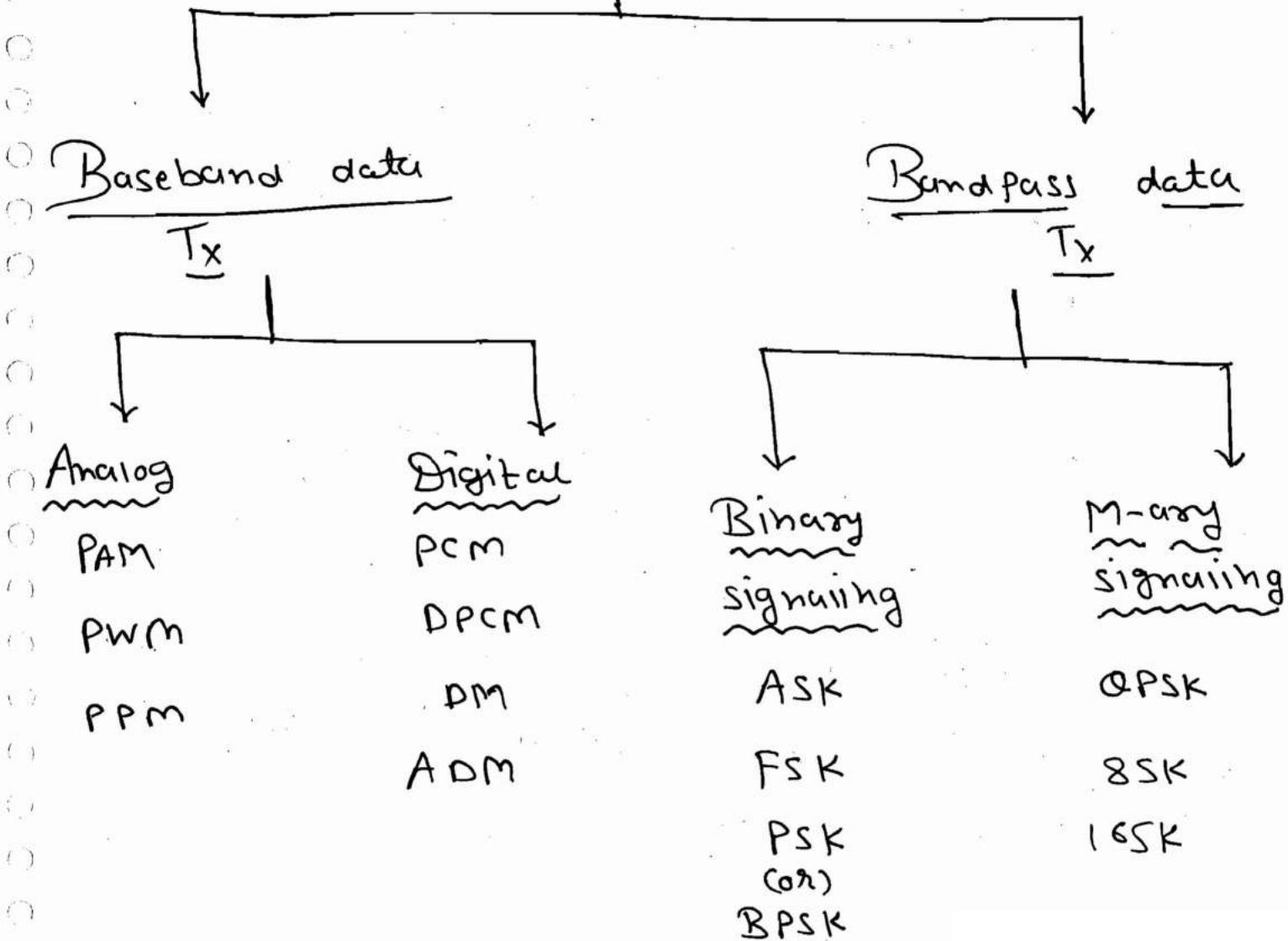


Digital Communication



* Review of Sampling Theorem:

=> In digital communications binary data is transmitted through the channel. To convert an analog signal into a digital signal, the signal should be sampled 1/s every T_s seconds. This samples are

to generate the binary data. At the receiver DAC is convert the binary info samples. Finally a LPF is used to deconstruct the signal from samples. But the signal deconstruction is possible only when the following condition is satisfied.

$$\frac{1}{T_s} \geq 2f_m \text{ Sample/sec.}$$

\Rightarrow The minimum sampling rate required to deconstruct the signal is called as the Nyquist rate.

$$\therefore \frac{1}{T_s} = 2f_m \text{ Sample/sec.} \leftarrow \text{H.B.}$$

\Rightarrow If the Sampling rate is greater than the Nyquist rate then the signal is over sampled.

$$\frac{1}{T_s} > 2f_m.$$

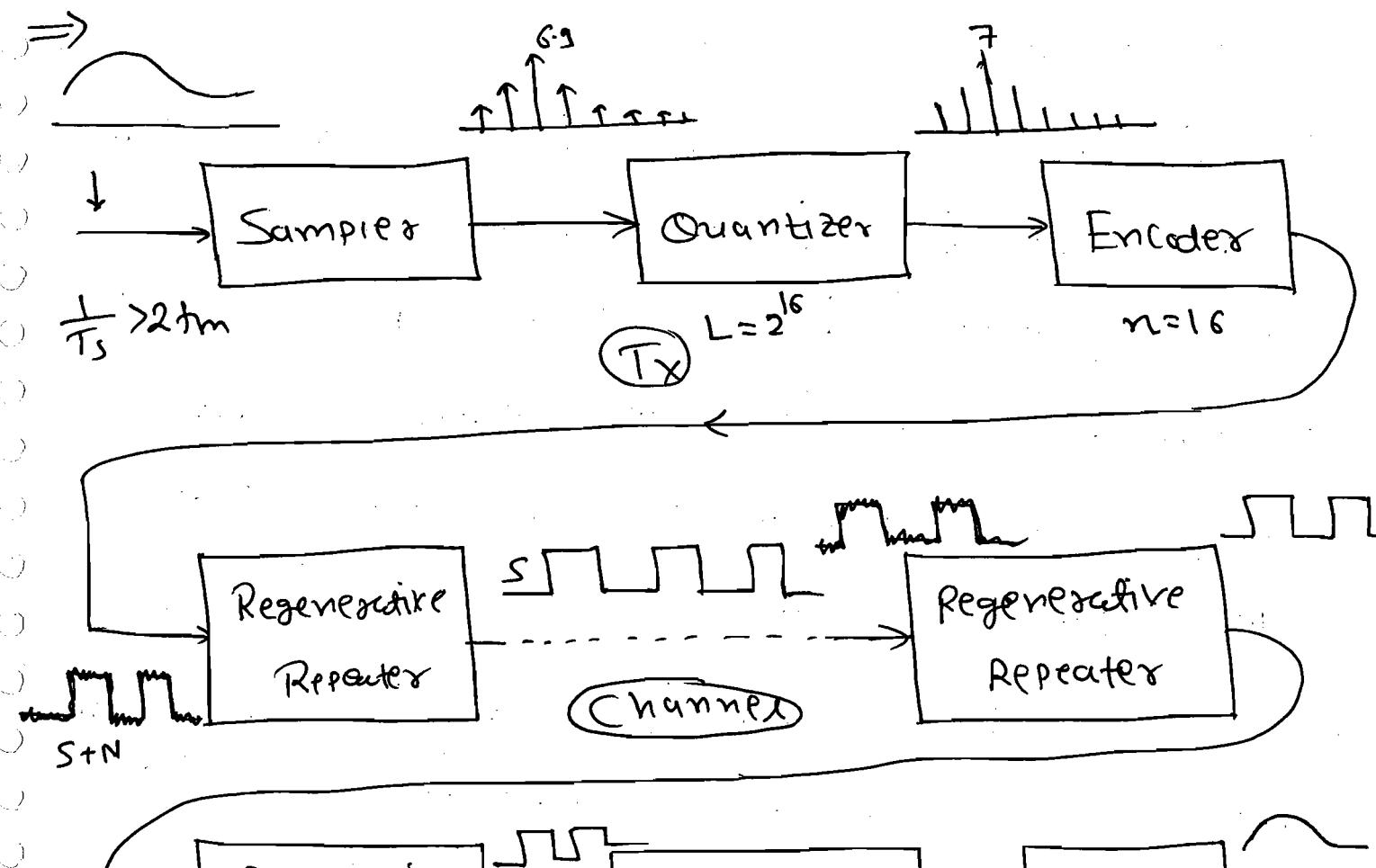
\Rightarrow If the Sampling rate is less than the Nyquist rate then the signal is under sampled and distortion will occur.

$$\frac{1}{T_s} < 2f_m$$

X

\Rightarrow The O/P of the LPF is envelope of the Sampled signal.

(1) Pulse Code Modulation (PCM):



⇒ Sampler Converts the continuous time signal into the discrete time signal but the signal should be over sampled ($\frac{1}{T_s} > 2f_m$).

⇒ In Quantizer each sample is rounded off to the nearest quantization level.

⇒ In Voice transmission in telephone system, the sampling rate used is 8000 sample/sec and each sample is encoded into 8 bits.

⇒ In Audio CD Recording, the Sampling rate used is 44,100 sample/sec and each sample is encoded into 16 bits.

⇒ Encoder output is the Binary data which is represented in the form of rectangular pulses.

⇒ When the binary data is transmitted through a channel, amplitude distortion

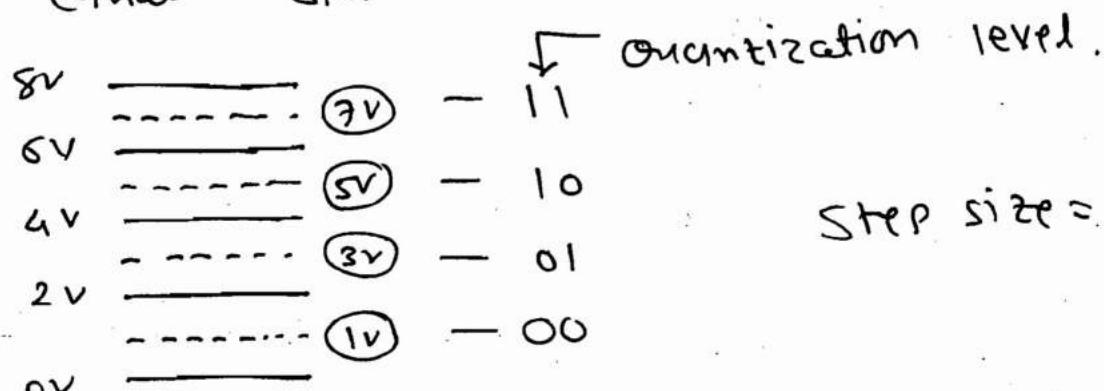
\Rightarrow Regenerative Repeaters are used to eliminate noise from the signal and noise present at the input of the receiver.

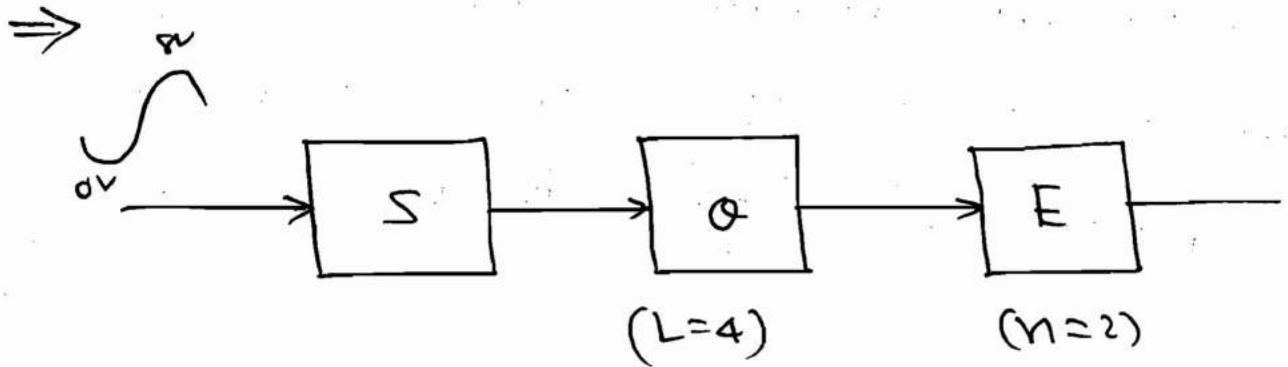
\Rightarrow The decoder is an analog converter which converts the binary data into Sample.

\Rightarrow The LPF is used to reconstruct the signal from the samples.

* Quantizer:

\Rightarrow Consider the 2 bit PCM System and assume that amplitude varies from 0 to 8V. The mux. Sampled value applied to the Quantizer is 8V and the min. value is 0V. This entire range is divided into 4 equal steps as shown in fig.





S	Q	E
1.3	1	00
5.9	5	10
8	7	11

⇒ Consider an n -bit PCM System

Where

→ n = no. of bits per Sample.

→ L = no. of Quantization level

$$L = 2^n \quad \leftarrow \text{H.B.}$$

$$\rightarrow \Delta = \text{Step size} = \frac{V_{\max} - V_{\min}}{L} \quad \leftarrow \text{H.B.}$$

$$\Delta = \frac{8-0}{4} = 2V$$

$$\rightarrow \Delta = \frac{V_{PP}}{L} \quad \leftarrow \text{H.B.}$$

→ ϱ_e = Quantization error.

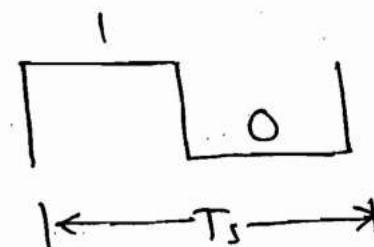
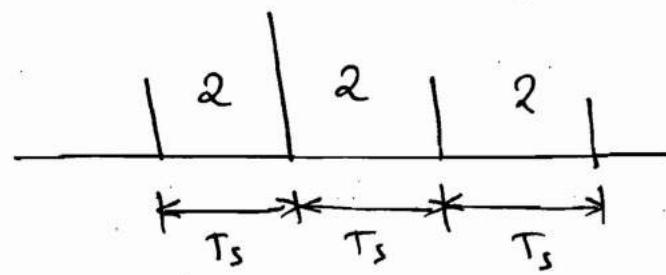
$$\varrho_e = \text{Sampled value} - \text{Quantized Value}$$

$$\Rightarrow [Q_e]_{\max} = \frac{\Delta}{2} \quad [Q_e \text{ varies from } -\frac{\Delta}{2} \text{ to } +\frac{\Delta}{2}]$$

↑
H.B.

$\Rightarrow T_b = \text{bit duration}$

$$T_b = \left(\frac{T_s}{n} \right) \text{ sec.} \quad \leftarrow \text{H.B.}$$



$$T_b = T_s/2$$

$$\Rightarrow R_b = \text{Bit rate} = \frac{\text{bits}}{\text{sec.}} \quad \leftarrow \text{H.B.}$$

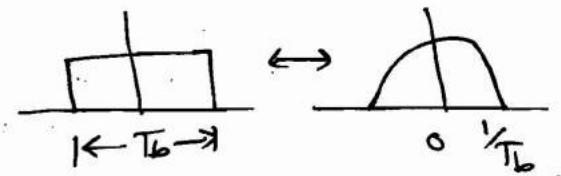
$$R_b = [\text{Sampling rate} \times n] \text{ bits/sec.} \quad - ①$$

$$= \frac{\text{Sample}}{\text{sec}} \times \frac{\text{bits}}{\text{sample}} = \text{bits/sec.}$$

$$\rightarrow R_b = \frac{1}{T_s} \times n = \frac{n}{T_s} \text{ bps.}$$

$$\xrightarrow{\text{H.B.}} R_b = \frac{1}{T_b} \text{ bps.} \quad - ②$$

$$\Rightarrow (BW)_{\max} = \left(\frac{1}{T_b} \right) \text{ Hz.}$$



Q-1 A signal whose amplitude varies from 0 to 10V is band limited to 4 kHz and transmitted through a channel using 5 bit PCM. The sampling rate is 50%. Higher than the Nyquist rate. Calculate all parameters of a PCM system.

Soln: $f_m = 4 \text{ kHz}$, $V_{\max} = 10V$
 $V_{\min} = 0V$.
 $n = 5 \text{ bit}$

Sampling rate = 1.5 of Nyquist rate
 $= 1.5 \times (2f_m)$
 $= \frac{3}{4} \times 2 \times 4000$

Sampling rate = 12000 sample/sec $\Rightarrow \frac{1}{T_s}$

$\rightarrow L = 2^n = 2^5 = 32 = \text{Quantization level.}$

$$\Delta = \frac{V_{\max} - V_{\min}}{L} = \frac{10 - 0}{32} = \frac{10}{32} \text{ V.}$$

$$\rightarrow [Q_e]_{\max} = \frac{\Delta}{2} = \frac{10}{64} \text{ Volts.}$$

$$T_b = \text{bit duration} = \left(\frac{T_s}{n} \right) \text{ sec}$$

$$T_b = \frac{1}{12000 \times 5}$$

$$R_b = 12000 \times 5$$

$$R_b = 60000 \text{ bits/sec}$$

$$R_b = 60 \text{ kbps}$$

$$(Bw)_{\max} = \frac{1}{T_b} = 60 \text{ kHz}$$

→ The minimum BW of the channel to transmit the PCM signal without any distortion is 60 kHz.

Q-2 A radio signal is Bandlimited to 4.5 MHz and Transmitted through a channel using PCM.

① Determine the Sampling rate if the signal is sampled at a rate of 20% higher than the Nyquist rate.

② Determine the bit rate if the no. of quantization level is 1024.

Soln:

$$f_m = 4.5 \text{ MHz}$$

$$\Rightarrow \text{Nyquist rate} = 2f_m = 9 \times 10^6 \text{ sample/sec.}$$

$$\rightarrow \text{Sampling rate} = 1.2 \times \text{Nyquist rate}$$

$$\text{Sampling rate} = 10.8 \times 10^6 \text{ sample/sec.}$$

$$② L = 1024$$

$$\therefore L = 2^n \Rightarrow n = 10$$

$$\text{bit rate } R_b = \left(\frac{1}{T_s} \times n \right)$$

$$\therefore R_b = 10.8 \times 10^6 \times 10$$

$$\therefore R_b = 10.8 \times 10^7$$

$$R_b = 108 \text{ MHz/sec.}$$

$$(Bw)_{\max} = 108 \text{ MHz.}$$

IMP

Q-3 A sinusoidal signal is band limited to 5 kHz and transmitted through a channel using PCM. The sampling rate is twice the Nyquist rate. The max. quantization error should be 0.1% of the peak signal amp. Determine the bit rate.

$$\text{Soln: } f_m = 5 \text{ kHz}$$

$$\Rightarrow \text{Nyquist rate} = 2f_m = 10,000 \text{ sample/sec.}$$

$$\Rightarrow \text{Sampling rate} = 2 \text{ (N.R.)}.$$

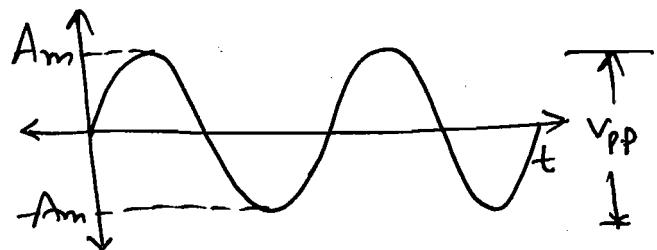
$$= 20,000 \text{ sample/sec.}$$

$$[\Phi_e]_{\max} = \frac{\Delta}{2} = \frac{V_{P-P}}{2L}$$

\Rightarrow Peak-value $V_p = A_m$

Peak-peak value

$$V_{p-p} = 2A_m.$$



$$[\alpha_e]_{\max} = \frac{0.1}{100} \times V_p.$$

$$\therefore \frac{V_{p-p}}{2L} = \frac{0.1}{100} \times V_p.$$

$$\therefore \frac{2A_m}{2L} = \frac{0.1}{100} \times A_m$$

$$\therefore L = 1000$$

$$\Rightarrow 2^n = L = 1000 \Rightarrow n = 10$$

$$R_b = \left(\frac{1}{T_s} \times n \right) \text{ bps}$$

$$\therefore R_b = 20,000 \times 10.$$

$$R_b = 0.2 \text{ mbps.} \Rightarrow B_{\max} = 0.2 \text{ MHz}$$

(Q-4) A signal $r(t) = 4(\cos 10^3 t \times 2\pi)$ is sampled at Nyquist rate and transmitted through a channel using 3 bit PCM.

① Calculate all parameters.

② If the sampled values are $3.8, 2.8, 2.1, 1.7, -0.5, -3.2, -4$.

Determine the quantization error and rounding error for each sample.

$$\text{Sol}^n: \textcircled{1} \quad V_p = Am = 4 \text{ V.}$$

$$V_{p-p} = 2 \times V_p = 8 \text{ V.}$$

$$V_{\max} = 4 \text{ V}, \quad V_{\min} = -4 \text{ V.}$$

$\rightarrow n = 3$ bit

$$L = 2^n = 2^3 = 8 \text{ level.}$$

$$\Delta = \frac{V_{p-p}}{L} = \frac{8}{8} = 1 \text{ V} = \text{step size.}$$

$$[\Delta e]_{\max} = \frac{\Delta}{2} = \frac{1}{2} = 0.5 \text{ V.}$$

$$\rightarrow f_m = 1 \text{ kHz}$$

Sampling rate = Nyquist rate = $2f_m$.

$$\therefore \text{Sampling rate} = \frac{1}{T_s} = 2000 \text{ sample/see.}$$

$$\rightarrow \text{Bit rate } R_b = \frac{1}{T_s} \times n$$

$$\Rightarrow R_b = 2000 \times 3 = 6000 \text{ bps}$$

$$R_b = 6 \text{ kbps.}$$

$$(B_w)_{\max} = \frac{1}{T_b} = 6 \text{ kHz.}$$

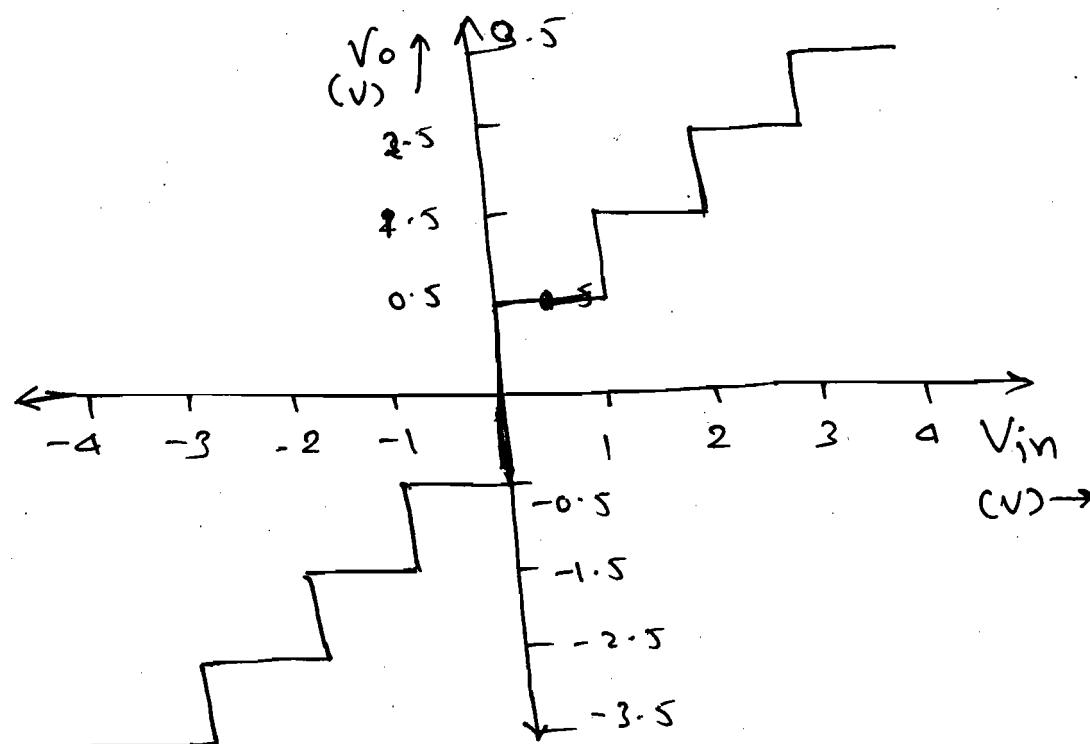
$$\rightarrow \text{Bit duration} = T_b = \frac{T_s}{n} = \frac{1}{6000} \text{ sec.}$$

(2) Sample values are 3.8, 2.8, 2.1, 1.3,

$S(V)$	$\alpha(V)$	$= S - \alpha$	E
3.8	3.5	0.3	111
2.1	2.5	-0.4	110
1.7	1.5	0.2	101
-0.5	-0.5	0	011
-3.2	-3.5	0.3	000
-4	-3.5	-0.5	000

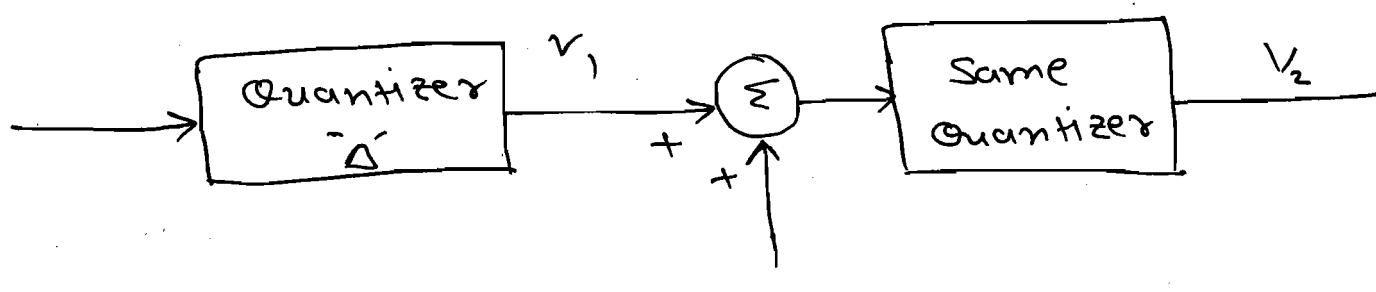
α -level	Encoder Out.
-3.5	000
-2.5	001
-1.5	010
-0.5	011
+0.5	100
+1.5	101
+2.5	110
+3.5	111

③ Transfer Chara.



Q-5 Consider a system as shown in fig.

Quantizer Step size = Δ

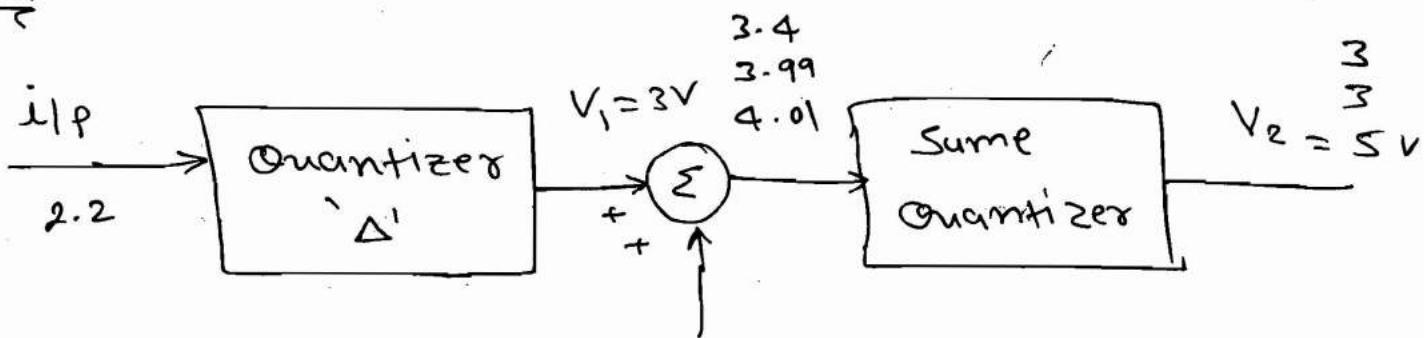


minimum value of K that

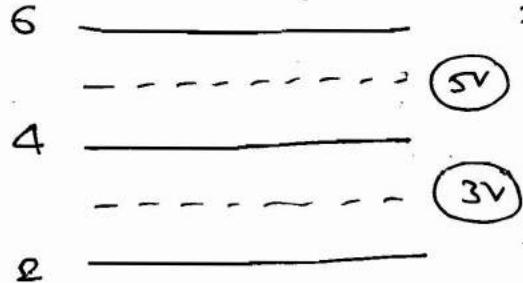
V_2 are different.

- (A) Δ
- (B) 2Δ
- (C) $\frac{\Delta}{2}$
- (D) Δ^2

Solⁿ:



assume $\Delta = 2$



$$= 1.01 \text{ let, } i/p = 2.2V$$

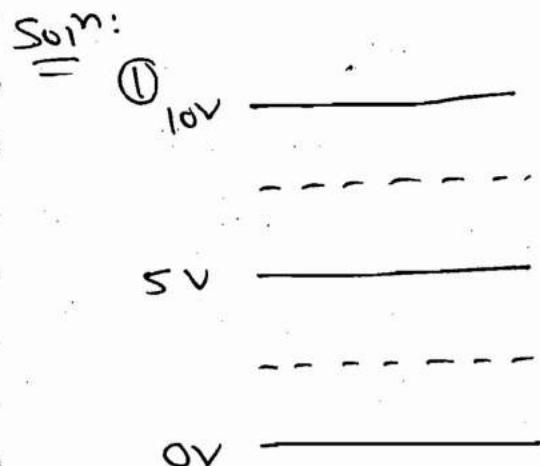
$$k = 1.01 = \frac{\Delta}{2}$$

So, Ans is $\frac{\Delta}{2}$.

Q-6 The i/p to the Quantizer varies from 0 to 10 volts.

① When the input lies bet'n 0 to 5V and from 5 to 10 ^{op is 7.5V}, sketch the transfer chara. and variation of Quantization error.

② When the i/p lies bet'n 0 to 5V, the op is 2V and when the i/p bet'n 5 to 10V the op is 9V. Sketch the chara. and variation of

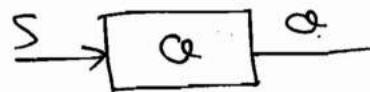


$$\text{Step size } \Delta = 5V$$

$$[\alpha_e]_{\max} = \frac{\Delta}{2} = 2.5V$$

α_e varies betw

$$\frac{\Delta}{2} \text{ to } -\frac{\Delta}{2} \text{ i.e. } 2.5V \text{ to } -2.5V$$



~~(2)~~ $\alpha_e = S - \alpha = i_l P - OLP$

S	0
0	2.5
1	2.5
2	2.5

