CUET (UG)

Mathematics Sample Paper - 05

Solved

Time Allowed: 50 minutes **Maximum Marks: 200**

General Instructions:

1. There are 50 questions in this paper.
2. Section A has 15 questions. Attempt all of them.
3. Attempt any 25 questions out of 35 from section B.
4. Marking Scheme of the test:

a. Correct answer or the most appropriate answer: Five marks (+5).
b. Any incorrectly marked option will be given minus one mark (-1).
c. Unanswered/Marked for Review will be given zero mark (0).

Section A

1. If A and B are square matrices of the same order then (A + B)(A - B) = ? **[5]**

a) None of these
b) $A^2 - AB + BA - B^2$
c) $(A^2 - B^2)$
d) $A^2 + AB - BA - B^2$

2. Rank of a non-zero matrix is always **[5]**

a) $\geqslant 1$
b) equal to 1
c) greater than 1
d) 0

3. $\begin{vmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{vmatrix} \begin{vmatrix} 3 \\ 4 \\ 5 \end{vmatrix} + 2 \begin{vmatrix} 4 \\ 5 \end{vmatrix}$ is equal to **[5]**

a) $\begin{vmatrix} 45 \\ 44 \end{vmatrix}$
b) $\begin{vmatrix} 44 \\ 43 \end{vmatrix}$
c) $\begin{vmatrix} 43 \\ 50 \end{vmatrix}$
d) $\begin{vmatrix} 43 \\ 45 \end{vmatrix}$

4. The interval of increase of the function $f(x) = x - e^x + \tan(\frac{2\pi}{7})$ is **[5]**

a) $(0, \infty)$
b) $(-\infty , 0)$
c) $(-\infty , 1)$
d) $(1,\infty)$

5. If the function $f(x) = 2\tan x + (2a + 1)\log_e |\sec x| + (a - 2)x$ is increasing on R, then **[5]**

a) $a = \frac{1}{2}$

b) $a \in R$

c) $a \in (\frac{1}{2}, \infty)$

d) $a \in (-\frac{1}{2}, \frac{1}{2})$

6. Slope of tangent to the curve $y = x^2 - x$ at the point where the line $y = 2$ cuts the curve in the first quadrant is **[5]**

a) 3

b) -3

c) -2

d) 2

7. $\int \frac{1}{x^2} e^{-1/x} dx = ?$ **[5]**

a) $-e^{-1/x} + C$

b) $e^{-1/x} + C$

c) None of these

d) $\frac{e^{-1/x}}{x} + C$

8. The derivative of $f(x) = \int_{x^2}^{x^3} \frac{1}{\log_e t} dt$, $(x > 0)$, is **[5]**

a) $\frac{1}{3 \ln x} - \frac{1}{2 \ln x}$

b) $\frac{1}{3 \ln x}$

c) $\frac{3x^2}{\ln x}$

d) $(\ln x)^{-1} \times (x - 1)$

9. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = ?$ **[5]**

a) None of these

b) 2

c) 0

d) -1

10. Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is **[5]**

a) 2

b) $\frac{17}{4}$

c) 3

d) 6

11. The number of arbitrary constants in the particular solution of a differential equation of third order are: **[5]**

a) 1

b) 3

c) 2

d) 0

12. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is **[5]**

a) not defined
b) 1
c) 2
d) 3

13. The comer points of the feasible region determined by a set of constraints (linear inequalities) are P(O, 5), Q(3, 5), R(5, 0) and S(4, 1) and the objective function is Z = ax + 2by where a, b > O. The condition on a and b such that the maximum Z occurs at Q and S is **[5]**

a) a - 2b = 0
b) a - 8b = 0
c) a - 5b = 0
d) a - 3b = 0

14. A couple has 2 children. What is the probability that both are boys, if it is known that one of them is a boy? **[5]**

a) $\frac{1}{3}$
b) $\frac{1}{4}$
c) $\frac{2}{3}$
d) $\frac{3}{4}$

15. Two numbers are selected at random from integers 1 through 9. If the sum is even, what is the probability that both numbers are odd? **[5]**

a) $\frac{5}{8}$
b) $\frac{1}{6}$
c) $\frac{4}{9}$
d) $\frac{2}{3}$

Section B

Attempt any 25 questions

16. Equivalence classes are **[5]**

a) trivial sets
b) mutually disjoint subsets
c) intersecting sets
d) power sets

17. If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$. Then, x = **[5]**

a) $\frac{1}{\sqrt{3}}$
b) $\frac{1}{\sqrt{2}}$
c) 2
d) 1

18. If A and B are two matrices of the order 3 × m and 3 × n, respectively, and m = n, then the order of matrix (5A – 2B) is **[5]**

a) 3 × 3 b) m × n

c) 3 × n d) m × 3

19. If $A = \frac{1}{3}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ satisfies $A^TA = I$, then x + y = **[5]**

a) -3 b) none of these

c) 0 d) 3

20. Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$. Then, the value of 5a + 4b + 3c + 2d + e is equal to **[5]**

a) none of these b) -16

c) 16 d) 0

21. If $\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = 16$, then the value of $\begin{vmatrix} p+q & a+x & a+p \\ q+y & b+y & b+q \\ x+z & c+z & c+r \end{vmatrix}$ is **[5]**

a) 8 b) 16

c) 32 d) 4

22. $\underset{x\to\infty}{Lt}\left(1+\frac{3}{x}\right)^x$ is equal to **[5]**

a) 3 e b) None of these

c) e^3 d) $e^{1/3}$

23. If y = (sin x) $\log^x$ then $\frac{dy}{dx} = ?$ **[5]**

a) $(\sin x)^{\log x} \cdot \left\{\frac{\cot x + \log \sin x}{x}\right\}$ b) $(\log x)(\sin x)^{(\log x - 1)}\cos x$

c) none of these d) $(\sin x)^{\log x} \cdot \left\{\frac{x \cot x \log x + \log \sin x}{x}\right\}$

24. The derivative of $\cos^{-1}\left(2x^2-1\right)$ w.r.t. $\cos^{-1} x$ is **[5]**

a) $1-x^2$

b) 2

c) $\frac{-1}{2\sqrt{1-x^2}}$

d) $\frac{2}{x}$

25. Let f(x) = $\cos^{-1}$(cos x) then f(x) is **[5]**

a) continuous at x = π and not differentiable at x = π

b) continuous at x = -π

c) differentiable at x = 0

d) differentiable at x = π

26. If y = $\tan^{-1}$ (sec x + tan x) then $\frac{dy}{dx} = ?$ **[5]**

a) None of these

b) $\frac{1}{2}$

c) 1

d) $\frac{-1}{2}$

27. Let f(x) = 2 $\sin^3$x - 3 $\sin^2$x + 12 sin x + 5, $0 \leq x \leq \frac{\pi}{2}$. Then f(x) is **[5]**

a) increasing in $[0, \frac{\pi}{2}]$

b) increasing in $[1, \frac{\pi}{4}]$ and decreasing in $[\frac{\pi}{4}, \frac{\pi}{2}]$

c) decreasing in $[0, \frac{\pi}{2}]$

d) increasing in $[0, \frac{\pi}{4}]$ and decreasing in $[\frac{\pi}{4}, \frac{\pi}{2}]$

28. The maximum value of $\left(\frac{\log x}{x}\right)$ is **[5]**

a) 1

b) e

c) $\frac{2}{e}$

d) $\left(\frac{1}{e}\right)$

29. If the function f(x) = x^3 - 9kx^2 + 27x + 30 is increasing on R, then **[5]**

a) 0 < k < 1

b) -1 < k < 1

c) k < - 1 or k > 1

d) -1 < k < 0

30. The function f(x) = x^x decreases on the interval **[5]**

a) (0, e)

b) (0, 1)

c) (1/e,e)

d) $(0, \frac{1}{e})$

31. $\int \frac{\cos 2x}{(\sin x+\cos x)^2} dx$ is equal to **[5]**

a) $\log|\sin x+\cos x|+C$

b) $\frac{-1}{\sin x+\cos x}+C$

c) $\frac{1}{(\sin x+\cos x)^2}$

d) $\log|\sin x-\cos x|+C$

32. $\int \frac{\sin x}{(1-\sin x)} dx=?$ **[5]**

a) x + cos x – sin x + C

b) - log |1 – sin x| + C

c) none of these

d) $-x-\frac{2}{\tan\frac{x}{2}+1}+C$

33. $\int \frac{dx}{\sqrt{2x-x^2}}=?$ **[5]**

a) $\sin^{-1}(x + 1) + C$

b) $\sin^{-1}(x - 1) + C$

c) $\sin^{-1}(x - 2) + C$

d) None of these

34. $\int_0^1 \sqrt{x(1-x)}$ dx equals **[5]**

a) $\frac{\pi}{2}$

b) $\frac{\pi}{16}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{8}$

35. The area of the region bounded by y = | x - 1| and y = 1 is **[5]**

a) 2

b) $\frac{1}{2}$

c) none of these

d) 1

36. Find the general solution of the differential equation $(e^x+e^{-x})\,dy-(e^x-e^{-x})\,dx=0$ **[5]**

a) $y=(e^x+e^{-x})+C$

b) $y=log(e^{2x}+e^{-x})+C$

c) $y=log|(e^x+e^{-x})|+C$

d) $y=log(e^{-2x}+e^{-x})+C$

37. The order of the differential equation of all circles of given radius a is: **[5]**

a) 4

b) 1

c) 2

d) 3

38. To form a differential equation from a given function **[5]**

a) Differentiate the function once and add values to arbitrary constants

b) Differentiate the function successively as many times as the number of arbitrary constants

c) Differentiate the function twice and eliminate the arbitrary constants

d) Differentiate the function once and eliminate the arbitrary constants

39. If $|\vec{a}| = 10$, $\vec{b} = 2$ and $\vec{a} \cdot \vec{b} = 12$, then what is the value of $|\vec{a} \times \vec{b}|$? **[5]**

a) 20

b) 24

c) 16

d) 12

40. If $\vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{b} = (\hat{i} - 3\hat{k})$ and then $|\vec{b} \times 2\vec{a}| =?$ **[5]**

a) $2\sqrt{23}$

b) $5\sqrt{17}$

c) $10\sqrt{3}$

d) $4\sqrt{19}$

41. In a hexagon ABCDEF $A\vec{B} = a, B\vec{C} = \vec{b}$ and $\overrightarrow{CD} = \vec{c}$. Then $\overrightarrow{AE} =$ **[5]**

a) $\vec{a} + 2\vec{b} + 2\vec{c}$

b) $2\vec{a} + \vec{b} + \vec{c}$

c) $\vec{a} + \vec{b} + \vec{c}$

d) $\vec{b} + \vec{c}$

42. The vectors $2\hat{i} + 3\hat{j} - 4\hat{k}$ and $a\hat{i} + b\hat{j} + c\hat{k}$ are perpendicular, if **[5]**

a) a = - 4, b = 4,c = -5

b) a = 2, b = 3, c = - 4

c) a = 4, b = 4, c = 5

d) a = 4, b=4, c = -5

43. If $\vec{a} = (\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{b} = (2\hat{i} + 3\hat{j} - 4\hat{k})$ then $|\vec{a} \times b| =?$ **[5]**

a) $\sqrt{174}$

b) $\sqrt{87}$

c) none of these

d) $\sqrt{93}$

44. The vector equation of the x-axis is given by **[5]**

a) $\vec{r} = \hat{j} + \hat{k}$

b) none of these

c) $\vec{r} = \hat{i}$

d) $\vec{r} = \lambda\hat{i}$

45. The angle θ between the planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by **[5]**

a) $\sin\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

b) $\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

c) $\tan\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

d) $\cot\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

46. Distance between two parallel planes 2x+y+2z=8 and 4x+2y+4z+5=0 is **[5]**

a) $\frac{5}{2}$

b) $\frac{3}{2}$

c) none of these

d) $\frac{7}{2}$

47. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is **[5]**

a) 4

b) 1

c) 2

d) 5

48. If in a binomial distribution n = 4, $P(X = 0) = \frac{16}{81}$, then P(X = 4) equals **[5]**

a) $\frac{1}{81}$

b) $\frac{1}{8}$

c) $\frac{1}{27}$

d) $\frac{1}{16}$

49. In a class, 60% of the students read mathematics, 25% biology and 15% both mathematics and biology. One student is selected at random. What is the probability that he reads mathematics, if it is known that he reads biology? **[5]**

a) $\frac{3}{8}$

b) $\frac{2}{5}$

c) $\frac{3}{5}$

d) $\frac{5}{8}$

50. Suppose a random variable X follows the binomial distribution with parameters n and p, where $0 < p < 1$. If $\frac{P(x=r)}{P(x=n-r)}$ is independent of n and r, then p equals **[5]**

a) $\frac{1}{7}$

b) $\frac{1}{3}$

c) $\frac{1}{2}$

d) $\frac{1}{5}$

Solutions

Section A

1.

(b) A^2 - AB + BA - B^2

Explanation: Since A and B are square matrices of same order.

$(A + B)(A - B) = A^2 - AB + BA - B^2$

2. **(a)** $\geqslant 1$

Explanation: Rank of a non zero matrix is always greater than or equal to 1.

3.

(c) $\begin{vmatrix} 43 \\ 50 \end{vmatrix}$

Explanation: $\begin{vmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{vmatrix}\begin{vmatrix} 3 \\ 4 \\ 5 \end{vmatrix} + 2\begin{vmatrix} 4 \\ 5 \end{vmatrix} = \begin{vmatrix} 35 \\ 40 \end{vmatrix} + \begin{vmatrix} 8 \\ 10 \end{vmatrix} = \begin{vmatrix} 43 \\ 50 \end{vmatrix}$

4.

(b) (-∞ , 0)

Explanation: $(-\infty, 0)$

f(x) = $x - e^x + \tan\left(\frac{2\pi}{7}\right)$

f(x) = 1- e^x

for f(x) to be increasing, we must have

f(x) > 0

$\Rightarrow 1 - e^x > 0$

$\Rightarrow e^x < 1$

= x < 0

$\Rightarrow x \in (-\infty, 0)$

so, f (x) is increasing on $(-\infty, 0)$

5. **(a)** a = $\frac{1}{2}$

Explanation: a = $\frac{1}{2}$

6. **(a)** 3

Explanation: We have y = x^2 - x $\Rightarrow \frac{dy}{dx}$ = 2x - 1

Slope of tangent m = $\frac{dy}{dx}$ = 2x - 1 ...(i)

Since the line y = 2 cuts the curve y = x^2 - x

⇒ 2 = x^2 - x ⇒ x^2 - x - 2 = 0

⇒ x^2 + x - 2x - 2 = 0 ⇒ x(x + 1) - 2(x + 1) = 0

⇒ (x + 1)(x - 2) = 0

⇒ x = -1 or 2

Point of intersection of the line y = 2 and the curve

y = x^2 - x are (-1, 2), (2, 2)

As point (2, 2) lies in first quadrant

∴ Slope of tangent at (2, 2) from (t) is m = 2 × 2 - 1 = 3

7.

(b) $e^{-1/x} + C$

Explanation: Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \sec^2 x dx = \tan x$

Therefore ,

Put $-\frac{1}{x} = t, \frac{1}{x^2}dx = dt$

$= \int e^t dt$

= e^t+c

$= e^{-\frac{1}{x}} + c$

8.

(d) (In x)$^{-1}$ × (x - 1)

Explanation: (In x)$^{-1}$ × (x - 1)

Using Newton Leibnitz formula

$f'(x) = \frac{1}{\log_e x^3}\left(3x^2\right) - \frac{1}{\log_e x^2}(2x)$

$= -\frac{332}{-315x} - \frac{2x}{24x}$

$= \frac{x^2}{\ln x} - \frac{x}{\ln x}$

$= \frac{1}{\ln x}x(x-1)$

(In x)$^{-1}$ x (x - 1)

9.

(b) 2

Explanation: cos x is an even function so,

$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2\int_{0}^{\frac{\pi}{2}} \cos x dx$

= 2 (1 - 0)

= 2

10.

(b) $\frac{17}{4}$

Explanation: Required area

$\int_{-2}^{1} x^3 dx = \int_{-2}^{0} x^3 dx + \int_{0}^{1} x^3 dx$

$= \left[\frac{x^4}{4}\right]_{-2}^{0} + \left[\frac{x^4}{4}\right]_{0}^{1}$

$= \left[0 - \frac{(-2)^4}{4}\right] + \left[\frac{1}{4} - 0\right]$

$= \frac{16}{4} + \frac{1}{4}$

$= \frac{17}{4}$

11.

(d) 0

Explanation: 0, because the particular solution is free from arbitrary constants.

12. **(a)** not defined

Explanation: In general terms for a polynomial the degree is the highest power. Degree of differential equation is defined as the highest integer power of highest order derivative in the equation

Here the differential equation is $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$

Now for degree to exist the given differential equation must be a polynomial in some differentials.

Here differentials mean $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ or .... $\frac{d^ny}{dx^n}$

The given differential equation is not polynomial because of the term $\sin\frac{dy}{dx}$ and hence degree of such a differential equation is not defined.

13.

(b) a - 8b = 0

Explanation: Given, Max. Z = ax + 2by

Max. value of Z on Q(3, 5) = Max. value of Z on S(4, 1)

$\Rightarrow$ 3a + 10b = 4a + 2b

$\Rightarrow$ a - 8b = 0

14. **(a)** $\frac{1}{3}$

Explanation: Let S = $\{B_1B_2, B_1G_2, G_1B_2, G_1G_2\}$.

Let A = event that both are boys and B = event that one of the two is a boy.

Then, A = $\{B_1B_2\}$, B = $\{B_1B_2, B_1G_2, G_1B_2\}$ and A $\cap$ B = $\{B_1B_2\}$

$P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{1}{3}$.Which is the required solution.

15. **(a)** $\frac{5}{8}$

Explanation: The sum will be even when; both numbers are either even or odd,

i.e. for both numbers to be even, the total cases $^5C_1 \times {}^4C_1$ (Both the numbers are odd)+$^4C_1 \times {}^3C_1$ (Both the numbers are even) = 32

The favourable number of cases will be,

Both odd, i.e. selecting numbers from 1, 3, 5, 7, or 9, i.e.

$^5C_1 \times {}^4C_1 = 20$

Thus, the probability that both numbers are odd will be

$= \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$

$\Rightarrow \frac{20}{32} = \frac{5}{8}$

Section B

16.

(b) mutually disjoint subsets

Explanation: An equivalence relation R gives a partitioning of the set A into mutually disjoint equivalence classes,i.e. union of equivalence classes is the set A itself. Any two equivalence classes i.e. subsets are either equal or disjoint.

17. **(a)** $\frac{1}{\sqrt{3}}$

Explanation: We have to find:

$$3\sin^{-1}\left(\frac{2x}{1+x^2}\right)-4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+2\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$$

Put x = $\tan\theta$

$$3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)-4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)=\frac{\pi}{3}$$

$$3\sin^{-1}(\sin2\theta)-4\cos^{-1}(\cos2\theta)+2\tan^{-1}(\tan2\theta)=\frac{\pi}{3}$$

$$3.2\theta-4.2\theta+2.2\theta=\frac{\pi}{3}\quad\Rightarrow 2\theta=\frac{\pi}{3}\quad\Rightarrow\theta=\frac{\pi}{6}$$

$$\therefore\ \tan^{-1}x=\frac{\pi}{6}\quad\Rightarrow\ x=\tan\left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}}$$

18.

(c) $3\times n$

Explanation: $A_{3\times m}$ and $B_{3\times n}$ are two matrices. If m = n then A and B same orders as $3\times n$ each so the order of (5A - 2B) should be same as $3\times n$.

19.

(b) none of these

Explanation: We have, $A=\frac{1}{3}\begin{bmatrix}1&1&2\\2&1&-2\\x&2&y\end{bmatrix}$

$$\Rightarrow A^T=\frac{1}{3}\begin{bmatrix}1&2&x\\1&1&2\\2&-2&y\end{bmatrix}$$

Now, $A^TA=I$

$$\Rightarrow\begin{bmatrix}x^2+5&2x+3&xy-2\\3+2x&6&2y\\xy-6&2y&y^2+8\end{bmatrix}=\begin{bmatrix}9&0&0\\0&9&0\\0&0&9\end{bmatrix}$$

The corresponding elements of two equal matrices are not equal.
Thus, the matrix A is not orthogonal.

20. **(a)** none of these

Explanation: We have,

$$\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x - x^2 & 0 & 0 \end{vmatrix}$$

[Applying $R_3 \rightarrow R_3 - R_2$]

$$\Rightarrow \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = (x - x^2)(12 - x^2)$$

$= 12x - x^3 - 12x^2 + x^4$

$\therefore$ a = 1, b = -1, c = -12, d = 12 and e = 0

$\therefore$ 5a + 4b + 3c + 2d + e = 5 - 4 - 36 + 24 + 0 = -11

21.

(c) 32

Explanation: $\begin{vmatrix} p+q & a+x & a+p \\ q+y & b+y & b+q \\ x+z & c+z & c+r \end{vmatrix}$

$C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2a+2p+q+x & a+x & a+p \\ 2b+2q+y+b & b+y & b+q \\ 2c+x+2z+r & c+z & c+r \end{vmatrix}$$

$$= 2\begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} + \begin{vmatrix} 2p+q+x & a & a \\ 2q+y+b & b & b \\ x+2z+r & c & c \end{vmatrix}$$

$$= 2\begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} + 0$$

$= 2 \times 16 = 32$

22.

(c) e^3

Explanation: $\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^x = \lim_{t \to 0^+} (1 + 3t)^{\frac{1}{t}} = \lim_{3t \to 0^+} \left[(1 + 3t)^{\frac{1}{3t}}\right]^3 = e^3$

23.

(d) $(\sin x)^{\log x} \cdot \left\{\frac{x\cot x\log x + \log \sin x}{x}\right\}$

Explanation: Given that y = (sinx) $^{\log_e x}$

Taking log both sides, we obtain

$\log_e y = \log_e x \times \log_e \sin x$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we obtain

$$\frac{1}{y}\frac{dy}{dx} = \log_e x \times \frac{1}{\sin x} \times \cos x + \log_e \sin x \times \frac{1}{x}$$

$$= \frac{x\cot x\log_e x + \log_e \sin x}{x}$$

Therefore $\frac{dy}{dx} = \frac{x\cot x\log_e x + \log_e \sin x}{x} \times y$

$$= \frac{x\cot x\log_e x + \log_e \sin x}{x}(\sin x)^{\log_e x}$$

24.

(b) 2

Explanation: let $u = \cos^{-1}\left(2x^2 - 1\right)$ and $v = \cos^{-1}x$

$$\therefore \frac{du}{dx} = \frac{-1}{\sqrt{1 - \left(2x^2 - 1\right)^2}}.4x = \frac{-4x}{\sqrt{1 - \left(4x^4 + 1 - 4x^2\right)}}$$

$$= \frac{-4x}{\sqrt{-4x^4 + 4x^2}} = \frac{-4x}{\sqrt{4x^2\left(1 - x^2\right)}}$$

$$= \frac{-2}{\sqrt{1 - x^2}}$$

and $\frac{dv}{dx} = \frac{-1}{\sqrt{1 - x^2}}$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{-2/\sqrt{1-x^2}}{-1\sqrt{1-x^2}} = 2.$$

Which is the required solution.

25. **(a)** continuous at x = π and not differentiable at x = π

Explanation: continuous at x = π and not differentiable at x = π

26.

(b) $\frac{1}{2}$

Explanation: Given that y = $\tan^{-1}$ (sec x + tan x)

Hence, $y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

Using $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta + \sin^2\theta = 1$

Hence, $y = \tan^{-1}\left(\frac{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}\right)$

$$= \tan^{-1}\left(\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right)$$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we obtain

$$y = \tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$$

Using $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x}$, we obtain

$$y = \tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) = \frac{\pi}{4}+\frac{x}{2}$$

Differentiating with respect to x, we

$$\frac{dy}{dx} = \frac{1}{2}$$

27. **(a)** increasing in $[0, \frac{\pi}{2}]$

Explanation: $f(x) = 2\sin^3 x - 3\sin^2 x + 12\sin x + 5$

$f'(x) = 6\sin^2 x\cos x - 6\sin x\cos x + 12\cos x$

$= 6\cos x(\sin^2 x - \sin x + 2\}$

$= 6\cos x\{\sin^2 x - 2\sin x \times \frac{1}{2} + \frac{1}{4} - \frac{1}{4} + 2\}$

$$= 6\cos x\left\{\left(\sin x - \frac{1}{2}\right)^2 + \frac{7}{4}\right\} \geq 0 \forall x \in \left[0, \frac{\pi}{2}\right]$$

$\therefore$ f(x) is increasing in $[0, \frac{\pi}{2}]$

28.

(d) $\left(\frac{1}{e}\right)$

Explanation: $\Rightarrow f(x) = \frac{\log x}{x}$

$$\therefore f'(x) = \frac{\log x - x\frac{1}{x}}{x^2}$$

$\Rightarrow$ f'(x) = log x - 1

$\Rightarrow$ substitute f'(x) = 0

We get x = e

F''(x) = $\frac{1}{x}$

Substitute x = e in f''(x)

$\frac{1}{e}$ is point of maxima

$\therefore$ The max value is $\frac{1}{e}$

29.

(b) -1 < k < 1

Explanation: -1 < k < 1

30.

(d) $(0, \frac{1}{e})$

Explanation: $(0, \frac{1}{e})$

Let y = x^x

$\Rightarrow \log(y) = x\log x$

$\Rightarrow \frac{1}{y} \times \frac{dy}{dx} = 1 + \log x$

$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$

Since the function is decreasing,

$\Rightarrow x^x x(1 + |\log x) < 0$

$\Rightarrow 1 + \log x < 0$

$\Rightarrow \log x < -1$

$\Rightarrow x < \frac{1}{e}$

Therefore, function is decreasing on $(0, \frac{1}{e})$

31. **(a)** $\log|\sin x + \cos x| + C$

Explanation: Given Integral is: $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$

Let $I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$

$= \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2} dx$

$= \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2} dx$

$= \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx$

Put sin x + cos x = t $\Rightarrow$ (cos x - sin x)dx = dt

$\Rightarrow \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx = \int \frac{dt}{t}$

= log |t| + C

= log |sin x + cos x| + C

32.

(d) $-x - \frac{2}{\tan \frac{x}{2} + 1} + C$

Explanation: Given

$\int \frac{\sin x}{1 - \sin x} dx$

$= -\int dx + \int \frac{dx}{1 - \sin x}$

$= -x + \int \frac{dx}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} - 2\sin \frac{x}{2} \cos \frac{x}{2}}$

$= -x + \int \frac{dx}{\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)^2}$

$= -x + \int \frac{\sec^2 \frac{x}{2} dx}{\left(\tan \frac{x}{2} - 1\right)^2}$

Let, $\tan \frac{x}{2} - 1 = z$

$\Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dz$

So,

$-x + \int \frac{2dz}{z^2}$

$$= -x - \frac{2}{z} + c$$

$$= -x - \frac{2}{\tan\frac{x}{2} + 1} + c$$

Which is the required solution.

33.

(b) $\sin^{-1}(x - 1) + C$

Explanation: The given integral is $\int \frac{dx}{\sqrt{2x - x^2}} = ?$

$$\sqrt{2x - x^2} = \sqrt{1 - \left(1 - 2x + x^2\right)} = \sqrt{1 - (x - 1)^2}$$

$$\therefore \quad I = \int \frac{dx}{\sqrt{1 - (x - 1)^2}} = \int \frac{dt}{\sqrt{1 - t^2}}, \text{ where } (x - 1) = t$$

$= \sin^{-1}t + C = \sin^{-1}(x - 1) + C$

34.

(d) $\frac{\pi}{8}$

Explanation: Let, $I = \int_0^1 \sqrt{x(1 - x)}dx$

$$= \int_0^1 \sqrt{x - x^2}dx$$

$$= \int_0^1 \sqrt{\frac{1}{4} - \left(x^2 - x + \frac{1}{4}\right)}dx$$

$$= \int_0^1 \sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}dx$$

$$= \left[\frac{\left(x - \frac{1}{2}\right)}{2}\sqrt{x - x^2} + \frac{1}{2} \times \frac{1}{4}\sin^{-1}(2x - 1)\right]_0^1$$

$$= \frac{1}{8}\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]_0^1$$

$= \frac{1}{8}\left[\frac{\pi}{2} + \frac{\pi}{2}\right]$

$= \frac{\pi}{8}$

35.

(d) 1

Explanation: Required area : $\left|\int_0^1 [(x-1)-(1-x)]dx\right| = 1$

36.

(c) $y = log|\left(e^x + e^{-x}\right)| + C$

Explanation: $(e^x + e^{-x})dy = (e^x - e^{-x})dx$

$\int dy = \int \frac{(e^x - e^{-x})}{(e^x + e^{-x})}dx$ Since $\int \frac{f'(x)}{f(x)}dx = lnf(x) + c$

$y = \log\left|(e^x + e^{-x})\right| + C$

37.

(c) 2

Explanation: Let the equation of given family be $(x - h)^2 + (y - k)^2 = a^2$. It has two arbitrary constants h and k. Therefore, the order of the given differential equation will be 2.

38.

(b) Differentiate the function successively as many times as the number of arbitrary constants

Explanation: We shall differentiate the function equal to the number of arbitrary constant so that we get equations equal to arbitrary constant and then eliminate them to form a differential equation

39.

(c) 16

Explanation: Given that, $|\vec{a}| = 10, |\vec{b}| = 2$ and

$\vec{a} \cdot \vec{b} = 12$

$\Rightarrow |\vec{a}||\vec{b}| \cos\theta = 12$

$\Rightarrow 10 \times 2 \times \cos\theta = 12$

$\Rightarrow \cos\theta = \frac{3}{5}$

$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

Now, $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta\vec{n}|$

$= |\vec{a}||\vec{b}||\sin\theta||\hat{n}| = 10 \cdot 2 \cdot 1 \cdot |\sin\theta|$

$= 10 \times 2 \times 1 \times |\frac{4}{5}| = 20 \times \frac{4}{5} = 4 \times 4 = 16$

40.

(d) $4\sqrt{19}$

Explanation: $2\vec{a} = (2\hat{i} - 4\hat{j} + 6\hat{k})$ and $\vec{b} = (\hat{i} - 3\hat{k})$

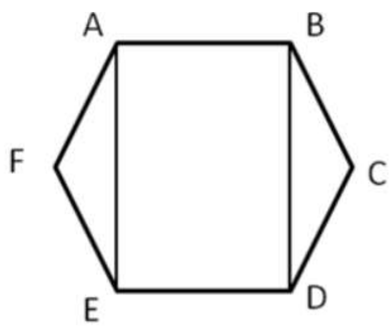
Now, $|\vec{b} \times 2\vec{a}| = \left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 3 \\ 2 & -4 & 6 \end{vmatrix}\right| = |-12\vec{i} - 12\vec{j} - 4\vec{k}|$

$= \sqrt{(144) + (144) + 16} = \sqrt{304}$

$= 4\sqrt{19}$

41.

(d) $\vec{b} + \vec{c}$

Explanation: 

In △BCD,

$\overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$

Given that $\overrightarrow{BC} = \vec{b}$, $\overrightarrow{CD} = \vec{c}$

And $\overrightarrow{BD}$ is parallel to $\overrightarrow{AE}$

$\Rightarrow \overrightarrow{AE} = \vec{b} + \bar{c}$

42.

(c) a = 4, b = 4, c = 5

Explanation: given the vectors $2\hat{i} + 3\hat{j} - 4\hat{k}$ and $a\hat{i} + b\hat{j} + c\hat{k}$ are perpendicular.

$\Longrightarrow$ 2a+3b-4c=0

now, from optional a=4,b=4 and c=5 are satisfies the above condition.

43.

(d) $\sqrt{93}$

Explanation: $\sqrt{93}$

$$(\vec{a} \times \vec{b}) = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 2 & 3 & -4 \end{vmatrix} = \; = (4-6)\hat{i} - (-4-4)\hat{j} + (3+2)\hat{k}$$

$= (-2)i + 8j + 5k$

$|\vec{a} \times \vec{b}| = \sqrt{4+64+25} = \sqrt{93}$

44.

(d) $\vec{r} = \lambda\hat{i}$

Explanation: Vector equation needs a fixed point and a parallel vector

For x -axis we take fixed point as origin.

And parallel vector is $\hat{i}$

Equation would be $\lambda\hat{i}$

45.

(b) $\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

Explanation: By definition , The angle θ between the planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by :

$$\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|.$$

46.

(d) $\frac{7}{2}$

Explanation: Given planes are

2x+2y+2z-8=0

and $2x+y+2z+\frac{5}{2}=0$

Distance between planes $=\frac{|c_1-c_2|}{\sqrt{a^2+b^2+c^2}}$.

$$= \left|\frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right| = \frac{\frac{21}{2}}{3} = \frac{7}{2}$$

47.

(c) 2

Explanation: Let X be the random variable which denote the number obtained on the die. Therefore, X = 1, 2 or 5 Therefore, the probability distribution of X is:

X	**1**	**2**	**5**
P(X)	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$
XP(X)	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$

Therefore, required mean = $\frac{3}{6}+\frac{4}{6}+\frac{5}{6}=2$

48. **(a)** $\frac{1}{81}$

Explanation: In the given binomial distribution, n = 4 and

$P(X=0) = \frac{16}{81}$

Binomial distribution is given by

$P(X=0) = {}^4C_0 p^0 q^{4-0} = q^4$

We know that $P(X=0) = \frac{16}{81}$

$\therefore q^4 = \frac{16}{81}$

$\Rightarrow q = \frac{2}{3}$

$\therefore p = 1-\frac{2}{3} = \frac{1}{3}$

Then, $P(X=4) = {}^4C_4 p^4 q^{4-4}$

$= \left(\frac{1}{3}\right)^4$

$= \frac{1}{81}$

49.

(c) $\frac{3}{5}$

Explanation: Given:

60% of the students read mathematics, 25% biology and 15% both mathematics and biology

That means,
Let the event A implies students reading mathematics,
Let the event B implies students reading biology,
Then, P(A) = 0.6
P(B) = 0.25
P(A ∩ B) = 0.15
We, need to find P(A/B) = P(A ∩ B)/ P(B)

$\Rightarrow \frac{0.15}{0.25} = \frac{3}{5}$

50.

(c) $\frac{1}{2}$

Explanation: $\because P(X = r) = {}^nC_r(p)^r(q)^{n-r}$

$= \frac{n!}{(n-r)!r!}(p)^r(1-p)^{n-r}[\because q = 1-p]\ldots(i)$

$P(X = 0) = (1-p)^n$

And $P(X = n-r) = {}^nC_{n-r}(p)^{n-r}(q)^{n-(n-r)}$

$= \frac{n!}{(n-r)!r!}(p)^{n-r}(1-p)^{-r}[\because q = 1-p]\left[\because {}^nC_r = {}^nC_{n-r}\right]\ldots$(ii)

Now, $\frac{P(x=r)}{P(x=n-r)} = \frac{\frac{n!}{(n-r)!r!}p^r(1-p)^{n-r}}{\frac{n!}{(n-r)!r!}p^{n-r}(1-p)^{+r}}$[using Eqs. (i) and (ii)]

$= \left(\frac{1-p}{p}\right)^{n-\gamma} \times \frac{1}{\left(\frac{1-p}{p}\right)^r}$

Above expression is independent of n and r, if $\frac{1-p}{p} = 1 \Rightarrow \frac{1}{p} = 2 \Rightarrow p = \frac{1}{2}$