Modern Maths

(i) Permutation & Combination

a. Factorial

Factorial of a natural number N is defined as the product of first N natural numbers. 'Factorial' is represented with a symbol '!' or '$\lfloor\;$' (not alphabet).

Example : $\lfloor 5$ or 5! = 1 × 2 × 3 × 4 × 5.

Example : $\frac{10!}{8!} = \frac{10 \times 9 \times 8!}{8!} = 10 \times 9 = 90$.

Note:
Factorial of zero is 1 (0! = 1)

b. Fundamental principle of counting

To make the counting simpler, there are two basic principles

1. Basic principle of multiplication

Suppose there are two ways of reaching railway station from your home and from railway station there are three ways of reaching airport. So the total number of ways of reaching airport from your home via railway station is 2 × 3 = 6.

Therefore, in general we can say that if a task can be done in ***x*** ways and the other task can be done in ***y*** ways, then the number of ways in which the two tasks can be in succession is ***x* × *y***.

Example :

There are 10 boys and 8 girls in a school. The class teacher wants to select one boy and one girl for the post of class monitors. In how many ways can he do this selection?

Solution :

For every boy he can select any one of the 8 girls. So for one boy he has 8 choices.

Hence, for 10 boys he has 10 × 8 = 80 choices, which is nothing but

$$^{10}C_1 \times {}^{8}C_1 = \frac{10!}{1!(10-1)!} \times \frac{8!}{1!\,(8-1)!} = 10 \times 8$$

2. Basic principle of addition

Suppose you are at a railway station and you want to go to either airport or your home. So you can do this task in 2 ways (if you are going home) + 3 ways (if you are going airport) = 5 ways.

So in general we can say that if a task can be done in ***x*** ways and the other task can be done in ***y*** ways, then either of the tasks can be done in ***x* + *y*** ways.

Example :

There are 10 boys and 8 girls in a school. The class teacher wants to select either a boy or a girl for the post of the class monitor. In how many ways can he do this selection?

Solution :

He can select one boy out of 10 boys in 10 ways.

He can select one girl out of 8 girls in 8 ways.

He can select either a boy or a girl in 10 + 8 = 18 ways.

Notes:

1. If all the tasks are correlated, then basic principle of multiplication is used.
2. If all the tasks are independent, then basic principle of addition is used.

Solved Examples

1. How many three-digit numbers are there?

Solution :

We know that there are 10 digits i.e. (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) which is used for the formation of the required numbers.

Now '0' cannot occupy the hundred's place, so 100's place can be filled in 9 ways.

Also 10's place can be filled in 10 ways and similarly unit's place can be filled in 10 ways.

Hence, the total number of three-digit numbers

= 9 × 10 × 10 = 900

2. How many three-digit numbers are there in which all the digits are distinct?

Solution :

100's place can be filled in 9 ways.

10's place can be filled in 9 ways.

Unit's place can be filled in 8 ways.

So the total number of three-digit numbers in which all the digits are distinct = 9 × 9 × 8 = 648

3. There are 5 multiple-choice questions in a test paper. First three questions have 4 answer choices each and other two questions have 5 answer choices each. How many sequences of answers are possible, if it is mandatory to attempt all the questions?

Solution :

Each one of the first three questions can be dealt in 4 ways, and each one of the last two questions can be dealt in 5 ways.

So the total number of different sequences are

$4 \times 4 \times 4 \times 5 \times 5 = 4^3 \times 5^2 = 1600$

4. How many even numbers less than 1000 can be formed by using the digits 2, 4, 3 and 5?

Solution :

All the numbers of one digit, two digits and three digits are less than 1000.

Let us take these cases one by one:

1. Single-digit even numbers are 2 and 4.

2. Number of two-digit even numbers:

Unit's place can be filled in 2 ways i.e. (by 2 and 4) because unit place digit must be an even number. Ten's place can be filled in 4 ways.

So the total number of two-digit even number = 2 × 4 = 8

3. Number of three-digit even numbers:

Unit's place can be filled in 2 ways.

Ten's place can be filled in 4 ways.

Hundred's place can be filled in 4 ways. So, the total number of three-digit even numbers = 2 × 4 × 4 = 32

Hence, the total number of three-digit even numbers (by using the digits 2, 4, 3 and 5) which are less than 1000 = 2 + 8 + 32 = 42.

c. Permutations (arrangements)

Suppose there are three persons A, B, and C for the post of president and vice-president of an organization and we have to select two persons to fill the two posts. The recruitments can be made in 6 ways - (A, B), (B, C), (A, C) (B, A), (C, B) and (C, A) where the format (x, y) represents *x* as president and *y* as vice-president.

The arrangements of a number of things taking some or all of them from a group at a time is called permutations.

Example: If there are *n* persons and we have to arrange *r* persons at a time, then the total number of permutations or arrangements is denoted by ${}^nP_r$ or by P(n, r), where ${}^nP_r = \frac{n!}{(n-r)!}$.

5. Find the number of all possible arrangements, such that two out of the four persons A, B, C and D are arranged in a row.

Solution :

The possible arrangements (**permutations**) are AB, BA, AC, CA, AD, DA, BC, CB, CD, DC, BD and DB which is 12. We can also say that out of 4 persons we have to arrange only 2 at a time, so the total number of permutations is ${}^4P_2 = \frac{4!}{(4-2)!}$

$= \frac{4!}{2!} = \frac{4 \times 3 \times 2}{2!} = 12$

6. In the above question, if all the persons are selected at a time, then how many arrangements are possible?

Solution :

We have to arrange 4 persons, so total number of permutations is

$${}^4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 4 \times 3 \times 2 \times 1 = 24.$$

7. How many different signals can be given, by raising any number of flags at a time from 4 flags of different colours? (No signal is formed if no flags are used).

Solution :

Signals can be given by either raising all or some of the flags at a time.

By raising 1 flag, number of signals that can be given = ${}^4P_1$

By raising 2 flags, number of signals that can be given = ${}^4P_2$

By raising 3 flags, number of signals that can be given = ${}^4P_3$

By raising 4 flags, number of signals that can be given = ${}^4P_4$

So, the total number of signals

$$= {}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$$

$$= \frac{4!}{(4-1)!} + \frac{4!}{(4-2)!} + \frac{4!}{(4-3)!} + \frac{4!}{(4-4)!}$$

= 4 + 12 + 24 + 24 = 64.

8. Find the number of ways in which 5 boys and 5 girls be seated in a row such that:

I. All the boys sit together and all the girls sit together.

II. Boys and girls sit alternately.

III. No two girls may sit together.

IV. All the girls are always together.

V. All the girls are never together.

Solution :

I. All the boys can be arranged in 5! ways and all the girls can be arranged in 5! ways. Now, we have two groups (boys and girls) and these 2 groups can be arranged in 2! ways. [boys–girls and girls–boys]

So total number of arrangements is 5! × 5! × 2! = 28800

II. Boys and girls sit alternately, this can be arranged like this

(B G B G B G B G B G)

or (G B G B G B G B G B)

In the first case boys can be arranged in 5! and girls can be arranged in 5! ways.

In the second case also, the number of arrangement is same as the first case

So the total number of arrangements = 5! × 5! + 5! × 5!

= 120 × 120 + 120 × 120

= 14,400 + 14,400 = 28,800 ways.

III. No two girls may sit together — In this case __B__B__B__B__B__ there are 6 spaces where a girl can find her seat.

5 girls can be arranged at these six spaces in $\frac{6!}{(6-5)!}$ = 6 × 5 × 4 × 3 × 2 × 1 = 720 ways and 5 boys can be arranged in $^5P_5 = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways.

So total number of arrangements

= 720 × 120 = 86,400.

IV. When all the girls are always together, treat them as one group. So now we have 5 boys and 1 group of 5 girls, and this can be permutated in 6! ways at the same time 5 girls in the group can be permutated in 5! ways. So the total number of permutations is 6! × 5! = 720 × 120 = 86,400.

V. All the girls are never together

Total number of arrangements of 5 boys and 5 girls is 10!

Number of arrangements in which all the girls are always together = 86,400

So number of arrangements in which all the girls are never together = total arrangements – number of arrangements when girls are always together = 10! – 6! × 5! = 35,42,400.

9. Find the number of permutations of the letters of the word FOLDER taking all the letters at a time?

Solution :

Number of letters in the word FOLDER is 6.

So the number of arrangements = $^6P_6 = 6!$.

Alternate method:

First place can be filled by any one of the six letters in 6 ways, the second place can be filled by any one of the five remaining letters in 5 ways, the third place can be filled by any one of the four remaining letters in 4 ways, and so on.

So the total number of arrangements is 6 × 5 × 4 × 3 × 2 × 1 = 720. (which is 6!)

10. How many four-digit numbers greater than 5000 can be formed by using the digits 4, 5, 6 and 7, if the repetition of the digits in the same number is not allowed.

Solution :

Total number of possible arrangements is $^4P_4 = 4!$

Total number of arrangements by using the digits 5, 6 and 7 = 3!

Hence, the total number of required arrangements is 4! – 3! = 24 – 6 = 18

Alternate method:

Thousand's place can be filled in 3 ways.

Hundred's place can be filled in 3 ways.

Ten's place can be filled in 2 ways.

Unit's place can be filled in 1 way.

So total number of arrangements

= 3 × 3 × 2 × 1 = 18

Permutation of alike things:

If there are three different coloured balls, then they can be arranged in $^3P_3 = 3!$ ways. But if all the three balls are of the same colour then there is only one way of arranging them.

So in general the number of arrangements of alike things = $\frac{\text{Number of arrangements of } x \text{ things}}{\text{Number of arrangements of } y \text{ things}}$

where x = Total number of elements and

y = Number of alike elements.

11. Find the number of permutations of the letters of the word STUDENT taking all the letters at a time?

Solution :

Number of letters in the word STUDENT = 7

Number of alike letters 'T' in the word = 2

So number of permutations = $\frac{^7P_7}{2!} = \frac{7!}{2!}$ = 2520.

12. Find the number of arrangements of the letters of the word 'MATHEMATICS' taking all the letters at a time.

Solution :

Number of letters in the word 'MATHEMATICS' = 11

Alike letters are 'M', 'A' and 'T' and are 2 in numbers. So the number of permutations

$= \frac{^{11}P_{11}}{2!\times 2!\times 2!} = \frac{11!}{2!\times 2!\times 2!}$ = 49,89,600

13. How many numbers greater than one million can be formed by using all the digits 4, 4, 5, 5, 5, 6 and 0 at a time?

Solution :

Here 4 is occurring 2 times and 5 is occurring 3 times.

Total number of arrangements = $\frac{^7P_7}{2!\times 3!}$

$= \frac{7!}{2!\times 3!} = \frac{7\times 6\times 5\times 4\times 3\times 2\times 1}{2\times 3\times 2} = 420$

Since the number beginning with 0 is not greater than one million, so the total number of numbers beginning with '0' has to be omitted which is

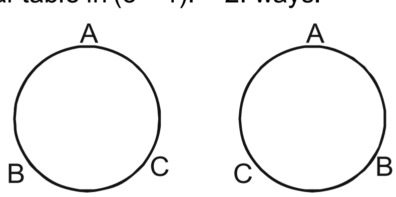
$= \frac{^6P_6}{2!\times 3!} = \frac{6!}{2!\times 3!} = \frac{6\times 5\times 4\times 3\times 2\times 1}{2\times 3\times 2} = 60$

Hence, the required number of numbers greater than one million is 420 – 60 = 360.

d. Circular permutations

If ***n*** persons are to be seated around a circular table, then they can be arranged in ***(n – 1)!*** ways.

Example: Three persons can be arranged around a circular table in (3 – 1)! = 2! ways.



Note:

It is assumed that persons or objects are at equidistant positions when they are arranged around a circle.

e. Combinations

Suppose there are three persons A, B and C contesting for the post of president and vice-president of an organization and we have to select two persons. We can select either (A, B) or (B, C) or (A, C) in 3 ways because here we are talking about the selection, not about the order. Whether 'A' is a president or 'B' is a vice-president or vice versa, doesn't matter.

Suppose there are 10 persons in a class and we have to select any 3 persons at a point regardless of the order, it is a case of ***combination (selection).***

If there are ***n*** things and we have to select some or all of them, it is called ***combinations.***

So out of ***n*** things we have to select r($1 \le r \le n$),

then the number of combinations = $^nC_r = \frac{n!}{(n-r)!r!}$

Difference between permutations and combinations.

Suppose that there are five persons A, B, C, D and E and we have to choose two persons at a time, then:

Permutations	Combinations
$^5P_2$	$^5C_2$
$= \frac{5!}{(5-2)!}$	$= \frac{5!}{(5-2)\times 2!}$
$= \frac{5!}{3!} = 5\times 4 = 20$	$= \frac{5!}{2!\times 3!} = \frac{5\times 4}{2} = 10$
^nP_r	^nC_r
$\frac{n!}{(n-r)!} = n(n-1)(n-2)$ ....$(n-r+1)$	$\frac{n!}{(n-r)!\ \ r!} = \frac{n}{r!}(n-1)$ .... $(n-r+1)$

So, it is clear that in permutations (arrangements) order matters but in combinations (selections) order does not matter.

Important: $^nP_r = {^nC_r} \times r!$

14. In a class there are 5 boys and 6 girls. How many different committees of 3 boys and 2 girls can be formed?

Solution :

Out of 5 boys we have to select 3 boys, and this can be done in $^5C_3$ ways.

Out of 6 girls we have to select 2 girls, and this can be done in ${}^{6}C_{2}$ ways.

So, selection of 3 boys and 2 girls can be done in ${}^{5}C_{3} \times {}^{6}C_{2}$ ways

[Basic rule of multiplication]

$$= \frac{5!}{3!(5-3)!} \times \frac{6!}{2!\ (6-2)!}$$

$$= \frac{5 \times 4}{2} \times \frac{6 \times 5}{2} = 10 \times 15 = 150 \text{ ways.}$$

15. There are 10 persons in a party and each of them shakes hands with the other. How many hand-shakes will take place in the party?

Solution :

It is very obvious that when two persons shake hands, it is counted as one handshake. So, we can say that there are 10 hands and every combination of 2 hands will give us one handshake.

So the number of handshakes = ${}^{10}C_{2}$

$$= \frac{10!}{2!(10-2)!} = \frac{10 \times 9 \times 8!}{2! \times 8!} = 45$$

16. For the post of Maths faculty in Career Launcher there are 6 vacant seats. Exactly 2 seats are reserved for MBAs. There are 10 applicants out of which 4 are MBAs. In how many ways the selection can be made?

Solution :

There are 4 MBAs and 6 other candidates.

So we have to select 2 out of 4 and the rest 4 out of 6 other candidates.

Hence the total number of ways of selection

$$= {}^{4}C_{2} \times {}^{6}C_{4} = \frac{4!}{2! \times (4-2)!} \times \frac{6!}{4!(6-4)!}$$

$$= \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} \times \frac{6 \times 5 \times 4!}{4! \times 2 \times 1}$$

= 6 × 15 = 90 ways.

17. There are 10 points out of which no three are collinear. By joining the points how many straight lines can be formed.

Solution :

By joining any two points we will get one line.

So the total number of lines formed = ${}^{10}C_{2}$

$$= \frac{10 \times 9 \times 8!}{2 \times (10-2)!} = \frac{10 \times 9 \times 8!}{2 \times 8!} = 45$$

18. Find the number of diagonals that can be drawn by joining the vertices of a decagon.

Solution :

In a decagon, there are 10 vertices and by joining any two vertices we will get one line.

So in a decagon total number of lines formed

$$= {}^{10}C_{2} = \frac{10!}{2!(10-2)!} = \frac{10 \times 9 \times 8!}{2! \times 8!} = 45$$

But out of these 45 lines, 10 lines will be the sides of the decagon. So total number of diagonals = 45 – 10 = 35.

19. In the above question, how many triangles can be formed?

Solution :

We know that in a triangle there are three vertices and by joining any three points we will get a triangle.

So number of triangles formed = ${}^{10}C_{3}$

$$= \frac{10 \times 9 \times 8 \times 7!}{3! \times (10-3)!} = \frac{10 \times 9 \times 8 \times 7!}{3! \times 7!} = 120.$$

20. There are 5 boys and 6 girls. A combination of 4 is to be selected so that it must consist of at-least one boy and one girl?

Solution :

The different possibilities are

I. 1 boy and 3 girls

II. 2 boys and 2 girls

III. 3 boys and 1 girl

I. Total number of combination is ${}^{5}C_{1} \times {}^{6}C_{3}$

II. Total number of combination is ${}^{5}C_{2} \times {}^{6}C_{2}$

III. Total number of combination is ${}^{5}C_{3} \times {}^{6}C_{1}$

So total number of combinations are

${}^{5}C_{1} \times {}^{6}C_{3} + {}^{5}C_{2} \times {}^{6}C_{2} + {}^{5}C_{3} \times {}^{6}C_{1}$ = 310

(ii) Probability

Sample space: It is the set all possible outcomes in an experiment.

For example: Number of elements in the sample space when a die is thrown is 6, i.e. any of the six numbers from 1 to 6 can appear at the top of the dice.

Event: An event is a set of outcomes (a subset of the sample space) to which a probability is assigned. Basically, when the sample space is finite, any subset of the sample space is an event

Definition of probability: Suppose a magician approaches you and says that he has a dice and if he throws that dice and a number greater than 5 comes on the top, he will give you Rs. 20 otherwise you will have to give Rs. 10 to him. What will you do? Here is an application of probability which deals with uncertainties. It has nothing to do with actual happenings. ***It just talks about the likelihood of happening of an event.***

To know about the probability in a better way just take the above example.

When a dice is thrown, then one of the numbers among 1, 2, 3, 4, 5 or 6 can come on its top face. So we can say there are a total of six possibilities. Out of these 6 numbers, greater than 5 is only one number and that is 6. So in your favour there is only 1 number that is 6, and against your (or your opponents) favour there are 5 numbers 1, 2, 3, 4 and 5. *This indicates that your chance of losing is five times than your chance of winning. It means this game is not in your favour.*

a. Probability of an event is defined as

$$\frac{\text{Number of outcomes favourable to the event}}{\text{Total number of possible outcomes}}$$

Like in the example given above probability of your winning = $\frac{1}{6}$ and probability of your losing = $\frac{5}{6}$

Note:

Probability of an event can't be less than 0 and more than 1.

b. Mutually exclusive events and addition law.

(A) Mutually exclusive events:

If two events are said to be mutually exclusive, then happening of one of the events precludes the happening of the other event. In other words, the events have no simultaneous occurence.

For example:

1. In rolling a dic:

E : – The event that the number is odd

F : – The event that the number is even

G : – The event that the number is a multiple of three.

2. In drawing a card from a deck of 52 cards:

E : – The event that it is a spade.

F : – The event that it is a club.

G : – The event that it is a king.

In the above 2 cases, events E and F are mutually exclusive but the events E and G are not mutually exclusive or disjoint since they may have common outcomes.

(B) Additional law of probability:

If A and B are two events, then the probability that either event A or event B will occur in a single trial is given by :

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Where, $P(A \cap B) = P(A) \times P(B)$

If the events are mutually exclusive, then

$P(A) \times P(B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$

Note:

Compare this with of set theory.

Similarly,

P (neither E nor F) = 1 – P(E or F).

c. Independent Events And Multiplication Law

(A) Independent Events:

Two events are independent if the occurence of one has no effect on the occurence of the other.

For example:

1. On rolling a dic and tossing a coin together:

E : – The event that number 6 turns up.

F : – The event that head turns up.

2. In shooting a target:

E : – Event that the first trial is missed.

F : – Event that the second trial is missed.

In both these cases events E and F are independent.

3. In drawing a card from a well–shuffled pack:

E : – Event that first card is drawn.

F : – Event that second card is drawn without replacing the first .

G : – Event that second card is drawn after replacing the first.

In this case, E and F are not independent but E and G are independent.

(B) Multiplication law of probability:

If the events E and F are independent, then **P(E and F) = P (E) × P (F)**

21. In a single throw of a die what is the probability that the number on the top is more than 2?

Solution :

In a die there are 6 faces numbered 1, 2, 3, 4, 5 and 6

So the total number of possible events = 1, 2, 3, 4, 5 and 6 = 6

and the total number of favourable events = 3, 4, 5 and 6 = 4

Hence, the required probability is $\frac{4}{6}=\frac{2}{3}$.

22. If two fair dice are thrown simultaneously then what is the probability that the sum of the numbers on the top faces is less than 4?

Solution :

Total number of possible events = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2) ... and so on. There will be 6 × 6 = 36 possible events.

Number of favourable events = (1, 1), (1, 2) and (2, 1) = 3 events

So, the required probability $=\frac{3}{36}=\frac{1}{12}$

23. If out of 20 numbers from 1 to 20 Mr X selects a number at random. What is the probability that this number will be a multiple of 4?

Solution :

Total number of possible events = 1, 2, 3, ..., 20 = 20 in number

Total number of favourable events = 4, 8, 12, 16 and 20 = 5 in number

So the required probability $=\frac{5}{20}=\frac{1}{4}$.

24. In example 23, what is the probability that this number will be a multiple of 4 or 7?

Solution :

Total number of possible events = 1, 2, …, 20 = 20 in number

Number divisible by 4 = 4, 8, 12, 16, 20 = 5 in number

Number divisible by 7 = 7 and 14 = 2 in number

Since from 1 to 20 there is no any number which is divisible by both 4 and 7, it is a case of mutually exclusive events.

So number of possible outcomes = 5 + 2 = 7.

Hence, the required probability is $=\frac{7}{20}$

25. In example 23, what is the probability that this number is divisible by 2 and 4?

Solution :

The total number of possible events = 20 in number.

Number divisible by 2 and 4 means the number should be divisible by 4 (LCM of 2 and 4 is 4) = 4, 8, 12, 16, 20 = 5 in number.

So the required probability is $\frac{5}{20}=\frac{1}{4}$.

26. In example 23, what is the probability that this number is divisible by 2 or 4?

Solution :

The total number of possible outcomes = 20 in number.

Number divisible by 2 = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 = 10 in number.

Number divisible by 4 = 4, 8, 12, 16 and 20 = 5 in number

There are certain numbers which are divisible by both 2 and 4, so it is not a case of mutually exclusive events.

Number divisible by both 2 and 4 are 4, 8, 12, 16 and 20 = 5 in number.

Hence, the required probability = P(A) + P(B) – P(C)

$=\frac{10}{20}+\frac{5}{20}-\frac{5}{20}=\frac{10}{20}=\frac{1}{2}$.

d. Conditional probability

Let A and B are two dependent events, then probability of occurence of event A when B has already occurred is given by $P(A\,|\,B)=\frac{P(A\cap B)}{P(B)}$

27. From a pack of 52 cards, 4 cards were picked one at a time.

- **a.** If the card picked is not replaced, find the probability that all the cards were aces.
- **b.** If the card picked was replaced, what is the probability that all the 4 pickings were aces?
- **c.** If the cards were picked all at a time, find the probability that all the 4 cards were aces.

Solution :

a. Probability that the first card is an ace is $\frac{^4C_1}{^{52}C_1}$.

Probabilities that the 2nd, 3rd and 4th cards are all aces are $\frac{^3C_1}{^{51}C_1}$, $\frac{^2C_1}{^{50}C_1}$ and $\frac{^1C_1}{^{49}C_1}$ respectively.

Hence, the total probability is

$$\frac{^4C_1}{^{52}C_1}\times\frac{^3C_1}{^{51}C_1}\times\frac{^2C_1}{^{50}C_1}\times\frac{^1C_1}{^{49}C_1}$$

$$=\frac{4\times3\times2\times1}{52\times51\times50\times49}=\frac{1}{^{52}C_4}.$$

b. With replacement, the probability is

$$\left(\frac{^4C_1}{^{52}C_1}\right)^4=\frac{1}{13^4}$$

c. If all the 4 cards were picked simultaneously, then the required probability is $\frac{^4C_4}{^{52}C_4} = \frac{1}{^{52}C_4}$.

Compare the cases (a) and (c). You would note that they are one and the same.

28. One card is drawn from a pack of 52 cards, each of the 52 cards being qually likely to be drawn. Find the probability that the card drawn is

i. a king,

ii. either red or king,

iii. red and a king.

Solution :

Out of 52 cards, one card can be drawn in $^{52}C_1$ ways. Therefore, exhaustive number of cases = $^{52}C_1$ = 52

i. There are 4 kings in a pack of cards, out of which one can be drawn in $^4C_1$. Therefore, favourable number of cases = $^4C_1$ = 4.

So, the required probability $= \frac{4}{52} = \frac{1}{13}$

ii. There are 28 cards in a pack of cards which are either a red or a king. Therefore, one can be drawn in $^{28}C_1$ ways. Therefore, favourable number of cases = $^{28}C_1$ = 28

So the required probability = $\frac{28}{52} = \frac{7}{13}$

iii. There are 2 cards which are red and king, i.e. red kings. Therefore, favourable number of cases = $^2C_1$ = 2.

So, the required probability $= \frac{2}{52} = \frac{1}{26}$

29. Three unbiased coins are tossed. What is the probability of getting the following?

i. All heads

ii. 2 heads

iii. Exactly 1 head

Solution :

If 3 coins are tossed together, we can obtain any one of the following as an outcome.

HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

So exhaustive number of cases = 8

i. All heads can be obtained in only one way, i.e. HHH.

So, the favourable number of cases = 1

Thus, the required probability $= \frac{1}{8}$

ii. Two heads can be obtained in any one of the following ways: HHT, THH, HTH. So favourable number of cases = 3. Thus, required probability $= \frac{3}{8}$

iii. Required probability = $\frac{3}{8}$. The probability of exactly 1 head is same as probability of exactly 1 tail (or 2 heads) since the coin is unbiased.

30. An urn contains 9 red, 7 white and 4 black balls. If 2 balls are drawn at random, find the probability that

i. both the balls are red,

ii. one ball is white.

Solution :

There are 20 balls in the bag out of which 2 balls can be drawn in $^{20}C_2$ ways. So the exhaustive number of cases = $^{20}C_2$ = 190

i. There are 9 red balls out of which 2 balls can be drawn in $^9C_2$ ways. Therefore, favourable number of cases = $^9C_2$ = 36.

So, the required probability $= \frac{36}{190} = \frac{18}{95}$

ii. There are 7 white balls out of which one white can be drawn in $^7C_1$ ways. One ball from the remaining 13 balls can be drawn in $^{13}C_1$ ways. Therefore, one white and one other colour ball can be drawn in $^7C_1 \times {}^{13}C_1$ ways.

So the favourable number of cases

$= {}^7C_1 \times {}^{13}C_1 = 91$

So, the required probability = $\frac{91}{190}$

Everything in the universe can be represented in the form of set or subset (subset is a part of the set). Like man is a subset of human beings which is again a subset of living beings, and so on.

Suppose A = All natural numbers; B = All odd numbers and C = All even numbers, then B and C are subsets of A.

(iii) Set Theory

Definition: A set is a well-defined collection of objects.

If A is a set and ‘a’ is an element of this set, we say that 'a' belongs to A or $a \in A$. A set ‘A’ which has a finite number of elements is called a finite set. The number of elements in a finite set is denoted by n(A).

The universal set is the set containing all the elements under the consideration.

The empty set or null set (ϕ) is the set which has no element.

If a is an element of set A, then we write $a \in A$ (read a belongs to A or a is a member of set A). If a does not belong to A, then we write $a \notin A$. It is assumed that either $a \in A$ or $a \notin A$ and the two possibilities are mutually exclusive.

Some important definitions:

Subset: If every element of A is an element of B, then A is called a subset of B and we write $A \subseteq B$. Every set is a subset of itself and the empty set is a subset of every set. A subset A of set B is called a proper subset of B if $A \neq B$ and we write $A \subset B$. If a set has n elements, then the number of its subsets = 2^n.

Superset: If A is a subset of B, then B is known as the superset of A and we write $B \supseteq A$.

Power set: Let A be a set. Then the collection or family of all subsets of A is called the power set of A and is denoted by P(A).

Example: Let A = {1, 2, 3}

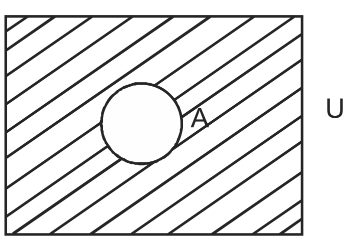
Then the subsets of A are ϕ , {1}, {2}, {3}, {1, 2} {1, 3}, {2, 3} and {1, 2, 3}.

Hence P(A) = {ϕ , {1}, {2}, {3}, {1, 2} {1, 3}, {2, 3} {1, 2, 3}}

Universal set: A set that contains all the sets in a given context is called the universal set, i.e. It is the super set of all the sets under consideration e.g. if A = {1, 2, 3} and B = {2, 4, 5, 6}, then a set of all natural numbers (N) can be taken as a universal set.

Introduction to Venn diagrams

The sets can be illustrated by means of Venn diagrams. A universal set U is represented by a rectangle and a subset by a circle within it.



Complement of a set

Let U and A be 2 sets such that $A \subseteq U$, then (U – A) is simply called the complement of A.

It is denoted by $\overline{A}$ or A'.

e.g. U is the set of natural numbers, the complement of odd numbers will be a set of even numbers.

The following letter sets are standard notations:

N : Set of natural numbers

Z : Set of integers

Q : Set of rational numbers.

R : Set of real numbers.

C : Set of complex numbers.

Example :

U = {1, 2, 3, 4}, A = {3}. What is the complement of A?

Solution :

A' = {1, 2, 4}

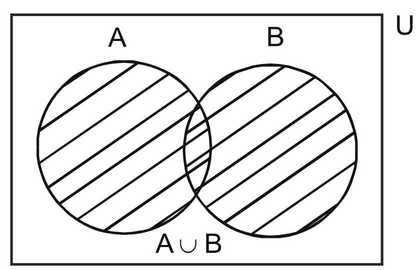
Remember the following

(i) $U' = \phi, \phi' = U$

(ii) (A')' = A

Union of Sets

If A and B are 2 sets, then the union of A and B, denoted by $A \cup B$, is the set of all elements which are **either** in A **or** in B or in both A and B.



Union of even and odd positive integers is a set of natural numbers.

Example :

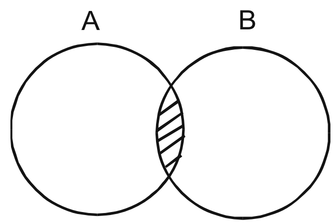
If A = {1, 2, 5, 7, 9} and B = {3, 8, 9, 2, 0}. Find $A \cup B$.

Solution :

$A \cup B$ = {0, 1, 2, 3, 5, 7, 8, 9}

Intersection of Sets

If A and B are sets, then the intersection of A and B, denoted by $A \cap B$, is the set of all elements which belong to **both** A **and** B.



$A \cap B$

e.g. Intersection of set of prime numbers and set of even numbers is a set having only one element, which is 2, i.e. = {2}.

Consider and verify the following identities:

i. $A \cup A = A, \quad A \cap A = A$

ii. $A \cup \phi = A, \quad A \cap \phi = \phi$

iii. $A \cup U = U, \quad A \cap U = A$

iv. $A \cup B = B \cup A, \quad A \cap B = B \cap A$

v. $A \cup A' = U, \quad A \cap A' = \phi$

Example :

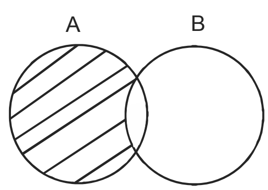
A = {1, 2, 3, 4, 5} and B = {3, 7, 9, 4}. Find $A \cap B$.

Solution :

$A \cap B$ = {3, 4}

If A and B have no elements in common, then they are called **disjoint sets**.

Difference of Sets



A–B

If A and B are sets, then the difference of A and B, written as A – B, is the set of all those elements of A which do not belong to B.

Note:

$A - B = A - A \cap B = A \cap B'$

Is A – B = B – A?

Find out for the sets A and B given in the example.

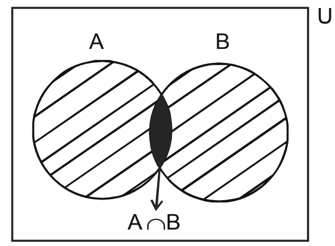
e.g. A = {1, 2, 3, 4, 5} and B = {3, 4, 6, 7},

find A – B.

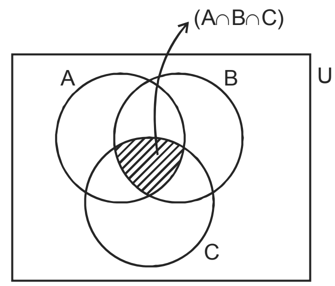
Solution: A – B = {1, 2, 5}

Venn Diagrams

For two sets, the following diagram is valid.



For three sets, the following diagram is valid.

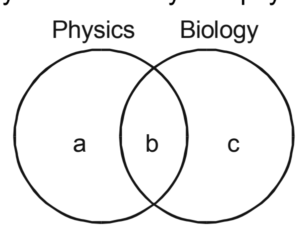


$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap C)$$

$$-n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$$

31. In a school, there are 100 students and every student studies at-least one subject. 60 students study physics and 80 students study biology.

How many students study both physics and biology?



Solution :

a + b = 60 and b + c = 80

So a + 2b + c = 140

It is given that a + b + c = 100

b = 140 – 100 = 40

By formulae:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$100 = 60 + 80 - n(A \cap B)$.

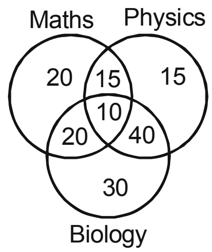
So $n(A \cap B) = 140 - 100 = 40$.

32. In the above question find the number of students who study only one subject.

Solution :

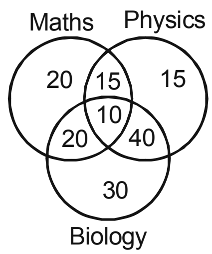
Number of students who study only one subject = Number of students who study only physics + number of students who study only biology = 20 + 40 = 60.

33. The given diagram indicates the number of students who passed in different subject combinations. Assuming that all the students appeared for the exam, passed in at-least one subject.



Find the percentage of students who passed in at-least two subjects.

Solution :



The number of students who passed in at-least two subjects

= 20 + 15 + 40 + 10

= 85

Total number of students = 20 + 15 + 15 + 20 + 10 + 40 + 30 = 150

So the percentage of students who passed in at-least two subjects = $\frac{85}{150} \times 100$

= 56.66 approx.

34. In a school there are 400 candidates appearing in an examination of three papers. 80% of the candidates passed in at-least one subject. Two hundred passed in physics, 100 passed in chemistry, 120 passed in biology, 40 passed in both physics and biology, 60 passed in both physics and chemistry and 50 passed in both biology and chemistry. How many passed in all three subjects?

Solution :

It is given that $n(A \cup B \cup C) = 320$ (80% of 400)

$n(A) = 200$, $n(B) = 100$, $n(C) = 120$, $n(A \cap B) = 40$, $n(B \cap C) = 60$, $n(C \cap A) = 50$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$\Rightarrow 320 = 200 + 100 + 120 - 40 - 60 - 50 + n(A \cap B \cap C)$

$\Rightarrow n(A \cap B \cap C) = 470 - 420 = 50.$

35. How many natural numbers not more than 100 are there, which are not divisible by 2, 3 or 5?

Solution :

Let us first calculate the total numbers not more than 100, which are divisible by 2, 3 or 5

Divisible by 2 $n(A) = \frac{100}{2} = 50$

Divisible by 3 $n(B) = \frac{100}{3} = 33$

Divisible by 5 $n(C) = \frac{100}{5} = 20$

Divisible by both 2 and 3, $n(A \cap B)$

$= \frac{100}{\text{LCM of 2 and 3}} = \frac{100}{6} = 16$

Divisible by both 3 and 5, $n(B \cap C)$

$= \frac{100}{\text{LCM of 3 and 5}} = \frac{100}{15} = 6$

Divisible by both 2 and 5, $n(A \cap C)$

$= \frac{100}{\text{LCM of 2 and 5}} = \frac{100}{10} = 10$

Divisible by 2, 3 and 5, $n(A \cap B \cap C)$

$= \frac{100}{\text{LCM of 2,3 and 5}} = \frac{100}{30} = 3$

So $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

= 50 + 33 + 20 – 16 – 6 – 10 + 3 = 74

So total number which are not more than 100, and are not divisible by neither of 2, 3 or 5 is

$100 - n(A \cup B \cup C) = 100 - 74 = 26.$

36. In a school, there are 200 candidates, 120 study English and 180 study Hindi. How many of them study both the subjects?

Solution :

The answer to the given question cannot be determined because we don't know whether every student studies at-least one of these subjects or not. There may be a case when certain students do not read English or Hindi, they study physics or some other subject.

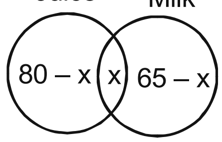
37. A survey of students show that 80% students like drinking juice and 65% students like drinking milk. What is the percentage of students according to survey who like both if it is known that each student drinks at-least one of the two drinks?

(a) 30% (b) 35%

(c) 45% (d) 55%

Solution :

Let x% students like drinking both juice and milk. Then,



$\therefore 80 - x + x + 65 - x = 100$

$\Rightarrow 145 - x = 100$

$\Rightarrow x = 45\%$.

Exercise

1. Find $^8P_6$
   (a) 33425 (b) 20160
   (c) 18972 (d) 6625
2. Find $^8C_6$
   (a) 33 (b) 32
   (c) 30 (d) 28
3. Find the number of ways in which the letters of the word BIHAR can be arranged.
   (a) 99 (b) 129
   (c) 120 (d) 125
4. Find the number of ways in which the letters of the word AMERICA can be rearranged.
   (a) 2519 (b) 2620
   (c) 1250 (d) 2500
5. Find the number of ways in which the letters of the word CALCUTTA can be rearranged.
   (a) 3000 (b) 5039
   (c) 5029 (d) 3029
6. How many 4-letter different words can be formed by using the letters a, b, c and d?
   (a) 296 (b) 346
   (c) 440 (d) 256
7. How many numbers can be formed by using the digits 2, 3, 4 and 5?(repetitions of digits is not allowed).
   (a) 50 (b) 60
   (c) 64 (d) 68
8. How many numbers greater than 4000 can be formed by using the digits 2, 3, 4 and 5? (repetitions of digits is not allowed).
   (a) 12 (b) 14
   (c) 20 (d) 24
9. How many numbers greater than 4000 can be formed by using the digits 2, 3, 4 and 5?
   (a) 120 (b) 130
   (c) 138 (d) 128
10. There are 3 roads from A to B, 4 roads from B to C and only 1 road from C to D. How many combinations of roads are there from A to D?
    (a) 12 (b) 15
    (c) 14 (d) 18
11. There are 5 questions in a question paper. In how many ways can a candidate solve at-least 1 question?
    (a) 30 (b) 31
    (c) 32 (d) 33
12. A coin is tossed 7 times, what is the probability that head appears odd number of times?
    (a) $\frac{1}{2}$ (b) $\frac{1}{4}$
    (c) 1 (d) None of these
13. If 4 dice and 3 coins are tossed simultaneously, the number of elements in the sample space is:
    (a) $2^4 \times 6^3$ (b) $6^4 \times 2^3$
    (c) 2156 (d) None of these

Directions for questions 14 to 16: A bag contains 6 white and 4 red balls.

14. Three balls are drawn one by one without replacement. What is the probability that all 3 balls are red?
    (a) $\frac{1}{10}$ (b) $\frac{1}{20}$
    (c) $\frac{1}{30}$ (d) $\frac{1}{120}$
15. Three balls are drawn one by one with replacement. What is the probability that 2 balls are white and 1 is red?
    (a) $\frac{54}{125}$ (b) $\frac{27}{154}$
    (c) $\frac{45}{125}$ (d) $\frac{15}{125}$

16. Three balls are drawn one by one without replacement. What is the probability that 2 are red and 1 is white?
(a) 0.1 (b) 0.2
(c) 0.3 (d) 0.4

Directions for questions 17 to 20: The probability that A will pass the examination is $\frac{1}{3}$, and the probability that B will pass the examination is $\frac{1}{2}$.

17. What is the probability that both will pass the examination?
(a) $\frac{1}{6}$ (b) 1
(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

18. What is the probability that only one person will secure the passing marks in the examination?
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

19. What is the probability that at-least one person will secure the passing marks in the examination?
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

20. What is the probability that none of them will secure the passing marks in the examination?
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

21. Two dice are thrown simultaneously. What is the probability that the product of the numbers on their top faces is less than 36?
(a) $\frac{1}{6}$ (b) $\frac{35}{36}$
(c) $\frac{23}{36}$ (d) $\frac{32}{36}$

22. Two cards are drawn together from a pack of 52 cards at random. What is the probability that both are spades?
(a) $\frac{^{4}C_2}{^{52}C_2}$ (b) $\frac{^{13}C_2}{^{52}C_2}$
(c) $\frac{^{26}C_2}{^{52}C_2}$ (d) $\frac{^{8}C_2}{^{52}C_2}$

23. In question no: 22, what is the probability that both are kings?
(a) $\frac{^{4}C_2}{^{52}C_2}$ (b) $\frac{^{13}C_2}{^{52}C_2}$
(c) $\frac{^{26}C_2}{^{52}C_2}$ (d) $\frac{^{8}C_2}{^{52}C_2}$

24. In question no. 22, what is the probability that one is a spade and one is a heart?
(a) $\frac{^{13}C_1 \times ^{13}C_1}{^{52}C_2}$ (b) $\frac{^{13}C_1 \times ^{26}C_1}{^{52}C_2}$
(c) $\frac{13}{52} \times \frac{13}{52}$ (d) $\frac{13 \times 26}{^{52}C_2}$

25. In question no: 22, what is the probability that exactly one is a king ?
(a) $\frac{^{52}C_1}{^{52}C_2}$ (b) $\frac{4}{^{58}C_2}$
(c) $\frac{^{4}C_1 \times ^{48}C_1}{^{52}C_2}$ (d) $\frac{1}{2}$

26. In a class of 60 boys, there are 45 boys who play cards and 30 boys who play carrom. Find how many boys play both the games (assuming that every boy plays either cards or carrom or both)?
(a) 15 (b) 17
(c) 20 (d) 21

27. In question no: 26, find the number of boys who play cards only.
(a) 25 (b) 30
(c) 32 (d) 35

28. In question no: 26, how many boys play carrom only?
(a) 10 (b) 12
(c) 15 (d) 20

29. Each student in a class of 40, studies at-least one of the subjects from English, Mathematics and Economics. 16 study English, 22 Economics and 26 Mathematics, 5 study English and Economics, 14 Mathematics and Economics, and 2 study all the three subjects.Find the number of students who study English and Mathematics.
(a) 7 (b) 10
(c) 17 (d) 27

30. In above question, find the number of students who study English and Mathematics but not Economics.
(a) 8 (b) 12
(c) 7 (d) 5

Answer Key

1. (b)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (d)	**10.** (a)
11. (b)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (a)	**16.** (c)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (c)
21. (b)	**22.** (b)	**23.** (a)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (a)	**30.** (d)

Explanations

1. b $^8P_6 = \frac{8!}{(8-6)!} = \frac{8\times7\times6\times5\times4\times3\times2!}{2!} = 20160$

2. d $^8C_6 = \frac{8!}{6!(8-6)!} = \frac{8\times7\times6!}{6!\times2!} = 28$

3. c Since there are 5 letters in the word so the total number of arrangement is $^5P_5$ = 5! that is 120.

4. a In the word AMERICA there are 7 letters and the letter ‘A’ is coming twice.
So the total number of rearrangement
$= \frac{7!}{2!} - 1 = \frac{7\times6\times5\times4\times3\times2\times1}{2\times1} - 1 = 2519$

Note: No. of Rearrangements = Total no. of arrangements – 1(initial arrangement)

5. b In the word CALCUTTA, there are 8 letters
Letter C comes twice
Letter A comes twice
Letter T comes twice
So the total number of rearrangements
$= \frac{8!}{2!\times2!\times2!} - 1 = 5039$

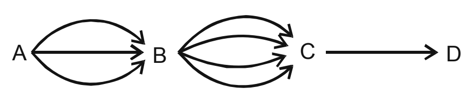
Note: No. of Rearrangements = Total no. of arrangements – 1 (initial arrangement)

6. d Letter *a* can be placed in all 4 positions.
Similarly, *b* can be placed in all 4 positions.
Similarly, *c* can be placed in all 4 positions.
Similarly, *d* can be placed in all 4 positions.
So the total number of arrangement is 4 × 4 × 4 × 4 = 256

7. c The number formed can be one-digit, two-digit, three-digit or four-digit number
Number of one-digit numbers formed = 4
Number of two-digit numbers formed $^4P_2 = \frac{4!}{2!} = 12$
Number of three-digit numbers formed $^4P_3 = 4! = 24$
Number of four-digit numbers formed $^4P_4 = 4! = 24$
Hence, total numbers formed = 4 + 12 + 24 + 24 = 64.

8. a Thousand’s place can be filled by using two digits i.e. 4 or 5
Hundred’s place can be filled in 3 ways.
Ten’s place can’t be filled in 2 ways.
Unit’s place can be filled in 1 way.
So total number of possible numbers = 2 × 3 × 2 × 1 = 12.

9. d Thousands place can be filled in two ways.
Hundred’s place can be filled with any of the 4 digits.
Ten’s place can be filled with any of the 4 digits.
Unit’s place can be filled with any of the 4 digits.
So the total numbers formed = 2 × 4 × 4 × 4 = 128.

10. a 

$= {}^3C_1 \times {}^4C_1 \times {}^1C_1 = \frac{3!}{1!(3-1)!} \times \frac{4!}{1!(4-1)!} \times \frac{1!}{1!(1-1)!}$
$= \frac{3!}{2!} \times \frac{4!}{3!} \times 1 = 3\times4\times1 = 12$ ways

11. b Either a candidate will solve the problem or he would not solve.
So the probability of solving is p = 1
and probability of not solving is q = 1
$(p+q)^r$ i.e. $(1+1)^5 = 2^5$
The number of ways of solving at least 1 question is $2^5 - 1 = 32 - 1 = 31$

Alternate method:
Number of ways of solving at least 1 question is candidate may solve 1 question out of 5.
or he may solve 2 out of 5

or he may solve 3 out of 5

or he may solve 4 out of 5

or he may solve 5 out of 5

i.e. $^5C_1 + ^5C_2 + ^5C_3 + ^5C_4 + ^5C_5$

$= \frac{5!}{1!} + \frac{5 \times 4}{2} + \frac{5 \times 4}{2} + 5 + 5 = 31$

12. a If Head appears odd number of times, then it can be 1, 3 or 5. So number of favourable ways

$= ^7C_1 + ^7C_3 + ^7C_5 + ^7C_7$

= 7 + 35 + 21 + 1 = 64 ways

Total number of ways = 2^7 = 128.

The required probability = $\frac{64}{128} = \frac{1}{2}$

13. b In every dice there are six spaces, so in 4 dice it is 6 × 6 × 6 × 6 = 6^4. Also in every coin there are two spaces, so in 3 coins it is 2 × 2 × 2. Hence, the total number of spaces = $6^4 \times 2^3$.

14. c Three red balls out of 4 red balls can be taken out without replacement in $^4C_1 \times ^3C_1 \times ^2C_1$ ways.

Total number of sample spaces is $^{10}C_1 \times ^9C_1 \times ^8C_1$

$\therefore$ Required probability = $\frac{4 \times 3 \times 2}{10 \times 9 \times 8} = \frac{1}{30}$.

15. a One red ball out of 4 can be taken out by $^4C_1$ ways and can come at three places. Two white balls can be taken out in $^6C_1 \times ^6C_1$ ways. (given as with replacement). So total favourable ways =

$3 \times ^4C_1 \times ^6C_1 \times ^6C_1$

Total number of sample spaces

$= ^{10}C_1 \times ^{10}C_1 \times ^{10}C_1$

Hence, required probability

$= \frac{3 \times ^4C_1 \times ^6C_1 \times ^6C_1}{^{10}C_1 \times ^{10}C_1 \times ^{10}C_1} = \frac{54}{125}$.

16. c The following combinations are possible. (WRR), or (RWR), or (RRW)

Hence, the probability

$= \left(\frac{6}{10} \times \frac{4}{9} \times \frac{3}{8}\right) + \left(\frac{4}{10} \times \frac{6}{9} \times \frac{3}{8}\right) + \left(\frac{4}{10} \times \frac{3}{9} \times \frac{6}{8}\right) = 0.3$

Solutions for questions 17 to 20:

Probability that A will pass in exam = $\frac{1}{3}$

$\therefore$ Probability that A will fail in exam = $\frac{2}{3}$

Probability that B will pass in exam = $\frac{1}{2}$

$\therefore$ Probability that B will fail in exam = $\frac{1}{2}$

17. a Probability that both will pass in the exam

= Probability that A will pass × probability that B will pass

$= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

18. b Probability that only one person will pass the exam

Here the possibility can be either A pass and B fails

or A fails and B pass

i.e (A pass and B fail) or (A fail and B pass)

$= \frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

19. d Probability that at-least one person will pass. Here the possibilities can be (A pass and B fails), or (A fails and B pass) or (Both A and B pass)

i.e. $= \frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} + \frac{1}{3} + \frac{1}{6} = \frac{2}{3}$

Alternate method:

1 – (None of them pass) i.e. (A fails and B fails)

$= 1 - \frac{2}{3} \times \frac{1}{2} = \frac{2}{3}$

20. c The probability that no one will pass

i.e. both A and B fail = $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

21. b If two dice are thrown, then total number of sample spaces = 36

Number of times when product of numbers on their top faces is less than 36.

(1 × 1), (1 × 2), (1 × 3), (1 × 4), (1 × 5), (1 × 6)
(2 × 1), (2 × 2), (2 × 3), (2 × 4), (2 × 5), (2 × 6)
(3 × 1), (3 × 2), (3 × 3), (3 × 4), (3 × 5), (3 × 6)
(4 × 1), (4 × 2), (4 × 3), (4 × 4), (4 × 5), (4 × 6)
(5 × 1), (5 × 2), (5 × 3), (5 × 4), (5 × 5), (5 × 6)
(6 × 1), (6 × 2), (6 × 3), (6 × 4), (6 × 5)

i.e. 35 times

$\therefore$ Required probability = $\frac{35}{36}$

Or

1 – probability of having product 36.

$= 1 - \frac{1}{36} = \frac{35}{36}$

22. b There are 13 spades in the pack.

Two spades out of 13 spades can be taken out in $^{13}C_2$ ways

Total number of sample spaces = $^{52}C_2$

$\therefore$ Required probability = $\frac{^{13}C_2}{^{52}C_2}$.

23. a There are 4 kings in the pack.

Two kings out of 4 kings can be drawn in $^4C_2$ ways.

$\therefore$ Required probability = $\frac{^4C_2}{^{52}C_2}$

24. a There are 13 spades and 13 hearts. One spade and one heart can be taken out in $^{13}C_1 \times ^{13}C_1$ ways.

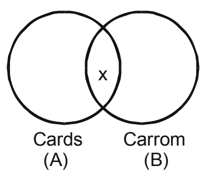
$\therefore$ Required probability = $\frac{^{13}C_1 \times ^{13}C_1}{^{52}C_2}$.

25. c There are 4 kings. One king can be taken out in $^4C_1$ ways.

Now out of remaining 48 cards, any one card can be taken out in $^{48}C_1$ ways.

$\therefore$ Red probability = $\frac{^4C_1 \times ^{48}C_1}{^{52}C_2}$.

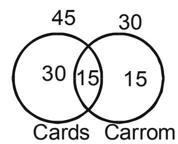
26. a Total number of students = 60



P(A $\cup$ B) = P(A) + P(B) – P(A $\cap$ B)

60 = 45 + 30 – x; x = 15.

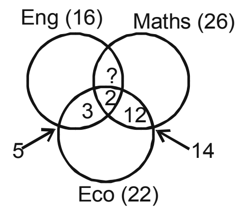
27. b



Cards only = 30.

28. c Carrom only = 15.

29. a



40 = 16 + 26 + 22 – 5 – 14 – x + 2; x = 7.

Note: x = 2 + ?

30. d English $\cap$ Maths $\cap$ Economics = 2

Only English $\cap$ Maths = 7 – 2 = 5.