CBSE Test Paper 05

Chapter 14 Oscillations

1. The displacement of a particle executing S.H.M is given by y=0.25 sin 200 t cm. The maximum speed of the particle is
   a. 200 cm/sec
   b. 0.25 cm/sec
   c. 100 cm/sec
   d. 50 cm/sec
2. For a particle executing simple harmonic motion represented by **x (t)= A cos** $(\omega t + \varphi)$, then the acceleration **a(t)** is given by
   a. a(t) = - $\omega^2 v(t)$
   b. a(t) = ω^2 x(t)
   c. a(t) = - 2 ω^2 x(t)
   d. a(t) = - ω^2 x(t)
3. The damped natural frequency of a Damped system is
   a. same as natural frequency
   b. none of these
   c. higher than natural frequency
   d. lower than natural frequency
4. Two pendulums of length I meter and 16 meters start vibrating one behind the other from the same stand. At some instant the two are in the mean position in the same phase. The time period of shorter pendulum is T. The minimum time after which the two threads of the pendulums will be one behind the other is
   a. T/4
   b. 4T/3
   c. T/3
   d. 2T/5
5. A particle executes S.H.M along a straight line with amplitude ‘A’. The potential energy is maximum when the displacement is

a. $\pm$ A
b. Zero
c. $\pm$ A$\sqrt{2}$
d. $\pm$ A/2

6. What is the phase relationship between displacement, velocity and acceleration in SHM?

7. Give some practical examples of S.H.M?

8. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?

9. A mass of 2 kg is suspended from a vertical spring. An additional force of 2.5 N stretched it by 1 cm.

   i. Calculate the force constant.
   ii. Calculate the frequency of oscillations if the spring is stretched by the given force and then released.

10. A point describes S.H.M. in a line 6 cm long. Its velocity, when passing through the centre of line is 18 cms. Find the time period.

11. A particle is executing SHM of amplitude A. At what displacement from the mean position does the energy become half kinetic and half opotential energy?

12. A 5 kg collar is attached to a spring of spring constant 500 Nm^{-1}. It slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 10.0 cm and released. Calculate

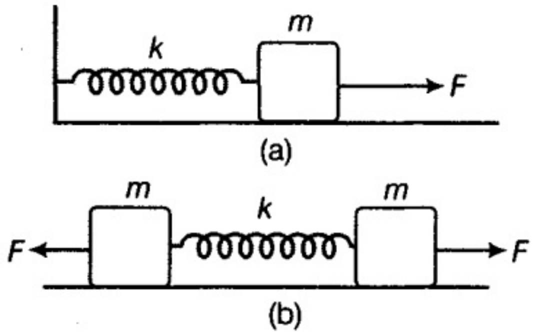
    i. the period of oscillation,
    ii. the maximum speed and
    iii. maximum acceleration of the collar.

13. Two particles execute SHM of the same amplitude and frequency along close parallel lines. They pass each other moving in opposite directions, each time their

displacements is half of their amplitudes. What is their phase difference?

14. A particle is moving with SHM in a straight line. When the distance of the particle from mean position has values x_1 and x_2 the corresponding values of velocities are v_1 and v_2. Show that the time period of oscillation is given by:$T = 2\pi \left[\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}\right]^{1/2}$

15. Fig. (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Fig. (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. (b) is stretched by the same force F



i. What is the maximum extension of the spring in both the cases?
ii. If the mass in Fig. (a) and the two masses in Fig. (b) are released, then what is the period of oscillation in each case?

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Answer

1. d. 50 cm/sec

   Explanation: Given y= 0.25 sin(200t) compare this equation with the standard equation y = A sin wt

   amplitude A = 0.25 cm

   w = 200

   v_{max} = wA

   v_{max} = 200 $\times$ 0.25

   v_{max} = 50 cm/s

2. d. a(t) = - ω^2 x(t)

   Explanation: Displacement of particle is given by

   x (t) = A cos (ωt + φ),

   then,velocity **v(t)**

   $v(t) = \frac{dx}{dt}$

   $v(t) = A\frac{d\cos(\omega t+\varphi)}{dt}$

   $v(t) = -A\,\omega\sin(\omega t + \varphi)$

   Now,acceleration **a(t)**

   $a(t) = \frac{dv}{dt}$

   $a(t) = -\omega A\frac{d\sin(\omega t+\varphi)}{dt}$

   $a(t) = -\omega^2 A\,cos(\omega t + \varphi)\ \ (\because\ x(t) = A\,cos(\omega t + \varphi))$

   $\therefore a(t) = -\omega^2 x(t)\ (-\omega A \to constant)$

3. d. lower than natural frequency

   Explanation: The damped natural frequency f_d is related to the natural frequency f_n by a relation

   $f_d = f_n\sqrt{1-\zeta^2}$

   where damping ratio ζ is <= 1

   so the f_d is less than the natural frequency

4. b. 4T/3

Explanation: Given lenght of 1st pendulum L_1=1 cm and that of 2nd pendulum L_2=16 cm as time period is given by

$T = 2\pi\sqrt{\frac{L}{g}}$ $\times$ period of 1st pendulum T $\times$period of 2nd pendulum will be T_2 = 4T

When two pendulums become again one behind the other the phase difference between them again become 2π.

$$(w_1 t - \ w_2 t) = 2\pi$$
$$\left(\frac{2\pi}{T_1} \ - \frac{2\pi}{T_2}\right) t = 2\pi$$
$$\left(\frac{2\pi}{T} - \frac{2\pi}{4T}\right) t = 2\pi$$
$$t = \frac{4T}{3}$$

5. a. $\pm$ A

Explanation: Potential energy in s.h.m. is given by

$$U = \ \frac{1}{2} kx^2$$

Maximum possible values of x is equal to amplitude $\pm$A. I.e. maximum displacement on both sides. Thus maximum P.E. is when x = $\pm$A.

6. In SHM, -The velocity leads the displacement by a phase 11.2 radians and acceleration leads the velocity by a phase $\frac{\pi}{2}$ radians.

7. Some practical examples of S.H.M. are:-

   i. A steel ball rolling in a curved dish.
   ii. Atoms vibrating in a crystal lattice.
   iii. Motion of helical spring.
   iv. A steel ruler clamped to a bench oscillates when its free end is displaced sideways.

8. Angular frequency of the piston, ω = 200 rad/ min.

Stroke = 1.0 m

Amplitude, $A = \frac{1.0}{2} = 0.5\text{m}$

The maximum speed (v_{max}) of the piston is given by the relation:

$$v_{max} = A\omega$$
$$= 200 \times 0.5 = 100 m / \min$$

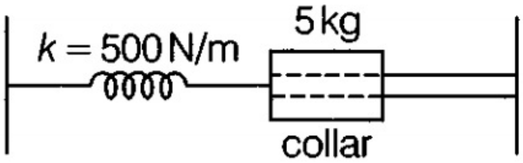
9. Here mass m = 2 kg, force F = 2.5 N and elongation x = 1 cm = 0.01 m.

   i. Force constant k = $\frac{F}{x}$ = $\frac{2.5\mathrm{N}}{0.01\mathrm{m}}$ = 250 N/m
   ii. Frequency of oscillations = $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ = $\frac{1}{2\times3.14}$ $\times$ $\sqrt{\frac{250}{2}}$
   = 1.78 Hz.

10. Here amplitude r = 6/2 = 3cm When y = 0, v = 18 cms^{-1}
    Now $v=\omega\sqrt{r^2-y^2} \Rightarrow 18=\omega\sqrt{3^2-0}$
    Or $3\omega=18 \Rightarrow \omega=6\mathrm{ras}s^{-1}$
    We know $T=\frac{2\pi}{\omega}=\frac{2\pi}{6}=1.047s$

11. According to the above problem, E_k = E_p [E_k and E_p are kinetic and potential energies respectively]
    $\therefore \quad \frac{1}{2}m\omega^2\left(A^2-x^2\right)=\frac{1}{2}m\omega^2x^2$
    $\Rightarrow$ A^2 - x^2 = x^2 or $2x^2$ = A^2
    $\Rightarrow x^2=\frac{A^2}{2}$ or $x=\pm\frac{A}{\sqrt{2}}$
    Thus, the energy will be half kinetic and half potential at a displacement of x = $\frac{A}{\sqrt{2}}$ on either side of the mean position.

12. The figure containing collar of 5 kg attached to a spring of spring constant 500 N /m



   i. Given, mass(m) = 5 kg, spring constant(k) = 500 N/m, amplitude(A) = 10 cm =0.1 m
   The period of oscillation is given by
   T = $\pi\sqrt{\frac{m}{k}}=2\pi\times\sqrt{\frac{5}{500}}$ = 0.628 s
   ii. Maximum speed of the collar, v_{max} = ωA(ω being angular velocity or frequency = $\sqrt{\frac{k}{m}}$)
   v_{max} = $\sqrt{\frac{k}{m}}\cdot A=\sqrt{\frac{500}{5}}\times 0.1$ =1 m/s
   iii. Maximum acceleration of the collar
   a_{max} = $\omega^2 A=\frac{k}{m}\cdot A=\frac{500}{5}\times 0.1$= 10 m/s^2 ($\omega=\sqrt{\frac{k}{m}}$ = angular velocity or

frequency)

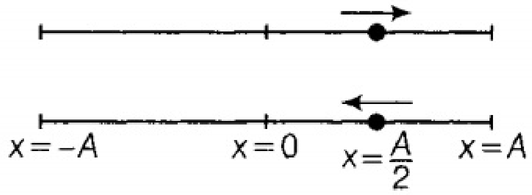
13. In SHM displacement is given by the equation, x = A $\sin(\omega t+\phi)$...(i), A being amplitude of the particle.

Velocity, v = $\frac{dx}{dt}=A\omega\cos(\omega t+\phi)$...(ii)

At t = 0, x = $\frac{A}{2}$, then from Eq. (i), $\frac{A}{2}=A\sin\phi$

or $\sin\phi=\frac{1}{2}=\sin\frac{\pi}{6}$ or $\sin\frac{5\pi}{6}$

$\therefore\phi=\frac{\pi}{6}$ or $\frac{5\pi}{6}$



If $\phi=\frac{\pi}{6}$, displacement and velocity of the particle executing SHM are positive. When $\phi=\frac{5\pi}{6}$, displacement is positive but the velocity is negative. Therefore, displacement-time equations of the two particles will be

$x_1=A\sin\left(\omega t+\frac{\pi}{6}\right)$

and $x_2=A\sin\left(\omega t+\frac{5\pi}{6}\right)$

Phase difference $\Delta\phi=\frac{5\pi}{6}-\frac{\pi}{6}=\frac{4\pi}{6}=\frac{2\pi}{3}$rad

14. If a = amplitude ; y = displacement; ω = angular frequency

v = Velocity, then

$v^2=\omega^2\left(a^2-y^2\right)$

For first case. $u_1^2=\omega^2\left(a^2-x_1^2\right)$ ....(1) (∵ velocity = u_1 ,Displacement = x_1)

For second case, $u_2^2=\omega^2\left(a^2-x_2^2\right)$ ....(2) (velocity = u_2 , Displacement = x_2)

Subtracting equation 2 from equation 1;

$u_1^2-u_2^2=\omega^2\left(a^2-x_1^2\right)-\omega^2\left(a^2-x_2^2\right)$

$u_1^2-u_2^2=\omega^2a^2-\omega^2x_1^2-\omega^2a^2+\omega^2x_2^2$

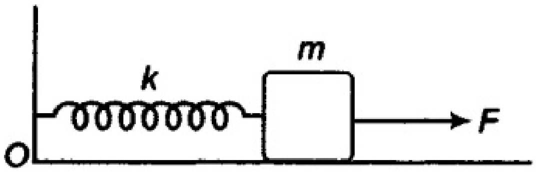
$u_1^2-u_2^2=\omega^2\left(x_2^2-x_1^2\right)$

Now, $\omega^2=\frac{u_1^2-u_2^2}{x_2^2-x_1^2}$

So, $\omega=\left[\frac{u_1^2-u_2^2}{x_2^2-x_1^2}\right]^{1/2}$

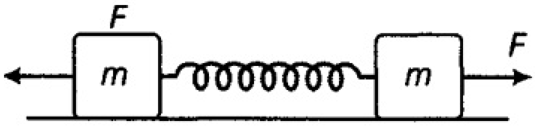
So, Time period $T = \frac{2\pi}{\omega} = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}$

15. i. For Case (a), as we know that the restoring force, F = -kx $\Rightarrow$ |F| = kx



So, x = $\frac{F}{k}$

Case (b)



If x' is the extension in the spring, then drawing free body diagram of either mass (as the system under applied force is under equilibrium).

kx' = F

$\therefore x' = \frac{F}{k}$

In both the cases, extension is the same $\left(\frac{F}{k}\right)$.

ii. The period of oscillation in case(a)

As, restoring force(F) = -kx

where, x = given extension

But from Newton's 2nd law of motion we know that, F = ma

$\therefore ma = -kx \Rightarrow a = -\left(\frac{k}{m}\right)x$.......(i)

$a \propto -x$

On comparing eq.(i) with a = -ω^2x, we get

$\omega = \sqrt{\frac{k}{m}}$ (angular frequency or velocity of the motion)

Period of oscillations, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Case (b)

The system is divided into two similar systems with spring divided in two equal halves, forming spring constant

k' = 2k

Hence, F = - k'x

Putting k' = 2k (on cutting a spring in two halves, its k doubles)

F = -2kx

But from Newton's 2nd law of motion, F = ma

∴ma = -2kx

$\Rightarrow a=-\left(\frac{2k}{m}\right)x$.........(ii)

On comparing Eq.(ii) with a = -ω^2x, we get angular frequency or velocity,

$\omega=\sqrt{\frac{2k}{m}}$

Hence the required period of oscillation of the geven question,

$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{2k}}$