 × 4)

= 2 (32 + 16 + 32)

= 160 Cm^2

Que 2. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Sol. The greatest diameter that a hemisphere can have = 7 cm = l

Radius of the hemisphere (R) = $\frac{7}{2}$ cm

$\therefore$ Surface area of the solid after surmounted hemisphere

$$= 6l^2 - \pi R^2 + 2\pi R^2 = 6l^2 + \pi R^2$$

$$= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 6 \times 49 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 294 + 38.5$$

$$= 332.5\ cm^2$$

Que 3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

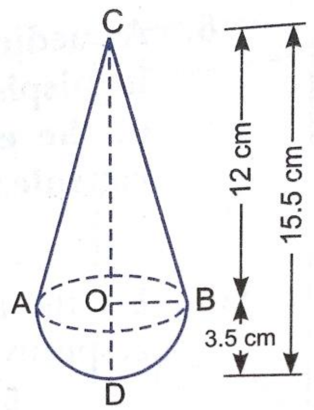

Fig. 13.10

Sol. We have,

Cd = 15.5 cm and OB = OD = 3.5 cm

Let r be the radius of the base of cone and h be the height of conical part of the toy.

Then, r = OB = 3.5 cm

h = OC = CD – OD = (15.5 – 3.5) cm = 12 cm

$$l = \sqrt{r^2 + h^2} = \sqrt{3.5^2 + 12^2}$$

$$= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5\ cm$$

Also, radius of the hemisphere, r = 3.5 cm

∴ Total surface area of the toy

= Surface area of cone + Surface area of hemisphere

$$= \pi rl + 2\pi r^2 = \pi r(l + 2r) = \frac{22}{7} \times 3.5(12.5 + 2 \times 3.5)$$

$$= \frac{22}{7} \times 3.5 \times 19.5 = 214.5\ cm^2$$

Que 4. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Sol. Here, we have

Edge of the cube = l = Diameter of the hemisphere

Therefore, radius of the hemisphere = $\frac{l}{2}$

∴ Surface area of the remaining solid after cutting out the hemispherical

$$\text{Depression} = 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$$

$$= 6l^2 + \pi \times \frac{l^2}{4} = \frac{l^2}{4}(24 + \pi)$$

Que 5. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m². (Note that the base of the tent will not be covered with canvas).

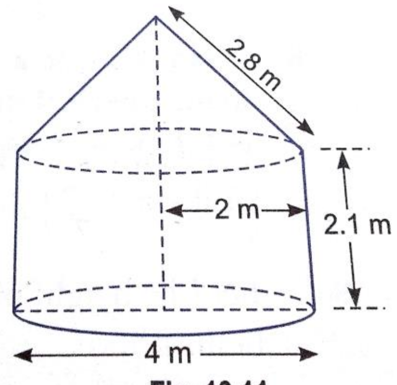

Fig. 13.11

Sol. We have,

Radius of cylindrical base = $\frac{4}{2} = 2m$

Height of cylindrical portion = 2.1 m

∴ Curved surface area of cylindrical portion = 2πrh

$$= 2 \times \frac{22}{7} \times 2 \times 2.1 = 26.4\ m^2$$

Radius of conical base = 2 m

Slant height of conical portion = 2.8 m

∴ Curved surface area of conical portion = πrl

$$= \frac{22}{7} \times 2 \times 2.8 = 17.6\ m^2$$

Now, total area of the canvas = (26.4 + 17.6) m^2 = 44 m^2

∴ Total cost of the canvas used = ₹ 500 × 44 = ₹ 22,000

Que 6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (Fig. 13.12). The length of the entire capsule is 5 mm. Find its surface area.

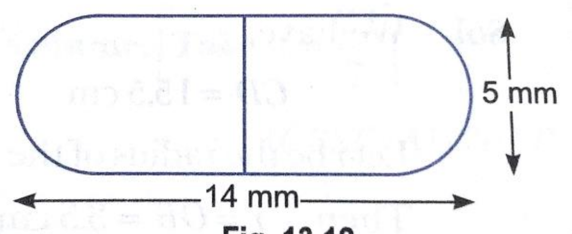

Fig. 13.12

Sol. Let the radius and height of the cylinder be r cm and h cm respectively. Then,

$r = \frac{5}{2} mm \; = 2.5 \, mm$

And $h = \left(14 - 2 \times \frac{5}{2}\right) \, mm = 9 \, mm$

Also, radius of hemisphere r = $\frac{5}{2} mm$

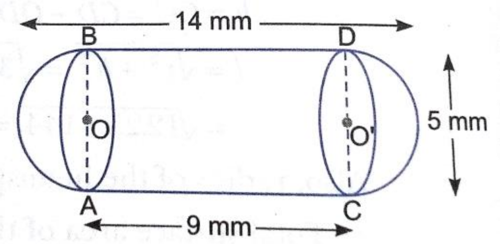

Fig. 13.13

Now, surface area of the capsule

= Curved surface of cylinder + Surface area of two hemispheres

= $2\pi rh + 2 \times 2\pi r^2 = 2\pi r \, (h + 2r)$

= $2 \times \frac{22}{7} \times \frac{5}{2} \times \left(9 + 2 \times \frac{5}{2}\right) \; = 2 \times \frac{22}{7} \times \frac{5}{2} \times 14 \; = 220 \, mm^2$

Que 7. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 13.14. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

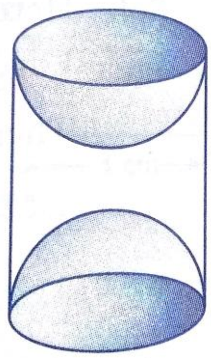
Fig. 13.14

Sol. We have, r = 3.5 cm and h = 10 cm

Total surface area of the article

= Curved surface area of cylinder + 2 × Curved surface area of hemisphere

= $2\pi rh + 2 \times 2\pi r^2 = 2\pi r \, (h \; + \; 2r)$

= $2 \times \frac{22}{7} \times 3.5 \times (10 + 2 \times 3.5) \; = 2 \times \frac{22}{7} \times 3.5 \times 17 \; = 374 \, cm^2$

Que 8. Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end (Fig. 13.15). The height of the cylinder is 1.45 m

and its radius is 30 cm. Find the total surface area of the bird-bath. $\left(\text{Take} \pi = \frac{22}{7}\right)$

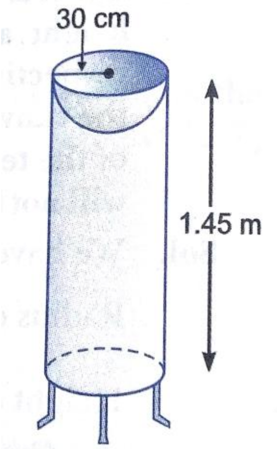

Fig. 13.15

Sol. Let h be height of the cylinder, and r be the common radius of the cylinder and hemisphere.
Then, the total surface area of the bird-bath

= Curved surface area of cylinder + Curved surface area of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 30\ (145 + 30)\ cm^2 = 33{,}000\ cm^2 = 3.3\ m^2$

Que 9. A juice seller was serving his customers using glasses as shown in Fig. 13.16. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14).

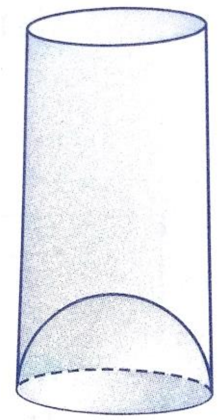

Fig. 13.16

Sol. Since, the inner diameter of the glass = 5 cm and height = 10 cm.

the apparent capacity of the glass = $\pi r^2 h$

$= (3.14 \times 2.5 \times 2.5 \times 10)\ cm^2 = 196.25\ cm^3$

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

i.e., it is less by $\frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5 \text{ cm}^3$

$= 32.71\ cm^3$

So, the actual capacity of the glass

= Apparent capacity of glass – Volume of the hemisphere

= (196.25 – 32.71 cm^3 = 163.54 cm^3

Que 10. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

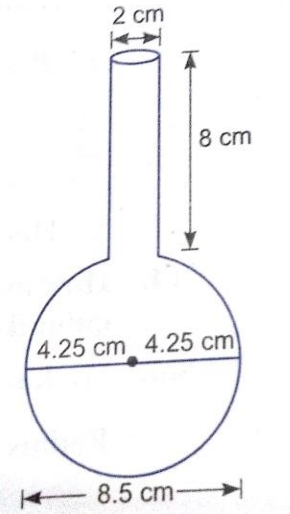

Fig. 13.17

Sol. We have,

Radius of cylindrical neck = 1 cm and height of cylindrical neck = 8 cm

Radius of spherical part = 4.25 cm

Now, Volume of spherical vessel = $\pi r^2 h + \frac{4}{3}\pi r^3$

$= \pi(1)^2 \times 8 + \frac{4}{3} \times \pi \times (4.25)^3$

$= 3.14 \times \left[8 + \frac{4}{3} \times (4.25)^3\right]$

$= 3.14 \times [8 + 102.354]$

$= 3.14 \times 110.354 = 346.514\ cm^3$

$\therefore$ The answer found by the child is incorrect.

Hence, the correct answer is 346.51 cm^3.

Que 11. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Sol. We have,

Radius of sphere = 4.2 cm, Radius of cylinder = 6 cm

Let h cm be the height of cylinder.

Now, since sphere is melted and recast into cylinder

∴ Volume of sphere = Volume of cylinder

i.e., $\frac{4}{3}\pi {r_1}^3 = \pi {r_2}^2 h$ $\Rightarrow \frac{4}{3} \times \pi \times (4.2)^3 = \pi \times (6)^2 \times h$

$\Rightarrow h = \frac{\frac{4}{3}\times\pi\times(4.2)^3}{\pi\times(6)^2} = \frac{\frac{4}{3}\times4.2\times4.2\times4.2}{36}$ $\Rightarrow h = 2.744\ cm$

Hence, height of the cylinder is 2.744 cm.

Que 12. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol. Let r be the radius of resulting sphere.

We have,

Volume of resulting sphere = Sum of the volumes of three gives spheres

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (6)^3 + \frac{4}{3}\pi \times (8)^3 + \frac{4}{3}\pi \times (10)^3$

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{4}{3}\pi[(16)^3 + (8)^3 + (10)^3]$ $\Rightarrow r^3 = (6)^3 + (8)^3 + (10)^3$

$\Rightarrow r^3 = 216 + 512 + 1000$ $\Rightarrow r^3 = 1728 = (12)^3$

∴ $r = 12\ cm$

Hence, the radius of the resulting sphere is 12 cm.

Que 13. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Sol. Here, radius of cylindrical well = $\frac{7}{2}$ m

Depth of cylindrical well = 20 m

Let H metre be the required height of the platform.

Now, the volume of the platform = Volume of the earth dugout from the cylindrical well.

$i.e., 22 \times 14 \times H = \pi r^2 h = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 20 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 770\ m^3$

$\Rightarrow H = \frac{770}{22\times14} = \frac{5}{2} = 2.5\ m$

∴ Height of the platform = 2.5 m

Que 14. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimension 5.5 cm × 3.5 cm?

Sol. We have,

Radius of coin = $\frac{1.75}{2} = 0.875\ cm$

And thickness i.e., height = 2 mm = $\frac{2}{10}$ cm = 0.2 cm

The shape of a coin will be like the shape of cylinder

$\therefore$ Volume of the coin = $\pi r^2 h = \frac{22}{7} \times 0.875 \times 0.875 \times 0.2$

Now, Volume of the cuboid = 5.5 × 10 × 3.5

$\therefore$ Number of coins required to form a cuboid = $\frac{Volume\ of\ the\ cuboid}{Volume\ of\ the\ coin}$

$$= \frac{5.5 \times 10 \times 3.5}{\frac{22}{7} \times 0.875 \times 0.875 \times 0.2} = 400$$

Que 15. A copper rod of a diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

Sol. The volume of the rod = $\pi \times \left(\frac{1}{2}\right)^2 \times\ 8cm^3\ = 2\pi cm^3$

The length of the new wire of the same volume = 18 m = 1800 cm

If r is the radius (in cm) of cross-section of the wire, its volume = $\pi \times r^2 \times 1800\ cm^3$

Therefore, $\pi \times r^2 \times 1800\ = 2\pi$

$i.e., r^2 = \frac{1}{900}$ $\qquad$ $i.e., r^2 = \frac{1}{30}$

So, the diameter of the cross section, i.e., the thickness of the wire is $\frac{1}{15}$ cm.

i.e., 0.67 mm (approx.).

Que 16. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameter of its to circular ends are 4 cm and 2 cm. Find the capacity of the glass.

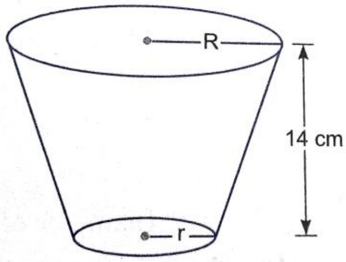

Fig. 13.18

Sol. We have, R = 2 cm, r = 1 cm, h = 14 cm

∴ Capacity of the glass = Volume of the frustum

$$= \frac{1}{2}\pi h(R^2 + r^2 + R)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14 \times [(2)^2 + (1)^2 + (2 \times 1)]$$

$$= \frac{44}{3} \times (4 + 1 + 2) = \frac{44}{3} \times 7 = \frac{308}{3} = 102\frac{2}{3}\ cm^3$$