CBSE Test Paper 04

Chapter 7 Integrals

1. $\int\limits_{-\pi/12}^{\pi/12} \frac{1}{\cos 2x} dx = .$

   a. $\frac{1}{2}\log 3$
   b. log 3
   c. $\frac{1}{3}\log 2$
   d. log9

2. $\int\limits_{-2}^{2} [x]\ dx$ is equal to, where [.] denotes Greatest Integer function.

   a. 0
   b. 2
   c. -2
   d. 4

3. If $\int f(x)dx = g(x) + C$ then

   a. $\frac{d}{dx}(g(x) + C) = f(x)$
   b. g (x) + f (x) =constant.
   c. g (x) = f (x)
   d. $\frac{d}{dx}(f(x)) = g(x)$

4. $\int \frac{x \cos x}{\sqrt{x \sin x + \cos x}} dx$ is equal to

   a. $\frac{1}{2}(x \sin x + \cos x)^{1/2} + C$
   b. $2(x\ \ \sin x + \cos\ \ x)^{1/2} + C$
   c. $\frac{3}{5}(x \cos x - \sin x)^{3/2} + C$
   d. $\frac{2}{3}(x \sin x + \cos x)^{3/2} + C$

5. $\int (\log(\log x) + (\log x)^{-1})$ dx is equal to

   a. x log (log x) $-\frac{x}{\log x}$ + C

b. $x - \frac{\log 2x-1}{x+2} + C$

c. x log (log x) + $\frac{x}{\log x}$ + C

d. x log (log x) + C

6. The intergral value of $\int_{-a}^{a} f(x)dx = 0$ ____.

7. The value of integral is $\int_0^1 \frac{dx}{e^x+e^{-x}}$ is ________.

8. The value of integral $\int \frac{1+\cos x}{x+\sin x} dx$______.

9. Evaluate $\int \frac{\sec^2 x}{3+\tan^2 x} dx$. **(1)**

10. Evaluate $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$.

11. $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos \alpha} dx$.

12. Integrate the function $\tan^{-1}x$.

13. $\int_{-1}^{3/2} |x \sin(\pi x)|\, dx$

14. Evaluate the following integral $\int \sqrt{x}\left(3x^2 + 2x + 3\right) dx$

15. Evaluate $\int \frac{\sin^6 x+\cos^6 x}{\sin^2 x \cos^2 x} dx$.

16. Evaluate $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$.

17. Show that $\int_0^a f(x) \,.\, g(x)\, dx = \int_0^a f(x)\, dx$
    If $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$.

18. $\int_0^{\pi} \frac{x dx}{a^2\cos^2 x+b^2\sin^2 x}$.

Solution

1. a. $\frac{1}{2}\log 3$

   Explanation: = $2\int_0^{\frac{\pi}{2}} \sec 2x dx = 2\left[\frac{\log|\tan\left(\frac{\pi}{4}+x\right)}{2}\right]_0^{\frac{\pi}{12}}$

   = $\log|\tan\frac{\pi}{3}| - \log|\tan\frac{\pi}{4}|$

   = $\log\sqrt{3} - \log 1 = \frac{1}{2}\log 3$

2. c. –2

   Explanation: $\int_{-2}^{2}[x]dx = \int_{-2}^{-1}[x]dx + \int_{-1}^{0}[x]dx + \int_{0}^{1}[x]dx + \int_{1}^{2}[x]dx$

   $\Longrightarrow \int_{-2}^{-1}(-2)dx + \int_{-1}^{0}(-1)dx + \int_0^1 0dx + \int_1^2 1dx$

   $\Longrightarrow [-2x]_{-2}^{-1} + [-x]_{-1}^{0} + [0]_0^1 + [x]_1^2 \Longrightarrow -2$

3. a. $\frac{d}{dx}(g(x)+C) = f(x)$

   Explanation: $\Rightarrow \frac{d}{dx}(g(x)+C) = \frac{d}{dx}\int f(x)dx = f(x)$

4. b. $2(x\ \sin x + \cos\ x)^{1/2} + C$

   Explanation: Substitute x sinx + cos x = t^2 $\Longrightarrow$ (x cos x)dx = 2tdt

   $\int 2dt = 2\,t + C \Longrightarrow 2(x\ sinx + cosx)^{1/2} + C$

5. d. x log (log x) + C

   Explanation: $\int log(logx)dx + \int \frac{1}{\log x}dx = \log(logx).\,x$

   $-\int \frac{1}{\log x}.\frac{1}{x}.\,xdx + \int \frac{1}{\log x}dx + C = \ x\ log(log\ x) + C$

6. True
7. $\tan^{-1} e - \frac{\pi}{4}$
8. $\log|x + \sin x| + C$
9. Let $I = \int \frac{\sec^2 x}{3+\tan^2 x}dx$

   Put tan x = t

   $\Rightarrow \sec^2 x dx = dt$

   $\therefore I = \int \frac{\sec^2 x}{3+\tan^2 x}dx = \int \frac{dt}{3+t^2}$

   $= \int \frac{dt}{(\sqrt{3})^2+t^2}$

   $= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C\left[\because \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C\right]$

$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\tan x}{\sqrt{3}}\right) + C$

10. $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$

$I = \int_0^1 \tan^{-1}\left(\frac{x+x-1}{1-x(x-1)}\right) dx$

$I = \int_0^1 \left[\tan^{-1}(x) + \tan^{-1}(x-1)\right] dx$ ...(1)

$I = \int_0^1 \left[\tan^{-1}(1-x) + \tan^{-1}(1-x-1)\right] dx [\because P_4]$

$I = \int_0^1 \left[-\tan^{-1}(x-1) - \tan^{-1}(x)\right] dx$...(2) $[\because tan^{-1}(-\theta) = -tan^{-1}\theta]$

(1) + (2)

2I = 0

I = 0

11. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} dx$

$= \int \frac{(2\cos^2 x - 1) - (2\cos^2\alpha - 1)}{(\cos x - \cos\alpha)} dx$

$= \int \frac{2(\cos x + \cos\alpha)(\cos x - \cos\alpha)}{(\cos x - \cos\alpha)} dx$

$= 2(\sin x + \cos\alpha . x) + c$

12. Let I = $\tan^{-1}$xdx

$= \int \left(\tan^{-1}x\right) .1dx$

$= \tan^{-1}x . x - \int \frac{1}{1+x^2} x . dx$

$= x\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2} dx$

$= x\tan^{-1}x - \frac{1}{2}\log\left|\left(1+x^2\right)\right| + c$

$\left[\because \int \frac{f'(x)}{f(x)} dx = \log|f(x)|\right]$

$= x\tan^{-1}x - \frac{1}{2}\log\left(1+x^2\right) + c$

13. Here, f(x) = |xsinπx| $= \begin{cases} xsin\ \pi x & \text{if } -1 \leqslant x \leqslant 1 \\ -xsin\ \pi x & \text{if } 1 \leqslant x \leqslant \frac{3}{2} \end{cases}$

Therefore $\int_{-1}^{3/2} |x\sin(\pi x)|\, dx = \int_{-1}^{1} x\sin\pi x dx + \int_1^{3/2} -x\sin\pi x dx$

$= \left[\frac{-x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2}\right]_{-1}^{1} - \left[\frac{-x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2}\right]_{1}^{3/2}$

$= \frac{2}{\pi} - \left[-\frac{1}{\pi^2} - \frac{1}{\pi}\right]$

$= \frac{3}{\pi} + \frac{1}{\pi^2}$

14. $\int \sqrt{x}\left(3x^2 + 2x + 3\right) dx$

$= \int x^{\frac{1}{2}}\left(3x^2 + 2x + 3\right) dx$

$= \int \left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right) dx$

$= \int 3x^{\frac{5}{2}}dx + \int 2x^{\frac{3}{2}}dx + \int 3x^{\frac{1}{2}}dx$

$= 3\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + 2\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

$= 3\frac{x^{\frac{7}{2}}}{\frac{7}{2}} + 2\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$

15. Let $I = \int \frac{\sin^6 x+\cos^6 x}{\sin^2 x\cos^2 x}dx$

$\Rightarrow I = \int \frac{(\sin^2 x)^3+(\cos^2 x)^3}{\sin^2 x\cos^2 x}dx$

$= \int \frac{\begin{bmatrix}(\sin^2 x+\cos^2 x)^3 \\ -3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)\end{bmatrix}}{\sin^2 x\cos^2 x}dx$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$= \int \frac{(1)^3-3\sin^2 x\cos^2 x}{\sin^2 x\cos^2 x}dx$

[∵sin²x + cos² x = 1]

$= \int \frac{1}{\sin^2 x\cos^2 x}dx - 3\int \frac{\sin^2 x\cos^2 x}{\sin^2 x\cos^2 x}dx$

$= \int \frac{\sin^2 x+\cos^2 x}{\sin^2 x\cos^2 x}dx - 3\int 1dx$

$= \int \left[\frac{\sin^2 x}{\sin^2 x\cos^2 x} + \frac{\cos^2 x}{\sin^2 x\cos^2 x}\right]dx - 3\int 1dx$

$= \int (\sec^2 x + \cot^2 x)\,dx - 3\int 1dx$

$= \int \sec^2 xdx + \int \cot^2 xdx - 3\int 1dx$

$= tanx - cotx - 3x + C$

16. According to the question, $I = \int_0^{2\pi} \frac{dx}{1+e^{\sin x}}$

$\Rightarrow I = \int_0^{2\pi} \frac{dx}{1+e^{\sin(2\pi-x)}} \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$\Rightarrow I = \int_0^{2\pi} \frac{dx}{1+e^{-\sin x}} = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x}+1}dx$

Adding Equations (i) and (ii),

$\Rightarrow I + I = \int_0^{2\pi} \frac{dx}{1+e^{\sin x}} + \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}}dx$

$\Rightarrow \quad 2I = \int_0^{2\pi} \frac{(1+e^{\sin x})}{(1+e^{\sin x})}dx$

$\Rightarrow \quad 2I = \int_0^{2\pi} 1dx = [x]_0^{2\pi} = 2\pi - 0$

$\therefore \quad I = \pi$

17. $I = \int_0^a f(x).g(x)\,dx$

$= \int_0^a f(a-x).g(a-x)\,dx \;\; [byP_4]$

$= \int_0^a f(x) . [4 - g(x)] dx$ [given]

$= \int_0^a 4f(x)\, dx - \int_0^a f(x) . g(x)\, dx$

$I = 4 \int_0^a f(x)\, dx - I$

$2I = 4 \int_0^a f(x)\, dx$

$I = 2 \int_0^a f(x)\, dx$

18. $I = \int_0^\pi \frac{x dx}{a^2\cos^2 x + b^2 \sin^2 x}$ ...(1)

$I = \int_0^\pi \frac{(\pi - x)}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)} dx$

$I = \int_0^\pi \frac{(\pi - x)}{a^2\cos^2 x + b^2\sin^2 x} dx$

$I = \int_0^\pi \frac{\pi\, dx}{a^2\cos^2 x + b^2\sin^2 x} - \int_0^\pi \frac{x\, dx}{a^2\cos^2 x + b^2\sin^2 x} dx$

$I = \pi \int_0^\pi \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} - I$

$2I = \pi \int_0^\pi \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$I = \frac{\pi}{2} \int_0^\pi \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$= \frac{\pi}{2} .2 \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

Dividing N and D by $\cos^2$x

$I = \pi \int_0^{\pi/2} \frac{\sec^2 x dx}{a^2 + b^2\tan^2 x}$

Put b tan x = t

$b \sec^2 x\, dx = dt$

$= \pi \int_0^\infty \frac{dt/b}{a^2 + t^2}$

$= \pi . \frac{1}{a} \left[\tan^{-1} \frac{t}{a}\right]_0^\infty \times \frac{1}{b}$

$= \frac{\pi}{ab} \left(\frac{\pi}{2}\right)$

$= \frac{\pi^2}{2ab}$