PRACTICE PAPER 15

MATHEMATICS

Time allowed: 45 minutes | **Maximum Marks: 200**

General Instructions: *As given in Practice Paper – 1.*

Section–A

Choose the correct option:

1. If matrix $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i \neq j \end{cases}$ then A^2 is equal to

(a) I (b) A (c) 0 (d) None of these

2. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x is equal to

(a) 6 (b) ± 6 (c) -6 (d) 0

3. If A is a square matrix, B is singular matrix of same order, then for a positive integer n, $(A^{-1}BA)^n$ equals to

(a) $A^{-n}B^nA^n$ (b) $A^nB^nA^{-n}$ (c) $A^{-1}B^nA$ (d) $n(A^{-1}BA)$

4. If $f(x) = x^n$, then the value of

$$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \frac{f'''(1)}{3!} \pm \ldots\ldots\ldots\ldots + \frac{(-1)^n f^n(1)}{n!} \text{ is}$$

(a) 1 (b) 0 (c) 2^n (d) 2

5. The point of intersection of the tangent drawn to the curve $x^2y = 1 - y$ at the points where it is meet by the curve $xy = 1 - y$, is given by

(a) $(0, -1)$ (b) $(1, 1)$ (c) $(0, 1)$ (d) $(-1, 0)$

6. If $\int (x^2 + 2x^4 + 3x^6)(1 + x^2 + x^4)^{1/2} dx = k(Ax^2 + Bx^4 + Cx^6)^p$ then

(a) $k = \frac{1}{3}, A = B = C = p$ (b) $k = \frac{1}{3}, A = B = C, p = \frac{3}{2}$

(c) $k = 3, p = 1/3, A = B = C$ (d) none of these

7. If $\int \sin^4 x\, e^{\log \cos x} dx = \frac{1}{k}\sin^p x + C$, then

(a) $k \neq p$ (b) $k = 3, p = 5$ (c) $k = p = 5$ (d) $k = p = -5$

8. If $f(x) = \int_0^x t \sin t \, dt$, then $f'(x)$ is

(a) $\cos x + x \sin x$ (b) $x \sin x$ (c) $x \cos x$ (d) $\sin x + x \cos x$

9. The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) \, dx$ is

(a) 0 (b) 2 (c) π (d) 1

10. Area lying between the curves $y^2 = 4x$ and $y = 2x$ is (in square units)

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

11. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$ are respectively

(a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4

12. The solution of $\frac{dy}{dx} = 2^{y-x}$ is

(a) $2^x + 2^y = C$ (b) $2^x - 2^y = C$ (c) $\frac{1}{2^x} - \frac{1}{2^y} = C$ (d) $\frac{1}{2^x} + \frac{1}{2^y} = C$

13. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of $Z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0), then

(a) $a = 2b$ (b) $2a = b$ (c) $a = b$ (d) $3a = b$

14. Consider the probability distribution of a random variable X

X	0	1	2	3	4
$P(X)$	0.1	0.25	0.3	0.2	0.15

then the variance of X is

(a) 1.4475 (b) 0.4575 (c) 1.5475 (d) None of these

15. An unbiased coin is tossed n times. If the probability of getting 5 heads is equal to the probability of getting 6 heads then probability of getting 3 heads, is

(a) ${}^{11}C_5\left(\frac{1}{2}\right)^5$ (b) ${}^{11}C_6\left(\frac{1}{2}\right)^6$ (c) ${}^{11}C_3\left(\frac{1}{2}\right)^{11}$ (d) $\frac{11}{1024}$

Section–B (B1)

16. The relation "greater than" denoted by > in the set of integers is

(a) Symmetric (b) Reflexive (c) Transitive (d) None of these

17. If R_1 and R_2 are symmetric relations in a set A, then $R_1 \cup R_2$ is

(a) Reflexive (b) Symmetric (c) Transitive (d) None of these

18. The function $f: R \to R$ defined by $f(x) = 4^x + 4^{|x|}$ is

(a) one-one and into (b) one-one and onto
(c) many one and into (d) many one and onto

19. Let * be binary operation on $\mathbb{R}$ (set of reals) such that $a * b = a + b - 2$ then $(\sqrt{3} * \sqrt{2}) * 2$ is equal to

(a) $\sqrt{3} + \sqrt{2}$ (b) $\sqrt{3} - \sqrt{2}$
(c) $\sqrt{3} + \sqrt{2} + 2$ (d) $\sqrt{3} + \sqrt{2} - 2$

20. The inverse of the function $f(x) = \frac{e^x - 2e^{-x}}{e^x + 2e^{-x}} + 1$ is

(a) $\log_{10}\frac{2x}{2-x}$ (b) $\log_{10}\frac{x}{2-x}$ (c) $\log_e\left(\frac{2x}{2-x}\right)^{1/2}$ (d) $\log\left(\frac{1}{2-x}\right)^{1/2}$

21. **The number of solution of the equation $\tan^{-1}\left(\frac{x}{3}\right)+\tan^{-1}\left(\frac{x}{2}\right)=\tan^{-1}x$ is**

(a) 2 (b) 3 (c) 0 (d) 1

22. **The greatest and least values of $(\sin^{-1}x)^3+(\cos^{-1}x)^3$ are respectively**

(a) $\frac{\pi^3}{8}$ and $\frac{\pi^3}{8}$ (b) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (c) $\frac{7\pi^3}{8}$ and $\frac{\pi^3}{32}$ (d) $\frac{\pi^3}{32}$ and $\frac{\pi^3}{8}$

23. **If $u=\cot^{-1}\sqrt{\tan\alpha}-\tan^{-1}\sqrt{\tan\alpha}$, then $\tan\left(\frac{\pi}{4}-\frac{u}{2}\right)$ is equal to**

(a) $\sqrt{\tan\alpha}$ (b) $\sqrt{\cot\alpha}$ (c) tan a (d) cot a

24. **If $\cos^{-1}x>\sin^{-1}x$, then**

(a) $\frac{1}{\sqrt{2}}<x\le 1$ (b) $0\le x<\frac{1}{\sqrt{2}}$ (c) $-1\le x<\frac{1}{\sqrt{2}}$ (d) $x>0$

25. **On using elementary column operation $C_2\to C_2-C_1$ in the following matrix equation $\begin{bmatrix}5&1\\2&0\end{bmatrix}=\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&1\\2&0\end{bmatrix}$, we get**

(a) $\begin{bmatrix}5&-4\\2&-2\end{bmatrix}=\begin{bmatrix}1&1\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&0\\2&-2\end{bmatrix}$ (b) $\begin{bmatrix}5&-4\\2&-2\end{bmatrix}=\begin{bmatrix}1&1\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&1\\2&0\end{bmatrix}$

(c) $\begin{bmatrix}5&-4\\2&-2\end{bmatrix}=\begin{bmatrix}1&2\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&0\\2&-2\end{bmatrix}$ (d) none of these

26. **If $A=\begin{bmatrix}\alpha&\beta\\\gamma&\delta\end{bmatrix}$ is such that $A^2=I$ (identity matrix) then**

(a) $1+\alpha^2+\beta\gamma=0$ (b) $1-\alpha^2+\beta\gamma=0$ (c) $1-\alpha^2-\beta\gamma=0$ (d) $1+\alpha^2-\beta\gamma=0$

27. **Which of the following is correct?**

(a) Determinant is a square matrix.
(b) Determinant is a number associated to a matrix.
(c) Determinant is a number associated to a square matrix.
(d) None of these

28. **For what values of a and b the system of equations**

$$2x+ay+6z=8$$
$$x+2y+bz=5$$
$$x+y+3z=4$$

has a unique solution?

(a) $a=2, b=3$ (b) $a\ne 2, b\ne 3$ (c) $a=-2, b=-3$ (d) None of these

29. ***Read the following statements.***

Statement I : If the function $f(x)$ defined by $f(x)=\begin{cases}\frac{\log(1+ax)-\log(1-bx)}{x} & ,\text{ if } x\ne 0\\ k & ,\text{ if } x=0\end{cases}$ is continuous at $x=0$, then $k=a+b$.

Statement II : We say function $f(x)$ is continuous at $x=a$, if $\lim_{x\to a^-}f(x)=\lim_{x\to a^+}f(x)\ne f(a)$

Choose the correct option:

30. If $f(x)$, $g(x)$, $h(x)$ are polynomials in x of degree 2 and $F(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$, then $F'(x)$ is equal to

(a) -1 (b) 2 (c) 0 (d) none of these

31. If g is inverse function of f and $f'(x) = \sin x$, then $g'(x)$ is

(a) $\sin(g(x))$ (b) $\sin^{-1} x$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\text{cosec}(g(x))$

32. The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ at $x = 0$

(a) is continuous.

(b) has removable discontinuity.

(c) has jump discontinuity.

(d) has oscillating discontinuity.

33. The global minimum value of $f(x) = x^4 - x^2 - 2x + 6$ is

(a) 6 (b) 8 (c) 4 (d) does not exists

34. $\int \frac{x^2 - 1}{(x^4 + 3x^2 + 1)\tan^{-1}\left(x + \frac{1}{x}\right)} dx$ equals

(a) $\tan^{-1}\left(x + \frac{1}{x}\right) + C$

(b) $\cot^{-1}\left(x + \frac{1}{x}\right) + C$

(c) $\log\left(x + \frac{1}{x}\right) + C$

(d) $\log\left[\tan^{-1}\left(x + \frac{1}{x}\right)\right] + C$

35. *Read the following statements.*

Statement I : The value of $\int_{\pi/6}^{\pi/4} \text{cosec}\, x\, dx$ is equal to $\log\left|\frac{\sqrt{2}-1}{2-\sqrt{3}}\right|$.

Statement II : If $f: [a, b] \longrightarrow \mathbb{R}$ and let $a < c < b$, then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Choose the correct option:

(a) Statement I is correct but statement II is not correct.

(b) Statement II is correct but statement I is not correct.

(c) Both statements I and II are correct.

(d) None of these

36. *Read the following statements.*

Statement I : $\int_a^b f(x)\, dx = \int_b^a f(x)dx$

Statement II : Let f be a continuous function defined on the closed interval $[a, b]$ and F is antiderivative of f.

Then $\int_a^b f(x)\, dx = [F(x)]_a^b = F(b) - F(a)$

Choose the correct option:

(a) Statement I is correct but statement II is not correct.

(b) Statement II is correct but statement I is not correct.

(c) Both statements I and II are correct.

(d) None of these

37. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is (in square units)

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

38. **The differential equation having $y = (\sin^{-1} x)^2 + A(\cos^{-1} x) + B$, where A and B are arbitrary constant, is**

(a) $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$ (b) $(1-x^2)\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$

(c) $(1-x)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ (d) None of these

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39. **The general solution of $\frac{dy}{dx} = 2xe^{x^2-y}$ is**

(a) $e^{x^2-y} = C$ (b) $e^{-y} + e^{x^2} = C$ (c) $e^y = e^{x^2} + C$ (d) $e^{x^2+y} = C$

40. **If $\vec{a}$ and $\vec{b}$ are unit vectors, then the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\,\vec{a} - \vec{b}$ to be a unit vector is**

(a) 30° (b) 45° (c) 60° (d) 90°

41. **If $|\vec{a}| = 3$ and $-1 \le k \le 2$, then $|k\vec{a}|$ lies in the interval**

(a) [0, 6] (b) [–3, 6] (c) [3, 6] (d) [1, 2]

42. **The position vector of the point which divides the joining of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3 : 1 is**

(a) $\frac{3\vec{a} - 2\vec{b}}{2}$ (b) $\frac{7\vec{a} - 8\vec{b}}{4}$ (c) $\frac{3\vec{a}}{4}$ (d) $\frac{5\vec{a}}{4}$

43. **If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then value of $|\vec{a} \times \vec{b}|$ is**

(a) 5 (b) 10 (c) 14 (d) 16

44. **Co-ordinates of the point where the line joining the points (– 3, 2, 4) and (3, 4, – 5) meets the ZX plane is**

(a) (– 9, 0, 13) (b) (9, 0, 13) (c) (– 9, 0, 12) (d) (9, 0, 12)

45. **If the distance of the point $A(8, 6, 10)$ from the x axis is α, then the value of α^2 is**

(a) 136 (b) 134 (c) 138 (d) None of these

46. **The shortest distance between the skew lines $\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$ is**

(a) 9 units (b) 10 units (c) 8 units (d) None of these

47. **The equation of the plane which is perpendicular to the plane $3x + 5y - 6z - 2 = 0$ and which contains the line of intersection of the planes $2x - 3y + z - 4 = 0$ and $x + y - 3z + 5 = 0$ is**

(a) $67x - 63y - 19z + 29 = 0$ (b) $67x - 63y + 19z - 29 = 0$

(c) $67x + 63y - 19z - 29 = 0$ (d) $67x - 63y - 19z - 29 = 0$

48. **A and B are two events $P(A \cup B) = \frac{5}{6}$ and $P(A \cap B) = \frac{1}{3}, P(\overline{B}) = \frac{1}{2}$, then the events A and B are**

(a) dependent (b) independent (c) mutually exclusive (d) none of these

49. **It is given that events A and B are such that $P(A) = \frac{1}{4}, P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Then $P(B)$ is**

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

50. **If $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B/A) + P(A/B)$ is equal to**

(a) $\frac{11}{4}$ (b) $\frac{11}{3}$ (c) $\frac{5}{12}$ (d) $\frac{7}{12}$