CBSE Test Paper 02
Chapter 7 System of Particles & Rotational

1. Two circular rings have their masses in the ratio 1:2 and their diameters in the ratio 2: 1. The ratio of their moments of inertia about their axes is **1**
   a. it is 1 : 2
   b. it is 4 : 1
   c. it is 2 : 1
   d. it is 1: 4
2. Angular acceleration vector is defined as **1**
   a. $\alpha = \frac{d^2\omega}{dt^2}$
   b. $\alpha = \frac{d\omega}{dt}$
   c. $\alpha = 2\frac{d\omega}{dt}$
   d. $\alpha = \frac{d^3\omega}{dt^3}$
3. The velocity of a particle on a body at a position vector r on a body rotating about an arbitrary axis with an angular velocity of ω is given by **1**
   a. $v = r \times \omega$
   b. $\vec{v} = \vec{\omega} \times \vec{r}$
   c. $v = \omega \times r\omega$
   d. $v = \omega \times rr$
4. Three thin uniform rods each of mass M and length L are placed along the three axis of a Cartesian coordinate system with one end of each rod at the origin. The M.I. of the system about z- axis is **1**
   a. $ML^2/3$
   b. ML^2
   c. $(2/3)ML^2$
   d. $ML^2/6$
5. The radius of gyration of a rod of mass 100 gm and length 100 cm about an axis passing through its edge and perpendicular to its length is given by **1**
   a. $\frac{100}{\sqrt{3}}$
   b. $\frac{50}{2\sqrt{3}}$

c. $\frac{50}{3\sqrt{2}}$
d. $\frac{100}{3\sqrt{3}}$

6. Three masses 3 kg, 4 kg and 5 kg are located at the corners of an equilateral triangle of side 1m. Locate the centre of mass of the system. **1**

7. How will you distinguish between a hardboiled egg and a raw egg by spinning it on a table top? **1**

8. A system is in stable equilibrium. What can we say about Its potential energy? **1**

9. An automobile moves on a road with a speed of 54 km h^{-1}. The radius of its wheels is 0.35 m. What is the average negative torque transmitted by its brakes to a wheel if the vehicle is brought to rest in 15s? The moment of inertia of the wheel about the axis of rotation is 3 kg m^2. **2**

10. How is it possible to describe the motion of a big system when Newton's laws of motion are applicable for individual particles of the system? **2**

11. What is the physical significance of the torque? **2**

12. State and briefly explain the laws of rotational motion. **3**

13. A cylinder of length 20 cm and radius 10 cm is rotating about its central axis at an angular speed of 100 rad s^{-1}. What tangential force will stop the cylinder at a uniform rate in 10 s? The moment of inertia of the cylinder about its axis of rotation is 0.8 kg-m^2. **3**

14. How is angular velocity related to linear velocity of a particle of the body in rotational motion ? Does pure rotation take place at uniform angular velocity or at uniform linear velocity? **3**

15. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10π rad s^{-1}. Which of the two will start to roll earlier? The co-efficient of kinetic friction is μ_k = 0.2. **5**

Answer

1. c. it is 2 : 1
   Explanation: $\frac{M_1}{M_2} = \frac{1}{2}$
   $\frac{R_1}{R_2} = \frac{2}{1}$
   $\frac{I_1}{I_2} = \frac{M_1 {R_1}^2}{M_2 {R_2}^2}$
   $\frac{I_1}{I_2} = \left(\frac{M_1}{M_2}\right)\left(\frac{R_1}{R_2}\right)^2 = \frac{1}{2} \times \left(\frac{2}{1}\right)^2 = \frac{1}{2} \times \frac{4}{1} = \frac{2}{1}$
   $I_1 : I_2 = 2 :1$

2. b. $\alpha = \frac{d\omega}{dt}$
   Explanation: As per the definition of angular accerlation that " it is the time rate of change of angular velocity" , therefore $\alpha = \frac{d\omega}{dt}$

3. b. $\vec{v} = \vec{\omega} \times \vec{r}$
   Explanation: If a particle is undergoing circular motion with an angular velocity $\vec{\omega}$ and the particle has a position vector $\vec{r}$ that is measured with respect to an origin that lies on the axis of rotation, then the velocity of the particle is $\vec{v} = \vec{\omega} \times \vec{r}$
   The reason for this (and not doing it the other way around) is purely convention. The way to remember is the right-hand rule. If you point your right thumb in the direction of $\vec{\omega}$ and then curl your fingers into a fist, your fingers will point in the direction of motion of the object.

4. c. (2/3) ML^2
   Explanation: Moment of inertia about an axis passing through edge
   $I = \frac{ML^2}{3}$
   Moment of inertia of system about z-axis
   $I = I_x + I_y + I_z$
   $I = \frac{ML^2}{3} + \frac{ML^2}{3} + 0$
   $I = \frac{2}{3} ML^2$

5. a. $\frac{100}{\sqrt{3}}$

Explanation: Moment of inertia of rod about an axis passing through its centre of gravity and perpendicular to its length

$I = \frac{Ml^2}{3}$

Moment of inertia of rod in terms of radius of gyration

I = Mk^2

M = 100 gm , l = 100 cm

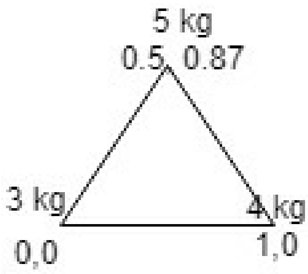
$Mk^2 = \frac{Ml^2}{3}$

$k = \sqrt{\frac{l^2}{3}} = \sqrt{\frac{100\times100}{3}}$

$k = \frac{100}{\sqrt{3}}cm$

6. Using Xcm = $\frac{M_1X_1+M_2X_2+M_3X_3}{M_1+M_2+M_3}$, and same for Y coordinate

(x, y) = (0.54 m, 0.36m)



7. Raw egg has liquid inside. When it is rotated, liquid shifts outward and increases moment of inertia (I=mr^2) than that for solid hard-boiled egg. Hence for same external torque, angular acceleration of raw egg will be small than that of hard-boiled egg due to which raw egg rotates slowly. (angular acceleration = torque/moment of inertia).

8. P.E is minimum.

9. $\alpha = \frac{w-w_0}{t} = \frac{1}{0.35}rads^{-2}$

$\tau = l\alpha$ = - 8.57 kgm^2s^{-2}

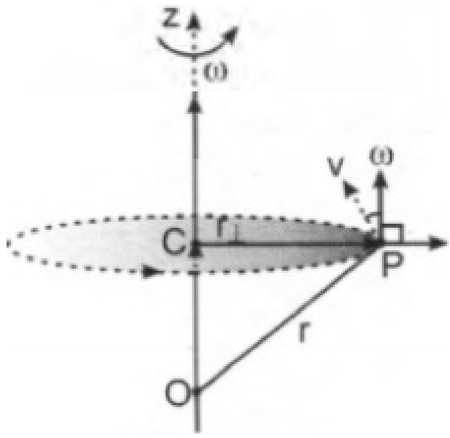
10. If in accordance with Newton's laws of motion, we form separate equations of motion of different individual particles, then the problem will become too much complicated. To solve the problem easily, we consider that all the external forces acting on the big body are acting at its centre of mass. Now by knowing the motion of centre of mass of

the body, we can study the motion of the big body as a whole.

11. Torque plays a very important role in rotational motion and is the rotational analog of force. For producing rotational motion or for altering the angular speed of a rotating body, we require a torque. In the absence of torque, a body will remain either at rest or will continue to rotate at a constant rate (clockwise or anticlockwise, etc.) about the axis of rotation.

12. The three laws of rotational motion are:
    First Law: It states that in absence of external torque, a body in rest remains static, but the body in rotation rotates with constant angular velocity.
    This inability of a rotating body to undergo any change in rotation by itself is called rotational inertia.
    Second Law: The time rate of change of angular momentum of a given rotating body is directly proportional to the applied external torque and takes place in the direction of torque.
    Explanation: Let a torque τ applied on a body and $\frac{dL}{dt}$ is the change in angular momentum of the body. Then, according to this law, $\vec{\tau} \propto \frac{d\overrightarrow{L}}{dt}$ and it finally leads to the relation $\vec{\tau} = I\vec{\alpha}$.
    Third Law: For every applied torque, there is an equal and opposite reaction torque.
    Explanation: Let the torque applied by the body A on B is $\overrightarrow{\tau_A}$ and the torque applied by the body B on A is $\overrightarrow{\tau_B}$, then according to this law, $\overrightarrow{\tau_A}$ = $-\overrightarrow{\tau_B}$

13. Given that the length of cylinder, l = 20 cm, Radius, r = 10 cm = 0.10 m , Angular speed, $\omega_0 = 100 \text{rads}^{-1}, \omega = 0$, time, t = 10 sec and Moment of inertia of the cylinder, I = 0.8 kg-m^2
    Therefore, Acceleration $\alpha = \frac{\omega - \omega_0}{t}$ = $\frac{0-100}{10}$ = - 10 rad s^{-2}
    $\therefore$ Torque, $\tau = I\alpha$ = $(0.8) \times (-10) = -8$ Nm
    If a tangential force F be acting on the cylinder, then torque of this force will be, τ = F $\times$ r
    $\Rightarrow$ F = $\frac{\tau}{r} = \frac{-8}{0.1}$ = - 80 N
    The negative sign shows that force is opposing the rotation of cylinder.

14. 

For a particle, moving in a circular path of radius r around the axis of rotation, the linear speed is given by,

$v = r\omega$, where ω is the angular velocity of a particle.

Vectorially, the linear velocity $\vec{v}$ is written as: $\vec{v} = \vec{\omega} \times \vec{r}$

The directions of $\vec{v}$ and $\vec{\omega}$ have been shown in Figure. ω is directed along the axis of rotation but the linear velocity v is perpendicular to both ω and rand is directed along the tangent to the circle described by the particle.Thus, for a given angular velocity ω, the linear velocity v of the particle is directly proportional to the distance of the particle from the centre of the circular path i.e.for a body in a uniform circular motion, the angular velocity is the same for all points in the body but linear velocity is different for different points of the body.

Pure rotation is characterised by all parts of the body having the same angular velocity at any instant of time.

15. Radii of the ring and the disc, r = 10 cm = 0.1 m

Initial angular speed, ω_z =10 π rad s^{-1}

Coefficient of kinetic friction, μ_k = 0.2

Initial velocity of both the objects, u = 0

Motion of the two objects is caused by frictional force. As per Newton's second law of motion, we have frictional force, $f = ma$

μ_k mg = ma

Where,

a = Acceleration produced in the objects

m = Mass

$\therefore a = \mu_k$ g ... (*i*)

As per the first equation of motion, the final velocity of the objects can be obtained as:

$v = u + at$

$= 0 + \mu_{\mathrm{k}}\mathrm{gt}$

$= \mu_{\mathrm{k}}\mathrm{gt}$... (*ii*)

The torque applied by the frictional force will act in a perpendicularly outward direction and cause a reduction in the initial angular speed.

Torque, T= - Iα

α = Angular acceleration

u_z mgr = -Iα

$\therefore a = \frac{-\mu_k mgr}{I}$ ....(iii)

Using the first equation of rotational motion to obtain the final angular speed:

$\omega = \omega_e + at$

$= \omega_x + \frac{-\mu_k mgr}{I} t$ ......(iv)

Rolling starts when linear velocity, $v = ru$

$\therefore v = r\left(\omega_0 - \frac{\mu_k gmrt}{I}\right)$ ......(*v*)

Equating equations (*ii*) and (*v*), we get:

$\mu_k gt = r\left(\omega_0 - \frac{\mu_k gmrt}{I}\right)$

$= r\omega_0 - \frac{\mu_i gmr^2 t}{I}$ ....(vi)

For the ring $I = mr^2$

$\therefore \mu_k gt = r\omega_0 - \frac{\mu_k gmr^2 t}{mr^2}$

$= r\omega_0 = u_k - \frac{u_k gmr^2 t}{mr^2}$

$2\mu_{\dot{k}} gt = r\omega_0$

$\therefore t_r = \frac{r\omega_0}{2\mu_k g}$

$= \frac{0.1\times10\times3.14}{2\times0.2\times9.8} = 0.80s$ ......(vii)

For the ring $I = \frac{1}{2}mr^2$

$\therefore \mu_k gt_d = r\omega_0 - \frac{\mu_k gmr^2 t}{\frac{1}{2}mr^2}$

$= r\omega_0 - 2\mu_k gt$

$3\mu_k gt_d = r\omega_0$

$\therefore t_d = \frac{r\omega_0}{3\mu_k g}$

$= \frac{0.1\times10\times3.14}{3\times0.2\times9.8} = 0.53s$ ......(viii)

Since$t_d > t_r$, the disc will start rolling before the ring.