SYSTEM OF LINEAR EQUATIONS (XII, R. S. AGGARWAL)

EXERCISE 8A (Pg. No.: 310)

Show that each one of the following systems of equations is inconsistent.

1. $x+2y=9;\ 2x+4y=7$

Sol. $A=\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}9\\ 7\end{bmatrix}$ $|A|=\begin{vmatrix}1 & 2\\ 2 & 4\end{vmatrix}=4-4=0$ $\therefore \text{adj} A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}'$

So the cofactor of the element of matrix A are $C_{11}=4, C_{12}=-2, C_{21}=-2, C_{22}=1$

$$\text{adj} A=\begin{bmatrix}4 & -2\\ -2 & 1\end{bmatrix}' \Rightarrow \text{adj} A=\begin{bmatrix}4 & -2\\ -2 & 1\end{bmatrix}$$

Now, $(\text{adj} A).B=\begin{bmatrix}4 & -2\\ -2 & 1\end{bmatrix}\begin{bmatrix}9\\ 7\end{bmatrix}=\begin{bmatrix}36-14\\ -18+7\end{bmatrix}=\begin{bmatrix}22\\ -11\end{bmatrix}\neq 0$

Hence, the system has no solution, it is inconsistent.

2. $2x+3y=5;\ 6x+9y=10$

Sol. $A=\begin{bmatrix}2 & 3\\ 6 & 9\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}5\\ 10\end{bmatrix}$ $\therefore \text{adj} A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}'$

So the cofactor of the element of matrix A are $C_{11}=9, C_{12}=-6, C_{21}=-3, C_{22}=2$

$$\text{adj} A=\begin{bmatrix}9 & -6\\ -3 & 2\end{bmatrix}' \Rightarrow \text{adj} A=\begin{bmatrix}9 & -3\\ -6 & 2\end{bmatrix}$$

Now, $\text{adj}(A)\times B=\begin{bmatrix}9 & -3\\ -6 & 2\end{bmatrix}\begin{bmatrix}5\\ 10\end{bmatrix}=\begin{bmatrix}45 & -30\\ -30 & +20\end{bmatrix}=\begin{bmatrix}15\\ -10\end{bmatrix}\neq 0$.

Hence, the system has no solution, it is inconsistent.

3. $4x-2y=3;\ 6x-3y=5$

Sol. $A=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}3\\ 5\end{bmatrix},$ $|A|=\begin{vmatrix}4 & -2\\ 6 & -3\end{vmatrix}=-12+12=0$ $\therefore \text{adj} A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}'$

So the cofactor of the element of matrix A are $C_{11}=-3, C_{12}=-6, C_{21}=2, C_{22}=4$

$$\text{adj } A=\begin{bmatrix}-3 & -6\\ 2 & 4\end{bmatrix}' \Rightarrow \text{adj } A=\begin{bmatrix}-3 & 2\\ -6 & 4\end{bmatrix}$$

Now, $(\text{adj} A).B=\begin{bmatrix}-3 & 2\\ -6 & 4\end{bmatrix}\begin{bmatrix}3\\ 5\end{bmatrix}=\begin{bmatrix}-9+10\\ -18+20\end{bmatrix}=\begin{bmatrix}1\\ 2\end{bmatrix}\neq 0$

Hence, the system has no solution, it is inconsistent.

4. $6x+4y=5;\ 9x+6y=8$

Sol. $A=\begin{bmatrix}6 & 4\\ 9 & 6\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}5\\ 8\end{bmatrix},$ $|A|=\begin{vmatrix}6 & 4\\ 9 & 6\end{vmatrix}=36-36=0$ $\therefore \text{adj} A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}'$

So the cofactor of the element of matrix A are $C_{11}=6, C_{12}=-9, C_{21}=-4, C_{22}=6$

$$\therefore \quad \text{adj } A=\begin{bmatrix} 6 & -9 \\ -4 & 6 \end{bmatrix}'=\begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}$$

Now, $(\text{adj } A).B=\begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix}=\begin{bmatrix} 30-32 \\ -45+48 \end{bmatrix}=\begin{bmatrix} -2 \\ 3 \end{bmatrix}\neq 0$.

Hence, the system has no solution, it is inconsistent.

5. $x+y-2z=5;\ x-2y+z=-2,\ -2x+y+z=4$

Sol. $A=\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix},\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B=\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix},$

$$|A|=\begin{vmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{vmatrix}=1\begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix}-1\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix}-2\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix}=0$$

$$\therefore \text{ adj } A=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=-3 & C_{12}=-3 & C_{13}=-3 \\ C_{21}=-3 & C_{22}=-3 & C_{23}=-3 \\ C_{31}=-3 & C_{32}=-3 & C_{33}=-3 \end{bmatrix}$

$$\Rightarrow \text{adj } A=\begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}' \quad \Rightarrow \text{adj } A=\begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}$$

Now, $(\text{adj } A).B=\begin{bmatrix} -3 & -3- & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}=\begin{bmatrix} -15+6-12 \\ -15+6-12 \\ -15+6-12 \end{bmatrix}=\begin{bmatrix} -21 \\ -21 \\ -21 \end{bmatrix}\neq 0$.

Hence, the system has no solution, it is inconsistent.

6. $2x-y+3z=1;\ 3x-2y+5z=-4;\ 5x-4y+9z=14$

Sol. $A=\begin{bmatrix} 2 & -1 & 3 \\ 3 & -2 & 5 \\ 5 & -4 & 9 \end{bmatrix},\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B=\begin{bmatrix} 1 \\ -4 \\ 14 \end{bmatrix},$

$$|A|=\begin{vmatrix} 2 & -1 & 3 \\ 3 & -2 & 5 \\ 5 & -4 & 9 \end{vmatrix}=2\begin{vmatrix} -2 & 5 \\ -4 & 9 \end{vmatrix}-(-1)\begin{vmatrix} 3 & 5 \\ 5 & 9 \end{vmatrix}+3\begin{vmatrix} 3 & -2 \\ 5 & -4 \end{vmatrix}=2(2)+(2)+3(-2)=4+2-6=0$$

$$\therefore \text{ adj } A=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=2 & C_{12}=-2 & C_{13}=-2 \\ C_{21}=-3 & C_{22}=3 & C_{23}=3 \\ C_{31}=1 & C_{32}=-1 & C_{33}=-1 \end{bmatrix}$

$$\operatorname{adj} A = \begin{bmatrix} 2 & -2 & -2 \\ -3 & 3 & 3 \\ 1 & -1 & -1 \end{bmatrix}' \Rightarrow \operatorname{adj} A = \begin{bmatrix} 2 & -3 & 1 \\ -2 & 3 & -1 \\ -2 & 3 & -1 \end{bmatrix}$$

Now, $(\operatorname{adj} A).B = \begin{bmatrix} 2 & -3 & 1 \\ -2 & 3 & -1 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -4 \\ 14 \end{bmatrix} = \begin{bmatrix} 2+12+14 \\ -2-12-14 \\ -2-12-14 \end{bmatrix} = \begin{bmatrix} 28 \\ -28 \\ -28 \end{bmatrix} \neq 0$

Hence, the solution has no solution, it is inconsistent.

7. $x+2y+4z=12$; $y+2z=-1$; $3x+2y+4z=4$

Sol. $A=\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B=\begin{bmatrix} 12 \\ -1 \\ 4 \end{bmatrix}$,

$$|A| = \begin{vmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} - 2\begin{vmatrix} 0 & 2 \\ 3 & 4 \end{vmatrix} + 4\begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix} = (4-4)-2(0-6)+4(0-3) = 0+12-12=0$$

$$\therefore \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=0 & C_{12}=6 & C_{13}=-3 \\ C_{21}=0 & C_{22}=-8 & C_{23}=4 \\ C_{31}=0 & C_{32}=-2 & C_{33}=1 \end{bmatrix}$

$$\therefore \operatorname{adj} A = \begin{bmatrix} 0 & 6 & -3 \\ 0 & -8 & 4 \\ 0 & -2 & 1 \end{bmatrix}' \Rightarrow \operatorname{adj} A = \begin{bmatrix} 0 & 0 & 0 \\ 6 & -8 & -2 \\ -3 & 4 & 1 \end{bmatrix}$$

Now, $(\operatorname{adj} A).B = \begin{bmatrix} 0 & 0 & 0 \\ 6 & -8 & -2 \\ -3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0-0+0 \\ 72+8-8 \\ -36-4+4 \end{bmatrix} = \begin{bmatrix} 0 \\ 72 \\ -36 \end{bmatrix} \neq 0$

Hence, the system has no solution, it is inconsistent.

8. $3x-y-2z=2$; $2y-z=-1$; $3x-5y=3$

Sol. $A=\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B=\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$,

$$|A| = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix} = 3\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} + 1\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} - 2\begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix}$$

$$= 3(0-5)+3-2(0-6) = -15+3+12=0$$

$\therefore$ adj $A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = -5 & C_{12} = -3 & C_{13} = -6 \\ C_{21} = 10 & C_{22} = 6 & C_{23} = 12 \\ C_{31} = 5 & C_{32} = 3 & C_{33} = 6 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} -5 & -3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

Now, $(\text{adj } A).B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & -3 \\ -6 & 12 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10-10+15 \\ -6-6+9 \\ -12-12+18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$

Hence, the system has no solution , it is inconsistent.

Solve each of the following systems of equations using matrix method.

9. $5x + 2y = 4$; $7x + 3y = 5$

Sol. $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$, $|A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 15 - 14 = 1 \Rightarrow |A| \neq 0$. Hence, A^{-1} exists.

$\therefore$ adj $A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

So the cofactor of the element of matrix A are, $C_{11} = 3, C_{21} = -2, C_{12} = -7, C_{22} = 5$

$$\text{adj } A = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \text{ and } |A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = (15 - 14) = 1$$

$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

Now, $AX = B \Rightarrow X = A^{-1}B \Rightarrow X = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 12-10 \\ -28+25 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Hence, $x = 2$ and $y = -3$, it is inconsistent.

10. $3x + 4y - 5 = 0$; $x - y + 3 = 0$

Sol. $A = \begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ $\quad \therefore$ adj $A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = -1 & C_{21} = -4 \\ C_{12} = -1 & C_{22} = 3 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} -1 & -1 \\ -4 & 3 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix} \text{ and } |A| = \begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} = (-3 - 4) = -7 \neq 0$$

and $A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{(-7)}\begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$

Now, $AX = B \Rightarrow X = A^{-1}B \Rightarrow X = \frac{1}{7}\begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 5 \\ -3 \end{bmatrix}$

$\Rightarrow X = \frac{1}{7}\begin{bmatrix} 5-12 \\ 5+9 \end{bmatrix} \Rightarrow X = \frac{1}{7}\begin{bmatrix} -7 \\ 14 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Hence, $x = -1$ and $y = 2$.

11. $x + 2y = 1;\ 3x + y = 4$

Sol. $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \quad \therefore\ \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = 1 & C_{21} = -2 \\ C_{12} = -3 & C_{22} = 1 \end{bmatrix}$

$\text{adj}\, A = \begin{bmatrix} 1 & -3 \\ -2 & 1 \end{bmatrix}' \Rightarrow \text{adj}\, A = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ and $|A| = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \Rightarrow |A| = (1-6) = (-5)$

and $A^{-1} = \frac{1}{|A|}(\text{adj}\, A) = \frac{1}{(-5)}\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix}$

Now $AX = B \Rightarrow X = A^{-1}B \Rightarrow X = \frac{1}{5}\begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} \Rightarrow X = \frac{1}{5}\begin{bmatrix} -1+8 \\ 3-4 \end{bmatrix}$

$\Rightarrow X = \frac{1}{5}\begin{bmatrix} 7 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7/5 \\ -1/5 \end{bmatrix}$. Hence, $x = \frac{7}{5}$ and $y = \frac{-1}{5}$.

12. $5x + 7y + 2 = 0;\ 4x + 6y + 3 = 0$

Sol. $A = \begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix},\ B = \begin{bmatrix} -2 \\ -3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ |A| = \begin{vmatrix} 5 & 7 \\ 4 & 6 \end{vmatrix} = (30-28) = 2$. Hence, A^{-1} exist.

$\therefore\ \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = 6 & C_{21} = -7 \\ C_{12} = -4 & C_{22} = 5 \end{bmatrix}$

$\text{adj}\, A = \begin{bmatrix} 6 & -4 \\ -7 & 5 \end{bmatrix}' = \begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$

$\therefore\ A^{-1} = \frac{1}{|A|}(\text{adj}\, A) = \frac{1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$

Now, $AX = B \Rightarrow X = A^{-1}B \Rightarrow X = \frac{1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix} \Rightarrow X = \frac{1}{2}\begin{bmatrix} -12+21 \\ 8-15 \end{bmatrix}$

$\Rightarrow X = \frac{1}{2}\begin{bmatrix} 9 \\ -7 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9/2 \\ -7/2 \end{bmatrix}$.

Hence, $x = \frac{9}{2}$ & $y = -\frac{7}{2}$

13. $2x - 3y + 1 = 0;\ x + 4y + 3 = 0$

Sol. $A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ B = \begin{bmatrix} -1 \\ -3 \end{bmatrix},\ |A| = \begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} = (8+3) = 11$. Hence, A^{-1} exist.

$\therefore\ \text{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = 4 & C_{21} = 3 \\ C_{12} = -1 & C_{22} = 2 \end{bmatrix}$

$\text{adj} A = \begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}' \Rightarrow \text{adj} A = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$

$\therefore\ A^{-1} = \frac{1}{|A|}(\text{adj} A) = \frac{1}{11}\begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$

Now, $AX = B \Rightarrow X = A^{-1}B \Rightarrow X = \frac{1}{11}\begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ -3 \end{bmatrix}$

$\Rightarrow X = \frac{1}{11}\begin{bmatrix} -4-9 \\ 1-6 \end{bmatrix} \Rightarrow X = \frac{1}{11}\begin{bmatrix} -13 \\ -5 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -13/11 \\ -5/11 \end{bmatrix}$. Hence, $x = \frac{-13}{11}$ and $y = \frac{-5}{11}$

14. $4x - 3y = 3$; $3x - 5y = 7$

Sol. $A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$, $|A| = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = (-20+9) = (-11)$. Hence, A^{-1} exist.

$\therefore\ \text{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = -5 & C_{21} = 3 \\ C_{12} = -3 & C_{22} = 4 \end{bmatrix}$

$\text{adj} A = \begin{bmatrix} -5 & -3 \\ 3 & 4 \end{bmatrix}' \Rightarrow \text{adj} A = \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj} A) = \frac{1}{(-11)}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}$

Now, $AX = B \Rightarrow X = A^{-1}B \Rightarrow X = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$\Rightarrow X = \frac{1}{11}\begin{bmatrix} 15-21 \\ 9-28 \end{bmatrix} \Rightarrow X = \frac{1}{11}\begin{bmatrix} -6 \\ -19 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6/11 \\ -19/11 \end{bmatrix}$. Hence, $x = \frac{-6}{11}$ and $y = \frac{-19}{11}$.

15. $2x + 8y + 5z = 5$; $x + y + z = -2$; $x + 2y - z = 2$

Sol. Let $A = \begin{bmatrix} 2 & 8 & 5 \\ 1 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$

$|A| = 2\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} - 8\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2(-3) - 8(-2) + 5(1) = -6 + 16 + 5 = 15 \Rightarrow |A| = 15$

Hence, A^{-1} exist.

$\therefore\ \text{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=-3 & C_{12}=2 & C_{13}=1 \\ C_{21}=18 & C_{22}=-7 & C_{23}=4 \\ C_{31}=3 & C_{32}=3 & C_{33}=-6 \end{bmatrix}$

$$\text{adj}\,A=\begin{bmatrix} -3 & 2 & 1 \\ 18 & -7 & 4 \\ 3 & 3 & -6 \end{bmatrix}' \Rightarrow \text{adj}\,A=\begin{bmatrix} -3 & 18 & 3 \\ 2 & -7 & 3 \\ 1 & 4 & -6 \end{bmatrix} \quad \therefore\ A^{-1}=\frac{1}{15}\begin{bmatrix} -3 & 18 & 3 \\ 2 & -7 & 3 \\ 1 & 4 & -6 \end{bmatrix}$$

$$\therefore\ AX=B \Rightarrow X=A^{-1}B \Rightarrow X=\frac{1}{15}\begin{bmatrix} -3 & 18 & 3 \\ 2 & -7 & 3 \\ 1 & 4 & -6 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix} \Rightarrow X=\frac{1}{15}\begin{bmatrix} -15-36+6 \\ 10+14+6 \\ 5-8-12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{15}\begin{bmatrix} -45 \\ 30 \\ -15 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix} \quad \therefore\ x=-3,\ y=2,\ z=-1$$

16. $x-y+z=1;\ 2x+y-z=2;\ x-2y-z=4$

Sol. $A=\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & -1 \end{bmatrix},\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B=\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

$$|A|=\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix} \Rightarrow |A|=1(-1-2)+1(-2+1)+1(-4-1)$$

$\Rightarrow |A|=-3-1-5 \Rightarrow |A|=(-9)$. Hence, A^{-1} exist.

$$\therefore\ \text{adj}\,A=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=-3 & C_{12}=1 & C_{13}=-5 \\ C_{21}=-3 & C_{22}=-2 & C_{23}=1 \\ C_{31}=0 & C_{32}=3 & C_{33}=3 \end{bmatrix}$

$$\therefore\ \text{adj}\,A=\begin{bmatrix} -3 & 1 & -5 \\ -3 & -2 & 1 \\ 0 & 3 & 3 \end{bmatrix}' \Rightarrow \text{adj}\,A=\begin{bmatrix} -3 & -3 & 0 \\ 1 & -2 & 3 \\ -5 & 1 & 3 \end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{|A|}(\text{adj}\,A)=\frac{1}{(-9)}\begin{bmatrix} -3 & -3 & 0 \\ 1 & -2 & 3 \\ -5 & 1 & 3 \end{bmatrix}=\frac{1}{9}\begin{bmatrix} 3 & 3 & 0 \\ -1 & 2 & -3 \\ 5 & -1 & -3 \end{bmatrix}$$

Now, $AX=B \Rightarrow X=A^{-1}B \Rightarrow X=\frac{1}{9}\begin{bmatrix} 3 & 3 & 0 \\ -1 & 2 & -3 \\ 5 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

$$\Rightarrow X=\frac{1}{9}\begin{bmatrix}3+6+0\\-1+4-12\\5-2-12\end{bmatrix} \quad \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{9}\begin{bmatrix}9\\-9\\-9\end{bmatrix} \quad \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-1\\-1\end{bmatrix}$$. Hence, $x=1$, $y=-1$ and $z=-1$.

17. $3x+4y+7z=4;\ 2x-y+3z=-3;\ x+2y-3z=8$

Sol. The given system of linear equations may be written in matrix form as $AX=B$ where

$$A=\begin{bmatrix}3&4&7\\2&-1&3\\1&2&-3\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \textbf{ and } B=\begin{bmatrix}4\\-3\\8\end{bmatrix}$$

Now, $|A|=\begin{vmatrix}3&4&7\\2&-1&3\\1&2&-3\end{vmatrix}$

$=3(3-6)-4(-6-3)+7(4+1)=-9+36+35=62\neq 0$

$C_{11}=(-1)^{1+1}\begin{vmatrix}-1&3\\2&-3\end{vmatrix}=3-6=-3$

$C_{12}=(-1)^{1+2}\begin{vmatrix}2&3\\1&-3\end{vmatrix}=-\{-6-3\}=9$

$C_{13}=(-1)^{1+3}\begin{vmatrix}2&-1\\1&2\end{vmatrix}=4+1=5$

$C_{21}=(-1)^{2+1}\begin{vmatrix}4&7\\2&-3\end{vmatrix}=-(-12-14)=26$

$C_{22}=(-1)^{2+2}\begin{vmatrix}3&7\\1&-3\end{vmatrix}=-8-7=-16$

$C_{23}=(-1)^{2+3}\begin{vmatrix}3&4\\1&2\end{vmatrix}=-(6-4)=-2$

$C_{31}=(-1)^{3+1}\begin{vmatrix}4&7\\-1&3\end{vmatrix}=12+7=19$

$C_{32}=(-1)^{3+2}\begin{vmatrix}3&7\\2&3\end{vmatrix}=-(9-14)=5$

$C_{33}=(-1)^{3+3}\begin{vmatrix}3&4\\2&-1\end{vmatrix}=(-3-8)=-11$

$$\therefore Adj\cdot A=\begin{bmatrix}-3&9&5\\26&-19&-2\\19&5&-11\end{bmatrix}=\begin{bmatrix}-3&26&19\\9&-16&5\\5&-2&-11\end{bmatrix}$$

$\therefore A^{-1}=\frac{1}{|A|}Adj\,A$

$$=\frac{1}{62}\begin{bmatrix}-3&26&19\\9&-16&5\\5&-2&-11\end{bmatrix}$$

$\therefore AX=B$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 4 \\ -3 \\ 8 \end{bmatrix} = \frac{1}{62}\begin{bmatrix} -12-78+152 \\ 36+48+40 \\ 20+6-88 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62}\begin{bmatrix} 62 \\ 124 \\ -62 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Equation the corresponding element we get $x = 1, y = 2, z = -1$

18. $x + 2y + z = 7$; $x + 3z = 11$; $2x - 3y = 1$

Sol. $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 1 \end{vmatrix} \Rightarrow |A| = 1(0+9) - 2(0-6) + 1(-3-0) \Rightarrow |A| = 9 + 12 - 3 \Rightarrow |A| = 18$$

Hence, A^{-1} exist.

$$\therefore \ \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = 9 & C_{12} = 6 & C_{13} = -3 \\ C_{21} = -3 & C_{22} = -2 & C_{23} = 7 \\ C_{31} = 6 & C_{32} = -2 & C_{33} = -2 \end{bmatrix}$

$$\text{adj}\, A = \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & -2 \end{bmatrix}' \Rightarrow \text{adj}\, A = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{1}{|A|}(\text{adj}\, A) = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

Now, $AX = B$, $X = A^{-1}B \Rightarrow X = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} \Rightarrow X = \frac{1}{18}\begin{bmatrix} 63-33+6 \\ 42-22-2 \\ -21+77-2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$. Hence, $x = 2$, $y = 1$ and $z = 3$.

19. $2x - 3y + 5z = 16$; $3x + 2y - 4z = -4$; $x + y - 2z = -3$

Sol. Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 16 \\ -4 \\ -3 \end{bmatrix}$

$$|A|=\begin{vmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{vmatrix}=2(-4+4)+3(-6+4)+5(3-2)=2(0)+3(-2)+5(1)=-6+5=-1$$

$\Rightarrow |A|=-1$. Hence, A^{-1} exist.

$$\therefore \text{adj}\,A=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix}C_{11}=0&C_{12}=2&C_{13}=1\\C_{21}=-1&C_{22}=-9&C_{23}=-5\\C_{31}=2&C_{32}=23&C_{33}=13\end{bmatrix}$

$$\text{adj}\,A=\begin{bmatrix}0&2&1\\-1&-9&-5\\2&23&13\end{bmatrix}'\Rightarrow \text{adj}\,A=\begin{bmatrix}0&-1&2\\2&-9&23\\1&-5&13\end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{|A|}(\text{adj}\,A)=-1\begin{bmatrix}0&-1&2\\2&-9&23\\1&-5&13\end{bmatrix}\Rightarrow A^{-1}=\begin{bmatrix}0&1&-2\\-2&9&-23\\-1&5&-13\end{bmatrix}$$

$$\therefore\ AX=B, X=A^{-1}B\Rightarrow X=\begin{bmatrix}0&1&-2\\-2&9&-23\\-1&5&-13\end{bmatrix}\begin{bmatrix}16\\-4\\-3\end{bmatrix}\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0-4+6\\-32-36+69\\-16-20+39\end{bmatrix}$$

$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$. Hence $x=2,\ y=1,\ z=3$

20. $x+y+z=4$; $2x-y+z=-1$; $2x+y-3z=-9$

Sol. Let $A=\begin{bmatrix}1&1&1\\2&-1&1\\2&1&-3\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}4\\-1\\-9\end{bmatrix}$

$$|A|=\begin{vmatrix}1&1&1\\2&-1&1\\2&1&-3\end{vmatrix}\Rightarrow |A|=(3-1)-(-6-2)+(2+2)\Rightarrow |A|=2+8+4\Rightarrow |A|=14$$

Hence, A^{-1} exist.

$$\therefore \text{adj}\,A=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix}C_{11}=2&C_{12}=8&C_{13}=4\\C_{21}=4&C_{22}=-5&C_{23}=1\\C_{31}=2&C_{32}=1&C_{33}=-3\end{bmatrix}$

$$\therefore \ \text{adj}\, A = \begin{bmatrix} 2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3 \end{bmatrix}' = \begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{1}{|A|}(\text{adj}\, A) \quad \Rightarrow A^{-1} = \frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$$

$$AX = B \ \Rightarrow \ X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \\ -9 \end{bmatrix} \ \Rightarrow X = \frac{1}{14}\begin{bmatrix} 8-4-18 \\ 32+5-9 \\ 16-1+27 \end{bmatrix} \ \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{14}\begin{bmatrix} -14 \\ 28 \\ 42 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}.$$ Hence $x = -1,\ y = 2,\ z = 3$

21. $2x - 3y + 5z = 11$; $3x + 2y - 4z = -5$; $x + y - 2z = -3$

Sol. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} \ \Rightarrow |A| = 2(-4+4) + 3(-6+4) + 5(3-2) \ \Rightarrow |A| = 0 - 6 + 5 \ \Rightarrow |A| = (-1)$$

Hence, A^{-1} exist.

$$\therefore \ \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = 0 & C_{12} = 2 & C_{13} = 1 \\ C_{21} = -1 & C_{22} = -9 & C_{23} = -5 \\ C_{31} = 2 & C_{32} = 23 & C_{33} = 13 \end{bmatrix}$

$$\text{adj}\, A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}' \quad \Rightarrow \text{adj}\, A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{1}{|A|}(\text{adj}\, A) = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now, $AX = B,\ X = A^{-1}B$

$$\Rightarrow X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \ \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} \ \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x=1$, $y=2$ and $z=3$

22. $x+y+z=1$; $x-2y+3z=2$; $5x-3y+z=3$

Sol. $A=\begin{bmatrix}1 & 1 & 1\\ 1 & -2 & 3\\ 5 & -3 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & 1 & 1\\ 1 & -2 & 3\\ 5 & -3 & 1\end{vmatrix} \Rightarrow |A|=1(-2+9)-1(1-15)+1(-3+10) \Rightarrow |A|=7+14+7 \Rightarrow |A|=28$$

Hence, A^{-1} exist.

$$\therefore\ \operatorname{adj} A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix}C_{11}=7 & C_{12}=14 & C_{13}=7\\ C_{21}=-4 & C_{22}=-4 & C_{23}=8\\ C_{31}=5 & C_{32}=-2 & C_{33}=-3\end{bmatrix}$

$$\therefore\ \operatorname{adj} A=\begin{bmatrix}7 & 14 & 7\\ -4 & -4 & 8\\ 5 & -2 & -3\end{bmatrix}' \Rightarrow \operatorname{adj} A=\begin{bmatrix}7 & -4 & 5\\ 14 & -4 & -2\\ 7 & 8 & -3\end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=\frac{1}{28}\begin{bmatrix}7 & -4 & 5\\ 14 & -4 & -2\\ 7 & 8 & -3\end{bmatrix}$$

Now, $AX=B$, $X=A^{-1}B$

$$\Rightarrow X=\frac{1}{28}\begin{bmatrix}7 & -4 & 5\\ 14 & -4 & -2\\ 7 & 8 & -3\end{bmatrix}\begin{bmatrix}1\\ 2\\ 3\end{bmatrix} \Rightarrow X=\frac{1}{28}\begin{bmatrix}7-8+15\\ 14-8-6\\ 7+16-9\end{bmatrix} \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{28}\begin{bmatrix}14\\ 0\\ 14\end{bmatrix} \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1/2\\ 0\\ 1/2\end{bmatrix}$$

Hence, $x=\frac{1}{2}$, $y=0$ and $z=\frac{1}{2}$

23. $x+y+z=6$; $x+2z=7$; $3x+y+z=12$

Sol. $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}6\\ 7\\ 12\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 1\end{vmatrix} \Rightarrow |A|=1(0-2)-1(1-6)+1(1-0) \Rightarrow |A|=-2+5+1 \Rightarrow |A|=4$$

Hence, A^{-1} exist.

$$\therefore\ \operatorname{adj} A=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=-2 & C_{12}=5 & C_{13}=1 \\ C_{21}=0 & C_{22}=-2 & C_{23}=2 \\ C_{31}=2 & C_{32}=-1 & C_{33}=-1 \end{bmatrix}$

$$\operatorname{adj} A=\begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}' \Rightarrow \operatorname{adj} A=\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=\frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

Now, $AX=B$, $X=A^{-1}B$

$$\Rightarrow X=\frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} \Rightarrow X=\frac{1}{4}\begin{bmatrix} -12+0+24 \\ 30-14-12 \\ 6+14-12 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x=3$, $y=1$ and $z=2$

24. $2x+3y+3z=5$; $x-2y+z=-4$; $3x-y-2z=3$

Sol. $A=\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B=\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$$|A|=\begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} \Rightarrow |A|=2(4+1)-3(-2-3)+3(-1+6) \Rightarrow |A|=10+15+15 \Rightarrow |A|=40$$

Hence, A^{-1} exist

$$\therefore \operatorname{adj} A=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=5 & C_{12}=5 & C_{13}=5 \\ C_{21}=3 & C_{22}=-13 & C_{23}=11 \\ C_{31}=9 & C_{32}=1 & C_{33}=-7 \end{bmatrix}$

$$\operatorname{adj} A=\begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}' \Rightarrow \operatorname{adj} A=\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=\frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

Now, $AX = B$, $X = A^{-1}B \quad \Rightarrow X = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$$\Rightarrow X = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$ and $z = -1$

25. $4x - 5y - 11z = 12$; $x - 3y + z = 1$; $2x + 3y - 7z = 2$

Sol. $A = \begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 12 \\ 1 \\ 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{vmatrix} \Rightarrow |A| = 4(21-3) + 5(-7-2) - 11(3+6) \Rightarrow |A| = 72 - 45 - 99$$

$\Rightarrow |A| = (-72)$. Hence, A^{-1} exist.

$$\therefore \text{ adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = 18 & C_{12} = 9 & C_{13} = 9 \\ C_{21} = -68 & C_{22} = -6 & C_{23} = -22 \\ C_{31} = -38 & C_{32} = -15 & C_{33} = -7 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} 18 & 9 & 9 \\ -68 & -6 & -22 \\ -38 & -15 & -7 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{(-72)}\begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix} = \frac{1}{72}\begin{bmatrix} -18 & 68 & 38 \\ -9 & 6 & 15 \\ -9 & 22 & 7 \end{bmatrix}$$

Now, $AX = B$, $X = A^{-1}B \quad \Rightarrow X = \frac{1}{72}\begin{bmatrix} -18 & 68 & 38 \\ -9 & 6 & 15 \\ -9 & 22 & 7 \end{bmatrix}\begin{bmatrix} 12 \\ 1 \\ 2 \end{bmatrix}$

$$\Rightarrow X = \frac{1}{72}\begin{bmatrix} -216+68+76 \\ -108+6+30 \\ -108+22+14 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{72}\begin{bmatrix} -72 \\ -72 \\ -72 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

Hence, $x = -1$, $y = -1$ and $z = -1$

26. $x - y + 2z = 7$; $3z + 4y - 5z = -5$; $2x - y + 3z = 12$

Sol. $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

Now, $|A|=\begin{vmatrix}1&-1&2\\3&4&-5\\2&-1&3\end{vmatrix}=1(12-5)+1(9+10)+2(-3-8)\quad=7+19-22=4\neq 0$

Hence A^{-1} **exist and system have unique solution**

$$C_{11}=(-1)^{1+1}\begin{vmatrix}4&-5\\-1&3\end{vmatrix}=12-5=7$$

$$C_{12}=(-1)^{1+2}\begin{vmatrix}3&-5\\2&3\end{vmatrix}=-(9+10)=-19$$

$$C_{13}=(-1)^{2+1}\begin{vmatrix}3&4\\2&-1\end{vmatrix}=+(-3-8)=-11$$

$$C_{21}=(-1)^{2+1}\begin{vmatrix}-1&2\\-1&3\end{vmatrix}=-(3+2)=1$$

$$C_{22}=(-1)^{2+2}\begin{vmatrix}1&2\\2&3\end{vmatrix}=+(3-4)=-1$$

$$C_{23}=(-1)^{2+3}\begin{vmatrix}1&-1\\2&-1\end{vmatrix}=-(-1+2)=-1$$

$$C_{31}=(-1)^{3+1}\begin{vmatrix}-1&2\\4&-5\end{vmatrix}=+(5-8)=-3$$

$$C_{32}=(-1)^{3+2}\begin{vmatrix}1&2\\3&-5\end{vmatrix}=-(-5-6)=11$$

$$C_{33}=(-1)^{3+3}\begin{vmatrix}1&-1\\3&4\end{vmatrix}=+(4+3)=7$$

$$\therefore adjA=\begin{bmatrix}7&-19&-11\\1&-1&-1\\-3&11&7\end{bmatrix}^T=\begin{bmatrix}7&1&-3\\-19&-1&11\\-11&-1&7\end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{|A|}adjA=\frac{1}{4}\begin{bmatrix}7&1&-3\\-19&-1&11\\-11&-1&7\end{bmatrix}$$

$$\because AX=B$$

$$\Rightarrow X=A^{-1}B$$

$$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{4}\begin{bmatrix}7&1&-3\\-19&-1&11\\-11&-1&7\end{bmatrix}\begin{bmatrix}7\\-5\\12\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{4}\begin{bmatrix}49-5-36\\-133+5+132\\-77+5+84\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{4}\begin{bmatrix}8\\4\\12\end{bmatrix}\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$$

Equation the corresponding elements, we get $x=2, y=1, z=3$

27. $6x-9y-20z=-4;\ 4x-15y+10z=-1;\ 2x-3y-5z=-1$

Sol. Let $A=\begin{bmatrix}6 & -9 & -20\\ 4 & -15 & 10\\ 2 & -3 & -5\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}-4\\ -1\\ -1\end{bmatrix}$

$|A|=\begin{vmatrix}6 & -9 & -20\\ 4 & -15 & 10\\ 2 & -3 & -5\end{vmatrix} \Rightarrow |A|=6(75+30)+9(-20-20)-20(-12+30)=6(105)+9(-40)-20(18)$

$\Rightarrow |A|=630-360-360=-90$. Hence, A^{-1} exist.

$\therefore\ \text{adj}\, A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix}C_{11}=105 & C_{12}=40 & C_{13}=18\\ C_{21}=15 & C_{22}=10 & C_{23}=0\\ C_{31}=-390 & C_{32}=-140 & C_{33}=-54\end{bmatrix}$

$\text{adj}\, A=\begin{bmatrix}105 & 40 & 18\\ 15 & 10 & 0\\ -390 & -140 & -54\end{bmatrix}' \Rightarrow (\text{adj}\, A)=\begin{bmatrix}105 & 15 & -390\\ 40 & 10 & -140\\ 18 & 0 & -54\end{bmatrix}$

$\therefore\ A^{-1}=\frac{1}{|A|}(\text{adj}\, A)=\frac{1}{-90}\begin{bmatrix}105 & 15 & -390\\ 40 & 10 & -140\\ 18 & 0 & -54\end{bmatrix}$

Now, $AX=B,\ X=A^{-1}B$

$\Rightarrow X=-\frac{1}{90}\begin{bmatrix}105 & 15 & -390\\ 40 & 10 & -140\\ 18 & 0 & -54\end{bmatrix}\begin{bmatrix}-4\\ -1\\ -1\end{bmatrix} \Rightarrow X=-\frac{1}{90}\begin{bmatrix}-420-15+390\\ -160-10+140\\ -72+0+54\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=-\frac{1}{90}\begin{bmatrix}-45\\ -30\\ -18\end{bmatrix} \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1/2\\ 1/3\\ 1/5\end{bmatrix}$. Hence $x=\frac{1}{2},\ y=\frac{1}{3},\ z=\frac{1}{5}$

28. $3x-4y+2z=-1;\ 2x+3y+5z=7;\ \ x+z=2$

Sol. $A=\begin{bmatrix}3 & -4 & 2\\ 2 & 3 & 5\\ 1 & 0 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}-1\\ 7\\ 2\end{bmatrix}$

$|A|=\begin{vmatrix}3 & -4 & 2\\ 2 & 3 & 5\\ 1 & 0 & 1\end{vmatrix} \Rightarrow |A|=3(3-0)+4(2-5)+2(0-3) \Rightarrow |A|=9-12-6 \Rightarrow |A|=(-9)$

Hence, A^{-1} exist.

$$\therefore \operatorname{adj} A=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=3 & C_{12}=3 & C_{13}=-3 \\ C_{21}=4 & C_{22}=1 & C_{23}=-4 \\ C_{31}=-26 & C_{32}=-11 & C_{33}=17 \end{bmatrix}$

$$\operatorname{adj} A=\begin{bmatrix} 3 & 3 & -3 \\ 4 & 1 & -4 \\ -26 & -11 & 17 \end{bmatrix}' \Rightarrow \operatorname{adj} A=\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

$$\therefore \quad A^{-1}=\frac{1}{|A|}(\operatorname{adj} A)=\frac{1}{(-9)}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}=\frac{1}{9}\begin{bmatrix} -3 & -4 & 26 \\ -3 & -1 & 11 \\ 3 & 4 & -17 \end{bmatrix}$$

Now, $AX=B \quad \Rightarrow X=A^{-1}B$

Now, $X=A^{-1}B=\frac{1}{9}\begin{bmatrix} -3 & 2 & -1 \\ 1 & 0 & -1 \\ -4 & 2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \quad \Rightarrow X=\frac{1}{9}\begin{bmatrix} -3+6+1 \\ 1+0+1 \\ -4+6+2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{9}\begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}$ Hence, $x=\frac{4}{9},\ y=\frac{2}{9},\ z=\frac{4}{9}$

29. $x+y-z=1;\ 3x+y-2z=3;\ x-y-z=-1$.

Sol. $A=\begin{bmatrix} 1 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}\ B=\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ and $AX=B \quad \Rightarrow X=A^{-1}B$

$$|A|=\begin{vmatrix} 1 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & -1 & -1 \end{vmatrix}=1\begin{vmatrix} 1 & -2 \\ -1 & -1 \end{vmatrix}-1\begin{vmatrix} 3 & -2 \\ 1 & -1 \end{vmatrix}-1\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix}$$

$=(-1-2)-(-3+2)-(-3-1)=-3+1+4=2$. $\therefore |A|\neq 0$. Hence, A^{-1} exist

$$\therefore \operatorname{adj} A=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are,

$C_{11}=(-1-2)=-3,\quad C_{21}=-(-1-1)=2,\quad C_{31}=(-2+1)=-1$

$C_{12}=-(-3+2)=1,\quad C_{22}=(-1+1)=0,\quad C_{32}=-(-2+3)=-1$

$C_{13}=(-3-1)=-4,\quad C_{23}=-(-1-1)=2,\quad C_{33}=(1-3)=-2$

$$\therefore \quad \operatorname{adj} A=\begin{bmatrix} -3 & 1 & -4 \\ 2 & 0 & 2 \\ -1 & -1 & -2 \end{bmatrix}'=\begin{bmatrix} -3 & 2 & -1 \\ 1 & 0 & -1 \\ -4 & 2 & -2 \end{bmatrix},\ A^{-1}=\frac{\operatorname{adj} A}{|A|}=\frac{1}{2}\begin{bmatrix} -3 & 2 & -1 \\ 1 & 0 & -1 \\ -4 & 2 & -2 \end{bmatrix}=\frac{1}{2}\begin{bmatrix} -3 & 2 & -1 \\ 1 & 0 & -1 \\ -4 & 2 & -2 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{2}\begin{bmatrix} -3 & 2 & -1 \\ 1 & 0 & -1 \\ -4 & 2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -3+6+1 \\ 1+0+1 \\ -4+6+2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}. \text{ So, } x = 2,\ y = 1,\ z = 2$$

30. $2x + y - z = 1;\ x - y + z = 2;\ 3x + y - 2z = -1$

Sol. $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 1 & -2 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 1 & -2 \end{vmatrix} \Rightarrow |A| = 2(2-1) - 1(-2-3) - 1(1+3) \Rightarrow |A| = 2+5-4 \Rightarrow |A| = 3$$

Hence, A^{-1} exist.

$$\therefore \text{ adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = 1 & C_{12} = 5 & C_{13} = 4 \\ C_{21} = 1 & C_{22} = -1 & C_{23} = 1 \\ C_{31} = 0 & C_{32} = -3 & C_{33} = -3 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} 1 & 5 & 4 \\ 1 & -1 & 1 \\ 0 & -3 & -3 \end{bmatrix}' \Rightarrow \text{adj } A = \begin{bmatrix} 1 & 1 & 0 \\ 5 & -1 & -3 \\ 4 & 1 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{3}\begin{bmatrix} 1 & 1 & 0 \\ 5 & -1 & -3 \\ 4 & 1 & -3 \end{bmatrix}. \text{ Now } AX = B,\ X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{3}\begin{bmatrix} 1 & 1 & 0 \\ 5 & -1 & -3 \\ 4 & 1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \Rightarrow X = \frac{1}{3}\begin{bmatrix} 1+2-0 \\ 5-2+3 \\ 4+2+3 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1,\ y = 2$ and $z = 3$

31. $x + 2y + z = 4;\ -x + y + z = 0;\ x - 3y + z = 4$

Sol. $x + 2y + z = 4$ ...... (i)

$-x + y + z = 0$ .... (ii)

$x - 3y + z = 4$ ...... (iii)

Let $A=\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 1\\ 1 & -3 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}4\\ 0\\ 4\end{bmatrix}$

Here $AX=B$,

$\Rightarrow A^{-1}AX=A^{-1}B$

$\Rightarrow IX=A^{-1}B$

$\Rightarrow X=A^{-1}B$ ...... (iv)

Now $|A|=\begin{vmatrix}1 & 2 & 1\\ -1 & 1 & 1\\ 1 & -3 & 1\end{vmatrix}=\begin{vmatrix}0 & 2 & 1\\ -2 & 1 & 1\\ 0 & -3 & 1\end{vmatrix}\{C_1\to C_1-C_3$

$=-(-2)\begin{vmatrix}2 & 1\\ -3 & 1\end{vmatrix}=2(2+3)=10$

Since $|A|\neq 0\Rightarrow A^{-1}$ exists

Now, co-factors of A,

$A_{11}=\begin{vmatrix}1 & 1\\ -3 & 1\end{vmatrix}=1+3=4$

$A_{12}=-\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}=-(-1-1)=2$

$A_{13}=\begin{vmatrix}-1 & 1\\ 1 & -3\end{vmatrix}=3-1=2$

$A_{21}=-\begin{vmatrix}2 & 1\\ -3 & 1\end{vmatrix}=-(2+3)=-5$

$A_{22}=\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix}=(1-1)=0$

$A_{23}=-\begin{vmatrix}1 & 2\\ 1 & -3\end{vmatrix}=-(-3-2)=5$

$A_{31}=\begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}=2-1=1$

$A_{32}=-\begin{vmatrix}1 & 1\\ -1 & 1\end{vmatrix}=-(1+1)=-2$

$A_{33}=\begin{vmatrix}1 & 2\\ -1 & 1\end{vmatrix}=(1+2)=3$

Now adj(A) $=\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix}=\begin{bmatrix}4 & -5 & 1\\ 2 & 0 & -2\\ 2 & 5 & 3\end{bmatrix}$

$A^{-1}=\frac{1}{|A|}$ Adj(A) $=\frac{1}{10}\begin{bmatrix}4 & -5 & 1\\ 2 & 0 & -2\\ 2 & 5 & 3\end{bmatrix}$

Now from (iv),

$X = A^{-1}B$

$$X = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ 8+0-8 \\ 8+0+12 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

Hence $x = 2, y = 0, z = 2$

32. $x - y - 2z = 3$; $x + y = 1$; $x + z = -6$

Sol. $A = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 3 \\ 1 \\ -6 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \Rightarrow |A| = 1(1-0) + 1(1-0) - 2(0-1) \Rightarrow |A| = 1+1+2 \quad \Rightarrow |A| = 4$$

Hence, A^{-1} exist.

$$\therefore \ \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = 1 & C_{12} = -1 & C_{13} = -1 \\ C_{21} = 1 & C_{22} = 3 & C_{23} = -1 \\ C_{31} = 2 & C_{32} = -2 & C_{33} = 2 \end{bmatrix}$

$$\therefore \ \text{adj}\, A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & -1 \\ 2 & -2 & 2 \end{bmatrix}' \Rightarrow \text{adj}\, A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & -2 \\ -1 & -1 & 2 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{1}{|A|}(\text{adj}\, A) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & -2 \\ -1 & -1 & 2 \end{bmatrix}$$

Now, $AX = B$, $X = A^{-1}B \quad \Rightarrow X = \frac{1}{4}\begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & -2 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ -6 \end{bmatrix}$

$\Rightarrow X = \frac{1}{4}\begin{bmatrix} 3+1-12 \\ -3+3+12 \\ -3-1-12 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -8 \\ 12 \\ -16 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$. Hence, $x = -2$, $y = 3$ and $y = -4$

Show that each one of the following systems of equations is inconsistent.

33. $5x-y=-7$; $2x+3z=1$; $3y-z=5$.

Sol. $A=\begin{bmatrix}5 & -1 & 0\\ 2 & 0 & 3\\ 0 & 3 & -1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}-7\\ 1\\ 5\end{bmatrix}$

$$|A|=\begin{vmatrix}5 & -1 & 0\\ 2 & 0 & 3\\ 0 & 3 & -1\end{vmatrix} \Rightarrow |A|=5(0-9)+1(-2-0)+0 \Rightarrow |A|=-45-2 \Rightarrow |A|=(-47)$$

Hence, A^{-1} exist.

$$\therefore \text{ adj } A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix}C_{11}=-9 & C_{12}=2 & C_{13}=6\\ C_{21}=-1 & C_{22}=-5 & C_{23}=-15\\ C_{31}=-3 & C_{32}=-15 & C_{33}=2\end{bmatrix}$

$$\text{adj } A=\begin{bmatrix}-9 & 2 & 6\\ -1 & -5 & -15\\ -3 & -15 & 2\end{bmatrix}' \Rightarrow \text{adj } A=\begin{bmatrix}-9 & -1 & -3\\ 2 & -5 & -15\\ 6 & -15 & 2\end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{|A|}(\text{adj } A)=\frac{1}{(-47)}\begin{bmatrix}-9 & -1 & -3\\ 2 & -5 & -15\\ 6 & -15 & 2\end{bmatrix}=\frac{1}{47}\begin{bmatrix}9 & 1 & 3\\ -2 & 5 & 15\\ -6 & 15 & -2\end{bmatrix}$$

Now $AX=B$, $X=A^{-1}B \Rightarrow X=\frac{1}{47}\begin{bmatrix}9 & 1 & 3\\ -2 & 5 & 15\\ -6 & 15 & -2\end{bmatrix}\begin{bmatrix}-7\\ 1\\ 5\end{bmatrix}$

$\Rightarrow X=\frac{1}{47}\begin{bmatrix}-63+1+15\\ 14+5+75\\ 42+15-10\end{bmatrix} \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{47}\begin{bmatrix}-47\\ 94\\ 47\end{bmatrix} \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-1\\ 2\\ 1\end{bmatrix}$. Hence, $x=-1$, $y=2$ and $z=1$

34. $x-2y+z=0$; $y-z=2$; $2x-3z=10$

Sol. $A=\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 2 & 0 & -3\end{bmatrix}$, $x=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}0\\ 2\\ 10\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 2 & 0 & -3\end{vmatrix} \Rightarrow |A|=1(-3+0)+2(0+2)+1(0-2) \Rightarrow |A|=-3+4-2 \Rightarrow |A|=(-1)$$

Hence, A^{-1} exist.

$$\therefore \text{ adj } A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=-3 & C_{12}=-2 & C_{13}=-2 \\ C_{21}=-6 & C_{22}=-5 & C_{23}=-4 \\ C_{31}=1 & C_{32}=1 & C_{33}=1 \end{bmatrix}$

$$\text{adj}\,A=\begin{bmatrix} -3 & -2 & -2 \\ -6 & -5 & -4 \\ 1 & 1 & 1 \end{bmatrix}' \Rightarrow \text{adj}\,A=\begin{bmatrix} -3 & -6 & 1 \\ -2 & -5 & 1 \\ -2 & -4 & 1 \end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{|A|}(\text{adj}\,A)=\frac{1}{(-1)}\begin{bmatrix} -3 & -6 & 1 \\ -2 & -5 & 1 \\ -2 & -4 & 1 \end{bmatrix}=\begin{bmatrix} 3 & 6 & -1 \\ 2 & 5 & -1 \\ 2 & 4 & -1 \end{bmatrix}$$

Now, $AX=B,\ X=A^{-1}B=\begin{bmatrix} 3 & 6 & -1 \\ 2 & 5 & -1 \\ 2 & 4 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 10 \end{bmatrix} \Rightarrow X=\begin{bmatrix} 0+12-10 \\ 0+10-10 \\ 0+8-10 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$

Hence, $x=2,\ y=0$ and $z=-2$

35. $x-y=3$; $2x+3y+4z=17$; $y+2z=7$

Sol. $A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix},\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B=\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$$|A|=\begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix} \Rightarrow |A|=1(6-4)+1(4-0)+0 \Rightarrow |A|=2+4 \Rightarrow |A|=6$$

Hence, A^{-1} exist.

$$\therefore\ \text{adj}\,A=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11}=2 & C_{12}=-4 & C_{13}=2 \\ C_{21}=2 & C_{22}=2 & C_{23}=-1 \\ C_{31}=-4 & C_{32}=-4 & C_{33}=5 \end{bmatrix}$

$$\text{adj}\,A=\begin{bmatrix} 2 & -4 & 2 \\ 2 & 2 & -1 \\ -4 & -4 & 5 \end{bmatrix}' \Rightarrow \text{adj}\,A=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$A^{-1}=\frac{1}{|A|}(\text{adj}\,A)=\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Now, $AX=B,\ X=A^{-1}B=\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\Rightarrow X=\frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$. Hence, $x=2,\ y=-1$ and $z=4$

36. $4x+3y+2z=60;\ x+2y+3z=45;\ 6x+2y+3z=70$

Sol. The system can be written as $AX=B \Rightarrow X=A^{-1}B$ ..... (i)

Where $A=\begin{bmatrix}4&3&2\\1&2&3\\6&2&3\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}60\\45\\70\end{bmatrix}$

$|A|=4(6-6)-3(3-18)+2(2-12)=0+45-20=25\neq 0$

For adj A

$A_{11}=6-6=0$ $\quad A_{21}=-(9-4)=-5$ $\quad A_{31}=(9-4)=5$

$A_{12}=-(3-18)=15$ $\quad A_{22}=(12-12)=0$ $\quad A_{32}=-(12-2)=-10$

$A_{13}=(2-12)=-10$ $\quad A_{23}=-(8-18)=10$ $\quad A_{33}=(8-3)=5$

$$\therefore adj\ A=\begin{bmatrix}0&15&-10\\-5&0&10\\5&-10&5\end{bmatrix}^T=\begin{bmatrix}0&-5&5\\15&0&-10\\-10&10&5\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{25}\begin{bmatrix}0&-5&5\\15&0&-10\\-10&10&5\end{bmatrix}=\frac{5}{25}\begin{bmatrix}0&-1&1\\3&0&-2\\-2&2&1\end{bmatrix}=\frac{1}{5}\begin{bmatrix}0&-1&1\\3&0&-2\\-2&2&1\end{bmatrix}$$

Now putting values in (i) we get

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{5}\begin{bmatrix}0&-1&1\\3&0&-2\\-2&2&1\end{bmatrix}\begin{bmatrix}60\\45\\70\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{5}\begin{bmatrix}0-45+70\\180+0-140\\-120+90+70\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{5}\begin{bmatrix}25\\40\\40\end{bmatrix}=\begin{bmatrix}5\\8\\8\end{bmatrix}$$

Hence $x=5, y=8, z=8$

37. If $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$. Find A^{-1}.

Using A^{-1}, solve the following system of equations $:2x-3y+5z=11;\quad 3x+2y-4z=-5;$ $x+y-2z=-3$

Sol. Let $|A|=\begin{vmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{vmatrix}=2(-4+4)+3(-6+4)+5(3-2)=2(0)+3(-2)+5(1)=-6+5=-1$

$\therefore\ |A|\neq 0,\ A^{-1}$ exist. $\quad\therefore$ adj $A=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}'$.

So the cofactor of the element of matrix A are $\begin{bmatrix} C_{11}=0 & C_{12}=2 & C_{13}=1 \\ C_{21}=-1 & C_{22}=-9 & C_{23}=-5 \\ C_{31}=2 & C_{32}=23 & C_{33}=13 \end{bmatrix}$

$$\operatorname{adj} A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \quad \Rightarrow A^{-1} = \frac{1}{|A|}(\operatorname{adj} A) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

Now, $AX = B$, $X = A^{-1}.B \Rightarrow X = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \Rightarrow X = -\begin{bmatrix} 0+5-6 \\ 22+45-69 \\ 11+25-39 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Hence. $x=1,\ y=2\ \&\ z=3$

38. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the following system of linear equations :

$2x+y+z=1;\ x-2y-z=\frac{3}{2};\ 3y-5z=9$

Sol. $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix} \Rightarrow |A| = 2(10+3)-1(-5+0)+1(3+0) \Rightarrow |A| = 26+5+3 \Rightarrow |A| = 34$$

Hence, A^{-1} exist. $\quad \therefore \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$

So the cofactor of the element of matrix A are. $\begin{bmatrix} C_{11}=13 & C_{12}=5 & C_{13}=3 \\ C_{21}=8 & C_{22}=-10 & C_{23}=-6 \\ C_{31}=1 & C_{32}=3 & C_{33}=-5 \end{bmatrix}$

$$\operatorname{adj} A = \begin{bmatrix} 13 & 5 & 3 \\ 8 & -10 & -6 \\ 1 & 3 & -5 \end{bmatrix}' \Rightarrow \operatorname{adj} A = \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|}(\operatorname{adj} A) = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$. Now, $AX = B,\ X = A^{-1}B = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$

$\Rightarrow X = \frac{1}{34}\begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -3/2 \end{bmatrix}$. Hence, $x=1,\ y=\frac{1}{2}$ and $z=\frac{-3}{2}$

39. If $A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$, find AB

Hence solve the system of equations $x-2y=10,\ 2x+y+3z=8$ and $-2y+z=7$

Sol. Given $A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$

$$A\cdot B=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

$$=\begin{bmatrix}7+4+0 & 2-2+0 & -6+6+0\\ 14-2-12 & 4+1+6 & -12-3+15\\ 0+4-4 & 0-2+2 & 0+6+5\end{bmatrix}=\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}=11\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=11I$$

$$\Rightarrow A\cdot\left(\frac{1}{11}\cdot B\right)=I$$

$$\Rightarrow A^{-1}=\frac{1}{11}B=\frac{1}{11}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

Given system of equitation are

$x-2y=10$

$2x+y+32=8$

$-2y+z=7$

This system may be written as $AX=C$ where

Where $A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ & $C=\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$

Now $AX=C$

$\Rightarrow X=A^{-1}C$

$$\Rightarrow X=\frac{1}{11}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{11}\begin{bmatrix}70+16-42\\ -20+8-21\\ -40+16+35\end{bmatrix}\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{11}\begin{bmatrix}44\\ -33\\ 11\end{bmatrix}=\begin{bmatrix}4\\ -3\\ 1\end{bmatrix}$$

Hence $x=4.,y=-3$ & $z=1$

40. Using matrices, solve the following system of equations :

$$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10,\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10,\ \frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$$

Sol. $A=\begin{bmatrix}2 & -3 & 3\\1 & 1 & 1\\3 & -1 & 2\end{bmatrix}$, $X=\begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix}$, $B=\begin{bmatrix}10\\10\\13\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & -3 & 3\\1 & 1 & 1\\3 & -1 & 2\end{vmatrix} \Rightarrow |A|=2(2+1)+3(2-3)+3(-1-3) \Rightarrow |A|=6-3-12 \quad \Rightarrow |A|=(-9)$$

Hence, A^{-1} exist. $\qquad \therefore \text{ adj}\, A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\C_{21} & C_{22} & C_{23}\\C_{31} & C_{32} & C_{33}\end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix}C_{11}=3 & C_{12}=1 & C_{13}=-4\\C_{21}=3 & C_{22}=-5 & C_{23}=-7\\C_{31}=-6 & C_{32}=1 & C_{33}=5\end{bmatrix}$

$$\text{adj}\, A=\begin{bmatrix}3 & 1 & -4\\3 & -5 & -7\\-6 & 1 & 5\end{bmatrix}' \quad \Rightarrow \text{adj}\, A=\begin{bmatrix}3 & 3 & -6\\1 & -5 & 1\\-4 & -7 & 5\end{bmatrix}$$

$$\therefore \quad A^{-1}=\frac{1}{|A|}(\text{adj}\, A)=\frac{1}{(-9)}\begin{bmatrix}3 & 3 & -6\\1 & -5 & 1\\-4 & -7 & 5\end{bmatrix}=\frac{1}{9}\begin{bmatrix}-3 & -3 & 6\\-1 & 5 & -1\\4 & 7 & -5\end{bmatrix}$$

Now, $AX=B, X=A^{-1}B=\frac{1}{9}\begin{bmatrix}-3 & -3 & 6\\-1 & 5 & -1\\4 & 7 & -5\end{bmatrix}\begin{bmatrix}10\\10\\13\end{bmatrix} \quad \Rightarrow X=\frac{1}{9}\begin{bmatrix}-30-30+78\\-10+50-13\\40+70-65\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix}=\frac{1}{9}\begin{bmatrix}18\\27\\45\end{bmatrix} \quad \Rightarrow \begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix}=\begin{bmatrix}2\\3\\5\end{bmatrix}$. Hence, $x=\frac{1}{2}$, $y=\frac{1}{3}$ and $z=\frac{1}{5}$

41. $\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4;\ \frac{2}{x}+\frac{1}{y}-\frac{3}{z}=0;\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2 \ (x,y,z\neq 0)$

Sol. Given equations are

$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4 \qquad$ ….. (i), $\frac{2}{x}+\frac{1}{y}-\frac{3}{z}=0 \quad$ …. (ii), $\quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2$ …. (iii)

Let $A=\begin{bmatrix}1 & -1 & 1\\2 & 1 & -3\\1 & 1 & 1\end{bmatrix}, X\begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\0\\2\end{bmatrix}$

Here $A\cdot X=B$

$\Rightarrow X=A^{-1}B \qquad$ …. (i)

Now, $|A|=\begin{vmatrix}1 & -1 & 1\\2 & 1 & -3\\1 & 1 & 1\end{vmatrix}$

$$=\begin{vmatrix}0 & 0 & 1\\ 5 & -2 & -3\\ 0 & 2 & 1\end{vmatrix}\begin{cases}C_1 \to C_1 - C_3\\ C_2 \to C_2 + C_3\end{cases}$$

$$=\begin{vmatrix}5 & -2\\ 0 & 2\end{vmatrix} \quad \{\text{expanding by } R_1$$

$$= 10$$

$\because |A| \neq 0 \quad \therefore A^{-1}$ exists

Co-factors of A :

$$A_{11}=\begin{vmatrix}1 & -3\\ 1 & 1\end{vmatrix}=1+3=4$$

$$A_{12}=-\begin{vmatrix}2 & -3\\ 1 & 1\end{vmatrix}=-(2+3)=-5$$

$$A_{13}=\begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}=2-1=1$$

$$A_{21}=-\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}=-(1-1)=2$$

$$A_{22}=\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix}=(1-1)=0$$

$$A_{23}=-\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}=-(1+1)=2$$

$$A_{31}=\begin{vmatrix}-1 & 1\\ 1 & -3\end{vmatrix}=3-1=2$$

$$A_{32}=-\begin{vmatrix}1 & 1\\ 2 & -3\end{vmatrix}=-(-3-2)=5$$

$$A_{33}=\begin{vmatrix}1 & -1\\ 2 & 1\end{vmatrix}=1+2=3$$

Now, $\text{Adj(A)}=\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix}=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & 2 & 3\end{bmatrix}$

$$A^{-1}=\frac{1}{|A|}\cdot adj(A)=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & 2 & 3\end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix}1/x\\ 1/y\\ 1/z\end{bmatrix}=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & 2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix} \quad =\frac{1}{10}\begin{bmatrix}16+0+4\\ -20+0+10\\ 4+0+6\end{bmatrix}=\frac{1}{10}\begin{bmatrix}20\\ -10\\ 10\end{bmatrix}=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}$$

$$\therefore \frac{1}{x}=2, y=-1 \;\&\; \frac{1}{z}=1$$

$$\Rightarrow x=\frac{1}{2}, y=1 \ \& \ z=1$$

42. The sum of three numbers is 2. If twice the second number is added to the sum of first and third, we get 1. on adding the sum of second and third numbers to five times the first, we get 6. Find the three numbers by using matrices.

Sol. Let these three number, first number be x, second number by y and third number be z.

$$x+y+z=2 \qquad \ldots(1)$$
$$x+2y+z=1 \qquad \ldots(2)$$
$$5x+y+z=6 \qquad \ldots(3)$$

$$A=\begin{bmatrix}1&1&1\\1&2&1\\5&1&1\end{bmatrix}, \ X=\begin{bmatrix}x\\y\\z\end{bmatrix}, \ B=\begin{bmatrix}2\\1\\6\end{bmatrix}$$

$$|A|=1\begin{vmatrix}2&1\\1&1\end{vmatrix}-1\begin{vmatrix}1&1\\5&1\end{vmatrix}+1\begin{vmatrix}1&2\\5&1\end{vmatrix} \Rightarrow |A|=(2-1)-1(1-5)+1(1-10) \Rightarrow |A|=1+4-9=-4$$

Hence, A^{-1} exist. $\qquad \therefore \text{adj } A=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}'$

So the cofactor of the element of matrix A are, $\begin{bmatrix}C_{11}=1&C_{12}=4&C_{13}=-9\\C_{21}=0&C_{22}=-4&C_{23}=4\\C_{31}=-1&C_{32}=0&C_{33}=1\end{bmatrix}$

$$(\text{adj } A)=\begin{bmatrix}1&4&-9\\0&-4&4\\-1&0&1\end{bmatrix}' \Rightarrow (\text{adj } A)=\begin{bmatrix}1&0&-1\\4&-4&0\\-9&4&1\end{bmatrix}$$

Now, $AX=B \Rightarrow X=A^{-1}B \Rightarrow X=-\frac{1}{4}\begin{bmatrix}1&0&-1\\4&-4&0\\-9&4&1\end{bmatrix}\begin{bmatrix}2\\1\\6\end{bmatrix} \Rightarrow X=-\frac{1}{4}\begin{bmatrix}2+0-6\\8-4+0\\-18+4+6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=-\frac{1}{4}\begin{bmatrix}-4\\4\\-8\end{bmatrix} \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-1\\2\end{bmatrix}$ Hence $x=1, \ y=-1, \ z=2$

43. The cost of 4 kg potato, 3 kg wheat and 2 kg rice is Rs 60. The cost of 1 kg potato, 2 kg wheat and 3 kg rice is Rs 45. The cost of 6 kg potato, 2 kg wheat and 3 kg rice is Rs 70. Find the cost of each item per kg by matrix method.

Sol. Let these three item, potato, wheat and rice be x, y, z respectively.

$$4x+3y+2z=60 \qquad \ldots(1)$$
$$x+2y+3z=45 \qquad \ldots(2)$$
$$6x+2y+3z=70 \qquad \ldots(3)$$

$$A=\begin{bmatrix}4&3&2\\1&2&3\\6&2&3\end{bmatrix}, \ X=\begin{bmatrix}x\\y\\z\end{bmatrix}, \ B=\begin{bmatrix}60\\45\\70\end{bmatrix}$$

$$|A|=4\begin{vmatrix}2&3\\2&3\end{vmatrix}-3\begin{vmatrix}1&3\\6&3\end{vmatrix}+2\begin{vmatrix}1&2\\6&2\end{vmatrix}\quad\Rightarrow|A|=4(6-6)-3(3-18)+2(2-12)$$

$\Rightarrow |A|=4(0)-3(-15)+2(-10) \quad \Rightarrow |A|=45-20=25$. Hence, A^{-1} exist.

$$\therefore\ \text{adj } A=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix}C_{11}=0&C_{12}=15&C_{13}=-10\\C_{21}=-5&C_{22}=0&C_{23}=10\\C_{31}=5&C_{32}=-10&C_{33}=5\end{bmatrix}$

$$\text{adj } A=\begin{bmatrix}0&15&-10\\-5&0&10\\5&-10&5\end{bmatrix}\quad\Rightarrow\text{adj } A=\begin{bmatrix}0&-5&5\\15&0&-10\\-10&10&5\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(\text{adj } A)\quad=\frac{1}{25}\begin{bmatrix}0&-5&5\\15&0&-10\\-10&10&5\end{bmatrix}$$

Now $AX=B,\ X=A^{-1}B \quad \Rightarrow X=\frac{1}{25}\begin{bmatrix}0&-5&5\\15&0&-10\\-10&10&5\end{bmatrix}\begin{bmatrix}60\\45\\70\end{bmatrix}$

$$\Rightarrow x=\frac{1}{25}\begin{bmatrix}0-225+350\\900+0-700\\-600+450+350\end{bmatrix}\quad\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{25}\begin{bmatrix}125\\200\\200\end{bmatrix}\quad\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5\\8\\8\end{bmatrix}$$

$\therefore\ x=\text{Rs. }5,\ y=\text{Rs. }8,\ z=\text{Rs. }8$

44. An amount of Rs. 5000 is put into three investments at 6%, 7% and 8% per annum respectively. The total annual income from these investments is Rs 358. If the total annual income from first two investments is Rs 70 more than the income from the third, find the amount of each investment by the matrix method.

Sol. Let these investment be Rs x, Rs y and Rs. z respectively.

$x+y+z=5000 \qquad \ldots(1) \qquad \Rightarrow \frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100}=358$

$6x+7y+8z=35800 \qquad \ldots(2) \qquad$ and $\frac{6x}{100}+\frac{7y}{100}=\frac{8z}{100}+70$

$6x+7y-8z=7000 \qquad \ldots(3)$

$$A=\begin{bmatrix}1&1&1\\6&7&8\\6&7&-8\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix},\ B=\begin{bmatrix}5000\\35800\\7000\end{bmatrix}$$

$$|A|=1\begin{vmatrix}7&8\\7&-8\end{vmatrix}-1\begin{vmatrix}6&8\\6&-8\end{vmatrix}+1\begin{vmatrix}6&7\\6&7\end{vmatrix}\quad\Rightarrow|A|=1(-56-56)-1(-48-48)+1(42-42)$$

$\Rightarrow |A|=(-112)-1(-96)+1(0) \quad \Rightarrow |A|=-112+96 \quad \Rightarrow |A|=-16$

$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}'$$

So the cofactor of the element of matrix A are, $\begin{bmatrix} C_{11} = -112 & C_{12} = 96 & C_{13} = 0 \\ C_{21} = 15 & C_{22} = -14 & C_{23} = -1 \\ C_{31} = 1 & C_{32} = -2 & C_{33} = 1 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix}' \quad \Rightarrow \text{adj } A = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) \quad \Rightarrow A^{-1} = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

Now, $AX = B \Rightarrow X = A^{-1}B \Rightarrow X = \frac{1}{-16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$

$$\Rightarrow X = -\frac{1}{16}\begin{bmatrix} -560000 + 537000 + 7000 \\ 480000 - 501200 - 14000 \\ 0 - 35800 + 7000 \end{bmatrix} \quad \Rightarrow X = -\frac{1}{16}\begin{bmatrix} -16000 \\ -35200 \\ -28800 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$$

Hence $x = 1000,\ y = 2200,\ z = 1800$

45. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award Rs. X each rs. Y each and rs. Z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs. 1600. school b wants to spend Rs. 2300 to award its 4,1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value of Rs. 900, using matrices, find the award money for each value. Apart from these three values suggest one more value which should be considered for award

Sol.

Number of student of school	Sincerity	Truthfulness	Helpfulness
A	3	2	1
B	4	1	3
One student for each value	1	1	1

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ where x, y and z are rupees mentioned as it is the question, for sincerity truthfulness and

Helpfulness respectively

$E = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$ is a matrix representing total award money for school A, B and for one prize for each

value

We can represent the given question in matrix multiplication as : DX = E

Or $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$

Solution of the matrix equation exist if $|D| \neq 0$

i.e. $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} = 3[1-3]-2[4-3]+1[4-1]$

$= -6-2+3 = -5$

Therefore, the solution of the matrix equation is $X = D^{-1}E$

To find $D^{-1}; D^{-1} = \frac{1}{|D|} adj(D)$

Cofactor matrix of D

$= \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$

Adjoint of $D = adj(D)$

$= \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$ {transpose of cofactor matrix}

$D^{-1} = \frac{1}{-6}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

Now, $X = D^{-1}E$

$= \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$

$x = 200, y = 300, z = 400$

Award can also be given for punctuality