Maharashtra State Board
Class X Mathematics - Geometry
Board Paper – 2017

Time: 2 hours **Maximum Marks: 40**

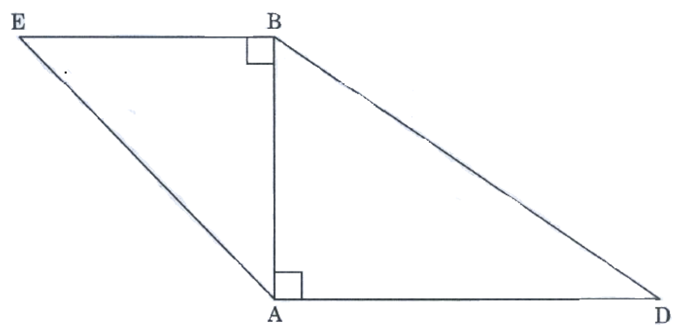
General Instructions:

(i) Solve all questions. Draw diagrams wherever necessary
(ii) Use of calculator is not allowed
(iii) Diagram is essential for writing the proof of the theorem.
(iv) Marks of constructions should be distinct. They should not be rubbed off.

---

1. Solve any five sub-equations **5**

(i) In the following figure, seg BE $\perp$ seg AB and seg BA $\perp$ seg AD. If BE = 6 and AD = 9 find $\frac{A(\Delta ABE)}{A(\Delta BAD)}$.



(ii) If two circles with radii 8 cm and 3 cm respectively touch internally, then find the distance between their centres.

(iii) Find the height of an equilateral triangle whose side is 6 units.

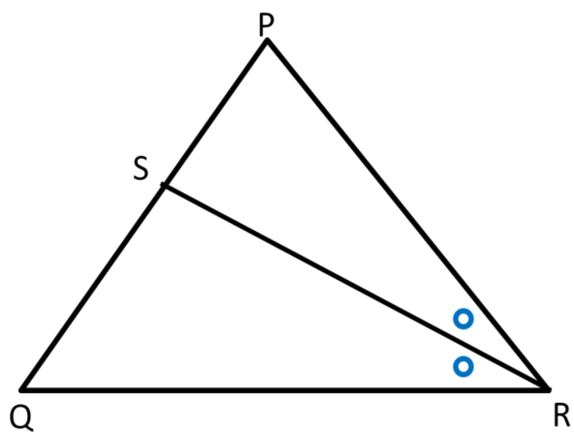
(iv) If the angle $\theta = -45^{\circ}$, find the value of tan θ.

(v) Find the slope and y - intercept of the line y = 3x – 5.

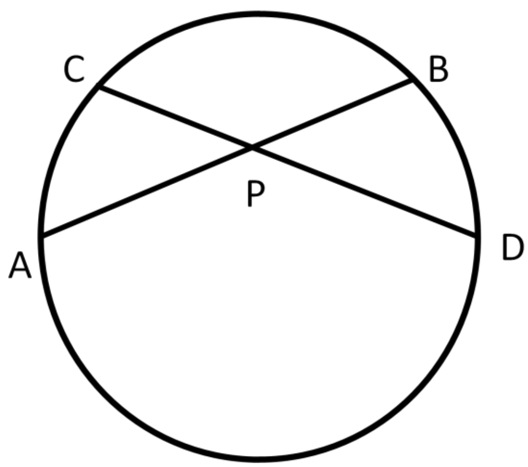
(vi) Find the circumferences of a circle whose radius is 7 cm.

2. Solve any four sub-questions **8**

(i) In Δ PQR, seg RS is the bisector of $\angle$PRQ, PS = 6, SQ = 8, PR = 12. Fin QR.



(ii) In the given figure PA = 10, PB = 2 and PC = 5. Find PD.



(iii) Draw $\angle$ ABC of measures 135^o and bisect it.

(iv) Find the sine ratio of θ in standard position whose terminal arm passes through (3,4)

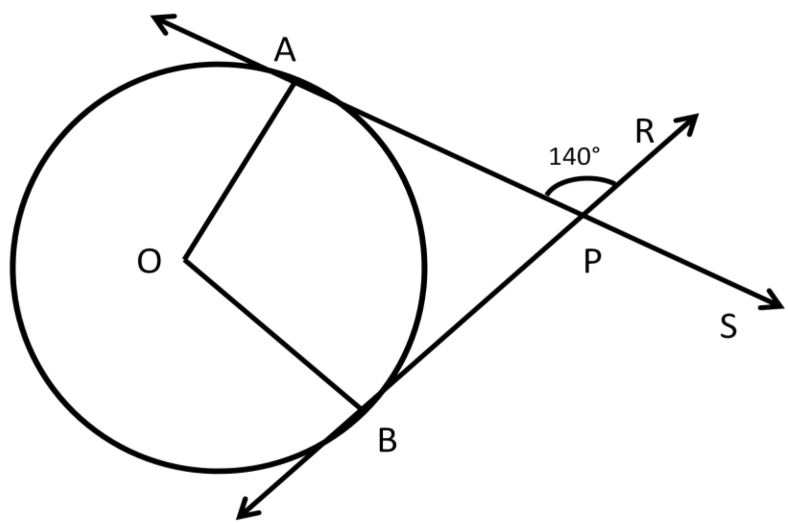
(v) Find the slope of the line passing through the points G(4,5) and H (-1,-2).

(vi) The dimensions of a cuboid in cm are $50 \times 18 \times 10$. Find its volume.

3. Solve any three sub-questions : **9**

(i) Prove that: If the angles of a triangle are $45^\circ - 45^\circ - 90^\circ$, then each of the perpendicular sides is $\frac{1}{\sqrt{2}}$ times the hypotenuse."

(ii) Find the angle between two radii at the centre of the circle as shown in the figure. Lines PA and PB are tangents to the circle at other ends of the radii and $\angle APR = 140^\circ$



(iii) Construct tangents to the circle from the point B, having radius 3.2 cm and centre 'C'. Point B is at a distance 7.6 cm from the centre.

(iv) From the top of a lighthouse, an observer looks at a ship and finds the angle of depression to be 60°. If the height of the lighthouse is 90 metres, then find how far is that ship from the lighthouse? $\left(\sqrt{3}=1.73\right)$

(v) The volume of a cube is 343 cm^3. Find its total surface area.

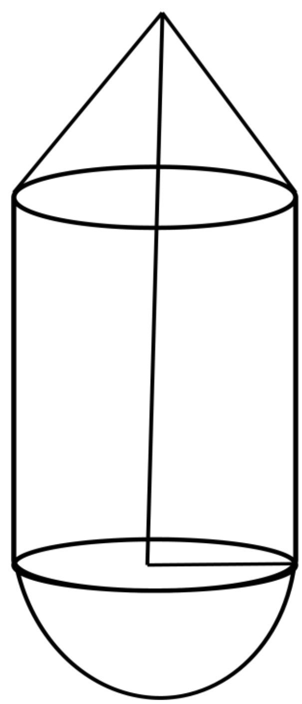
4. Solve any two sub questions : **8**

(i) Prove that "The opposite angles of a cyclic quadrilateral are supplementary".

(ii) Eliminate θ, if

$$x = 3 \operatorname{cosec} \theta + 4 \cot \theta$$
$$y = 4 \operatorname{cosec} \theta - 3 \cot \theta$$

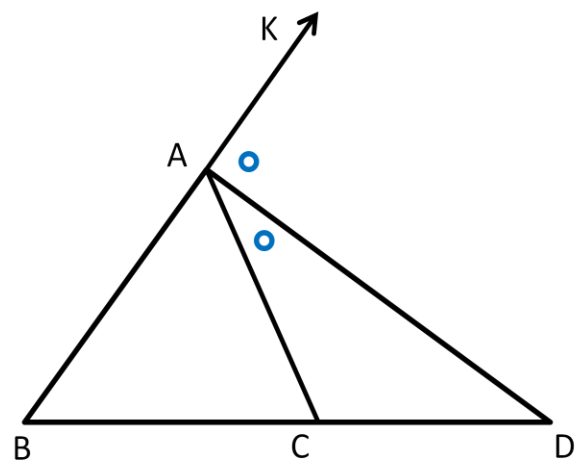
(iii) A toy is a combination of a cylinder, hemisphere and a cone, each with radius 10 cm as shown in the figure. Height of the conical part is 10 cm and total height is 60 cm. Find the total surface area of the toy.
(π=3.14, $\sqrt{2}$=1.41)



5. Solve any two sub-questions : **10**

(i) In the given figure, AD is the bisector of the exterior $\angle A$ of $\triangle ABC$. Seg AD intersects the side BC produced in D. Prove that :

$$\frac{BD}{CD} = \frac{AB}{AC}$$



(ii) Construct the circumcircle and incircle of an equilateral $\triangle XYZ$ with side 6.5 cm and centre O. Find the ratio of the radii of incircle and circumcircle.

(iii) A (5, 4), B (-3,-2) and C (1,-8) are the vertices of a triangle ABC. Find the equation of median AD and line parallel to AB passing through point C.

Maharashtra State Board
Class X Mathematics - Geometry
Board Paper – 2017 Solution

1.

i. Given that seg BE $\perp$ seg AB and seg BA $\perp$ seg AD

In ΔABE and ΔBAD,

Base for the both triangles is same.

Hence,

$$\frac{A(\Delta ABE)}{A(\Delta BAD)} = \frac{BE}{AD}$$

$$= \frac{6}{9} = \frac{2}{3}$$

ii. Given that the two circles touch eachother internally.

So, distance between their centres $= 8 - 3 = 5$ cm.

iii. Given that side of an equilateral triangle is 6 units.

Height of an equilateral triangle

$$= \frac{\sqrt{3}}{2} \times \text{side}$$

$$= \frac{\sqrt{3}}{2} \times 6$$

$$= 3\frac{\sqrt{3}}{2} \text{ units}$$

iv. the angle $\theta = -45^\circ$

then,

$$\tan(-45^\circ) = -\tan 45^\circ = -1$$

v. Given line is $y = 3x - 5$.

Comparing with $y = mx + c$,

where m is slope of the line and c is y – intercept of the line.

$m = 3$ and $c = -5$

vi. The circumferences of a circle whose radius is 7 cm is,

$$2\pi r = 2 \times \frac{22}{7} \times 7$$

$$= 44 \text{ cm}$$

2.

i. In ΔPQR, seg RS is the bisector of ΔPRQ,

Hence, using angle bisector theoem,

$$\frac{PR}{QR} = \frac{PS}{SQ}$$

Put $PS = 6, SQ = 8, PR = 12$,

$$\frac{12}{QR} = \frac{6}{8}$$

$$QR = 12 \times \frac{8}{6}$$

$QR = 16$ units

ii. From the given diagram,

$$PA \times PB = PC \times PD$$

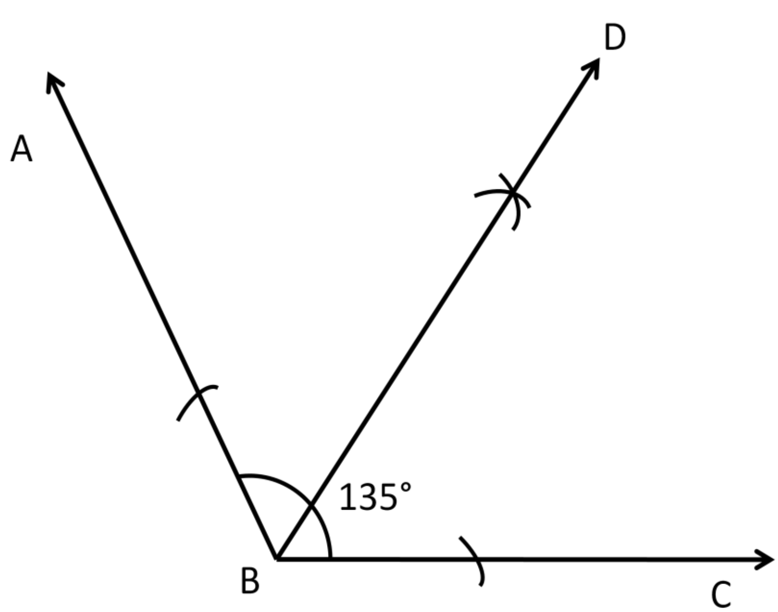
Given that $PA = 10, PB = 2$ and $PC = 5$,

$$10 \times 2 = 5 \times PD$$

$$\frac{10 \times 2}{5} = PD$$

$PD = 4$ units

iii.



iv. The standard position whose terminal arm passes through $(3,4)$.

then $x = 3$ and $y = 4$.

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{3^2 + 4^2}$

$r = 5$ units

Hence, $\sin\theta = \frac{y}{r} = \frac{4}{5}$

v. The points on the line are $G(4,5)$ and $H\ (-1,-2)$.

Let, $G(x_1, y_1) = (4,5)$ and $H(x_2, y_2) = (-1,-2)$

The slope of the line GH is,

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-2-5}{-1-4}$

$m = \frac{-7}{-5}$

$m = \frac{7}{5}$

vi.

The dimensions of a cuboid in cm are $50 \times 18 \times 10$.

Volume of cuboid

$= l \times b \times h$

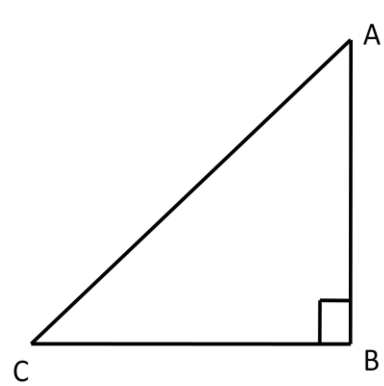
$= 50 \times 18 \times 10$

$= 900\,\text{cm}^3$

The volume of cuboid is 900 cm^3.

3.

i.



$\text{Given : In } \Delta ABC, \angle A = \angle C = 45^o \text{ and } \angle D = 90^o$

$\text{To Prove : } AB = BC = \frac{1}{\sqrt{2}} AC$

Proof :

In ΔABC,

$\angle A = \angle C = 45^o$ .....(Given)

$\therefore AB = BC$ .....(i)(Side opposite to congruent angles)

In ΔABC,

$AC^2 = AB^2 + BC^2$ ......(By Pythagoras theorem)

$AC^2 = AB^2 + AB^2$ ....(From (i))

$AC^2 = 2AB^2$

$\therefore \frac{1}{2} AC^2 = AB^2$

$\therefore \frac{1}{\sqrt{2}} AC = AB$ ......(By taking square root)

$AB = BC = \frac{1}{\sqrt{2}} AC$ ......(From (i) and (ii))

$\therefore$ If the angles of a triangle are $45^o - 45^o - 90^o$, then each of the perpendicular sides is $\frac{1}{\sqrt{2}}$ times the hypotenuse.

ii.

Given that lines PA and PB are tangents to the circle at other ends of the radii and $\angle APR = 140^o$

$\angle APR + \angle APB = 180^o$ .....(Linear pairs)

$140^o + \angle APB = 180^o$

$\angle APB = 180^o - 140^o$

$\angle APB = 40^o$

Consider, □AOPB,

$\angle OAP = OBP = 90^o$ .....(PA and PB are tangents)

$\angle OAP + OBP = 180^o$

Hence,

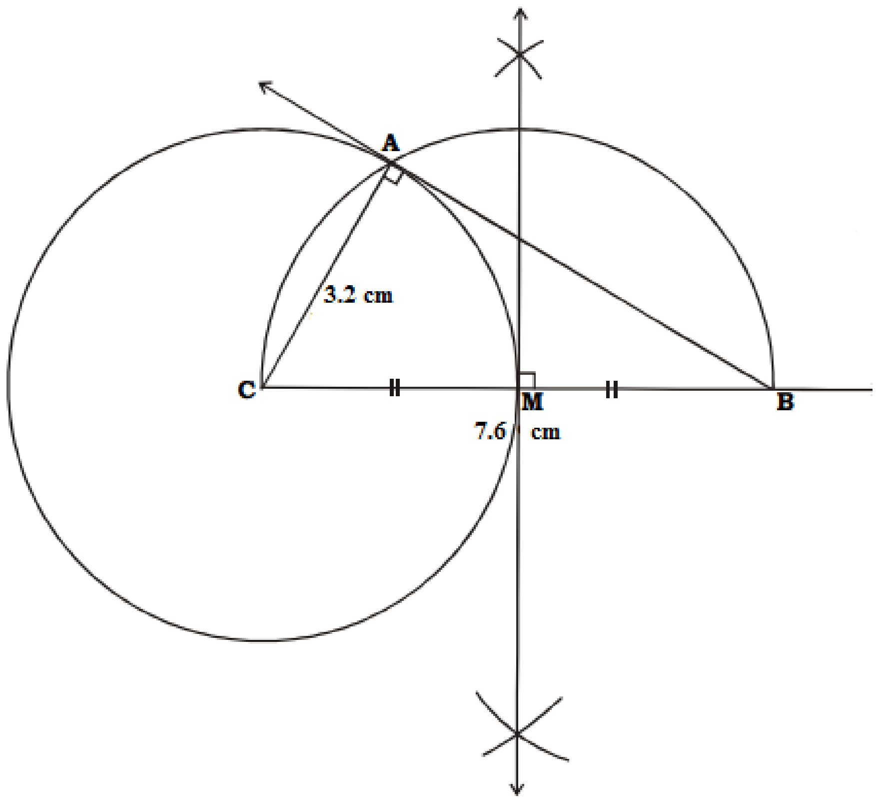
$\angle APB + \angle AOB = 180^o$

$40^o + \angle AOB = 180^o$

$\angle AOB = 140^o$

Angle between two radii at the centre of the circle is 140^o.

iii.



Steps of construction :

1. Draw a circle with radius 3.2 cm. Let C be the centre of the circle.

2. Take a point B such that CB = 7.6 cm.

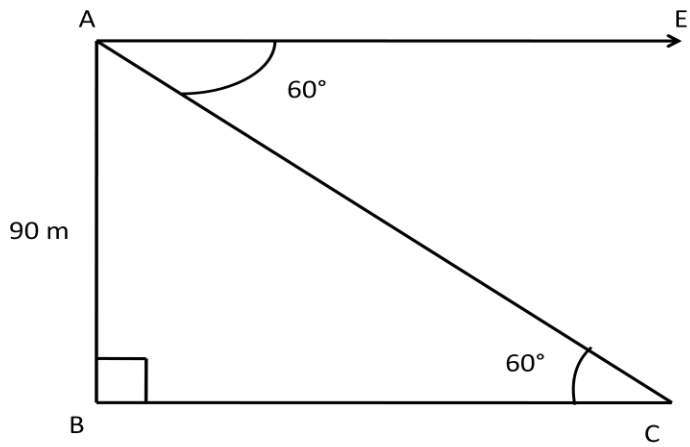
3. Draw perpendicular bisector of seg CB and mark the midpoint of seg CB as ‘M’.

4. With ‘M’ as a centre and radius MP draw a semicircle .

5. Let ‘A’ be the point of intersection of semicircle and the circle.

6. Draw a line joining B and A. Line BA is the required tangent.

iv.



Let AB be the height of lighthouse.

$\Rightarrow AB = 90\,m$ .....(Given)

The point 'C' such that $\angle ACB = 60^o$.

$\tan 60^o = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{90}{BC}$

$\Rightarrow BC = \frac{90}{\sqrt{3}}$

$\Rightarrow BC = 30\sqrt{3}$

$\Rightarrow BC = 51.9\,m.$

The ship is 51.9 m is away from lighthouse.

v.

Volume of a cube is 343 cm^3.

Volume of cube $= 343$

$a^3 = 7^3$

$a = 7\,cm$

Total surface area of a cube $= 6s^2$

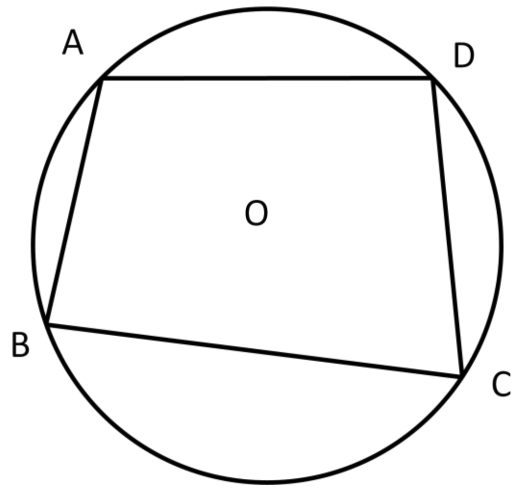
$= 6 \times 7 \times 7$

$= 294\,cm^2$

Total surface area of a cube is 294 cm^2.

4.

i.



Given : □ABCD is cyclic quadrilateral.

To prove : $\angle BAD + \angle BCD = 180^\circ$ and $\angle ABC + \angle ADC = 180^\circ$

Proof :

Arc BCD is intercepted by the inscribed $\angle BAD$.

$\therefore \angle BAD = \frac{1}{2} m(\text{arcBCD})$ .....(1)(Inscribed angle theorem)

Arc BAD is intercepted by the inscribed $\angle BCD$.

$\therefore \angle BCD = \frac{1}{2} m(\text{arcDAB})$ .....(2)(Inscribed angle theorem)

From (1) and (2), we get

$$\angle BAD + \angle BCD = \frac{1}{2}[m(\text{arcBCD}) + m(\text{arcDAB})]$$

$$= \frac{1}{2} \times 360^\circ$$

$$= 180^\circ$$

Again, as the sum of the measures of angles of a quadrilateral is 360°.

$$\therefore \angle ADC + \angle ABC = 360^\circ - (\angle BAD + \angle BCD)$$

$$= 360^\circ - 180^\circ$$

$$= 180^\circ$$

Hence, the opposite angles of a cyclic quadrilateral are supplementary.

ii.

$\text{Let } x = 3\operatorname{cosec}\theta + 4\cot\theta \quad .....(i)$

$y = 4\operatorname{cosec}\theta - 3\cot\theta \quad ......(ii)$

Multiplying by 4 and 3 to (i) and (ii) respectively,

$4x = 12\operatorname{cosec}\theta + 12\cot\theta \quad ....(iii)$

$3y = 12\operatorname{cosec}\theta - 12\cot\theta \quad ....(iv)$

Consider,

Subtracting (iv) from (iii),

$4x - 3y = 24\cot\theta$

$$\cot\theta = \frac{4x-3y}{24}$$

$$\cot^2\theta = \left(\frac{4x-3y}{24}\right)^2 \quad .....(v)$$

Adding (iii) and (iv),

$4x + 3y = 24\operatorname{cosec}\theta$

$$\operatorname{cosec}\theta = \frac{4x+3y}{24}$$

$$\operatorname{cosec}^2\theta = \left(\frac{4x+3y}{24}\right)^2 \quad ....(vi)$$

Using $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\left(\frac{4x+3y}{24}\right)^2 - \left(\frac{4x-3y}{24}\right)^2 = 1$$

$$(4x+3y)^2 - (4x-3y)^2 = 24^2$$

$$(4x+3y)^2 - (4x-3y)^2 = 576$$

iii.

radius of the cylinder, hemisphere and cone = 10 cm

height of the conical part = 10 cm

total height = 60 cm

Height of the conical part(h) = 10 cm

Height of the hemisperical part = its radius = 10 cm

So, height if the cylindrical part $(h_1) = 60 - 10 - 10 = 40$ cm

$l^2 = r^2 + h^2$

$\Rightarrow l^2 = 10^2 + 10^2$

$\Rightarrow l^2 = 200$

$\Rightarrow l = 10\sqrt{2}$ cm $= 10 \times 1.41 = 14.1$ cm

Total surface area of the toy

= curved surface area of the cone

+curved surface area of the cylinder

+curved surface area of the hemisphere

$= \pi rl + 2\pi rh + 2\pi r^2$

$= \pi r(l + 2h + 2r)$

$= 3.14 \times 10(14.1 + 2 \times 40 + 2 \times 10)$

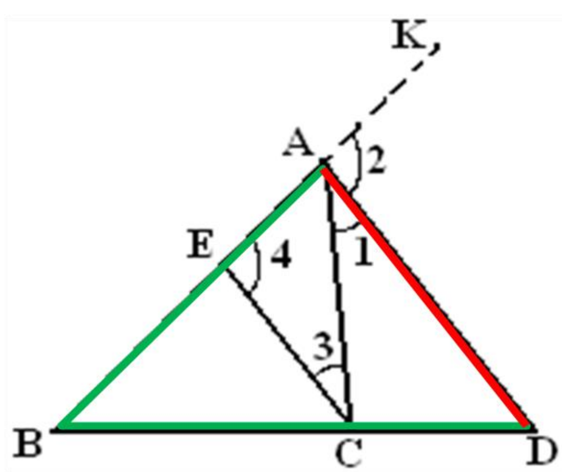
$= 31.4(114.1)$

$= 3582.74 \text{ cm}^2$

Hence, the total surface area of the toy is 3582.74 cm^2.

5.

(i)



Given : AD is the bisector of the exterior $\angle A$ and intersects BC produced in D.

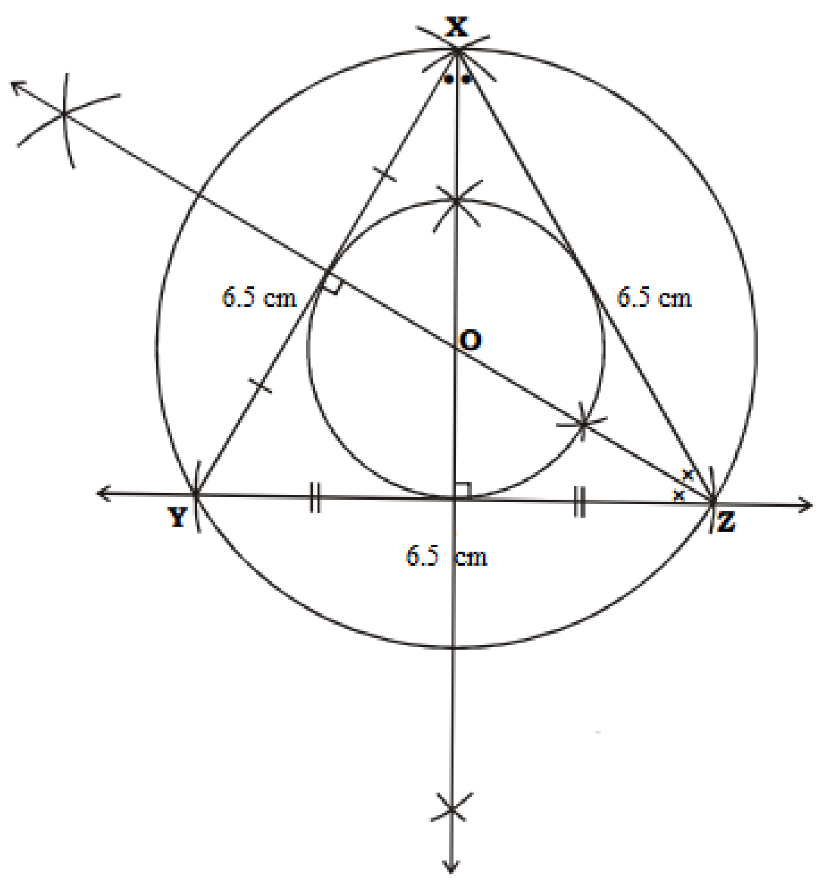
Prove that : $\frac{BD}{CD} = \frac{AB}{AC}$

Construction : Draw CE || DA meeting AB in E.

Proof :

CE || DA	......(By construction)
$\angle 1 = \angle 3$	......(Alternate interior angle)
$\angle 2 = \angle 4$	......(Corresponding angles since CE||DA and BK is a transversal
AD is a bisector of $\angle A$	......(Given)
$\angle 1 = \angle 2$	......(AD is the bisector of the exterior $\angle A$)
$\angle 3 = \angle 4$	.....(Since $\angle 1 = \angle 3$)
AE = AC	......(If angles are equal then side opposite to them are also equal
$\Rightarrow \frac{BD}{CD} = \frac{AB}{EA}$	......[By Basic proportionality theorem(EC || AD)]
$\Rightarrow \frac{BD}{CD} = \frac{AB}{AC}$	.......[Since AE = EC]

(ii)



Steps of construction :

1. Construct an equilateral ΔABC.
2. Draw perpendicular bisectors of any two sides of ΔABC at point O.
3. Draw a circle with centre O and radius OA.
4. This circle is the circumcircle of ΔABC.
5. Next draw the angle bisector of any two angles of the ΔABC.
6. Draw a circle with centre O and radius equal to the distance from the centre to the sides.
7. This is the incircle of ΔABC.

(iii)

AD is given to be the median on BC.

So, it divides BC in two halves.

D(x, y) = mid - point of BC

$$\Rightarrow D(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\Rightarrow D(x,y) = \left(\frac{-3+1}{2}, \frac{-2-(-8)}{2}\right)$$

$$\Rightarrow D(x,y) = (-1,3)$$

Using the slope - point form,

$$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-4}{-1-5} = \frac{1}{6}$$

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 3 = \frac{1}{6}(x - (-1))$$

$$\Rightarrow 6y - 18 = x + 1$$

$$\Rightarrow x - 6y = -19$$

which is the required equation of median AD.

Since the line is parallel to AB, slope of AB = slope of the line

$$\Rightarrow \text{slope of the line} = \frac{-2-4}{-3-5} = \frac{3}{4}$$

Using $y - y_1 = m(x - x_1)$

$$\Rightarrow y - (-8) = \frac{3}{4}(x - 1)$$

$$\Rightarrow 4y + 32 = 3x - 3$$

$$\Rightarrow 3x - 4y = -35$$

which is the equation of the required line.