

1. જો $\sin\theta + \operatorname{cosec}\theta = 2$ હોય, તો $\sin^2\theta + \operatorname{cosec}^2\theta = \dots$

(A) 1

(B) 4

(C) 2

(D) આ પૈકી એકપણ નહીં

જવાબ (C) 2

➡ અહીં $\sin\theta + \operatorname{cosec}\theta = 2$

$$\therefore \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta \cdot \operatorname{cosec}\theta = 4$$

$$\therefore \sin^2\theta + \operatorname{cosec}^2\theta = 4 - 2$$

$$\therefore \sin^2\theta + \operatorname{cosec}^2\theta = 2$$

2. જો $f(x) = \cos^2x + \sec^2x$ હોય, તો નીચેના પૈકી સત્ય હોય.

(A) $f(x) < 1$

(B) $f(x) = 1$

(C) $2 < f(x) < 1$

(D) $f(x) \geq 2$

જવાબ (D) $f(x) \geq 2$

➡ અહીં $f(x) = \cos^2x + \sec^2x$

આપણે જાણીએ છીએ કે $AM \geq GM$

$$\therefore \frac{\cos^2x + \sec^2x}{2} \geq \sqrt{\cos^2x \cdot \sec^2x}$$

$$\therefore \cos^2x + \sec^2x \geq 2$$

$$\therefore f(x) > 2 \quad [\because \cos x \cdot \sec x = 1]$$

3. જો $\tan\theta = \frac{1}{2}$ અને $\tan\phi = \frac{1}{3}$ હોય તો $\theta + \phi$ નું મૂલ્ય =

(A) $\frac{\pi}{6}$

(B) π

(C) 0

(D) $\frac{\pi}{4}$

જવાબ (D) $\frac{\pi}{4}$

➡ અહીં $\tan\theta = \frac{1}{2}$ અને $\tan\phi = \frac{1}{3}$ છે.

$$\text{હવે } \tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi}$$

$$\therefore \tan(\theta + \phi) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}$$

$$\therefore \tan(\theta + \phi) = \frac{\frac{3+2}{6}}{\frac{6-1}{6}} = \frac{5}{5} = 1$$

$$\therefore \tan(\theta + \phi) = \tan\frac{\pi}{4}$$

$$\therefore \theta + \phi = \frac{\pi}{4}$$

4. નીચેના પૈકી વિધાન અસત્ય છે.

(A) $\sin\theta = -\frac{1}{5}$

(B) $\cos\theta = 1$

(C) $\sec\theta = \frac{1}{2}$

(D) $\tan\theta = 20$

જવાબ (C) $\sec\theta = \frac{1}{2}$

➡ આપણે જાણીએ છીએ કે વિધેય $\sec\theta$ નો વિસ્તાર ગણ = $R - (-1, 1)$

$\therefore \sec\theta$ નું મૂલ્ય કદાપી $\frac{1}{2}$ હોય નહીં.

5. $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$ નું મૂલ્ય =

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) એકપણ નહીં.

જવાબ (B) 1

➡ આપેલ વિસ્તરણ = $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$
 $= \tan 1^\circ \tan 2^\circ \dots \tan 45^\circ \cdot \tan(90^\circ - 44^\circ) \tan(90^\circ - 43^\circ) \dots \tan(90^\circ - 1^\circ)$
 $= \tan 1^\circ \cdot \cot 1^\circ \cdot \tan 2^\circ \cdot \cot 2^\circ \dots \tan 89^\circ \cdot \cot 89^\circ$
 $= 1 \cdot 1 \dots 1 \cdot 1 = 1$

6. $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$ નું મૂલ્ય =

(A) 1

(B) $\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}$

(D) 2

જવાબ (C) $\frac{\sqrt{3}}{2}$

➡ $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \frac{\cos^2 15^\circ - \sin^2 15^\circ}{\cos^2 15^\circ + \sin^2 15^\circ}$
 $= \frac{\cos 30^\circ}{1}$
 $= \frac{\sqrt{3}}{2}$

7. $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ$ નું મૂલ્ય =

(A) $\frac{1}{\sqrt{2}}$

(B) 0

(C) 1

(D) -1

જવાબ (B) 0

➡ આપેલ બહુપદી = $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ$
 $= \cos 1^\circ \cos 2^\circ \dots \cos 90^\circ \dots \cos 179^\circ$ [$\because \cos 90^\circ = 0$]
 $= 0$

8. જો $\tan \theta = 3$ અને θ ત્રીજા ચરણમાં હોય, તો $\sin \theta = \dots$

(A) $\frac{1}{\sqrt{10}}$

(B) $-\frac{1}{\sqrt{10}}$

(C) $-\frac{3}{\sqrt{10}}$

(D) $\frac{3}{\sqrt{10}}$

જવાબ (C) $-\frac{3}{\sqrt{10}}$

➡ આપેલ $\tan \theta = 3$
 $\therefore \sec^2 \theta = 1 + \tan^2 \theta$
 $\therefore \sec \theta = \sqrt{1 + 9} = \pm \sqrt{10}$
 $\therefore \sec \theta = -\sqrt{10}$
 $\therefore \cos \theta = -\frac{1}{\sqrt{10}}$
 $\therefore \sin \theta = \pm \sqrt{1 - \frac{1}{10}} = \pm \sqrt{\frac{9}{10}} = \pm \sqrt{\frac{3}{10}}$ ($\because$ ત્રીજા ચરણમાં છે.)
 $\therefore \sin \theta = -\frac{3}{\sqrt{10}}$

9. $\tan 75^\circ - \cot 75^\circ$ નું મૂલ્ય =

(A) $2\sqrt{3}$

(B) $2 + \sqrt{3}$

(C) $2 - \sqrt{3}$

(D) 1

જવાબ (A) $2\sqrt{3}$

➡ આપેલ બહુપદી = $\frac{\tan 75^\circ}{\sin 75^\circ} - \frac{\cot 75^\circ}{\cos 75^\circ}$

$$\begin{aligned}
&= \frac{\cos 75^\circ}{\sin 75^\circ} - \frac{\sin 75^\circ}{\cos 75^\circ} \\
&= \frac{\sin^2 75^\circ - \cos^2 75^\circ}{\sin 75^\circ \cdot \cos 75^\circ} \\
&= \frac{-2\cos 150^\circ}{\sin 150^\circ} \\
&= \frac{-2\cos(90^\circ + 60^\circ)}{\sin(90^\circ + 60^\circ)} \\
&= \frac{+2\sin 60^\circ}{\cos 60^\circ} \\
&= \frac{2 \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\sqrt{3}
\end{aligned}$$

10. નીચેના પૈકીસત્ય છે.

(A) $\sin 1^\circ > \sin 1$

(B) $\sin 1^\circ < \sin 1$

(C) $\sin 1^\circ = \sin 1$

(D) $\sin 1^\circ = \frac{\pi}{18^\circ} \sin 1$

જવાબ (B) $\sin 1^\circ < \sin 1$

➡ આપણે જાણીએ છીએ કે, જો θ એ વધતું હોય તો $\sin \theta$ પણ વધતું વિધેય થાય.
 $\sin 1^\circ < \sin 1$ [$\because 1 \text{ rad} = 57^\circ 30'$]

11. જો $\tan \alpha = \frac{m}{m+1}$ અને $\tan \beta = \frac{1}{2m+1}$ હોય, તો $(\alpha + \beta) = \dots\dots\dots$

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

જવાબ (D) $\frac{\pi}{4}$

➡ અહીં $\tan \alpha = \frac{m}{m+1}$ અને $\tan \beta = \frac{1}{2m+1}$ આપેલ છે.

$$\text{અહીં } \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$\therefore \tan(\alpha + \beta) = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right) \cdot \left(\frac{1}{2m+1}\right)}$$

$$\therefore \tan(\alpha + \beta) = \frac{m(2m+1) + m+1}{(m+1)(2m+1) - m}$$

$$\therefore \tan(\alpha + \beta) = \frac{2m^2 + m + m + 1}{2m^2 + 2m + m + 1 - m}$$

$$\therefore \tan(\alpha + \beta) = \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} \Rightarrow \tan(\alpha + \beta) = 1$$

$$\therefore \alpha + \beta = \frac{\pi}{4}$$

12. $3\cos x + 4\sin x + 8$ નું ન્યુનતમ મૂલ્ય છે.

(A) 5

(B) 9

(C) 7

(D) 3

જવાબ (D) 3

➡ આપેલ બહુપદી $3\cos x + 4\sin x + 8$

હવે $y = 3\cos x + 4\sin x + 8$ લેતાં,

$$\therefore y - 8 = 3\cos x + 4\sin x$$

$$\therefore y - 8 \text{ નું લઘુત્તમ મૂલ્ય} = -\sqrt{9+16}$$

$$\therefore y - 8 = -5 \Rightarrow y = -5 + 8$$

$$\therefore y = 3$$

આપેલ બહુપદી $3 \cos x + 4 \sin x + 8$ નું ન્યુનતમ મૂલ્ય 3 છે.

13. $\tan 3A - \tan 2A - \tan A$ નું મૂલ્ય થાય.

(A) $\tan 3A \tan 2A \tan A$

(B) $-\tan 3A \tan 2A \tan A$

(C) $\tan A \tan 2A - \tan 2A \tan 3A - \tan 3A \tan A$

(D) આ પૈકી એકપણ નહીં.

જવાબ (A) $\tan 3A \tan 2A \tan A$

➡ અહીં $3A = A + 2A$

$$\therefore \tan 3A = \tan(A + 2A)$$

$$\therefore \tan 3A = \frac{\tan A + \tan 2A}{1 - \tan A \cdot \tan 2A}$$

$$\therefore \tan A + \tan 2A = \tan 3A - \tan 3A \cdot \tan 2A \cdot \tan A$$

$$\therefore \tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$$

14. $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$ નું મૂલ્ય છે.

(A) $2 \cos \theta$

(B) $2 \sin \theta$

(C) 1

(D) 0

જવાબ (D) 0

➡ આપેલ બહુપદી, $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$

$$= \sin 45^\circ \cdot \cos \theta + \cos 45^\circ \cdot \sin \theta - \cos 45^\circ \cdot \cos \theta - \sin 45^\circ \cdot \sin \theta$$

$$= \frac{1}{\sqrt{2}} \cdot \cos \theta + \frac{1}{\sqrt{2}} \cdot \sin \theta - \frac{1}{\sqrt{2}} \cdot \cos \theta - \frac{1}{\sqrt{2}} \cdot \sin \theta$$

$$= 0$$

15. $\cot\left(\frac{x}{4} + \theta\right) \cot\left(\frac{x}{4} - \theta\right)$ નું મૂલ્ય =

(A) -1

(B) 0

(C) 1

(D) નું અસ્તિત્વ નથી.

જવાબ (C) 1

➡ આપેલ બહુપદી $\cot\left(\frac{\pi}{4} + \theta\right) - \cot\left(\frac{\pi}{4} - \theta\right)$

$$= \left(\frac{\cot \frac{\pi}{4} \cot \theta - 1}{\cot \frac{\pi}{4} + \cot \theta} \right) \cdot \left(\frac{\cot \frac{\pi}{4} \cot \theta + 1}{\cot \theta - \cot \frac{\pi}{4}} \right)$$

$$= \left(\frac{\cot \theta - 1}{\cot \theta + 1} \right) \cdot \left(\frac{\cot \theta + 1}{\cot \theta - 1} \right)$$

$$= 1$$

16. $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi) = \dots\dots\dots$

(A) $\sin 2(\theta + \phi)$

(B) $\cos 2(\theta + \phi)$

(C) $\sin 2(\theta - \phi)$

(D) $\cos 2(\theta - \phi)$

જવાબ (B) $\cos 2(\theta + \phi)$

➡ આપેલ બહુપદી $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$$= \cos 2\theta \cdot \cos 2\phi + \sin(\theta - \phi + \theta + \phi) \cdot \sin(\theta - \phi - \theta - \phi)$$

$$= \cos 2\theta \cdot \cos 2\phi - \sin 2\theta \cdot \sin 2\phi$$

$$= \cos(2\theta + 2\phi) = \cos 2(\theta + \phi)$$

17. $\cos 12^\circ + \cos 84^\circ + \cos 150^\circ + \cos 132^\circ$ નું મૂલ્ય =

(A) $\frac{1}{2}$

(B) 1

(C) $-\frac{1}{2}$

(D) $\frac{1}{8}$

જવાબ (C) $-\frac{1}{2}$

➡ $\cos 12^\circ + \cos 84^\circ + \cos 150^\circ + \cos 132^\circ$

$$\cos 12^\circ + \cos 150^\circ + \cos 84^\circ + \cos 132^\circ$$

$$= 2\cos\left(\frac{12^\circ + 150^\circ}{2}\right) \cdot \cos\left(\frac{12^\circ - 150^\circ}{2}\right) + 2\cos\left(\frac{84^\circ + 132^\circ}{2}\right) \cdot \cos\left(\frac{84^\circ - 132^\circ}{2}\right)$$

$$= 2\cos 84^\circ \cos 72^\circ + 2\cos 108^\circ \cos 24^\circ$$

$$\begin{aligned}
&= 2\cos 84^\circ \cos(90^\circ - 18^\circ) + 2\cos(90^\circ + 18^\circ) \cdot \cos 24^\circ \\
&= 2\cos 84^\circ \sin 18^\circ - 2\sin 18^\circ \cdot \cos 24^\circ \\
&= 2\cos 18^\circ (\cos 84^\circ - \cos 24^\circ) \\
&= 2\sin 18^\circ \cdot 2\sin\left(\frac{84^\circ + 24^\circ}{2}\right) \cdot \sin\left(\frac{84^\circ - 24^\circ}{2}\right) \\
&= -4\sin 18^\circ \cdot \sin 54^\circ \sin 30^\circ \\
&= -4\left(\frac{\sqrt{5}-1}{4}\right) \cdot \cos 36^\circ \cdot \frac{1}{2} \\
&= -(\sqrt{5}-1)\left(\frac{\sqrt{5}+1}{4}\right) \cdot \frac{1}{2} = -\left(\frac{5-1}{8}\right) = \frac{-4}{8} = \frac{-1}{2}
\end{aligned}$$

18. જો $\tan A = \frac{1}{2}$ અને $\tan B = \frac{1}{3}$ હોય, તો $\tan(2A + B) = \dots\dots\dots$

- (A) 1 (B) 2 (C) 3 (D) 4

જવાબ (C) 3

➡ અહીં $\tan A = \frac{1}{2}$ અને $\tan B = \frac{1}{3}$ આપેલ છે.

$$\text{હવે } \tan(2A + B) = \frac{\tan 2A + \tan B}{1 - \tan 2A \cdot \tan B}$$

$$\text{અહીં } \tan 2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$$

$$\tan(2A + B) = \frac{\frac{4}{3} + \frac{1}{3}}{1 - \frac{4}{3} \cdot \frac{1}{3}} = \frac{\frac{4}{3} + \frac{1}{3}}{\frac{9-4}{9}} = \frac{\frac{5}{3}}{\frac{5}{9}} = 3$$

19. $\sin = \frac{\pi}{10}$ $\sin = \frac{13\pi}{10}$ નું મૂલ્ય = $\dots\dots\dots$

- (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $-\frac{1}{4}$ (D) 1

જવાબ (C) $-\frac{1}{4}$

➡ આપેલ બહુપદો $\sin\left(\frac{\pi}{10}\right)\sin\left(\frac{13\pi}{10}\right) = \sin\frac{\pi}{10}\sin\left(\pi + \frac{3\pi}{10}\right)$

$$= -\sin\frac{\pi}{10}\sin\frac{3\pi}{10} = -\sin 18^\circ \cdot \sin 54^\circ$$

$$= -\sin 18^\circ \cdot \cos 36^\circ$$

$$= -\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)$$

$$= -\left(\frac{5-1}{16}\right) = -\frac{1}{4}$$

20. $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ નું મૂલ્ય = $\dots\dots\dots$

- (A) 1 (B) 0 (C) $\frac{1}{2}$ (D) 2

જવાબ (B) 0

➡ આપેલ બહુપદો $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$

$$= 2\cos\left(\frac{50^\circ + 70^\circ}{2}\right) \cdot \sin\left(\frac{50^\circ - 70^\circ}{2}\right) + \sin 10^\circ$$

$$= -2 \cos 60^\circ \sin 10^\circ + \sin 10^\circ$$

$$= -2 \cdot \frac{1}{2} \sin 10^\circ + \sin 10^\circ = 0$$

21. જો $\sin\theta + \cos\theta = 1$ હોય, તો $\sin(2\theta) = \dots\dots\dots$

(A) 1

(B) $\frac{1}{2}$

(C) 0

(D) -1

જવાબ (C) 0

➡ આપેલ બહુપદી $\sin\theta + \cos\theta = 1$

બંને બાજુ વર્ગ કરતાં,

$$\sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta = 1$$

$$\therefore 1 + \sin 2\theta = 1$$

$$\therefore \sin 2\theta = 0$$

22. જો $\alpha + \beta = \frac{\pi}{4}$ હોય, તો $(1 + \tan\alpha)(1 + \tan\beta) = \dots\dots\dots$

(A) 1

(B) 2

(C) -2

(D) મળશે નહીં.

જવાબ (B) 2

➡ અહીં $\alpha + \beta = \frac{\pi}{4}$

$$\text{હવે } (1 + \tan\alpha)(1 + \tan\beta) = 1 + \tan\alpha + \tan\beta + \tan\alpha \tan\beta$$

$$\text{આપણે જાણીએ છીએ કે, } \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$

$$\therefore 1 = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} \left(\because \alpha + \beta = \frac{\pi}{4} \Rightarrow \tan(\alpha + \beta) = \tan\frac{\pi}{4} = 1 \right)$$

$$\therefore \tan\alpha + \tan\beta = 1 - \tan\alpha \tan\beta$$

પરિણામ (1) પરથી,

$$(1 + \tan\alpha)(1 + \tan\beta) = 1 + 1 - \tan\alpha \cdot \tan\beta + \tan\alpha \cdot \tan\beta = 2$$

23. જો $\sin\theta = \frac{-4}{5}$ અને θ ત્રીજા ચરણમાં હોય તો $\cos\frac{\theta}{2} = \dots\dots\dots$

(A) $\frac{1}{5}$

(B) $-\frac{1}{\sqrt{10}}$

(C) $-\frac{1}{5}$

(D) $\frac{1}{\sqrt{10}}$

જવાબ (C) $-\frac{1}{5}$

➡ $\sin\theta = \frac{-4}{5}$ આપેલ છે.

$$\therefore \cos\theta = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25 - 16}{25}} = \pm \frac{3}{5}$$

$$\therefore \cos\theta = \frac{-3}{5} \quad (\because \theta \text{ ત્રીજા ચરણમાં છે.})$$

$$\therefore 2\cos^2\frac{\theta}{2} - 1 = \frac{-3}{5}$$

$$\therefore 2\cos^2\frac{\theta}{2} = 1 - \frac{3}{5}$$

$$\therefore 2\cos^2\frac{\theta}{2} = \frac{2}{5}$$

$$\therefore \cos\frac{\theta}{2} = \pm \frac{1}{\sqrt{5}}$$

$$\therefore \cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}} \quad (\because \theta \text{ ત્રીજા ચરણમાં છે.})$$

24. અંતરાલ $[0, 2\pi]$ માં સમીકરણ $\tan x + \sec x = 2\cos x$ ના ઉકેલની સંખ્યા $\dots\dots\dots$ છે.

(A) 0

(B) 1

(C) 2

(D) 3

જવાબ (C) 2

➡ આપેલ $\tan x + \sec x = 2\cos x$ છે.

$$\therefore \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$

$$\therefore 1 + \sin x = 2\cos^2 x$$

$$\therefore 1 + \sin x = 2(1 - \sin^2 x)$$

$$\therefore 1 + \sin x = 2 - 2\sin^2 x$$

$$\therefore 2\sin^2 x + \sin x - 1 = 0$$

$$\therefore 2\sin^2 x + 2\sin x - \sin x - 1 = 0$$

$$\therefore 2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$$

$$\therefore (\sin x + 1)(2\sin x - 1) = 0$$

$$\therefore \sin x + 1 = 0 \text{ અથવા } (2\sin x - 1) = 0$$

$$\therefore \sin x = -1, \sin x = \frac{1}{2}$$

$$\therefore x = \frac{3\pi}{2}, x = \frac{\pi}{6}$$

આમ, અંતરાલ $[0, 2\pi]$ માં સમીકરણના ઉકેલની સંખ્યા બે છે.

25. $\sin\left(\frac{\pi}{18}\right) + \sin\left(\frac{\pi}{9}\right) + \sin\left(\frac{2\pi}{9}\right) + \sin\left(\frac{5\pi}{18}\right)$ નું મૂલ્ય =

(A) $\sin\left(\frac{7\pi}{18}\right) + \sin\left(\frac{4\pi}{9}\right)$ (B) 1

(C) $\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{3\pi}{7}\right)$ (D) $\cos\left(\frac{\pi}{9}\right) + \sin\left(\frac{\pi}{9}\right)$

જવાબ (A) $\sin\left(\frac{7\pi}{18}\right) + \sin\left(\frac{4\pi}{9}\right)$

➡ $\sin\left(\frac{\pi}{18}\right) + \sin\left(\frac{\pi}{9}\right) + \sin\left(\frac{2\pi}{9}\right) + \sin\left(\frac{5\pi}{18}\right)$

$$= \sin(10^\circ) + \sin(20^\circ) + \sin(40^\circ) + \sin(50^\circ)$$

$$= \sin 50^\circ + \sin 10^\circ + \sin 40^\circ + \sin 20^\circ$$

$$= \sin 130^\circ + \sin 10^\circ + \sin 140^\circ + \sin 20^\circ$$

$$= 2\sin 70^\circ \cos 60^\circ + 2\sin 80^\circ \cdot \cos 60^\circ$$

$$\left[\because \sin x + \sin y = 2\sin \frac{x+y}{2} \cdot \cos \frac{x-y}{2} \right]$$

$$= 2 \cdot \frac{1}{2} \sin 70^\circ + 2 \cdot \frac{1}{2} \sin 80^\circ \left[\because \cos 60^\circ = \frac{1}{2} \right]$$

$$= \sin 70^\circ + \sin 80^\circ = \sin \frac{7\pi}{18} + \sin \frac{4\pi}{9}$$

26. જો A બીજા ચરણમાં હોય અને $3\tan A + 4 = 0$ હોય તો $2\cot A - 5\cos A + \sin A$ નું મૂલ્ય =

(A) $\frac{-53}{10}$

(B) $\frac{23}{10}$

(C) $\frac{37}{10}$

(D) $\frac{7}{10}$

જવાબ (B) $\frac{23}{10}$

➡ $3\tan A + 4 = 0$ આપેલ છે.

$$\therefore 3\tan A = -4$$

$$\therefore \tan A = \frac{-4}{3}$$

$$\therefore \cot A = \frac{-3}{4}$$

$$\text{હવે } \sec A = \sqrt{1 + \frac{16}{9}} = \sqrt{\frac{25}{9}} = \pm \frac{5}{3}$$

$$\therefore \sec A = \frac{-5}{3}$$

[A બીજા ચરણમાં છે.]

$$\therefore \cos A = \frac{-3}{5}$$

$$\text{તથા } \sin A = \sqrt{1 - \frac{9}{25}} = \frac{\sqrt{25-9}}{25} = \pm \frac{4}{5}$$

$$\therefore \sin A = \frac{4}{5} \quad [A \text{ બીજા ચરણમાં છે.}]$$

$$\begin{aligned} \therefore 2\cot A - 5\cos A + \sin A &= 2\left(\frac{-3}{4}\right) - 5\left(\frac{-3}{5}\right) + \frac{4}{5} \\ &= \frac{-6}{4} + 3 + \frac{4}{5} \\ &= \frac{-30 + 60 + 16}{20} = \frac{46}{20} \\ &= \frac{23}{10} \end{aligned}$$

$$27. \quad \cos^2 48^\circ - \sin^2 12^\circ \text{ નું મૂલ્ય} = \dots\dots\dots$$

$$(A) \quad \frac{\sqrt{5} + 1}{8}$$

$$(B) \quad \frac{\sqrt{5} - 1}{8}$$

$$(C) \quad \frac{\sqrt{5} + 1}{5}$$

$$(D) \quad \frac{\sqrt{5} + 1}{2\sqrt{2}}$$

$$\text{જવાબ (A) } \frac{\sqrt{5} + 1}{8}$$

$$\begin{aligned} \Rightarrow \cos^2 48^\circ - \sin^2 12^\circ \\ &= \cos(48^\circ + 12^\circ) \cdot \cos(48^\circ - 12^\circ) \quad (\because \cos^2 \alpha - \sin^2 \beta = \cos(\alpha + \beta) \cdot \cos(\alpha - \beta) \text{ છે.}) \\ &= \cos 60^\circ \cdot \cos 36^\circ \\ &= \frac{1}{2} \cdot \left(\frac{\sqrt{5} + 1}{4}\right) \\ &= \frac{\sqrt{5} + 1}{8} \end{aligned}$$

$$28. \quad \text{જો } \tan \alpha = \frac{1}{7} \text{ અને } \tan \beta = \frac{1}{3} \text{ હોય, તો } \cos 2\alpha = \dots\dots\dots$$

$$(A) \quad \sin 2\beta$$

$$(B) \quad \sin 4\beta$$

$$(C) \quad \sin 3\beta$$

$$(D) \quad \cos 2\beta$$

$$\text{જવાબ (B) } \sin 4\beta$$

$$\Rightarrow \text{અહીં } \tan \alpha = \frac{1}{7} \text{ અને } \tan \beta = \frac{1}{3} \text{ આપેલ છે.}$$

$$\begin{aligned} \text{હવે } \cos 2\alpha &= \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} = \frac{1 - \frac{1}{49}}{1 + \frac{1}{49}} = \frac{\frac{48}{49}}{\frac{50}{49}} \\ &= \frac{48}{50} = \frac{24}{25} \end{aligned}$$

$$\therefore \cos 2\alpha = \frac{24}{25} \quad \dots\dots\dots(i)$$

$$\text{તથા } \sin(4\beta) = \frac{2 \tan(2\beta)}{1 + \tan^2 2\beta} \quad \dots\dots\dots(ii)$$

$$\begin{aligned} \text{અને } \tan(2\beta) &= \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \times \frac{1}{3}}{1 - \frac{1}{9}} \\ &= \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{2 \times 9}{3 \times 8} = \frac{3}{4} \end{aligned}$$

હવે પરિણામ (ii) પરથી,

$$\sin 4\beta = \frac{2 \times \frac{3}{4}}{1 + \frac{9}{16}} = \frac{\frac{6}{4}}{\frac{25}{16}} = \frac{6 \times 16}{4 \times 25}$$

$$\therefore \sin 4\beta = \frac{24}{25}$$

$$\therefore \sin 4\beta = \cos 2\alpha$$

$$\therefore \cos 2\alpha = \sin 4\beta \quad (\text{પરિણામ (i) પરથી})$$

29. જો $\tan \theta = \frac{a}{b}$ હોય, તો $b \cos 2\theta + a \sin 2\theta = \dots\dots\dots$

(A) a

(B) b

(C) $\frac{a}{b}$

(D) આ પૈકી એકપણ નહીં.

જવાબ (B) b

➡ અહીં $\tan \theta = \frac{a}{b}$ આપેલ છે.

$$\begin{aligned} \therefore b \cos 2\theta + a \sin 2\theta &= b \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + a \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \\ &= b \left(\frac{1 - \frac{a^2}{b^2}}{1 + \frac{a^2}{b^2}} \right) + a \left(\frac{\frac{2a}{b}}{1 + \frac{a^2}{b^2}} \right) \\ &= b \left(\frac{b^2 - a^2}{b^2 + a^2} \right) + \frac{2a^2 b}{a^2 + b^2} \\ &= \frac{b}{a^2 + b^2} [b^2 - a^2 + 2a^2] \\ &= \frac{(a^2 + b^2)b}{(a^2 + b^2)} = b \end{aligned}$$

30. x વાસ્તવિક સંખ્યા હોય તો $\cos \theta = x + \frac{1}{x}$ નીચેના પૈકી $\dots\dots\dots$ સત્ય હોય.

(A) θ એ લઘુકોણ હોય.

(B) θ એ કાટકોણ હોય.

(C) θ એ ગુરુકોણ હોય.

(D) θ નું કોઈપણ મૂલ્ય નહીં મળે.

જવાબ (D) θ નું કોઈપણ મૂલ્ય નહીં મળે.

➡ અહીં $\cos \theta = x + \frac{1}{x}$

$$\therefore \cos \theta = \frac{x^2 + 1}{x}$$

$$\therefore x^2 - x \cos \theta + 1 = 0$$

$$x \text{ ની વાસ્તવિક કિંમત માટે, } (-\cos \theta)^2 - 4 \times 1 \times 1 > 0$$

$$\cos^2 \theta > 4$$

$$\cos \theta > \pm 2 \quad [\because -1 \leq \cos \theta \leq 1]$$

જે શક્ય નથી.

$\therefore \theta$ નું કોઈપણ મૂલ્ય મળે નહીં.