

OBJECTIVE - I

Sol 1. C

\vec{A} , unit vector along the radial direction + \vec{B} , unit vector along the tangential direction

angle between \vec{A} & \vec{B} is 90° .

$$\text{So } \vec{A} \cdot \vec{B} = AB \cos q = AB \cos 90^\circ = 0$$

Sol 2. C

\vec{A} , unit vector along the radial direction + \vec{B} , unit vector along the away from the axis.

angle between \vec{A} & \vec{B} is 90° .

$$\text{So } \vec{A} \cdot \vec{B} = |A| |B| \cos q = 0$$

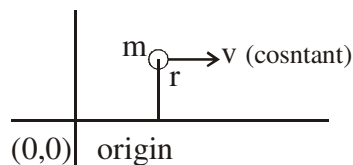
Sol 3. B

Angular momentum w.r.t. origin

$$= m (\vec{r} \times \vec{v})$$

$$= mvr \otimes$$

$$= \text{Constant}$$



Sol 4. 'w' is independent of r but velocity is dependent upon 'r'.

$$w = \frac{v}{r}$$

$$v = wr$$

$$\backslash \quad v \propto r$$

Sol 5. Angular velocity 'w' is same for both the wheel.

$$v_A = wR$$

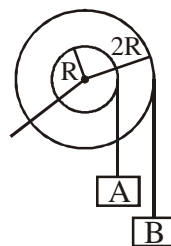
$$v_B = w2R$$

$$x = v_A t = wRt \quad \dots\dots\dots (1)$$

$$y = v_B t = w(2R)t \quad \dots\dots\dots (2)$$

From equation (1) & (2) we get

$$y = 2x$$



Sol 6. C

The resultant force on a particle is in the vertical direction not in horizontal or intersecting the axis. Because body is rotating uniformly along the vertical axis in an inertial frame.

Sol 7. B

Body is rotating non uniformly along the vertical axis is horizontal and skew with the axis.

Sol 8. A

$$\vec{\Gamma} = \vec{r} \times \vec{F}$$

$$= r F \sin q$$

\vec{F} is along the position vector \vec{r} so angle between r & F is 0.

$$\vec{\Gamma} = rF \sin 0^\circ = 0$$

$$\text{P} \quad \vec{r} \cdot \vec{\Gamma} = 0 \text{ and } \vec{F} \cdot \vec{\Gamma} = 0$$

Sol 9. D

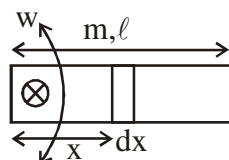
$$dm = \frac{m}{\ell} dx$$

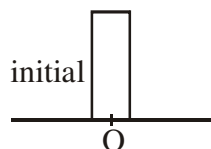
Centripetal force is

$$\text{P} \quad \int_0^\ell \frac{m}{\ell} w^2 x dx$$

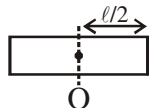
$$= \frac{m}{\ell} w^2 \frac{x^2}{2} \Big|_0^\ell$$

$$= \frac{1}{2} m w^2 \ell$$



Sol 10. C

Centre of mass of the rod remain constant along the y-axis.



The lower end will remain at a distance $l/2$ from O.

Sol 11. C

Thickness 't' { Q I for disc is $\frac{mr^2}{2}$ }

$$m_A = \rho \pi r^2 t$$

$$I_A = \frac{m_A r^2}{2} = \frac{\rho \pi r^2 t \cdot r^2}{2} = \frac{\rho \pi r^4 t}{2} \dots\dots\dots (1)$$

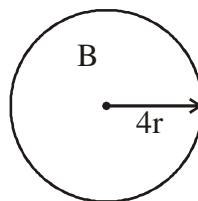
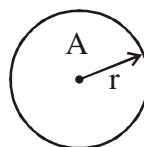
Thickness 't/4'

$$m_B = \rho \pi (4r)^2 t/4 = 4 \rho \pi r^2 t$$

$$I_B = \frac{m_B (4r)^2}{2} = \frac{64 \rho \pi r^4 t}{2} \dots\dots\dots (2)$$

from (1) & (2) we get

$$I_B > I_A$$

**Sol 12. A**

$$t_A = t_B$$

$$I_A \mu_A = I_B \mu_B$$

$$\therefore I_B > I_A$$

$$\therefore v_A > v_B$$

$$(\because I = I \mu)$$

I \propto moments of inertial

μ \propto angular acceleration

t \propto Torque

Sol 13. A

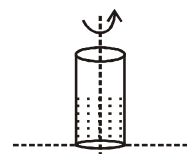
Moment of inertia

$$I = m r^2$$

distance of the particle of the water is increase.

$$I = \mu r^2$$

So I is increase.

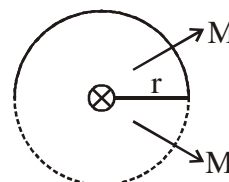
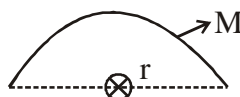
**Sol 14. A**

Use the symmetricity condition

$$I = M r^2$$

$$I = 2 M r^2$$

So the moment of inertia of uniform semicircular wire is $= \frac{I}{2} = M r^2$



Sol 15. A

$$I = mr^2$$

density of Iron > density of aluminium

So mass of Iron > mass of aluminium

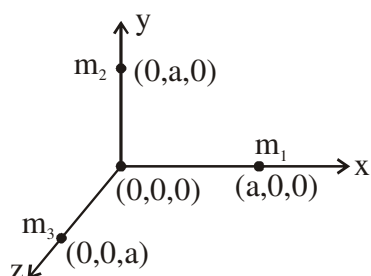
$$I_2 > I_1$$

Sol 16. D

$$I_x = m_2 a^2 + m_3 a^2 = 0.20 \quad \dots\dots\dots (1)$$

$$I_y = m_1 a^2 + m_3 a^2 = 0.20 \quad \dots\dots\dots (2)$$

$$I_z = m_1 a^2 + m_2 a^2 \quad \dots\dots\dots (3)$$

 I_z cannot be deduced with this information or solving equation (1) & (2).**Sol 17. N = Mg cos q**

Block move with uniform velocity

$$f = Mg \sin q$$

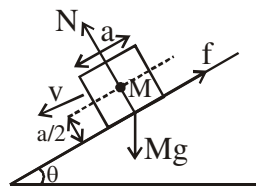
Net torque on the block is zero.

$$\tau_{N\odot} + \tau_{f\odot} = 0$$

$$t_N = t_f$$

$$= Mg \sin q \cdot a/2$$

$$= 1/2 Mg a \sin q$$

**Sol 18. B**

By angular momentum conservation

initial angular momentum = final angular momentum

$$I\omega = I'\omega'$$

$$Mr^2\omega = (Mr^2 + 2mr^2)\omega'$$

$$\omega' = \frac{\omega M}{M + 2m}$$

Here I is the moment of inertia of circular ring.

 I' is the moment of inertia of system (circular ring + two particle)Here moment of inertia of each particle is ' mr^2 ' about the centre of the circular ring.**Sol 19. C**

Angular momentum about the axis of rotation is remain unchanged.

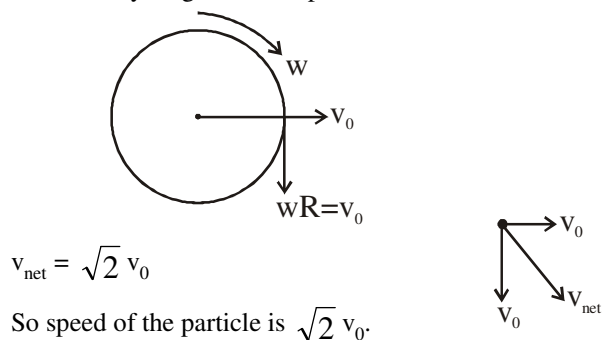
$$I_1\omega_1 = I_2\omega_2$$

If he stretched his arms, I is increase, because of distance of some mass of body increase ($I = mr^2$). That causes angular velocity is decrease.

If he folds his arms, I isdecreases & angular velocity is increase.

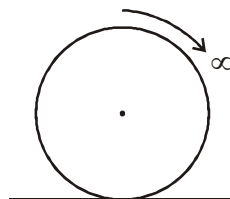
Sol 20. C

The velocity diagram of the particle which is same level of centre of wheel is



Sol 21. A

Causes of friction force wheel is move along the surface. So we can say that frictional force acting on the wheel by the surface is along the velocity of the wheel.



Sol 22. D

w \propto petrol input / second

"on a friction less road" \Rightarrow angular velocity of the engine $w = 0$, It move with uniform velocity.

The linear velocity of the scooter is remain same.

Sol 23. D

A solid sphere, a hollow sphere and a disc is placed at the top of smooth incline. At friction less surface angular velocity of there is zero and acceleration at the incline plane is same equal to $g \sin \theta$. So we can say that all will take same time to reaches the bottom of inclined plane.

Sol 24. D

Since linear acceleration is same for all ($a = Mg \sin \theta - m Mg \cos \theta$) as they have same mass 'M' and same 'm' Hence, all will reach the bottom simultaneously.

Hence (D)

Sol 25. B

For all the bodies, torque is same .

Hence, angular momentum (L) is also same.

$$\text{Now, K.E.} = \frac{1}{2}mv^2 + \frac{L^2}{2I}$$

Linear velocity 'v' is same for all as same force acts on them.

Therefore more value of moment of inertia implies lesser kinetic energy.

Among all, the hollow sphere has the maximum moment of inertia $I = \left(\frac{2}{3}MR^2\right)$.

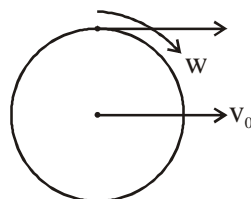
Hence (B).

Sol 26. B

$$v_0 + wR = 2v_0$$

$$\Rightarrow l = v_0 t$$

$$\text{Length passes through the Hand} = 2v_0 t = 2l$$



OBJECTIVE - II

Sol 1. BD

The axis of rotation of a purely rotating body may pass through the centre of mass or may pass through a particle of the body.

Sol 2. B

Angular momentum $L = I\omega$

$$\text{or } \frac{d\vec{L}}{dt} = \vec{\Gamma}_{\text{ext}}$$

where $\vec{\Gamma}_{\text{ext}}$ is the total torque on the system due to all the external forces acting on the system.

Sol 3. BCD

Angular momentum = $m(\vec{r} \times \vec{v})$

about P is zero because $\vec{r} = 0$

about Q is non zero = mvl

Sol 4. AB

$$\vec{\Gamma}_{\text{ext}} = \frac{d\vec{L}}{dt}$$

$$0 = \frac{d\vec{L}}{dt} \quad \{F_{\text{ext}} = 0\}$$

\vec{L} (Angular momentum) is remain constant.

$F_{\text{ext}} = 0,$

$$F_{\text{ext}} = \frac{d\vec{P}}{dt} = 0$$

\vec{P} (Linear momentum) is remain constant.

Sol 5. C

By parallel axis theorem

$$I_B = I_A + Id^2$$

$$I_B > I_A$$

Sol 6. B

If sphere is rotating about a diameter, the particle on the diameter mentioned above do not have any linear acceleration.

Sol 7. D

$$\text{Torque} = \vec{r} \times \vec{F} \quad (\text{Torque is depend on force \& length of rod})$$

$$= IF \odot \quad (\text{upwards direction})$$

$$\tau = I\mu$$

$$\mu = \tau/I \quad (\mu \odot \text{ angular acceleration})$$

If pivoted end is change then the position of moment of inertia is shift along vertical axis.

$$\text{angular momentum} = I\omega$$

Sol 8. ACD

The speed of 'O' is v_0 pure rolling $v_0 = wR$

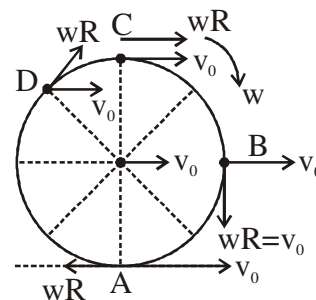
So speed of the particle A is zero.

The speed of C is $v_0 + wR = 2v_0$ m/s

The speed of C is $\sqrt{(v_0)^2 + (wR)^2} + 2v_0(wR)\cos 45^\circ$

The speed of D is $\sqrt{2v_0^2 + 2v_0^2 \times \frac{1}{\sqrt{2}}}$

$$v_0 = \sqrt{2 + \sqrt{2}} \text{ m/s}$$

**Sol 9. C**

Acceleration of both sphere on the cline plane is

$$a_{\text{com}} = \frac{g \sin \theta}{1 + I_{\text{com}} / MR^2}$$

$$I_{\text{com}} \text{ for first solid sphere is } = \frac{2}{5} M_1 R_1^2$$

$$\text{So } a_{\text{com}} = \frac{g \sin \theta}{1 + \frac{2/5 M_1 R_1^2}{M_1 R_1^2}} = \frac{5}{7} g \sin \theta$$

The acceleration of Both the sphere is same. So we can say that both sphere will reach to bottom together.

Sol 10. B

Acceleration on the inclined plane is

$$a_{\text{com}} = \frac{g \sin \theta}{1 + I_{\text{com}} / MR^2}$$

$$I_{\text{com}} \text{ for hollow sphere } = \frac{2}{3} MR^2$$

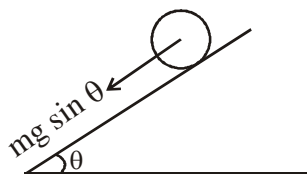
$$a_{\text{com}} \text{ of hollow sphere } = \frac{g \sin \theta}{1 + 2/3} = \frac{3}{5} g \sin \theta$$

$$I_{\text{com}} \text{ for hollow sphere } = \frac{2}{5} MR^2$$

$$a_{\text{com}} \text{ of hollow sphere } = \frac{g \sin \theta}{1 + 2/5} = \frac{5}{7} g \sin \theta$$

$a_{\text{com}} \text{ of solid sphere } > a_{\text{com}} \text{ of hollow sphere}$

The solid sphere reaches the bottom with greater speed.

Sol 11. B

A sphere cannot roll on a smooth inclined surface.

Sol 12. ABC

Engine force apply on the rear wheels in back ward direction so friction force oppose it that causes friction force on the rear wheels is in the forward direction.

Friction force oppose the motion of the particle, Here front wheel freely rotated in forward direction so friction force on the front wheel is in the backward direction.

Due to friction force 'car' is move, so we can say that friction force on the front wheels has larger magnitude than the friction on the front wheels.

Sol 13. C

Acceleration of the sphere down the plane is 'a'.

$$f_r = I\mu \quad \left(Q\mu = \frac{a}{r} \right)$$

$$f_r = \frac{2}{5} mr^2 \cdot \left(\frac{a}{r} \right)$$

$$f = \frac{2}{5} ma \quad \dots\dots\dots (1)$$

$$mg \sin q - f = ma \quad \dots\dots\dots (2)$$

from (1) & (2)

$$a = \frac{5}{7} g \sin q$$

$$f = \frac{2}{7} mg \sin q \quad \dots\dots\dots (3)$$

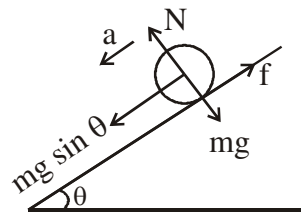
$$\text{here } mg \sin q > f \quad \dots\dots\dots (4)$$

The normal force is equal to $mg \cos q$, as there is no acceleration perpendicular to the incline. The maximum friction that can act is, therefore $\mu mg \cos q$, where μ is the coefficient of static friction. Thus, for pure rolling

$$\mu mg \cos q > \frac{2}{7} mg \sin q$$

$$\mu > \frac{2}{7} \tan q \quad \dots\dots\dots (5)$$

From equation (4) & (5) we conclude that sphere will translate and rotate about the centre.



Sol 14. AB

The force of friction tries to decrease the linear velocity & increases the angular velocity.

Sol 15. A

$$a = g \tan q \text{ (Given)}$$

Component of pseudo force in inclined plane is $= ma \cos q$

$$\begin{aligned} &= mg \tan q \cos q \\ &= mg \sin q \end{aligned}$$

Net force on the inclined plane direction is

$$\begin{aligned} &= mg \sin q - ma \cos q \\ &= mg \sin q - mg \sin q \\ &= 0 \end{aligned}$$

So we can say that

If the sphere is set in pure rolling on the incline, it will continue pure rolling.

