18 PRACTICE PAPER

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: As given in Practice Paper – 1.

Section–A

Choose the correct option:

1. **Let** $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ **then** A^4 **is equal to**

(*a*) $\begin{bmatrix} 4a & 0 & 0 \\ 0 & 4a & 0 \\ 0 & 0 & 4a \end{bmatrix}$ (*b*) $\begin{bmatrix} a^4 & 0 & 0 \\ 0 & a^4 & 0 \\ 0 & 0 & a^4 \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 0 & a^4 \\ 0 & a^4 & 0 \\ a^4 & 0 & 0 \end{bmatrix}$ (*d*) $\begin{bmatrix} a^4 & a^4 & a^4 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

2. **The value of the determinant** $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$ **is**

(*a*) $x^3 - x^2 + 2$ (*b*) $x^3 - x^2 - 2$ (*c*) $x^3 + x^2 - 2$ (*d*) None of these

3. **The solution of system of equations** $x + (\sin\alpha)y = 1$ **and** $(\sin\alpha)x + 4y = 2$ **satisfying** $x \geq \frac{4}{5}$ **and** $y \leq \frac{1}{2}$, **then**

(*a*) $\alpha \in \left[\frac{\pi}{4}, \frac{\pi}{3}\right]$ (*b*) $\alpha \in \left[0, \frac{\pi}{6}\right]$ (*c*) $\alpha \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (*d*) None of these

4. **If** $y = x^x$ **then** $\frac{d^2y}{dx^2}$ **is**

(*a*) $x^x\left\{(1 + \log x)^2 - \frac{1}{x}\right\}$ (*b*) $x^x\left\{(1 + \log x)^2 + \frac{1}{x}\right\}$ (*c*) 0 (*d*) $x^x\left\{(1 - \log x)^2 + \frac{1}{x}\right\}$

5. **If** x **is real, then the minimum value of** $x^2 - 8x + 17$ **is**

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 2

6. ***Read the following statements.***

Statement I : The value of $\int_2^8 \frac{\sqrt{10 - x}}{\sqrt{x} + \sqrt{10 - x}} dx$ is equal to 6.

Statement II : Let f be a continuous function on the closed interval $[a, b]$ and let $A(x)$ be the area function. Then $A'(x) = f(x)\ \forall x \in [a, b]$.

Choose the correct option:

(*a*) Statement I is correct but statement II is not correct.

(*b*) Statement II is correct but statement I is not correct.

(*c*) Both statements I and II are correct.

(*d*) None of these

7. $\int_{0}^{1.5}[x^2]dx$, where [ ] denotes the greatest integer function, is equal to

(a) $2+\sqrt{2}$ (b) $\sqrt{2}-2$ (c) $2-\sqrt{2}$ (d) $\sqrt{2}-3$

8. The value of $\int \frac{dx}{x^2+2x+2}$ equals

(a) $\tan^{-1}(x+1)+C$ (b) $\cot^{-1}(x+1)+C$ (c) $\tan^{-1}(x+2)+C$ (d) $\cot^{-1}x+C$

9. The value of $\int \frac{dx}{\sqrt{5x^2-2x}}$ is

(a) $\frac{1}{5}\log\left|x-\frac{1}{5}+\sqrt{x^2-\frac{2x}{5}}\right|+C$ (b) $\frac{1}{\sqrt{5}}\log\left|\left(x-\frac{1}{5}\right)+\sqrt{x^2-\frac{2x}{5}}\right|+C$

(c) $\frac{1}{2\sqrt{5}}\log\left|\left(x+\frac{1}{5}\right)+\sqrt{x^2-\frac{x}{5}}\right|+C$ (d) None of these

10. The area of the region included between $y^2=9x$ and $y=x$ is

(a) 27 sq. units (b) $\frac{27}{2}$ sq. units (c) 20 sq. units (d) 8 sq. units

11. Solution of the equation $x^2y-x^3\frac{dy}{dx}=y^4\cos x$, when $y(0)=1$ is

(a) $y^3=3x^3\sin x$ (b) $x^3=3y^3\sin x$ (c) $x^3=y^3\sin x$ (d) None of these

12. Order and degree of differential equation $\frac{d^2y}{dx^2}=\left[y+\left(\frac{dy}{dx}\right)^2\right]^{1/4}$ are

(a) 4 and 2 (b) 1 and 2 (c) 1 and 4 (d) 2 and 4

13. The objective function $Z=4x+3y$ can be maximised subject to constraints $3x+4y\le 24, 8x+6y\le 48, x\le 5, y\le 6, x, y\ge 0$

(a) at only one point (b) at two points only

(c) at an infinite number of points (d) none of these

14. A discrete random variable X has probability distribution given below

X	0.5	1	1.5	2
$P(X)$	k	k^2	$2k^2$	k

then the value of k is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

15. A die is thrown $2n$ times, the probability that the number greater than 4 appears at least once in $2n$ throws is

(a) $\left(\frac{1}{3}\right)^{2n}$ (b) $1-\left(\frac{1}{3}\right)^{2n}$ (c) $\frac{3^{2n}-2^{2n}}{3^{2n}}$ (d) none of these

Section–B(B1)

16. Let f, g be the functions $f=\{(1,5),(2,6),(3,4)\}$, $g=\{(4,7),(5,8),(6,9)\}$ then gof is equal to

(a) $\{(1,5),(2,9),(3,7)\}$ (b) $\{(1,8),(2,9),(3,7)\}$

(c) $\{(3,7)\}$ (d) $\{(1,8),(2,9)\}$

17. Let $*$ be binary operation on $\mathbb{R}$ defined by $a*b=a+b-\sqrt{2}$ then the value of $(\sqrt{3}*\sqrt{2})$ is

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt{3}$ (d) 3

18. Let $f:\mathbb{N}\longrightarrow Y$ be a function defined as $f(x)=4x+3$ where $Y=\{y\in\mathbb{N} \mid y=4x+3, \text{ for some } x\in\mathbb{N}\}$ then its inverse is

(a) $g(y)=\frac{y-3}{4}$ (b) $g(y)=\frac{3y+4}{3}$ (c) $g(y)=4+\frac{y+3}{4}$ (d) $g(y)=\frac{y+3}{4}$

19. **Let A and B be finite sets containing m and n elements respectively. The number of relations that can be defined from A to B is**

(a) 2^{mn} (b) 2^{m+n} (c) mn (d) 0

20. **If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is**

(a) Reflexive (b) Transitive (c) Symmetric (d) None of these

21. **The number of real solution of the equation $\tan^{-1}\sqrt{x^2 - 3x + 2} + \cos^{-1}\sqrt{4x - x^2 - 3} = \pi$ is**

(a) one (b) two (c) zero (d) infinite

22. **If $\alpha = \tan^{-1}\left\{\tan\left(\frac{5\pi}{4}\right)\right\}$ and $\beta = \tan^{-1}\left\{-\tan\left(\frac{2\pi}{3}\right)\right\}$ then**

(a) $4\alpha = 3\beta$ (b) $3\alpha = 4\beta$ (c) $\alpha = \beta$ (d) none of these

23. **Domain of $\cos^{-1}[x]$ is**

(a) $[-2, 1]$ (b) $(-1, 1)$ (c) $[-1, 2)$ (d) None of these

24. **If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{\pi}{2}$, then the value of $x^2 + y^2 + z^2 + 2xyz$ equals**

(a) 2 (b) 0 (c) -1 (d) 1

25. **If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\frac{x}{\pi} \\ \sin^{-1}\frac{x}{\pi} & \cot^{-1}\pi x \end{bmatrix}$, $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\frac{x}{\pi} \\ \sin^{-1}\frac{x}{\pi} & -\tan^{-1}\pi x \end{bmatrix}$ then $A - B$ is equal to**

(a) 1 (b) 0 (c) $2I$ (d) $\frac{1}{2}I$

26. **If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then A^TA is**

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) None of these

27. **The value of the determinant $\begin{vmatrix} \text{cosec}^2\theta & \cot^2\theta & 1 \\ \cot^2\theta & \text{cosec}^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix}$ is**

(a) 4 (b) -4 (c) 2 (d) 0

28. **Let $a, b, c \in \mathbb{R}^+$ then the following system of equation in x, y, z given by**
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1$ and $\frac{-x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

(a) No solution (b) Unique solution

(c) Infinitely many solution (d) Finitely many solutions

29. **The value of p and q for which the function**

$$f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$$ **is continuous for all $x \in R$, are**

(a) $p = \frac{1}{2}, q = \frac{3}{2}$ (b) $p = \frac{5}{2}, q = \frac{7}{2}$

(c) $p = -\frac{3}{2}, q = \frac{1}{2}$ (d) none of these

30. **Let $f(x) = x^{3/2} - \sqrt{x^3 + x^2}$ then**

(*a*) LHD at $x = 0$ exists but RHD at $x = 0$ does not exists

(*b*) $f(x)$ is differentiable at $x = 0$

(*c*) RHD at $x = 0$ exists but LHD at $x = 0$ does not exists

(*d*) None of these

31. **Number of points at which $f(x) = \dfrac{1}{\log|x|}$ is discontinuous is**

(*a*) 2 (*b*) 3 (*c*) 1 (*d*) 4

32. **If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, $a > 0$ and $-1 < t < 1$, then $\dfrac{dy}{dx}$ is**

(*a*) $\dfrac{y}{x}$ (*b*) $\dfrac{x}{y}$ (*c*) $\dfrac{-y}{x}$ (*d*) None of these

33. **The point at which the normal to the curve $y = 2x^2 - 2x + 7$ has a slope $\dfrac{1}{6}$ is**

(*a*) $(-1, -11)$ (*b*) $(1, -11)$ (*c*) $(-1, 11)$ (*d*) $(-1, -9)$

34. ***Read the following statements.***

Statement I : The value of $\int \dfrac{x^9}{(4x^2+1)^6} dx$ is equal to $\dfrac{1}{10}\left(4 + \dfrac{1}{x^2}\right)^{-5} + C.$

Statement II : $\int \dfrac{dx}{x\sqrt{x^4 - 1}}$ is equal to $\sec^{-1}(x^2) + C.$

Choose the correct option:

(*a*) Statement I is correct but statement II is not correct.

(*b*) Statement II is correct but statement I is not correct.

(*c*) Both statements I and II are correct.

(*d*) None of these

35. ***Read the following statements.***

Statement I : $\int f'(x)dx = f(x) + C$

Statement II : $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$

Choose the correct option:

(*a*) Statement I is correct but statement II is not correct.

(*b*) Statement II is correct but statement I is not correct.

(*c*) Both statements I and II are correct.

(*d*) None of these

36. **The value of $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+\pi)]\,dx$ is**

(*a*) $\dfrac{\pi}{2}$ (*b*) $\dfrac{\pi^4}{32}$ (*c*) $\dfrac{\pi^4}{8}$ (*d*) $-\dfrac{\pi}{2}$

37. **The area of the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$ is**

(*a*) $\dfrac{9}{2}$ sq. units (*b*) 4 sq. units (*c*) 2 sq. units (*d*) None of these

38. **The general solution of the differential equation $x(1 + y^2)dx + y(1 + x^2)dy = 0$ is**

(*a*) $(1 + x^2)(1 + y^2) = 0$ (*b*) $(1 + x^2)(1 + y^2) = C$ (*c*) $(1 + x^2) = C(1 + y^2)$ (*d*) $(1 + y^2) = C(1 + x^2)$

39. **$y = e^{-x}(A\cos x + B\sin x)$, where A and B are arbitrary constants is solution of**

(*a*) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = 0$ (*b*) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$ (*c*) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = 0$ (*d*) $\dfrac{d^2y}{dx^2} + 2y = 0$

40. The vector $\vec{a}+\vec{b}$ bisects the angle between non-collinear vectors $\vec{a}$ and $\vec{b}$, if

(a) $\vec{a}$ and $\vec{b}$ are equal vectors (b) $\vec{a}$ and $\vec{b}$ are unequal vectors

(c) $\vec{a}$ and $\vec{b}$ are orthogonal (d) none of these

41. The value of k, for which $|k\vec{a}| < |\vec{a}|$ and $k\vec{a}+\frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true if

(a) $k \neq \frac{1}{4}$ (b) $k \in]-1, 1[\; k \neq \frac{-1}{2}$ (c) 0 (d) none of these

42. The value of the expression $|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2$ is

(a) $\vec{a}\cdot\vec{b}$ (b) $|\vec{a}|.|\vec{b}|$ (c) $|\vec{a}|^2|\vec{b}|^2$ (d) $(\vec{a}\cdot\vec{b})$

43. If $\vec{a}$ is any non-zero vector, then $(\vec{a}\cdot\hat{i})\hat{i}+(\vec{a}\cdot\hat{j})\hat{j}+(\vec{a}\cdot\hat{k})\hat{k}$ is equal to

(a) $\vec{a}\cdot\vec{b}$ (b) $\vec{a}$ (c) 0 (d) none of these

44. The distance of the point A (4, 3, 2) from the line $\frac{x}{2}=\frac{y-2}{6}=\frac{z+3}{3}$ measured parallel to the plane $2x+2y+3z-5=0$ is

(a) $\sqrt{35}$ (b) $\sqrt{33}$ (c) $\sqrt{34}$ (d) None of these

45. The equation of plane through the line of intersection of the planes $x+2y-z=3$ and $-3x+5y+4z+9=0$ and parallel to the line $\frac{x-3}{4}=\frac{y-1}{2}=\frac{z-5}{5}$ is

(a) $9x+7y+10z-27=0$ (b) $9x+7y-10z-27=0$

(c) $9x-7y-10z-27=0$ (d) $9x+7y-10z+27=0$

46. The value of λ so that the lines $\frac{1-x}{3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x+1}{3\lambda}=\frac{y-1}{1}=\frac{6-z}{7}$ are perpendicular to each other is

(a) -2 (b) -3 (c) 2 (d) None of these

47. The x coordinate of point, which divides the line joining the points (2, 3, 4) and (3, −4, 7) in the ratio $\lambda : 1$ is $\frac{21}{8}$, the value of λ equals

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{5}{3}$ (d) None of these

48. Two aeroplane I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by II plane is

(a) 0.2 (b) 0.7 (c) 0.06 (d) 0.14

49. The mean and variance of a binomial distribution are 4 and 2 respectively. The probability of two success is

(a) $\frac{128}{256}$ (b) $\frac{219}{256}$ (c) $\frac{37}{256}$ (d) $\frac{28}{256}$

50. A coin is tossed n times, the probability that head will turn up on even number of times is

(a) $\frac{n+1}{2n}$ (b) $\frac{n}{n+1}$ (c) $\frac{1}{2}$ (d) 2^{n-1}