BCECE
Engineering Entrance Exam
Solved Paper 2009

PHYSICS

1. A rectangular coil of 100 turns and size 0.1×0.05 m is placed perpendicular to a magnetic field of 0.1 T. The induced emf when the field drops to 0.05 T in 0.05 s is
(a) 0.5 V (b) 1.0 V
(c) 1.5 V (d) 2.0 V

2. A paramagnetic substance is placed in a weak magnetic field and its absolute temperature T is increased. As a result, its magnetisation
(a) increases in proportion to T
(b) decreases in proportion to $\frac{1}{T}$
(c) increases in proportion to T^2
(d) decreases in proportion to $\frac{1}{T^2}$

3. The current through a bulb is increased by 1%. Assuming that the resistance of the filament remains unchanged the power of the bulb will
(a) increase by 1% (b) decrease by 1%
(c) increase by 2% (d) decrease by 2%

4. You are given two resistances R_1 and R_2. By using them singly, in series and in parallel, you can obtain four resistances of 1.5 Ω , 2 Ω, 6 Ω and 8 Ω. The values of R_1 and R_2 are
(a) 1 Ω, 7 Ω (b) 1.5 Ω, 6.5 Ω
(c) 3 Ω, 5 Ω (d) 2 Ω, 6 Ω

5. A wire of length l carrying a current I A is bent into a circle. The magnitude of the magnetic moment is
(a) $\frac{Il^2}{2\pi}$ (b) $\frac{Il^2}{4\pi}$
(c) $\frac{l^2I}{2\pi}$ (d) $\frac{l^2I}{4\pi}$

6. A parallel beam of white light is reflected from a thin wedge-shaped film. The colour of the fringe at the edge of the wedge will be
(a) white (b) red
(c) black (d) violet

7. An object is placed at the focus of a convex mirror. If its focal length is 20 cm, the distance of mirror is
(a) 10 cm (b) 20 cm
(c) 40 cm (d) None of these

8. A convex lens of glass ($\mu = 1.5$) has a focal length of 8 cm when placed in air. What is the focal length of lens when it is immersed in water $\left(\mu = \frac{4}{3}\right)$?
(a) 4 cm (b) 8 cm
(c) 16 cm (d) 32 cm

9. An LCR series circuit, connected to a source E, is at resonance. Then
(a) the voltage across R is zero
(b) the voltage across R equals applied voltage
(c) the voltage across C is zero
(d) the voltage across L equals applied voltage

10. What will be the colour of the sky as seen from the earth if there were no atmosphere ?
(a) Black (b) Blue
(c) Orange (d) Red

11. In a single slit diffraction experiment, the width of the slit is made double its original width. Then the central maximum on the diffraction pattern will become
(a) narrower and fainter
(b) narrower and brighter
(c) broader and fainter
(d) broader and brighter

12. An α-particle and a deuteron projected with equal kinetic energies describe circular paths of radii r_1 and r_2 respectively in a uniform magnetic field. The ratio r_1/r_2 is
(a) 1 (b) 2
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

13. The energy in monochromatic X-rays of wavelength 1 Å is roughly equal to
(a) 2×10^{-15} J (b) 2×10^{-16} J
(c) 2×10^{-17} J (d) 2×10^{-18} J

14. The de-Broglie wavelength of a neutron at 927°C is λ . What will be its wavelength at 27°C ?
(a) $\frac{\lambda}{2}$ (b) λ
(c) 2λ (d) 4λ

15. A telescope has an objective of focal length 100 cm and an eye-piece of focal length 5 cm. What is the magnifying power of the telescope when it is in normal adjustment ?
(a) 0.2 (b) 2.0
(c) 20 (d) 200

16. An isotropic source of 2 candela produces a light flux equal to
(a) 2π lumen (b) 4π lumen
(c) 6π lumen (d) 8π lumen

17. No photoelectrons are emitted from a metal if the wavelength of the light exceeds 600 nm. The work function of the metal is approximately equal to
(a) 3×10^{-16} J (b) 3×10^{-19} J
(c) 3×10^{-20} J (d) 3×10^{-22} J

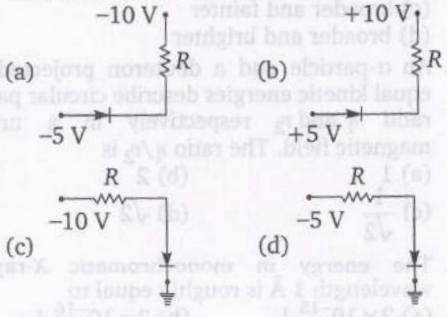
18. In Rutherford's α-particle experiment with thin gold foil, 8100 scintillations per minute are observed at an angle of 60°. The number of scintillations per minute at an angle of 120° will be
(a) 900 (b) 2025
(c) 32400 (d) 4050

19. Which of the following gates has the truth table ?

A	B	X
0	0	1
1	0	1
0	1	1
1	1	0

(a) NAND (b) NOR
(c) XOR (d) AND

20. Which of the junction diodes shown below are forward biased ?



21. A uniform rod of mass m and length l is held inclined at an angle of 60° with the vertical. What will be the potential energy of the rod in this position ?
(a) Zero (b) $\frac{mgl}{4}$
(c) $\frac{mgl}{2}$ (d) mgl

22. A sphere rolls down an inclined plane without slipping. What fraction of its total energy is rotational ?
(a) $\frac{2}{7}$ (b) $\frac{3}{7}$
(c) $\frac{4}{7}$ (d) $\frac{5}{7}$

23. From the top of a building 40 m tall, a boy projects a stone vertically upwards with an initial velocity 10 ms^{-1} such that it eventually falls to the ground. After how long will the stone strike the ground ? Take $g = 10\ ms^{-2}$
(a) 1 s (b) 2 s
(c) 3 s (d) 4 s

24. A 150 m long train is travelling from east to west at a speed of 20 ms^{-1}. A bird is flying from west to east at a speed of 5 ms^{-1}. How long will the bird take to cross the train ?
(a) 6 s (b) 7.5 s
(c) 10 s (d) 30 s

25. An elastic spring has a length l_1 when it is stretched with a force of 2 N and a length of l_2 when it is stretched with a force of 3N. What will be the length of the spring if it is stretched with force of 5N ?
(a) $(l_1 + l_2)$ (b) $\frac{1}{2}(l_1 + l_2)$
(c) $(3l_2 - 2l_1)$ (d) $(3l_1 - 2l_2)$

26. The density of water at the surface of ocean is ρ. If the bulk modulus of water is B, what is the density of ocean water at a depth where the pressure is np_0, where p_0 is the atmospheric pressure ?
(a) $\frac{\rho B}{B-(n-1)p_0}$ (b) $\frac{\rho B}{B+(n-1)p_0}$
(c) $\frac{\rho B}{B-np_0}$ (d) $\frac{\rho B}{B+np_0}$

27. What is the torque of force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at point whose position vector is $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$?
(a) $14\hat{i} - 38\hat{j} + 16\hat{k}$ (b) $4\hat{i} + 4\hat{j} + 6\hat{k}$
(c) $-14\hat{i} + 38\hat{j} - 16\hat{k}$ (d) $-21\hat{i} + 3\hat{j} + 5\hat{k}$

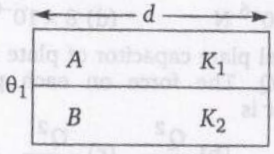
28. Two satellites of masses $3M$ and M orbit the earth in circular orbits of radii r and $3r$ respectively. The ratio of their speeds is
(a) 1 : 1 (b) $\sqrt{3}$: 1
(c) 3 : 1 (d) 9 : 1

29. The dimensions of Reynolds's constant are
(a) $[M^0L^0T^0]$ (b) $[ML^{-1}T^{-1}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[ML^{-2}T^{-2}]$

30. The surface tension of soap solution is σ. What is the work done in blowing soap bubble of radius r?
(a) $\pi r^2\sigma$ (b) $2\pi r^2\sigma$
(c) $4\pi r^2\sigma$ (d) $8\pi r^2\sigma$

31. The ratio of rms speed of O_2 to H_2 is
(a) $\frac{1}{4}$ (b) 4 (c) 2 (d) $\frac{1}{2}$

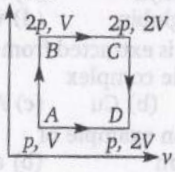
32. Two rods A and B of different materials are welded together as shown in figure. If their thermal conductivities are K_1 and K_2 the thermal conductivity of the composite rod will be



(a) $2(K_1 + K_2)$ (b) $\frac{3}{2}(K_1 + K_2)$
(c) $(K_1 + K_2)$ (d) $\frac{1}{2}(K_1 + K_2)$

33. The wavelength of the radiation emitted by a body depends upon
(a) the nature of its surface
(b) the area of its surface
(c) the temperature of its surface
(d) All the above factors

34. An ideal monoatomic gas is taken around the cycle $ABCDA$ as shown in the p-V diagram. The work done during the cycle is given by



(a) $\frac{1}{2}pV$ (b) pV
(c) $2pV$ (d) $4pV$

35. An ideal gas at pressure p is adiabatically compressed so that its density becomes n times the initial value. The final pressure of the gas will be $\left(\gamma = \frac{C_p}{C_V}\right)$
(a) $n^{\gamma}p$ (b) $n^{-\gamma}p$
(c) $n^{(\gamma-1)}p$ (d) $n^{(1-\gamma)}p$

36. The displacement of a travelling wave is given by
$$y = 2\cos 2\pi(10t - 0.008x + 0.35)$$
where, x and y are in centimetre and t in second. What is the phase difference between oscillatory motion at two points separated by a distance of 4 m ?
(a) 2π (b) 4π (c) 6π (d) 8π

37. A bimetallic strip consists of brass and iron when it is heated it bends into an arc with brass on the convex and iron on the concave side of the arc. This happens because
(a) brass has a higher specific heat capacity than iron
(b) density of brass is more than that of iron
(c) it is easier to bend an iron strip than a brass strip of the same size
(d) brass has a higher coefficient of linear expansion than iron

38. Three sound waves of equal amplitudes have frequencies $(\nu - 1)$, ν, $(\nu + 1)$. They superpose to give beats. The number of beats produced per second will be
(a) ν (b) $\nu/2$
(c) 2 (d) 1

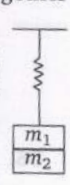
39. Which of the following functions does not represent a stationary wave ? Here a, b and c are constants.
(a) $y = a\cos bx \sin ct$
(b) $y = a\sin bx \cos ct$
(c) $y = a\sin(bx + ct)$
(d) $y = a\sin(bx + ct) + a\sin(bx - ct)$

40. The velocity of sound is greatest in
(a) water (b) air
(c) vacuum (d) steel

41. Due to Doppler effect, the shift in wavelength observed is 0.1 Å, for a star producing wavelength 6000 Å. The velocity of recession of star will be
(a) 2.5 kms^{-1} (b) 10 kms^{-1}
(c) 5 kms^{-1} (d) 20 kms^{-1}

42. A body executing linear simple harmonic motion has a velocity of 3 ms^{-1} when its displacement is 4 cm and a velocity of 4 ms^{-1} when its displacement is 3 cm. What is the amplitude of oscillation ?
(a) 5 cm (b) 7.5 cm
(c) 10 cm (d) 12.5 cm

43. Two masses m_1 and m_2 are suspended together by a massless spring of force constant k, as shown in figure. When the masses are in equilibrium, mass m_1 is removed without disturbing the system. The angular frequency of oscillation of mass m_2 is



(a) $\sqrt{\frac{k}{m_2}}$ (b) $\sqrt{\frac{k}{m_1}}$
(c) $\sqrt{\frac{km_1}{m_2^2}}$ (d) $\sqrt{\frac{km_2}{m_1^2}}$

44. The intensity level of a sound wave is defined by an arbitrary scale. The zero of the scale is taken at the sound wave intensity
(a) 1×10^{-10} Wm^{-2}
(b) 1×10^{-12} Wm^{-2}
(c) 1×10^{-14} Wm^{-2}
(d) 1×10^{-16} Wm^{-2}

45. A pendulum bob of mass m carrying a charge q is at rest with its string making an angle θ with the vertical in a uniform horizontal electric field E. The tension in the string is
(a) $\frac{mg}{\sin\theta}$ and $\frac{qE}{\cos\theta}$ (b) $\frac{mg}{\cos\theta}$ and $\frac{qE}{\sin\theta}$
(c) $\frac{qE}{mg}$ (d) $\frac{mg}{qE}$

46. A galvanometer of resistance 10 Ω gives full-scale deflection when 1 mA current passes through it. The resistance required to convert it into a voltmeter reading upto 2.5 V is
(a) 24.9 Ω
(b) 249 Ω
(c) 2490 Ω
(d) 24900 Ω

47. The current inside an electrolytic cell is carried by
(a) positive ions
(b) negative ions
(c) both positive and negative ions
(d) electrons

48. The electric potential V (in volt) varies with x (in metre) according to the relation $V = (5 + 4x^2)$. The force experienced by a negative charge of 2×10^{-6} C located at $x = 0.5$ m is
(a) 2×10^{-6} N (b) 4×10^{-6} N
(c) 6×10^{-6} N (d) 8×10^{-6} N

49. A parallel plate capacitor of plate area A has a charge Q. The force on each plate of the capacitor is
(a) $\frac{2Q^2}{\varepsilon_0 A}$ (b) $\frac{Q^2}{\varepsilon_0 A}$ (c) $\frac{Q^2}{2\varepsilon_0 A}$ (d) zero

50. In Millikan's oil drop experiment, an oil drop is observed to move vertically upwards. The upward motion of the drop is due to
(a) gravity (b) viscosity
(c) buoyancy (d) electric field

CHEMISTRY

51. Which one of these is an important polysaccharide present in plants ?
(a) Glucose (b) Glycogen
(c) Sucrose (d) Starch

52. Cannizaro reaction would be given by
(a) C_6H_5CHO (b) CH_3COCH_3
(c) CH_3CHO (d) $C_2H_5COC_2H_5$

53. $CH_3C{\equiv}CH \xrightarrow[+Hg^{2+},\ 60°C]{H_2SO_4} ?$
(a) CH_3CHO (b) CH_3COOH
(c) CH_3COCH_3 (d) CH_3CH_2COOH

54. In which of the following can peroxide effect operate ?
(a) $CH_3CH_2CH{=}CH_2 + HCl$
(b) $CH_3CH_2CH{=}CH_2 + HBr$
(c) $CH_3CH{=}CH{\cdot}CH_3 + HBr$
(d) $CH_3CH_2CH{=}CH_2 + HI$

55. The term 'quaternary structure' is used in the context of
(a) carbohydrates (b) vitamins
(c) proteins (d) nucleic acids

56. Aspirin is
(a) antibiotic (b) sedative
(c) tranquilizer (d) antipyretic

57. The following is a coordination compound of iron
(a) chlorophyll (b) Mohr's salt
(c) haemoglobin (d) vitamin B_{12}

58. This metal is extracted from its ore by formation of a cyanide complex
(a) Ag (b) Cu (c) Al (d) Zn

59. Smoke is an example of
(a) emulsion (b) aerosol
(c) gel (d) foam

60. Propan-2-ol + ethanoic acid $\longrightarrow$?
(a) $(CH_3)_2CHCOOCH_3$
(b) $CH_3COOCH(CH_3)_2$
(c) $CH_3COOCH_2CH_3$
(d) $(CH_3)_2CHCOOCH_2CH_3$

61. Which of the following gives RNC when treated with $CHCl_3$ and KOH ?
(a) RNH_2 (b) R_2NH
(c) $R_4N^+Cl^-$ (d) R_3N

62. Which of the following is a soap ?
(a) Sodium acetate
(b) Magnesium butyrate
(c) Sodium palmitate
(d) Palmitic acid

63. The C—C bond dissociation energy in kcal/mol is
(a) 8.1 (b) 0.81 (c) 81 (d) 810

64. A compound has four sp^3 hybridised covalent bonds. Its shape is
(a) linear (b) square planar
(c) octahedral (d) tetrahedral

65. The resonance energy of benzene in kcal/mol is about
(a) ≈ 20 (b) ≈ 40
(c) ≈ 60 (d) ≈ 80

66. 1 mole of CO_2 contains
(a) 6×10^{23} atoms of C
(b) 6×10^{23} atoms of O
(c) 18×10^{23} molecules of CO_2
(d) 3 g-atoms of CO_2

67. Rutherford's famous experiment with α-particles used this metal
(a) Ni (b) Au
(c) Fe (d) Zn

68. The charge on an electron in Coulombs is
(a) 1.602×10^{-19} (b) 1.062×10^{-19}
(c) 1.620×10^{-19} (d) 1.006×10^{-19}

69. The number of subshells for a shell with principal quantum number n is
(a) n (b) $n+1$
(c) $n-1$ (d) $2n^2$

70. Which of the following pairs of atomic numbers represents s-block elements ?
(a) 7, 15 (b) 6, 12
(c) 9, 17 (d) 3, 11

71. For a system in equilibrium
(a) $\Delta G = 0$ (b) $\Delta G > 0$
(c) $\Delta G < 0$ (d) $\Delta H = 0$

72. Which of the following will produce the highest rise in temperature ?
(a) 67 mL of 1 M NaOH + 33 mL of 0.5 M H_2SO_4
(b) 33 mL of 1 M NaOH + 67 mL of 0.5 M H_2SO_4
(c) 40 mL of 1 M NaOH + 60 mL of 0.5 M H_2SO_4
(d) 50 mL of 1 M NaOH + 50 mL of 0.5 M H_2SO_4

73. The standard for atomic mass is
(a) ^1_1H (b) $^{12}_6C$
(c) $^{14}_6C$ (d) $^{16}_8O$

74. Which of the following involves absorption of energy ?
(a) $Cl + e^- \longrightarrow Cl^-$
(b) $O^- + e^- \longrightarrow O^{2-}$
(c) $O + e^- \longrightarrow O^-$
(d) $S + e^- \longrightarrow S^-$

75. The successive ionisation energies of an element are 800, 2,000, 3,600, 25,000 and 32,000 kJ/mol. The number of valence electrons are
(a) 5 (b) 2
(c) 3 (d) 4

76. The first order rate constant for the decomposition of N_2O_5 is $6.2 \times 10^{-4}\ s^{-1}$. The half-life in seconds is nearly
(a) 1117.7 (b) 111.7
(c) 223 (d) 160

77. The rate constant of a reaction is $1 \times 10^{-2}\ mol^{-2}\ L^2\ s^{-1}$. The order of the reaction is
(a) 0 (b) 1
(c) 2 (d) 3

78. The oxidation number of O in OF_2 is
(a) -2 (b) $+1$
(c) $+2$ (d) -1

79. The correct relationship is
(a) $K_C = n \times K_p$ (b) $K_p = K_C(RT)^{\Delta n_g}$
(c) $K_p = n \times K_C$ (d) $K_C - K_p = (RT)^{\Delta N}$

80. For the reaction, $2B + A \rightleftharpoons C$, the equilibrium constant is
(a) $\frac{[A][B]^3}{[C]}$ (b) $\frac{[C]}{[A][2B]}$
(c) $\frac{[C]}{[A][B]^2}$ (d) $\frac{[A][B]}{[C]}$

81. A buffer contains equal concentrations of X^- and HX. The K_a for HX is 10^{-8}. The pH of the buffer is
(a) 3 (b) 8
(c) 7 (d) 11

82. The metal which gives H_2 on treatment with acid as well as NaOH is
(a) Fe (b) Cu
(c) Zn (d) Hg

83. Which is isoelectronic with hydride ion ?
(a) He (b) He^+
(c) Li (d) H^+

84. A solution of Na in liquid NH_3 is
(a) green (b) blue
(c) pink (d) yellow

85. The oxide which gives H_2O_2 on treatment with dilute acid is
(a) PbO_2 (b) TiO_2
(c) MnO_2 (d) Na_2O_2

86. During the electrolysis of fused NaCl, this happens at cathode
(a) oxidation of Na^+ (b) reduction of Na^+
(c) reduction of Cl^- (d) oxidation of Cl^-

87. Which of the following is a coloured gas ?
(a) N_2O (b) NO (c) H_2 (d) NO_2

88. This compound gives off O_2 on moderate heating
(a) HgO (b) ZnO
(c) CaO (d) Al_2O_3

89. Plaster of Paris is hardened by
(a) liberating CO_2
(b) hydration
(c) dehydration
(d) changing into $CaCO_3$

90. The least ionic chloride is formed by
(a) Mg (b) Ca (c) Be (d) Sr

91. Aluminium bronze has
(a) 90% Cu (b) 50% Ni
(c) 90% Sn (d) 50% Cu

92. In the thermite welding, this is used
(a) Ca (b) Al + Fe
(c) Fe (d) Al + Fe_2O_3

93. K_2CO_3 cannot be prepared by Solvay-ammonia process because
(a) K_2CO_3 is fairly soluble in water
(b) it has no water of crystallisation
(c) $KHCO_3$ is highly soluble in water
(d) K_2CO_3 decomposes in H_2O

94. Glauber's salt is
(a) $Na_2SO_4 \cdot 10H_2O$ (b) $FeSO_4 \cdot 7H_2O$
(c) $ZnSO_4 \cdot 7H_2O$ (d) $MgSO_4 \cdot 7H_2O$

95. The salt responsible for permanent hardness of H_2O is
(a) Na_2SO_4 (b) $MgHCO_3$
(c) NaCl (d) $MgCl_2$

96. Which of the following carbonium ion is most stable ?
(a) $C_6H_5\overset{+}{C}H_2$ (b) $(CH_3)_3\overset{+}{C}$
(c) $(CH_3)_2\overset{+}{C}H$ (d) $CH_3\overset{+}{C}H_2$

97. Which of the following reacts in presence of anhydrous $AlCl_3$ to give acetophenone ?
(a) $C_6H_6 + CH_3COCH_3$
(b) $C_6H_5OH + CH_3COCH_3$
(c) $C_6H_5OH + CH_3COOH$
(d) $C_6H_6 + CH_3COCl$

98. The isomers which can be interconverted through rotation around a single bond are
(a) conformers (b) diastereoisomers
(c) enantiomers (d) position isomers

99. Phenol is the component of this fraction upon fractional distillation of petroleum
(a) light oil (b) middle oil
(c) heavy oil (d) pitch

100. Ethyl chloride on treatment with alcoholic alkali gives
(a) C_2H_6 (b) CH_3CHO
(c) C_2H_5OH (d) $CH_2{=}CH_2$

MATHEMATICS

101. The area of the figure bounded by $y^2 = 2x + 1$ and $x - y = 1$ is
(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{11}{3}$

102. The order of the differential equation of all tangent lines to the parabola $y = x^2$ is
(a) 1 (b) 2
(c) 3 (d) 4

103. $\int \frac{dx}{\sqrt{(1-x)(x-2)}}$ is equal to
(a) $\sin^{-1}(2x-3) + C$ (b) $\sin^{-1}(2x+5) + C$
(c) $\sin^{-1}(3-2x) + C$ (d) $\sin^{-1}(5-2x) + C$

104. $\int \frac{(\sec^2 x - 7)}{\sin^7 x} dx$ is equal to
(a) $\frac{\tan x}{\sin^7 x} + C$ (b) $\frac{\cos x}{\sin^7 x} + C$
(c) $\frac{\sin x}{\cos^7 x} + C$ (d) $\frac{\sin x}{\tan^7 x} + C$

105. $\int_{-3}^{2} \{|x+1| + |x+2| + |x-1|\} dx$ is equal to
(a) $\frac{31}{2}$ (b) $\frac{35}{2}$ (c) $\frac{37}{2}$ (d) $\frac{39}{2}$

106. The variance of first n natural numbers is
(a) $\frac{n^2+1}{12}$ (b) $\frac{n^2-1}{12}$
(c) $\frac{(n+1)(2n+1)}{6}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$

107. A particular solution of $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, $y(0) = 0$ is
(a) $e^{3x} + 3e^{-4y} = 4$ (b) $4e^{3x} - 3e^{-4y} = 3$
(c) $3e^{3x} + 4e^{-4y} = 7$ (d) $4e^{3x} + 3e^{-4y} = 7$

108. The equation of the curve whose subnormal is equal to a constant 'a' is
(a) $y = ax + b$ (b) $y^2 = 2ax + 2b$
(c) $ay^2 - x^3 = a$ (d) None of these

109. If the standard deviation of a variable X is σ, then the standard deviation of $\frac{aX+b}{c}$ (a, b, c are constants) is

(a) $\frac{a}{c}\sigma$ (b) $\left|\frac{a}{c}\right|\sigma$

(c) $\left|\frac{c}{a}\right|\sigma$ (d) $\frac{c}{a}\sigma$

110. If $\sin\alpha, \cos\alpha$ are the roots of the equation $ax^2+bx+c=0, (c\neq 0)$, then

(a) $a^2-b^2+2ac=0$

(b) $(a+c)^2=b^2-c^2$

(c) $a^2+b^2-2ac=0$

(d) $(a-c)^2=b^2+c^2$

111. If α, β, γ are such that $\alpha+\beta+\gamma=2$, $\alpha^2+\beta^2+\gamma^2=6$, $\alpha^3+\beta^3+\gamma^3=8$, then $\alpha^4+\beta^4+\gamma^4$ is equal to

(a) 7 (b) 12

(c) 18 (d) 36

112. The equation $\bar{z}=\bar{a}+\frac{r^2}{(z-a)}, r>0$ represents

(a) an ellipse

(b) a parabola

(c) a circle

(d) a straight line through point $\bar{a}$

113. If $\frac{3}{2+\cos\theta+i\sin\theta}=a+ib$, then $[(a-2)^2+b^2]$ is equal to

(a) 0 (b) 1

(c) –1 (d) 2

114. The value of

$$4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$$ is

(a) $1-i\sqrt{3}$ (b) $-1+i\sqrt{3}$

(c) $i\sqrt{3}$ (d) $-i\sqrt{3}$

115. If in the expansion of $(a-2b)^n$, the sum of 4th and 5th term is zero, then the value of $\frac{a}{b}$ is

(a) $\frac{n-4}{5}$ (b) $\frac{2(n-4)}{5}$

(c) $\frac{5}{n-4}$ (d) $\frac{5}{2(n-4)}$

116. If $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is such that $|A|=0$ and $A^2-(a+d)A+kI=0$, then k is equal to

(a) $b+c$ (b) $a+d$

(c) $ab+cd$ (d) zero

117. The sum of series $1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+\ldots$ is equal to

(a) $2(n-1)+\frac{1}{2^{n-1}}$ (b) $2n-\frac{1}{2^n}$

(c) $2+\frac{1}{2^n}$ (d) $2n-1+\frac{1}{2^n}$

118. The number of meaningful solutions of $\log_4(x-1)=\log_2(x-3)$ is

(a) 3 (b) 2

(c) 1 (d) zero

119. The number of ways of arranging letters of the word HAVANA so that V and N do not appear together is

(a) 60 (b) 80

(c) 100 (d) 120

120. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six is

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$

(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

121. The expression

$\cos^2(A-B)+\cos^2 B-2\cos(A-B)\cos A\cos B$

is independent of

(a) A (b) B

(c) both A and B (d) neither A nor B

122. If the system of equations $ax+y=3$, $x+2y=3$, $3x+4y=7$ is consistent, then value of a is equal to

(a) 2 (b) 1

(c) –1 (d) 0

123. If $\begin{vmatrix}x-1 & 5x & 7\\ x^2-1 & x-1 & 8\\ 2x & 3x & 0\end{vmatrix}=ax^3+bx^2+cx+d$,

then c is equal to

(a) –1 (b) 12

(c) 15 (d) 17

124. If $a, b, c>0$, the minimum value of $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ is

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$

125. If $\tan 1^\circ \tan 2^\circ \ldots \tan 89^\circ = x^2-8$, then the value of x can be

(a) ± 1 (b) ± 2

(c) ± 3 (d) ± 4

126. If $\sin(x+3\alpha)=3\sin(\alpha-x)$, then

(a) $\tan x=\tan\alpha$ (b) $\tan x=\tan^2\alpha$

(c) $\tan x=\tan^3\alpha$ (d) $\tan x=3\tan\alpha$

127. In a ΔABC, $2a^2 + 4b^2 + c^2 = 4ab + 2ac$, then $\cos B$ is equal to

(a) 0 (b) $\frac{1}{8}$

(c) $\frac{3}{8}$ (d) $\frac{7}{8}$

128. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. If A and B are independent events, then $P(B)$ is

(a) $\frac{3}{7}$ (b) $\frac{4}{7}$

(c) $\frac{5}{7}$ (d) $\frac{6}{7}$

129. A bag contains 4 brown and 5 white balls. A man pulls two balls at random without replacement. The probability that the man gets both the balls of the same colour is

(a) $\frac{5}{108}$ (b) $\frac{1}{6}$

(c) $\frac{5}{18}$ (d) $\frac{4}{9}$

130. The value of expression $\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ}$ is equal to

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{4}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

131. An equation of a line through the point (1, 2) whose distance from the point (3, 1) has the greatest value is

(a) $y = 2x$ (b) $y = x + 1$

(c) $x + 2y = 5$ (d) $y = 3x - 1$

132. A line is drawn through the point $P(3, 11)$ to cut the circle $x^2 + y^2 = 9$ at A and B. Then, $PA \cdot PB$ is equal to

(a) 9 (b) 121

(c) 205 (d) 139

133. If the tangent at point P on the circle $x^2 + y^2 + 6x + 6y - 2 = 0$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then length PQ is

(a) 4 (b) $2\sqrt{5}$

(c) 5 (d) $3\sqrt{5}$

134. In a triangle ABC, $a : b : c = 4 : 5 : 6$. The ratio of the radius of the circumcircle to that of the incircle is

(a) $\frac{15}{4}$ (b) $\frac{11}{5}$

(c) $\frac{16}{7}$ (d) $\frac{16}{3}$

135. If $4\sin^{-1} x + \cos^{-1} x = \pi$, then x is equal to

(a) $-\frac{1}{2}$ (b) 0

(c) $\frac{1}{2}$ (d) 1

136. The value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is equal to

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$

(c) $\frac{1}{16}$ (d) $\frac{1}{4}$

137. The vectors $\vec{a} = \hat{i} + \hat{j} + m\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + (m+1)\hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + m\hat{k}$ are coplanar, if m is equal to

(a) 1
(b) 4
(c) 3
(d) no value of m for which vectors are coplanar

138. If $|\vec{a}| = 2, |\vec{b}| = 3, |\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then the value of $\vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} + \vec{a}\cdot\vec{b}$ is equal to

(a) $\frac{19}{2}$ (b) $-\frac{19}{2}$

(c) $\frac{29}{2}$ (d) $-\frac{29}{2}$

139. The line $x + y = 6$ is a normal to the parabola $y^2 = 8x$ at the point

(a) (18, –12) (b) (4, 2)

(c) (2, 4) (d) (8, 8)

140. If $F_1 = (3, 0)$, $F_2 = (-3, 0)$ and P is any point on the curve $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ is equal to

(a) 6 (b) 8 (c) 10 (d) 12

141. If e and e_1 are the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e^2} + \frac{1}{e_1^2}$ is equal to

(a) –1 (b) 0 (c) 1 (d) 2

142. $\lim_{x \to 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$ is equal to

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

143. If $y = \cos^{-1}(\cos x)$, then $y'(x)$ is equal to

(a) 1 for all x
(b) –1 for all x
(c) 1 in 2nd and 3rd quadrant
(d) –1 in 3rd and 4th quadrant

144. If the vectors $\vec{AB} = -3\hat{i} + 4\hat{k}$ and $\vec{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a ΔABC, then the length of the median through A is
(a) $\sqrt{14}$ (b) $\sqrt{18}$
(c) $\sqrt{29}$ (d) 4

145. The period of the function $f(x) = \text{cosec}^2 3x + \cot 4x$ is
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) π

146. The graph of the function $f(x) = \log_a (x + \sqrt{x^2 + 1})$ is symmetric about
(a) x-axis (b) origin
(c) y-axis (d) the line $y = x$

147. The difference between the greatest and least values of the function $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$ is
(a) $\frac{2}{3}$ (b) $\frac{8}{7}$
(c) $\frac{3}{8}$ (d) $\frac{9}{4}$

148. If $f(x) = \begin{cases} ax^2 + b, & b \neq 0, x \leq 1 \\ bx^2 + ax + c, & x > 1 \end{cases}$
then $f(x)$ is continuous and differentiable at $x = 1$ if
(a) $c = 0, a = 2b$
(b) $a = b, c \in R$
(c) $a = b, c = 0$
(d) $a = b, c \neq 0$

149. Let $g(x)$ be the inverse of function $f(x)$ and $f'(x) = \frac{1}{1 + x^3}$, then $g'(x)$ is equal to
(a) $\frac{1}{1 + \{g(x)\}^3}$ (b) $\frac{1}{1 + \{f(x)\}^3}$
(c) $1 + \{g(x)\}^3$ (d) $1 + \{f(x)\}^3$

150. If θ is the semi vertical angle of a cone of maximum volume and given slant height, then $\tan\theta$ is given by
(a) 2 (b) 1
(c) $\sqrt{2}$ (d) $\sqrt{3}$

Answers

PHYSICS

1. (a)	2. (b)	3. (c)	4. (d)	5. (d)	6. (d)	7. (b)	8. (d)	9. (b)	10. (a)
11. (c)	12. (d)	13. (a)	14. (a)	15. (c)	16. (d)	17. (b)	18. (a)	19. (a)	20. (a)
21. (c)	22. (a)	23. (d)	24. (a)	25. (c)	26. (a)	27. (c)	28. (b)	29. (a)	30. (d)
31. (a)	32. (d)	33. (c)	34. (b)	35. (a)	36. (c)	37. (d)	38. (c)	39. (c)	40. (d)
41. (c)	42. (a)	43. (a)	44. (b)	45. (b)	46. (c)	47. (c)	48. (d)	49. (c)	50. (d)

CHEMISTRY

51. (d)	52. (a)	53. (c)	54. (b)	55. (c)	56. (d)	57. (c)	58. (a)	59. (d)	60. (b)
61. (a)	62. (c)	63. (c)	64. (d)	65. (b)	66. (a)	67. (b)	68. (a)	69. (c)	70. (d)
71. (a)	72. (b)	73. (b)	74. (b)	75. (c)	76. (a)	77. (d)	78. (c)	79. (b)	80. (c)
81. (b)	82. (c)	83. (a)	84. (b)	85. (d)	86. (b)	87. (d)	88. (a)	89. (b)	90. (c)
91. (a)	92. (d)	93. (c)	94. (a)	95. (d)	96. (b)	97. (d)	98. (a)	99. (b)	100. (d)

MATHEMATICS

101. (*)	102. (a)	103. (a)	104. (a)	105. (*)	106. (b)	107. (c)	108. (b)	109. (b)	110. (a)
111. (c)	112. (c)	113. (b)	114. (c)	115. (*)	116. (d)	117. (a)	118. (c)	119. (b)	120. (a)
121. (b)	122. (a)	123. (d)	124. (b)	125. (c)	126. (c)	127. (d)	128. (c)	129. (d)	130. (b)
131. (a)	132. (b)	133. (c)	134. (c)	135. (c)	136. (a)	137. (d)	138. (d)	139. (c)	140. (c)
141. (c)	142. (c)	143. (a)	144. (b)	145. (d)	146. (b)	147. (d)	148. (a)	149. (c)	150. (c)

Note : '*' Option is not correct.

Solutions

PHYSICS

1. The induced emf

$$e = -N\frac{\Delta\phi}{\Delta t}$$

or $$e = -\frac{N(B_2 - B_1)A}{\Delta t}$$

or $$e = -\frac{100\,(0.1 - 0.05)\times 0.1\times 0.05}{0.05}$$

$$e = 0.5\,\text{V}$$

2. The magnetisation

$$I \propto \frac{1}{T}$$

As temperature is increased the magnetisation decreases.

3. Power $P = I^2R$

$$\therefore \quad P \propto I^2$$

Then $$\frac{P_1}{P_2} = \frac{I^2}{\left(\frac{101}{100}\right)^2 I^2}$$

Percentage increase in power

$$\left(\frac{P_2}{P_1} - 1\right)\% = \left(\frac{101\times 101}{100\times 100} - 1\right)\%$$

$$= 2\%$$

4. Let $R_1 + R_2 = 8\Omega$ ...(i)

and $\frac{R_1R_2}{R_1 + R_2} = 1.5\,\Omega$ ...(ii)

By using Eqs. (i) and (ii,) we obtain

$$R_1 = 6\,\Omega \text{ and } R_2 = 2\Omega$$

5. If the radius of circle is r, then

$$2\pi r = l$$

$$\text{Area} = \pi r^2 = \frac{l^2}{4\pi}$$

Magnetic moment $= IA = \frac{Il^2}{4\pi}$

7. f is +ve for convex mirror.

Therefore, for convex mirror

$$\frac{1}{v} - \frac{1}{20} = +\frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{10}$$

or $$v = 10\text{ cm}$$

8. $f_{\text{water}} = 4\times f_{\text{air}}$

$$= 4\times 8 = 32\,\text{cm}$$

9. Equation of voltage $E = \sqrt{V_R^2 + (V_L - V_C)^2}$

At resonance (series circuit)

$$V_L = V_C \Rightarrow E = V_R$$

ie, whole applied voltage appeared across the resistance.

10. As there is no atmosphere, so no scattering of colours of white light occurs, hence the sky will appear black.

12. Given, $K_\alpha = K_d = K$, $q_\alpha = q = q_d$, $m_\alpha = 2m_d$

For α-particle $r_1 = \frac{\sqrt{2m_1K}}{Bq}$

For deuteron $r_2 = \frac{\sqrt{2m_2K}}{Bq}$

$$\therefore \quad \frac{r_1}{r_2} = \sqrt{2} \qquad \left(\because m_2 = \frac{m_1}{2}\right)$$

13. Energy $E = \frac{hc}{\lambda} = \frac{6.6\times 10^{-34}\times 3\times 10^8}{10^{-10}}$

$$\approx 2\times 10^{-15}\,\text{J}$$

14. de-Broglie wavelength $\lambda \propto \frac{1}{\sqrt{T}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{273 + 927}{273 + 27}} = \sqrt{\frac{1200}{300}} = 2$$

or $$\lambda_2 = \frac{\lambda}{2}$$

15. The magnifying power of the telescope in normal adjustment

$$m = \frac{f_o}{f_e} = \frac{100}{5} = 20$$

16. Given, $L = 2\,\text{cd}$

The light flux $\phi = 4\pi L$

$$= 4\pi\times 2$$

$$\phi = 8\pi \text{ lumen}$$

17. The work function

$$W = \frac{hc}{\lambda}$$

$$= \frac{6.6\times 10^{-34}\times 3\times 10^8}{600\times 10^{-9}}$$

$$W = 3\times 10^{-19} \qquad \text{(approximately)}$$

18. Number of α-particles scattered through angle θ

$$N \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

or $$\frac{N_1}{N_2} = \frac{\left(\sin^4 \frac{\theta_2}{2}\right)}{\left(\sin^4 \frac{\theta_1}{2}\right)}$$

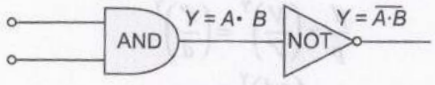
or $$\frac{8100}{N_2} = \frac{\left(\sin^4 \frac{120}{2}\right)}{\left(\sin^4 \frac{60}{2}\right)}$$

$$\frac{8100}{N_2} = \frac{\sin^4 60}{\sin^4 30}$$

$$\Rightarrow N_2 = \frac{8100 \times \frac{1}{16}}{\frac{9}{16}}$$

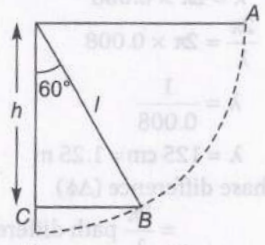
or $$N_2 = 900$$

19. The given truth table is for a NAND gate.



20. For diode to be operated in forward biasing, its *p*-side should be at higher potential than *n*-side. In all given options, (a) satisfies the required condition.

21. Vertical height = $h = l \cos 60°$



and potential energy = mgh

$$= mgl \cos 60° = \frac{mgl}{2}$$

22. When a body rolls down an inclined plane, it makes rotational as well as translational motions. Thus, it is associated with rotational and translational kinetic energies. Hence, total kinetic energy of sphere

$$K = K_{\text{rot.}} + K_{\text{trans}}$$

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$

$$= \frac{1}{2}\frac{2}{5} mr^2 \left(\frac{v^2}{r^2}\right) + \frac{1}{2} mv^2$$

$$= \frac{1}{5} mv^2 + \frac{1}{2} mv^2 = \frac{7}{10} mv^2$$

Also, $$K_{\text{rot.}} = \frac{1}{2} I\omega^2 = \frac{1}{5} mv^2$$

Hence, $$\frac{K_{\text{rot.}}}{K} = \frac{\frac{1}{5} mv^2}{\frac{7}{10} mv^2} = \frac{2}{7}$$

23. $$S = ut + \frac{1}{2} gt^2$$

or $$40 = -10t + \frac{10}{2} t^2$$

or $$t^2 - 2t - 8 = 0$$

or $$(t + 2)(t - 4) = 0$$

$\Rightarrow t = 4$ s as t can't be negative.

24. Relative velocity of bird with respect to train

$$= 5 - (-20)$$

$$= 5 + 20 = 25 \text{ ms}^{-1}$$

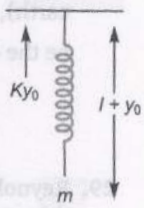
Time taken by bird

$$= \frac{d}{v_{\text{rel}}} = \frac{150}{25} = 6 \text{ s}$$

25. In equilibrium position for a spring reaction.

$$ky_0 = mg$$

where y_0 is the extension in spring.



Let l be the length of the spring.

Then for l_1

Extension = $(l_1 - l)$

$\therefore$ $$k(l_1 - l) = 2 \quad \ldots(i)$$

Similarly for l_2

$$k(l_2 - l) = 3$$

Eliminating k from Eqs. (i) and (ii), we get

$$l = 3l_1 - 2l_2$$

Moreover adding Eqs. (i) and (ii), we get

$$k[l_1 + l_2 - 2l] = 5$$

Substituting the value of l in above equation we get

$$k[5l_2 - 5l_1] = 5$$

$5l_2 - 5l_1$ is the extension for 5 N force

$\therefore$ Total length = $5l_2 - 5l_1 + l$

$$= 3l_2 - 2l_1$$

26. When pressure is increased then density will decrease.

Thus, the density of ocean water at a depth

$$\rho' = \frac{\rho}{1 - \frac{dp}{B}}$$

Here, $dp = np_0 - p_0$

$$= p_0(n-1)$$

$$\therefore \quad \rho' = \frac{\rho B}{B - (n-1)p_0}$$

27. Torque $\tau = \vec{F} \times \vec{r}$

$$= (-3\hat{i} + \hat{j} + 5\hat{k}) \times (7\hat{i} + 3\hat{j} + \hat{k})$$

or $$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ 7 & 3 & 1 \end{vmatrix}$$

$$= -14\hat{i} + 38\hat{j} - 16\hat{k}$$

28. Orbital velocity $v = \sqrt{\frac{GM}{r}}$

where, M = mass of the planet

r = radius of the orbit

Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit. For a given planet (here earth), greater the radius of orbital, lesser will be the orbital velocity of the satellite $\left(v \propto \frac{1}{\sqrt{r}}\right)$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3}{1}}$$

29. Reynold's number is a pure number which determines the nature of flow of liquid through a pipe. It is a dimensionless quantity.

Reynold's number is defined as the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid.

30. Work done in blowing a soap bubble $W = \sigma \times 8\pi r^2$ as bubble has two free surfaces.

31. Root mean square velocity

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

For constant temperature $v_{rms} \propto \frac{1}{\sqrt{M}}$

So, $$\frac{(v_{rms})_{O_2}}{(v_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$= \sqrt{\frac{2}{32}} = \frac{1}{4}$$

32. Two slabs each of length d and same areas and thermal conductivities K_1 and K_2 are connected in parallel, then

Heat current $H = H_1 + H_2$

$$\frac{K(2A)(\theta_1 - \theta_2)}{d} = \frac{K_1 A(\theta_1 - \theta_2)}{d} + \frac{K_2 A(\theta_1 - \theta_2)}{d}$$

$$\Rightarrow \quad K = \frac{K_1 + K_2}{2}$$

33. The wavelength of radiation emitted by a body depends upon the temperature of its surface.

According to Wien's law $\lambda_m T$ = constant, on heating up to ordinary temperatures, only long wavelength (red) radiation is emitted. As the temperature rises, shorter wavelengths are also emitted in more and more quantity. Hence, the colour of radiation emitted by the hot wire shifts from red to yellow, then the blue and finally to white.

34. Work done = area of closed pV diagram

$$= (2V - V) \times (2p - p) = pV$$

35. Volume of the gas $V = \frac{m}{d}$ and using

pV^γ = constant we get

$$\frac{p'}{p} = \left(\frac{V}{V'}\right)^\gamma = \left(\frac{d'}{d}\right)^\gamma$$

$$p' = \left(\frac{nd}{d}\right)^\gamma p = n^\gamma p$$

36. Given, $y = 2\cos(20\pi t - 2\pi \times 0.008x + 0.7\pi)$

Comparing with general equation

$$y = a\cos(\omega t \pm kx \pm \phi_0)$$

we get $k = 2\pi \times 0.008$

$$\frac{2\pi}{\lambda} = 2\pi \times 0.008 \qquad \left[\text{As } k = \frac{2\pi}{\lambda}\right]$$

$$\Rightarrow \quad \lambda = \frac{1}{0.008}$$

$$\lambda = 125 \text{ cm} = 1.25 \text{ m}$$

Now, phase difference ($\Delta\phi$)

$$= \frac{2\pi}{\lambda} \text{ path difference } (\Delta x)$$

$$= \frac{2\pi}{1.25} \times 4 = 6\pi$$

37. Two strips of equal lengths but of different materials (different coefficient of linear expansion) when joined together, is called bimetallic strip, and can be used in thermostat to break or make electrical contact. This strip has the characterstic property of bending on heating due to unequal linear expansion of the two metals.

The strip will bend with metal of greater α on outer side *ie*, convex side.

38. If $(\nu - 1)$, ν, $(\nu + 1)$ be the frequencies of the three waves and a be the amplitude of each then $y_1 = a \sin 2\pi (\nu - 1)t$, $y_2 = a \sin 2\pi \nu t$ and $y_3 = a \sin 2\pi (\nu + 1)t$

Resultant displacement due to all three waves is

$$y = y_1 + y_2 + y_3$$
$$= a \sin 2\pi\nu t + a[\sin 2\pi (\nu - 1)t + \sin 2\pi (\nu + 1)t]$$
$$= a \sin 2\pi\nu t + a[2 \sin 2\pi \nu t \cos 2\pi t]$$
$$= a[2 \cos 2\pi t + 1] \sin 2\pi\nu t$$
$$= a' \sin 2\pi \nu t \text{ with } a' = a[1 + 2 \cos 2\pi t]$$

So, $I \propto (a')^2 \propto a^2 (1 + 2 \cos 2\pi t)^2$

For I to be max. or min.

$$\frac{dI}{dt} = 0 \Rightarrow \frac{d}{dt}(1 + 2 \cos 2\pi t)^2 = 0$$

ie, $2(1 + 2 \cos \pi t)(2 \sin 2\pi t) \times 2\pi = 0$

$\sin 2\pi t = 0$ or $1 + 2 \cos 2\pi t = 0$

So, if $1 + 2 \cos 2\pi t = 0 \Rightarrow 2\pi t = 2\pi n \pm \frac{2\pi}{3}$ with $n = 0, 1, 2, \ldots$

$t = \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{5}{3}, \ldots$ and for these value of t

$\cos 2\pi t = -\left(\frac{1}{2}\right)$, $I = 0$, *ie,* I is minimum and if $\sin 2\pi t = 0$

$2\pi t = n\pi, n = 0, 1, 2, \ldots \Rightarrow t = 0, \frac{1}{2}, 1, \frac{3}{2}, 2 \ldots$

I is therefore $9a^2, a^2, 9a^2, a^2$

ie, intensity is maximum (with two different values)

ie, number of beats per sec is two.

39. Two superimposing waves are incident wave $y_1 = a \sin (\omega t - kx)$ and reflected wave $y_2 = a \sin (\omega t + kx)$

Then by principle of superposition

$$y = y_1 + y_2$$
$$= a[\sin (\omega t - kx) + \sin \omega t + kx)]$$
$$\Rightarrow \quad y = 2a \cos kx \sin \omega t$$

Therefore, option (c) doesnot represent a stationary wave.

40. Velocity of sound in any elastic medium is given by $v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\text{Elasticity of the medium}}{\text{Density of the medium}}}$

As solids are most elastic while gases least *ie,* $E_S > E_L > E_G$. So, the velocity of sound is maximum in solids and minimum in gases.

Hence, $v_{\text{steel}} > v_{\text{water}} > v_{\text{air}}$

$5000 \text{ ms}^{-1} > 1500 \text{ ms}^{-1} > 330 \text{ ms}^{-1}$

41. The spectral shift $(z) = \frac{\lambda - \lambda_0}{\lambda_0}$

Also, $z = \frac{\text{Velocity of recession of star } (v)}{\text{Velocity of light } (c)}$

$$\therefore \quad v = \frac{\lambda - \lambda_0}{\lambda_0} \times c$$
$$= \frac{0.1}{6000} \times 3 \times 10^8 = 5000 \text{ ms}^{-1}$$
$$= 5 \text{ km s}^{-1}$$

42. Velocity of SHM $v = \omega\sqrt{a^2 - y^2}$

$$\Rightarrow \quad 9 = \omega^2 [a^2 - 16] \quad \ldots(i)$$

Similarly, $16 = \omega^2 [a^2 - 9]$ $\ldots(ii)$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{16}{9} = \frac{[a^2 - 9]}{[a^2 - 16]}$$
$$\Rightarrow \quad a^2 = 25$$

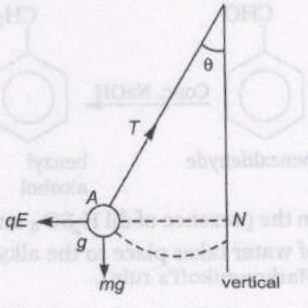
or $a = 5 \text{ cm}$

43. With mass m_2 alone, the angular frequency $\omega = \sqrt{\frac{k}{m_2}}$

44. The minimum intensity that can be heard called threshold of hearing $(I_0) = 10^{-12} \text{ Wm}^{-2}$ at 1 kHz.

45. A pendulum bob of weight mg and carrying a charge q is suspended in the electric field E. Suppose that it comes to rest at point A, so that it makes an angle θ with the vertical as shown.

At point A, the bob is acted upon by the following three forces.



(i) Weight mg acting vertically downward.

(ii) Tension T in the string along AS

(iii) Electrostatic force qE on the bob along horizontal.

Since, the bob is in equilibrium under the action of the forces mg, T and qE, these forces can be represented by the sides SN, AS and NA of triangle ANS.

Therefore, $\frac{mg}{SN} = \frac{qE}{NA} = \frac{T}{AS}$

$\Rightarrow qE = T \sin\theta$

and $mg = T\cos\theta$

46. Required series resistance

$$R = \frac{V}{I_g} - G$$

$$= \frac{2.5}{1\times10^{-3}} - 10$$

$$= 2490\ \Omega$$

47. In an electrolytic cell, the current is carried out inside the electrolyte by both positive and negative ions.

48. Given, $V = 5 + 4x^2$

$$\frac{dV}{dr} = E$$

$$\therefore \quad \frac{dV}{dx} = 8x = E$$

or $E = 8\times0.5 = 4\ \text{Vm}^{-1}$

Now, $F = qE$

$$= 2\times10^{-6}\times4$$

$$= 8\times10^{-6}\ \text{N}$$

49. Force on one plate due to another is

$$F = QE = Q\times\frac{\sigma}{2\varepsilon_0}$$

$$= Q\left[\frac{Q}{2A\varepsilon_0}\right]$$

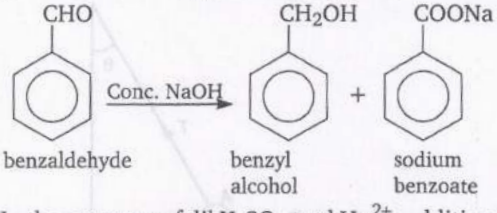
$$= \frac{Q^2}{2A\varepsilon_0}$$

50. In Millikan's oil drop experiment the electric field is applied between the plates, the negatively charged oil drop carrying a charge (q) will experience electrostatic force of attraction in upward direction. If the magnitude of electrostatic force is greater than that of gravitational force, the oil drop will start moving in upward direction.

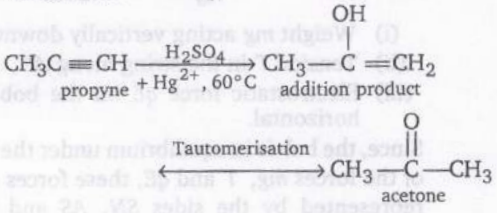
CHEMISTRY

51. Starch is found in seeds and tubers of almost all plants. It is the form in which glucose is stored for later use, *ie*, it is the main source of energy. Hence, it is obvious that starch is an important polysaccharide, found in plants.

52. Cannizaro reaction is given by only those aldehydes in which α-H atom is not present. Among the given carbonyl compounds, C_6H_5CHO (benzaldehyde) due to lack of α-H atom, gives Cannizaro reaction as



53. In the presence of dil H_2SO_4 and Hg^{2+}, addition of water takes place to the alkyne (according to Markownikoff's rule).



54. When HBr reacts with unsymmetrical alkene, peroxide effect (*ie*, addition of HBr opposite to the Markownikoff's rule) can operate, since it is only HBr for that both the propagation steps (the steps involved during the reaction) are exothermic.

$$CH_3CH_2CH{=}CH_2 + \overset{\delta+\ \delta-}{H\ Br} \xrightarrow{\text{Peroxide}} CH_3CH_2CH_2CH_2Br$$

55. The term 'quaternary structure' is used in proteins. Infact, quaternary structure refers to the aggregation of protein monomers. A good example of protein that have such structure is haemoglobin.

56. Aspirin is used to bring down the body temperature in case of high fever and to relief pain, so it is an antipyretic as well as analgesic.

57. Haemoglobin, contains two part *viz*, haem and globular protein. Haem part of haemoglobin contains one Fe^{2+} ion (iron) coordinated in a square planar array with the four N-atoms of a porphyrin molecule.

Chlorophyll is a coordination compound of Mg, vitamin B_{12} is a coordination compound of Co while Mohr's salt is a double salt, not a coordination compound.

58. Gold (Au) and silver (Ag) are extracted by treating their ores with dilute NaCN solution in the presence of air. During this process, a cyanide complex is formed which gives pure metal when treated with more electropositive metal like Zn.

$$4Ag + 8CN^- + 2H_2O + O_2 \longrightarrow 4[Ag(CN)_2]^- + 4OH^-$$

$$2[Ag(CN)_2]^- + Zn \longrightarrow [Zn(CN)_4]^{2-} + \underset{\text{silver}}{2Ag}$$

This process is called Mac Arthur Forest process.

59. In smoke, the dispersed phase is solid and dispersion medium is gas, *ie*, a solid is dispersed in gas. Hence, it is an example of solid foam.

60. $\underset{\text{ethanoic acid}}{CH_3COOH} + \underset{\text{propan-2-ol}}{HOCH(CH_3)_2} \xrightarrow{H_2SO_4} \underset{\textit{iso}\text{-propyl ethanoate}}{CH_3COOCH(CH_3)_2}$

This reaction is called esterification as ester is the final product.

61. Only primary amines, when treated with chloroform and alkali (KOH), gives isocyanide (*R*NC), the bad smelling compound. This reaction is called carbylamine reaction.
Among the given amines, RNH_2 is a primary amine, so forms *R*NC when treated with $CHCl_3$ and KOH.

$$RNH_2 + CHCl_3 + 3KOH \longrightarrow \underset{\text{alkyl } \textit{iso}\text{-cyanide}}{RNC} + 3KCl + 3H_2O$$

62. Soaps are sodium or potassium salts of higher fatty acids like palmitic acid, stearic acid, lauric acid etc. Hence, sodium palmitate ($C_{15}H_{31}COONa$) is a soap.

63. Bond dissociation energy is the energy that is required to break one mole of a particular type of bond. For C—C bond the amount of bond dissociation energy is 81 kcal/mol (or 347 kJ/mol).

64. The shape of the compound having four sp^3 hybridised covalent bond is tetrahedral as there is only bond pair-bond pair repulsion.

65. The energy difference between the observed and calculated value of heat of hydrogenation is 36 kcal/mol. This energy is called resonance energy. Hence, resonance energy of benzene in kcal/mol is about 40.

66. 1 mole of $CO_2 = 6 \times 10^{23}$ atoms of C
$= 2 \times 6 \times 10^{23}$ atoms of O
$= 6 \times 10^{23}$ molecules of CO_2
$= 44$ g-atoms of CO_2

67. In the famous α-ray scattering experiment, Rutherford bombarded, a thin foil of gold (Au) with α-particles and found that most of the α-particles passed undeflected and a few returned back. By this experiment, he found that in centre every atom contains a nucleus.

68. The charge on an electron $= 1.602 \times 10^{-19}$C. This was determined by Mullikan in his oil drop experiment.

69. Principal quantum number also represents the shell of an electron.
$\because$ Shell of an electron $= n$
$\therefore$ Number of subshells in that shell $= n - 1$

70. *s*-block elements contain either 1 or 2 electrons in their outer most shell.

$3 = 1s^2, 2s^1$

$11 = 1s^2, 2s^2, 2p^6, 3s^1$

From the above electronic configuration, it is clear that elements having atomic number 3 and 11 belong to *s*-block. Since, both contains one electron in outer most shell, these belong to alkali metals.

71. If a system is in equilibrium, its Gibbs free energy change is zero, *ie*, $\Delta G = 0$.

72. The combination, in which concentration of H^+ is maximum, will produce the highest rise in temperature.

(a) Conc. of $OH^- = \dfrac{67 \times 1 - 33 \times 1}{67 + 33}$

[$\because N = 2M$ for H_2SO_4]

$= 0.34 = 3.4 \times 10^{-1}$

$[H^+][OH^-] = 1 \times 10^{-14}$

$\therefore \quad [H^+] = \dfrac{1 \times 10^{-14}}{3.4 \times 10^{-1}} = 2.94 \times 10^{-14}$

(b) $[H^+] = \dfrac{67 \times 1 - 33 \times 1}{67 + 33} = 0.34$

(c) $[H^+] = \dfrac{60 \times 1 - 40 \times 1}{60 + 40} = 0.20$

(d) Since concentration and volume of acid and base is equal, the solution is neutral *ie*, $[H^+] = 10^{-7}$

Hence, mixture given in option (b) will produce the highest rise in temperature.

73. Atomic mass is a number that indicates as how many times an atom of an element is heavier as compared with 1/12 of the mass of an atom of carbon-12 ($^{12}_{6}C$). Hence, standard for atomic mass is $^{12}_{6}C$.

74. Most of the atoms have a natural tendency to accept an electron, so energy is released when first electron is added. But when a second electron is added to an atom of small size, the added electron experience greater repulsion from the previous one. To overcome this repulsion and to push in second electron, the energy has to supplied. Thus, the process,

$O^- + e^- \longrightarrow O^{2-}$ involves absorption of energy.

75. Since there is a sudden increase in the ionisation energy after third ionisation (*ie*, after the removal of third electron) the number of valence electrons is 3.

76. For a first order reaction,

$$\text{half-life period, } t_{1/2} = \frac{0.693}{k} = \frac{0.693}{6.2 \times 10^{-4}\ s^{-1}}$$

$$= 1117.7\ s$$

77. For n order,

the units of rate constant $= mol^{1-n}L^{n-1}s^{-1}$.

where, n = order of reaction

Given, units of rate constant $= mol^{-2}L^2s^{-1}$

On comparing the units of rate constants

$$1 - n = -2$$

$$\therefore \quad n = 3$$

or $n - 1 = 2$

$n = 3$

∴The order of the reaction is three.

78. Let the oxidation state of O in OF_2 is x.

$$OF_2$$

$$x + (-1) \times 2 = 0$$

$$x - 2 = 0$$

$$\therefore \quad x = +2$$

79. K_p and K_c are related as

$$K_p = K_c(RT)^{\Delta n_g}$$

where, $\Delta n_g = n_p - n_r$

n_p = number of gaseous products

n_r = number of gaseous reactants

80. For the reaction,

$2B + A \rightleftharpoons C$

Rate of forward reaction $= k_f[A][B]^2$

Rate of backward reaction $= k_b[C]$

At equilibrium,

Rate of forward reaction = Rate of backward reaction

$$k_f[A][B]^2 = k_b[C]$$

$$\frac{k_f}{k_b} = \frac{[C]}{[A][B]^2} = K_c$$

81. Given, $[X^-] = [HX]$

For acidic buffer solution,

$$pH = pK_a + \log\frac{[X^-]}{[HX]}$$

$$pH = -\log 10^{-8} + 0$$

$$\therefore \quad pH = 8$$

82. Zinc (Zn) being a moderately reactive metal, gives H_2 with acids as well as NaOH.

$Zn + 2HCl \longrightarrow ZnCl_2 + H_2$

$Zn + NaOH \longrightarrow \underset{\text{sodium zincate}}{Na_2ZnO_2} + H_2$

83. Isoelectronic species have same number of electrons.

In hydride ion (H^-),

no. of electrons $= 1 + 1 = 2$

In He,

no. of electrons $= 2$

∴H^- is isoelectronic with He.

84. When alkali metals are dissolved in liquid ammonia, a blue coloured solution is obtained. This solution is paramagnetic in nature.

$$Na^+ + (x+y)NH_3 + e^- \longrightarrow \underset{\substack{\text{ammoniated}\\\text{sodium}}}{[Na(NH_3)_x]^+} + \underset{\substack{\text{ammoniated}\\\text{electrons}\\\text{(blue)}}}{[e(NH_3)_y]^-}$$

85. Only true peroxides gives H_2O_2, when treated with dilute acids. Among the given compounds, Na_2O_2 is a true peroxide, so gives H_2O_2 with dilute acid.

$$\underset{\substack{\text{sodium}\\\text{peroxide}}}{Na_2O_2} + 2HCl \longrightarrow 2NaCl + \underset{\substack{\text{hydrogen}\\\text{peroxide}}}{H_2O_2}$$

86. During the electrolysis of fused NaCl, following reactions take place :

$NaCl \longrightarrow Na^+ + Cl^-$

At anode $Cl^- + e^- \xrightarrow{\text{Oxidation}} Cl$

At cathode

$Na^+ \xrightarrow{\text{Reduction}} Na + e^-$

87. Nitrous oxide (N_2O), nitric oxide (NO) and hydrogen (H_2), all are colourless gases while nitrogen dioxide (NO_2) is a reddish brown gas.

88. Oxides of less electropositive metals (like HgO) on moderate heating gives off O_2.

$$2HgO \xrightarrow{\text{High temperature}} 2Hg + O_2$$

89. When water is added to plaster of Paris, a substance, called gypsum, that becomes hard, is formed. Thus, plaster of Paris is hardened by hydration.

$$\underset{\text{plaster of Paris}}{CaSO_4 \cdot \frac{1}{2}H_2O} + 1\frac{1}{2}H_2O \longrightarrow \underset{\text{(hard)}}{CaSO_4 \cdot 2H_2O}$$

90. According to Fajan's rule,

$$\text{ionic character} \propto \frac{1}{\text{covalent character}}$$

$$\propto \text{size of cation or metal}$$

Since, Be is smallest among the given metals, it forms least ionic chloride.

91. Aluminium bronze is an alloy of 10-12% aluminium with 90-88% Cu. It has more tensile strength than ordinary bronze and is used for making utensils, jewellery etc.

92. In thermite welding, a mixture of aluminium powder and iron oxide (Fe_2O_3) in the ratio of 1 : 3 is used. This mixture is called thermite mixture.

93. Solvay-ammonia process involves the formation of bicarbonate and bicarbonate of potassium, *ie*, $KHCO_3$, is highly soluble in water. That's why K_2CO_3 cannot be prepared by Solvay-ammonia process.

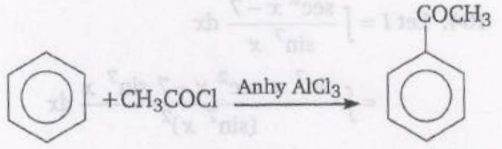
94. Glauber's salt is sodium sulphate deca hydrate *ie*, $Na_2SO_4 \cdot 10H_2O$.

95. Chlorides and sulphates of calcium and magnesium are responsible for permanent hardness of water.

96. As the number of alkyl group attached to the positively charged carbon atom increases, the number of hyperconjugative or no bond contributing structures increases and thus, the stability increases.

Benzyl carbonium ion is more stable than secondary carbocation due to resonance stabilisation but less stable than tertiary carbocation.

97. Acetophenone is obtained when benzene reacts with acetyl chloride in the presence of anhydrous $AlCl_3$.



acetophenone

98. Conformers are inter converted through rotation around a single bond. Such isomerism is termed as conformational isomerism.

99. When petroleum (coal tar) is subjected to fractional distillation, phenol is present in middle oil or carbolic oil fraction (at 443-503).

100. Alcoholic alkali acts as dehydrohalogenating agent.

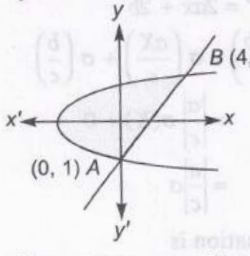
$$\underset{\text{ethyl chloride}}{C_2H_5Cl} \xrightarrow[-KCl\ -H_2O]{\text{Alc. (KOH)}} \underset{\text{ethylene}}{CH_2{=}CH_2}$$

MATHEMATICS

101. Given curves are

$$y^2 = 2x + 1$$

and $x - y = 1$



Points of intersection are $A(0, -1)$ and $B(4, 3)$.

$$\text{Area} = \int_{-1}^{3}(1+y)\,dy - \int_{-1}^{3}\left(\frac{y^2-1}{2}\right)dy$$

$$= \left[y + \frac{y^2}{2}\right]_{-1}^{3} - \left[\frac{1}{2}\left(\frac{y^3}{3} - y\right)\right]_{-1}^{3}$$

$$= \left[3 + \frac{9}{2} - \left(-1 + \frac{1}{2}\right)\right] - \frac{1}{2}\left[9 - 3 - \left(-\frac{1}{3} + 1\right)\right]$$

$$= 8 - \frac{8}{3} = \frac{16}{3}$$

102. Given curve is $y = x^2$

For this curve there is only one tangent line *ie*, x-axis ($y = 0$)

$$\therefore \quad \frac{dy}{dx} = 0$$

Hence, order is 1.

103. Let $I = \int \frac{dx}{\sqrt{(1-x)(x-2)}}$

$$= \int \frac{dx}{\sqrt{-x^2+3x-2}} = \int \frac{dx}{\sqrt{\frac{1}{4}-\left(x-\frac{3}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{\left(x-\frac{3}{2}\right)}{\frac{1}{2}}\right) + C$$

$$= \sin^{-1}(2x-3) + C$$

104. Let $I = \int \frac{\sec^2 x - 7}{\sin^7 x}\, dx$

$$= \int \frac{\sin^7 x \sec^2 x - 7\sin^7 x}{(\sin^7 x)^2}\, dx$$

$$= \int \frac{d}{dx}\left(\frac{\tan x}{\sin^7 x}\right) dx = \frac{\tan x}{\sin^7 x} + C$$

105. Let $I = \int_{-3}^{2} (|x+1| + |x+2| + |x-1|)\, dx$

Again let $f(x) = |x+1| + |x+2| + |x-1|$

$$= \begin{cases} -(x+1)-(x+2)-(x-1), & -3 < x \le -2 \\ -(x+1)+x+2-(x-1), & -2 < x \le -1 \\ 1+x+x+2-(x-1) & -1 < x \le 0 \\ 1+x+x+2-(x-1) & 0 \le x < 1 \\ 1+x+x+2+x-1 & 1 \le x < 2 \end{cases}$$

$$= \begin{cases} -3x-2, & -3 < x \le -2 \\ -x+2, & -2 < x \le -1 \\ x+4, & -1 \le x < 1 \\ 3x+2, & 1 \le x < 2 \end{cases}$$

$$\therefore I = \int_{-3}^{-2} (-3x-2)\, dx + \int_{-2}^{-1} (-x+2)\, dx + \int_{-1}^{1} (x+4)\, dx + \int_{1}^{2} (3x+2)\, dx$$

$$= \left[-\frac{3x^2}{2} - 2x\right]_{-3}^{-2} + \left[-\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{x^2}{2} + 4x\right]_{-1}^{1} + \left[\frac{3x^2}{2} + 2x\right]_{1}^{2}$$

$$= \left[-6 + 4 - \left(-\frac{27}{2} + 6\right)\right] + \left[-\frac{1}{2} - 2 - (-2-4)\right] + \left[\frac{1}{2} + 4 - \left(\frac{1}{2} - 4\right)\right] + \left[6 + 4 - \left(\frac{3}{2} + 2\right)\right]$$

$$= \frac{11}{2} + \frac{7}{2} + 8 + \frac{13}{2}$$

$$= \frac{31}{2} + 8 = \frac{47}{2}$$

106. $\bar{x} = \frac{1+2+3+\ldots+n}{n}$

$$= \frac{(n+1)}{2}$$

$$\therefore \quad \sigma^2 = \frac{\Sigma (x_i)^2}{n} - (\bar{x})^2$$

$$= \frac{\Sigma n^2}{n} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n^2-1}{12}$$

107. Given, $\log\left(\frac{dy}{dx}\right) = 3x + 4y$

$$\Rightarrow \quad \frac{dy}{dx} = e^{3x} e^{4y}$$

$$\Rightarrow \quad e^{-4y}\, dy = e^{3x}\, dx$$

On integrating both sides, we get

$$-4e^{-4y} = 3e^{3x} + c$$

At $x = 0, y = 0,$

$$-4 = 3 + c \Rightarrow c = -7$$

$\therefore$ Solution is $4e^{-4y} + 3e^{3x} = 7$

108. Since, length of subnormal $= a$

$$\Rightarrow \quad y\frac{dy}{dx} = a$$

$$\Rightarrow \quad y\, dy = a\, dx$$

On integrating both sides, we get

$$\frac{y^2}{2} = ax + b,$$

where b is a constant of integration

$$\Rightarrow \quad y^2 = 2ax + 2b$$

109. $\sigma\left(\frac{aX+b}{c}\right) = \sigma\left(\frac{aX}{c}\right) + \sigma\left(\frac{b}{c}\right)$

$$= \left|\frac{a}{c}\right| \sigma(X) + 0$$

$$= \left|\frac{a}{c}\right| \sigma$$

110. Given equation is

$$ax^2 + bx + c = 0$$

$$\therefore \quad \sin\alpha + \cos\alpha = -\frac{b}{a}$$

and $\sin\alpha\cos\alpha = \frac{c}{a}$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \frac{b^2}{a^2}$

and $\sin\alpha\cos\alpha = \frac{c}{a}$

$\Rightarrow 1 + \frac{2c}{a} = \frac{b^2}{a^2}$

$\Rightarrow a^2 + 2ac = b^2$

$\Rightarrow a^2 - b^2 + 2ac = 0$

111. Given, $\alpha+\beta+\gamma=2$, $\alpha^2+\beta^2+\gamma^2=6$, $\alpha^3+\beta^3+\gamma^3=8$

Now, $(\alpha+\beta+\gamma)^2 = 2^2$

$\Rightarrow \alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)=4$

$\Rightarrow 2(\alpha\beta+\beta\gamma+\gamma\alpha) = 4-6=-2$

Also, $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma$

$=(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)$

$\Rightarrow 8-3\alpha\beta\gamma = 2[6-(-1)]$

$\Rightarrow 8-3\alpha\beta\gamma = 14$

$\Rightarrow \alpha\beta\gamma = 8-14$

$\Rightarrow \alpha\beta\gamma = -2$

Now, $\alpha^4+\beta^4+\gamma^4 = (\alpha^2+\beta^2+\gamma^2)^2 - 2\Sigma\alpha^2\beta^2$

$=(\alpha^2+\beta^2+\gamma^2)^2 - 2[(\Sigma\beta\gamma)^2 - 2\alpha\beta\gamma\Sigma\alpha]$

$=(6)^2 - 2[(-1)^2 - 2(-2)2]$

$=36-2[9]$

$=36-18$

$=18$

112. Given, $\bar{z} = \bar{a} + \frac{r^2}{z-a}, r>0$

$\Rightarrow \bar{z}(z-a) = \bar{a}(z-a) + r^2$

$\Rightarrow z\bar{z} - a\bar{z} - \bar{a}z + a\bar{a} + r^2 = 0$

This represents the equation of a circle.

113. Given, $\frac{3}{2+\cos\theta+i\sin\theta} = a+ib$

$\Rightarrow \frac{3[(2+\cos\theta)-i\sin\theta]}{(2+\cos\theta)^2+\sin^2\theta} = a+ib$

$\Rightarrow \frac{3[2+\cos\theta-i\sin\theta]}{5+4\cos\theta} = a+ib$

$\Rightarrow a = \frac{3(2+\cos\theta)}{5+4\cos\theta}, b = -\frac{3\sin\theta}{5+4\cos\theta}$

$\therefore (a-2)^2+b^2 = \left(\frac{6+3\cos a}{5+4\cos\theta}-2\right)^2 + \frac{9\sin^2\theta}{(5+4\cos\theta)^2}$

$=\frac{(-4-5\cos\theta)^2+9\sin^2\theta}{(5+4\cos\theta)^2}$

$=\frac{16+25\cos^2\theta+40\cos\theta+9\sin^2\theta}{(5+4\cos\theta)^2}$

$=\frac{16+16\cos^2\theta+40\cos\theta+9}{(5+4\cos\theta)^2}$

$=\frac{(5+4\cos\theta)^2}{(5+4\cos\theta)^2} = 1$

114. $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$

$=4+5(\omega)^{334}+3(\omega)^{365}$

$=4+5(\omega)^{333}\omega+3(\omega)^{363}\omega^2$

$=4+5\omega+3\omega^2$

$=4+3(\omega+\omega^2)+2\omega$

$=4+3(-1)+2\left(\frac{-1+i\sqrt{3}}{2}\right)$

$=1-1+i\sqrt{3}$

$=i\sqrt{3}$

115. $T_4 = {}^nC_3(a)^{n-3}(-2b)^3$

and $T_5 = {}^nC_4(a)^{n-4}(-2b)^4$

$\because T_4 + T_5 = 0$ (given)

$\therefore {}^nC_3(a)^{n-3}(-2b)^3 + {}^nC_4(a)^{n-4}(-2b)^4 = 0$

$\Rightarrow (a)^{n-4}(-2b)^3[a\,{}^nC_3 + {}^nC_4(-2b)] = 0$

$\Rightarrow \frac{a}{b} = \frac{2\cdot{}^nC_4}{{}^nC_3}$

$=2\cdot\frac{n(n-1)(n-2)(n-3)}{4\cdot3\cdot2\cdot1}\times\frac{3\cdot2\cdot1}{n(n-1)(n-2)}$

$=\frac{n-3}{2}$

116. Given, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$=\begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix}$

$\therefore A^2-(a+d)A+kI = 0$

$\Rightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix} - \begin{bmatrix} a^2+ad & ab+bd \\ ac+dc & ad+d^2 \end{bmatrix}$

$+\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} bc-ad+k & 0 \\ 0 & bc-ad+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On equating, we get

$$bc - ad + k = 0$$

$$\Rightarrow \quad k = ad - bc \qquad \ldots(i)$$

Also, $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$$

$$\Rightarrow \quad ad - bc = 0$$

$\therefore$ From Eq. (i), $k = 0$

117. Let $S = 1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \ldots$

$$= 1 + \frac{(4-1)}{2} + \frac{(8-1)}{4} + \frac{(16-1)}{8} + \frac{(32-1)}{16} + \ldots$$

$$= 1 + 2 - \frac{1}{2} + 2 - \frac{1}{4} + 2 - \frac{1}{8} + 2 - \frac{1}{16} + \ldots$$

$$= 1 + 2(n-1) - \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots (n-1)\right]$$

$$= 1 + 2(n-1) - \left[\frac{\frac{1}{2}\left(1 - \frac{1}{2^{n-1}}\right)}{1 - \frac{1}{2}}\right]$$

$$= 1 + 2(n-1) - 1 + \frac{1}{2^{n-1}}$$

$$= 2(n-1) + \frac{1}{2^{n-1}}$$

118. $\log_4 (x-1) = \log_2 (x-3)$

$$\Rightarrow \log_{2^2} (x-1) = \log_2 (x-3)$$

$$\Rightarrow \frac{1}{2}\log_2 (x-1) = \log_2 (x-3)$$

$$\Rightarrow (x-1) = (x-3)^2$$

$$\Rightarrow x^2 - 7x + 10 = 0$$

$$\Rightarrow (x-2)(x-5) = 0$$

$$\Rightarrow x = 2, 5$$

But at $x = 2$ given equation is not satisfied.
Hence, meaningful solution is 1.

119. Given word is *HAVANA*(3*A*, 1*H*, 1*N*, 1*V*)
Total number of ways arranging the given word

$$= \frac{6!}{3!} = 120$$

Total number of words in which *N*, *V* together

$$= \frac{5!}{3!} \times 2! = 40$$

$\therefore$ Required number of ways $= 120 - 40 = 80$

120. Let *E* be the event that a six occurs and *A* be the event that man reports that it is a six.

$$\therefore \quad P(E) = \frac{1}{6}, P(E') = \frac{5}{6}, P\left(\frac{A}{E}\right) = \frac{3}{4}$$

and $$P\left(\frac{A}{E'}\right) = \frac{1}{4}$$

Using Bay's theorem

$$P\left(\frac{E}{A}\right) = \frac{P(E) \cdot P\left(\frac{A}{E}\right)}{P(E) \cdot P\left(\frac{A}{E}\right) + P(E')\, P\left(\frac{A}{E'}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}$$

121. $\cos^2(A-B) + \cos^2 B - 2\cos(A-B)\cos A \cos B$

$$= \cos^2 (A-B) + \cos^2 B - \cos(A-B)$$
$$[\cos(A+B) + \cos(A-B)]$$

$$= \cos^2 B - \cos(A-B)\cos(A+B)$$

$$= \cos^2 B - \frac{1}{2}[\cos 2A + \cos 2B]$$

$$= \cos^2 B - \frac{1}{2}[2\cos^2 B - 1 + \cos 2A]$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2A = \frac{1}{2} - \frac{1}{2}(2\cos^2 A - 1)$$

$$= 1 - \cos^2 A = \sin^2 A$$

Hence, it is independent of *B*.

122. The intersection point of $x + 2y = 3$ and $3x + 4y = 7$ is (1, 1).
For consistent, point (1, 1) satisfies the equation

$$ax + y = 3$$

$$\therefore \quad a + 1 = 3$$

$$\Rightarrow \quad a = 2$$

123. Given,

$$\begin{vmatrix} x-1 & 5x & 7 \\ x^2-1 & x-1 & 8 \\ 2x & 3x & 0 \end{vmatrix} = ax^3 + bx^2 + cx + d$$

$$\text{LHS} = (x-1)(0-24x) - 5x(0-16x)$$
$$+ 7(3x^3 - 3x - 2x^2 + 2x)$$

$$= -24x^2 + 24x + 80x^2 + 21x^3 - 14x^2 - 7x$$

$$= 21x^3 + 42x^2 + 17x$$

$$\therefore \; 21x^3 + 42x^2 + 17x$$
$$= ax^3 + bx^2 + cx + d$$

$$\Rightarrow \quad c = 17$$

124. Using AM ≥ GM

$$\frac{\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}}{3} \geq \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}} \quad \ldots(i)$$

Again using AM ≥ GM

$$\frac{a+b}{2} \geq \sqrt{ab}, \frac{b+c}{2} \geq \sqrt{bc}, \frac{c+a}{2} \geq \sqrt{ca}$$

$$\Rightarrow \quad (a+b)(b+c)(c+a) \geq 8\,abc$$

$$\Rightarrow \quad \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}} \leq \frac{1}{2}$$

∴From Eq. (i)

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \geq \frac{3}{2}$$

125. Given, $\tan 1^\circ \tan 2^\circ \ldots \tan 89^\circ = x^2 - 8$

$\Rightarrow \tan 1^\circ \tan 2^\circ \ldots \tan 44^\circ \tan 45^\circ \cot 44^\circ \ldots \cot 2^\circ \cot 1^\circ = x^2 - 8$

$$\Rightarrow \quad 1 = x^2 - 8 \Rightarrow x^2 = 9$$

$$\Rightarrow \quad x = \pm 3$$

126. Given, $\frac{\sin(x+3\alpha)}{\sin(\alpha - x)} = 3$

Applying componendo and dividendo, we get

$$\frac{\sin(x+3\alpha)+\sin(\alpha-x)}{\sin(x+3\alpha)-\sin(\alpha-x)} = \frac{3+1}{3-1}$$

$$\Rightarrow \quad \frac{2\sin 2\alpha \cos(\alpha+x)}{2\cos 2\alpha \sin(\alpha+x)} = 2$$

$$\Rightarrow \quad \frac{\tan 2\alpha}{\tan(\alpha+x)} = 2$$

$$\Rightarrow \quad \frac{2\tan\alpha}{1-\tan^2\alpha} \times \frac{(1-\tan\alpha\tan x)}{(\tan\alpha+\tan x)} = 2$$

$\Rightarrow \tan\alpha - \tan^2\alpha \tan x = \tan\alpha + \tan x - \tan^3\alpha - \tan^2\alpha \tan x$

$\Rightarrow \tan x = \tan^3\alpha$

127. $2a^2 + 4b^2 + c^2 = 4ab + 2ac$

$\Rightarrow a^2 + (2b)^2 - 4ab + a^2 + c^2 - 2ac = 0$

$$\Rightarrow \quad (a-2b)^2 + (a-c)^2 = 0$$

$$\Rightarrow \quad a = 2b = c$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{c^2 + c^2 - \left(\frac{c}{2}\right)^2}{2 \times c \times c} = \frac{2c^2 - \frac{c^2}{4}}{2c^2}$$

$$\Rightarrow \quad \cos B = \frac{7}{8}$$

128. Since, $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$[\because P(A \cap B) = P(A)P(B)]$

$$\Rightarrow \quad 0.8 = 0.3 + P(B) - 0.3P(B)$$

$$\Rightarrow \quad 0.7P(B) = 0.5$$

$$\Rightarrow \quad P(B) = \frac{5}{7}$$

129. ∴Required probability $= \frac{{}^4C_2 + {}^5C_2}{{}^9C_2}$

$$= \frac{6+10}{36} = \frac{16}{36}$$

$$= \frac{4}{9}$$

130. $\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ}$

$$= \frac{1}{\cos 70^\circ} - \frac{1}{\sqrt{3}\sin 110^\circ}$$

$$= \frac{\sqrt{3}\sin 110^\circ - \cos 70^\circ}{\sqrt{3}\sin 110^\circ \cos 70^\circ}$$

$$= \frac{\sqrt{3}\sin(180^\circ - 70^\circ) - \cos 70^\circ}{\sqrt{3}\sin(180 - 70^\circ)\cos 70^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}\sin 70^\circ - \frac{1}{2}\cos 70^\circ}{\frac{\sqrt{3}}{2}\sin 70 \cos 70^\circ}$$

$$= \frac{\cos 30^\circ \sin 70^\circ - \sin 30^\circ \cos 70^\circ}{\frac{\sqrt{3}}{2} \cdot \frac{1}{2}\sin 140^\circ}$$

$$= \frac{\sin(70^\circ - 30^\circ)}{\frac{\sqrt{3}}{4}\sin(180^\circ - 40^\circ)}$$

$$= \frac{\sin 40^\circ}{\frac{\sqrt{3}}{4}\sin 40^\circ} = \frac{4}{\sqrt{3}}$$

131. Equation of any line through (1, 2) is

$$y - 2 = m(x-1)$$

$$\Rightarrow \quad y - mx + m - 2 = 0$$

The distance of line $y = 2x$ from the point (3, 1) is greatest.

∴Required line in $y = 2x$.

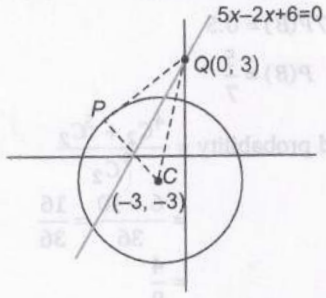
132. Let $S_1 \equiv x^2 + y^2 = 9$

$$PA \cdot PB = (\sqrt{S_1})^2$$

$$= \left(\sqrt{(3)^2 + (11)^2 - 9}\right)^2 = 121$$

133. $x^2 + y^2 + 6x + 6y - 2 = 0$

Centre $(-3, -3)$, radius $= \sqrt{9 + 9 + 2} = \sqrt{20}$



Now, $QC = \sqrt{(-3)^2 + 6^2} = \sqrt{45}$

In right ΔCPQ $\quad PQ = \sqrt{45 - 20} = 5$

134. Let sides of the triangle are $4x$, $5x$, $6x$.

$$s = \frac{4x + 5x + 6x}{2} = \frac{15}{2}x$$

$$\Delta = \sqrt{\frac{15}{2}x\left(\frac{15}{2}x - 4x\right)\left(\frac{15}{2}x - 5x\right)\left(\frac{15}{2}x - 6x\right)}$$

$$= \sqrt{\frac{15}{2}x \times \frac{7}{2}x \times \frac{5}{2}x \times \frac{3}{2}x}$$

$$= \frac{15\sqrt{7}x^2}{4}$$

Circumradius, $R = \dfrac{4x \times 5x \times 6x}{4 \times \dfrac{15\sqrt{7}x^2}{4}}$

$$= \frac{8}{\sqrt{7}}x$$

Inradius, $\quad r = \dfrac{\dfrac{15\sqrt{7}}{4}x^2}{\dfrac{15}{2}x} = \frac{\sqrt{7}}{2}x$

$$\frac{R}{r} = \frac{\dfrac{8x}{\sqrt{7}}}{\dfrac{\sqrt{7}x}{2}} = \frac{16}{7}$$

135. $4\sin^{-1}x + \cos^{-1}x = \pi$

$$\Rightarrow \quad 4\sin^{-1}x + \left(\frac{\pi}{2} - \sin^{-1}x\right) = \pi$$

$$\Rightarrow \quad 3\sin^{-1}x = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}x = \frac{\pi}{6} \Rightarrow x = \sin\frac{\pi}{6} = \frac{1}{2}$$

136. $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right) = x$ (let)

$$\Rightarrow \quad \cos^{-1}\frac{1}{8} = 2\cos^{-1}x$$

$$= \cos^{-1}(2x^2 - 1)$$

$$\Rightarrow \quad \frac{1}{8} = 2x^2 - 1$$

$$\Rightarrow \quad 2x^2 = \frac{1}{8} + 1 = \frac{9}{8}$$

$$\Rightarrow \quad x^2 = \frac{9}{16}$$

$$\Rightarrow \quad x = \sqrt{\frac{9}{16}} = \frac{3}{4} \qquad [\because 0 \le x \le 1]$$

137. Given vectors are coplanar, if

$$\begin{vmatrix} 1 & 1 & m \\ 1 & 1 & m+1 \\ 1 & -1 & m \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 0 & 0 & -1 \\ 1 & 1 & m+1 \\ 1 & -1 & m \end{vmatrix} = 0 \qquad [R_1 \to R_1 - R_2]$$

$$\Rightarrow \quad -1(-1-1) = 0$$

$$\Rightarrow \quad 2 \neq 0$$

$\therefore$ No value of m for which vectors are coplanar.

138. Given, $\vec{a} + \vec{b} + \vec{c} = 0$

$$\Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = 0$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow 4 + 9 + 16 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = \frac{-29}{2}$$

139. Here $a = 2$, $m = -1$

$\therefore$ Required point is $(am^2, -2am) = (2, 4)$

140. Given curve is $16x^2 + 25y^2 = 400$

$$\Rightarrow \quad \frac{x^2}{25} + \frac{y^2}{16} = 1$$

Here $\quad a = 5, b = 4$

F_1 and F_2 are focus.

$\therefore \quad PF_1 + PF_2 = 2a = 10$

141. Let the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Eccentricity, $e = \sqrt{\frac{a^2 + b^2}{a^2}}$

$\Rightarrow \quad \frac{1}{e^2} = \frac{a^2}{(a^2 + b^2)}$

Conjugate hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Eccentricity, $e_1 = \sqrt{\frac{a^2 + b^2}{b^2}}$

$\Rightarrow \quad \frac{1}{e_1^2} = \frac{b^2}{a^2 + b^2}$

Now, $\frac{1}{e^2} + \frac{1}{e_1^2} = \frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2} = 1$

142. $\lim_{x \to 0} \frac{1 - \cos^3 x}{x \sin x \cos x} = \lim_{x \to 0} \frac{2(1 - \cos^3 x)}{x \sin 2x}$

[Using L'Hospital Rule]

$= \lim_{x \to 0} \frac{2[-3\cos^2 x(-\sin x)]}{\sin 2x + x \cos 2x \cdot 2}$

$= \lim_{x \to 0} \frac{6\cos^2 x \sin x}{\sin 2x + 2x \cos 2x}$

$= \lim_{x \to 0} \frac{6[-2\cos x \sin^2 x + \cos^3 x]}{2\cos 2x + 2[-x \sin 2x \cdot 2 + \cos 2x]}$

$= \lim_{x \to 0} \frac{6[-2\cos x \sin^2 x + \cos^3 x]}{2\cos 2x - 4x \sin 2x + 2\cos 2x}$

$= \frac{6}{2+2} = \frac{3}{2}$

143. $y = \cos^{-1}(\cos x)$

$y' = \frac{-1}{\sqrt{1 - \cos^2 x}}(-\sin x)$

$= \frac{\sin x}{\sqrt{\sin^2 x}} = 1 \ \forall x$

144. Position vector of median $\vec{AD} = \frac{\vec{AB} + \vec{AC}}{2}$

$= \frac{(-3+5)\hat{i} + (0-2)\hat{j} + (4+4)\hat{k}}{2}$

$= \hat{i} - \hat{j} + 4\hat{k}$

$\therefore |\vec{AD}| = \sqrt{1^2 + (-1)^2 + (4)^2} = \sqrt{18}$

145. $f(x) = \text{cosec}^2 3x + \cot 4x$

Period of $\text{cosec}^2 3x$ is $\frac{\pi}{3}$ and $\cot 4x$ is $\frac{\pi}{4}$.

$\therefore$ Period of $f(x) = \text{LCM of}\left(\frac{\pi}{3} \text{ and } \frac{\pi}{4}\right)$

$= \frac{\text{LCM of } (\pi, \pi)}{\text{HCF of } (3, 4)}$

$= \frac{\pi}{1} = \pi$

146. $f(x) = \log_a (x + \sqrt{x^2 + 1})$ is symmetric about origion.

147. $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$

$\Rightarrow f'(x) = \sin x + \sin 2x - \sin 3x$

Put $\quad f'(x) = 0$

$\Rightarrow 2\sin\frac{3x}{2}\cos\frac{x}{2} = 2\sin\frac{3x}{2}\cos\frac{3x}{2}$

$\Rightarrow \quad \sin\frac{3x}{2} = 0, \cos\frac{3x}{2} = \cos\frac{x}{2}$

$\Rightarrow \quad x = \frac{2n\pi}{3}, \frac{3x}{2} = 2n\pi \pm \frac{x}{2}$

$\Rightarrow \quad x = 0, \frac{2\pi}{3}, x = 0,$

At $\quad x = 0,$

$f(x) = 1 + \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$

At $\quad x = \frac{2\pi}{3}$

$f(x) = -\frac{1}{2} - \frac{1}{4} - \frac{1}{3}$

$= -\frac{13}{12}$

$\therefore \quad$ Difference $= \frac{7}{6} + \frac{13}{12}$

$= \frac{27}{12}$

$= \frac{9}{4}$

148. $f(x)$ is continuous at $x = 1$

$\therefore \quad f(1) = \lim_{x \to 1^+} f(x)$

$\Rightarrow \quad a + b = b + a + c$

$\Rightarrow \quad c = 0$

Also, $f(x)$ is differentiable at $x = 1$

$Lf'(x) = Rf'(x)$

$\therefore \quad \frac{d}{dx}(ax^2+b)=\frac{d}{dx}(bx^2+ax+c)$

$\Rightarrow \quad 2ax=2bx+a$

At $x=1$

$\Rightarrow \quad 2a=2b+a$

$\Rightarrow \quad a=2b$

149. Let $f(x)=y$

$\Rightarrow \quad x=f^{-1}(y)$

$\Rightarrow \quad g(y)=x$

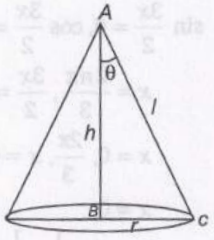
Now, $g'(f(x))=\frac{1}{f'(x)}, \forall x$

$=1+x^3$

$\Rightarrow \quad g'(y)=1+\{g(y)\}^3$

$\Rightarrow \quad g'(x)=1+\{g(x)\}^3$

150. Volume of cone, $V=\frac{\pi}{3}r^2h$



$\Rightarrow \quad V=\frac{\pi}{3}r^2\sqrt{l^2-r^2}$

On differentiating w.r.t., r we get

$$\frac{dV}{dr}=\frac{\pi}{3}\left[2r\sqrt{l^2-r^2}+\frac{r^2}{2\sqrt{l^2-r^2}}(-2r)\right]$$

Put $\frac{dV}{dr}=0$

$\Rightarrow \quad 2r(\sqrt{l^2-r^2})-\frac{r^3}{\sqrt{l^2-r^2}}=0$

$\Rightarrow \quad r[2(l^2-r^2)-r^3]=0$

$\Rightarrow \quad 2l^2-3r^2=0$

$\Rightarrow \quad r=\pm l\sqrt{\frac{2}{3}}$

$\therefore$ At $r=l\sqrt{\frac{2}{3}}, \frac{d^2V}{dr^2}<0$, maxima

$\therefore \quad h=\sqrt{l^2-\frac{2}{3}l^2}=\frac{l}{\sqrt{3}}$

In ΔABC, $\tan\theta=\frac{r}{h}$

$=\frac{l\sqrt{\frac{2}{3}}}{\frac{l}{\sqrt{3}}}=\sqrt{2}$