Sample Question Paper - 33
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours **Maximum Marks : 40**

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. If $x = 2$ and $x = 3$ are roots of the equation $3x^2 - 2kx + 2m = 0$, then find the value of k and m.

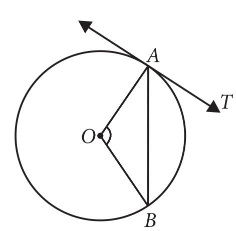
2. If a, b, c are in A.P., prove that $a^2 + c^2 - 2bc = 2a(b - c)$.

OR

If a, $2(a + 5)$ and $2(4a - 5)$ are in A.P., then find the value of a.

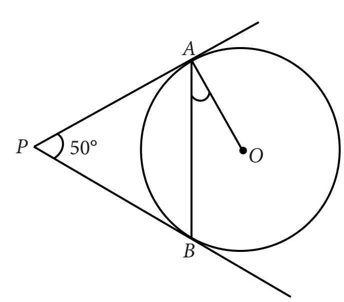
3. A polygon of n sides has $\frac{n(n-3)}{2}$ diagonals. How many sides a polygon has with 54 diagonals?

4. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 100°$, then find the value of $\angle BAT$.



OR

In the given figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$. Write the measure of $\angle OAB$.



5. Two cubes, each of side 4 cm are joined end to end. Find the surface area of the resulting cuboid.

6. If mean of 5 observations x, $x + 3$, $x + 6$, $x + 9$ and $x + 12$ is 11, then find the value of x.

SECTION - B

7. Draw two concentric circles of radii 2 cm and 5 cm. Taking a point on outer circle, construct the pair of tangents to the other. Also, measure the length of a tangent.

8. If mode of the following series is 54, then find the value of f.

Class- interval	0-15	15-30	30-45	45-60	60-75	75-90
Frequency	3	5	f	16	12	7

9. An observer, 1.8 m tall is 40.2 m away from a 42 m high tower. Determine the angle of elevation of the top of the tower from his eye.

OR

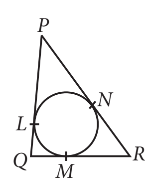
From the top of a tower 50 m high, the angle of depression of the top of a pole is 45° and from the foot of the pole, the angle of elevation of the top of the tower is 60°. Find the height of the pole if the pole and tower stand on the same plane.

10. If the median of the distribution given below is 27. Find the value of x and y.

Class-interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	20	14	y	8	68

SECTION - C

11. In the given figure, a circle is inscribed in a triangle PQR with $PQ = 10$ cm, $QR = 8$ cm and $PR = 12$ cm. Find the lengths QM, RN and PL.



12. A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cone.

OR

The $\left(\frac{3}{4}\right)^{\text{th}}$ part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

Case Study - 1

13. Jack is much worried about his upcoming assessment on A.P. He was vigorously practicing for the exam but unable to solve some questions. One of these questions is as shown below.

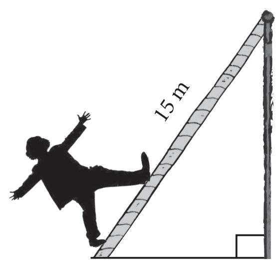
If the 3rd and the 9th terms of an A.P. are 4 and – 8 respectively, then help Jack in solving the problem.

(i) Which term of the A.P. is –160 ?

(ii) What is the 75th term of the A.P.?

Case Study - 2

14. A circus artist is climbing through a 15 m long rope which is highly stretched and tied from the top of a vertical pole to the ground as shown below.



Based on the above information, answer the following questions.

(i) Find the height of the pole, if angle made by rope to the ground level is 45°.

(ii) If the angle made by the rope to the ground level is 45°, then find the distance between artist and pole at ground level.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Since, $x = 2$ and $x = 3$ are roots of the equation $3x^2 - 2kx + 2m = 0$

$\therefore\ 3 \times 2^2 - 2k \times 2 + 2m = 0$

and $3 \times 3^2 - 2k \times 3 + 2m = 0$

$\Rightarrow\ 12 - 4k + 2m = 0$ and $27 - 6k + 2m = 0$

$\Rightarrow\ 12 = 4k - 2m$ and $27 = 6k - 2m$

Solving these two equations, we get

$k = 15/2$ and $m = 9$.

2. Given, a, b, c are in A.P.

$\Rightarrow\ b - a = c - b \Rightarrow 2b = c + a \Rightarrow b = \dfrac{c+a}{2}$

To prove : $a^2 + c^2 - 2bc = 2a(b - c)$

Substituting, $b = \dfrac{c+a}{2}$ in L.H.S. and R.H.S., we get

$$\text{L.H.S.} = a^2 + c^2 - 2\left(\frac{c+a}{2}\right)c$$

$$= a^2 + c^2 - c^2 - ac = a^2 - ac = a(a - c)$$

$$\text{R.H.S.} = 2a\left(\frac{c+a}{2} - c\right) = 2a\left(\frac{c+a-2c}{2}\right) = a(a-c)$$

$\Rightarrow$ L.H.S. = R.H.S.

OR

Given, a, $2(a + 5)$ and $2(4a - 5)$ are in A.P.

$\therefore\ a + 2(4a - 5) = 2 \times 2(a + 5)$

$\Rightarrow\ a + 8a - 10 = 4a + 20$

$\Rightarrow\ 9a - 10 = 4a + 20 \Rightarrow 5a = 30 \Rightarrow a = 6$

3. Given, when number of sides is n, then the number of diagonals is $\dfrac{n(n-3)}{2}$.

It is given that the number of diagonals = 54

$\Rightarrow\ \dfrac{n(n-3)}{2} = 54 \Rightarrow n^2 - 3n = 108$

$\Rightarrow\ n^2 - 3n - 108 = 0 \Rightarrow n^2 - 12n + 9n - 108 = 0$

$\Rightarrow\ n(n - 12) + 9(n - 12) = 0$

$\Rightarrow\ (n - 12)(n + 9) = 0$

$\Rightarrow\ n = 12$ or $n = -9 \Rightarrow n = 12$

($\because\ n \neq -9$, as number of sides cannot be negative)

$\therefore$ The number of sides of the polygon is 12.

4. Given, $\angle AOB = 100°$

Now, $OA = OB\ \Rightarrow\ \angle OAB = \angle OBA$ ... (i)

In ΔAOB, $\angle AOB + \angle OAB + \angle OBA = 180°$

$\Rightarrow\ 100° + \angle OAB + \angle OAB = 180°$ [Using (i)]

$\Rightarrow\ 2\angle OAB = 80° \Rightarrow \angle OAB = 40°$

Now, $\angle OAT = 90°$ [$\because$ Tangent at any point of a circle is perpendicular to the radius through point of contact]

Thus, $\angle BAT = \angle OAT - \angle OAB = 90° - 40° = 50°$

OR

Here, $PA = PB$

($\because$ Tangents drawn from external point are equal)

$\Rightarrow\ \angle ABP = \angle BAP = x$(say)

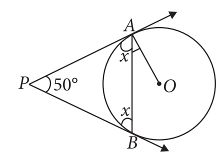
($\because$ Angles opposite to equal sides are equal)

In ΔAPB, $50° + x + x = 180°$

$\Rightarrow\ 2x = 130°$

$\Rightarrow\ x = 65°$

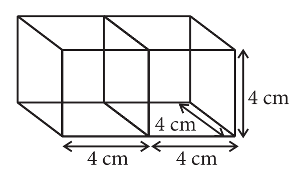
Now, $\angle OAP = 90°$



($\because$ Tangent is perpendicular to the radius through the point of contact)

$\therefore\ \angle OAB = 90° - 65° = 25°$

5. If two cubes each of side 4 cm are joined end to end, then the length (l), breadth (b) and height (h) of the resulting cuboid will be 8 cm, 4 cm and 4 cm, respectively.



$\therefore$ Surface area of the resulting cuboid

$= 2\ (lb + bh + lh)$

$= 2\ (8 \times 4 + 4 \times 4 + 8 \times 4)$

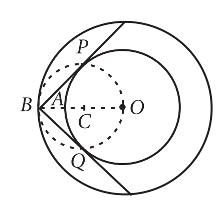
$= 2 \times (32 + 16 + 32) = 160\ \text{cm}^2$

6. $$\text{Mean} = \frac{x + (x+3) + (x+6) + (x+9) + (x+12)}{5}$$

$\Rightarrow\ 11 = \dfrac{5x + 30}{5}$ [$\because$ Mean = 11]

$\Rightarrow\ 55 = 5x + 30 \Rightarrow 5x = 25 \Rightarrow x = 5$

7. Steps of construction:



Step-I : Take a point O and draw a circle of radius OA = 2 cm and also from the same point O draw another circle of radius OB = 5 cm.

Step-II : By taking midpoint of OB as C draw another circle of radius $OC = BC$, the circle intersect the circle having radius 2 cm at P and Q.

Step-III : Now, join BP and BQ to get the tangents from a point B on the circle of radius 5 cm.

We find $BP = BQ = \sqrt{21}$ cm.

[$\because\ BP = \sqrt{OB^2 - OP^2} = \sqrt{5^2 - 2^2} = \sqrt{21}$ cm]

8. Here, given mode is 54, which lies in the interval 45-60. Therefore, the modal class is 45-60.

$\therefore$ $l = 45, f_1 = 16, f_0 = f, f_2 = 12$ and $h = 15$

$\because$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

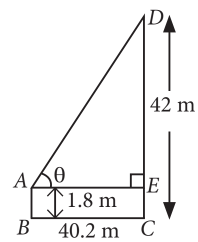
$\therefore 54 = 45 + \left(\dfrac{16 - f}{2 \times 16 - f - 12}\right) \times 15 \Rightarrow 9 = \dfrac{16 - f}{20 - f} \times 15$

$\Rightarrow$ $9(20 - f) = 15(16 - f) \Rightarrow 180 - 9f = 240 - 15f$

$\Rightarrow$ $6f = 240 - 180 = 60$ $\Rightarrow$ $f = 10$

Hence, required value of f is 10.

9. Let AB be the observer of height 1.8 m and CD be the tower of height 42 m.



Let θ be the angle of elevation of the top of the tower from the eye of the observer.

Here, $ED = CD - CE$

$= CD - AB$ $[\because AB = CE]$

$= 42 - 1.8 = 40.2$ m

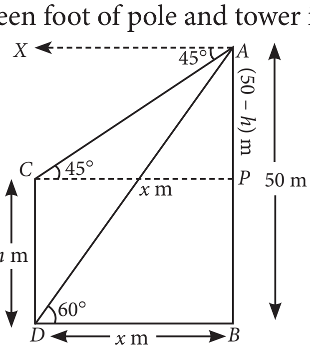
In right ΔADE, $\tan\theta = \dfrac{ED}{AE} = \dfrac{40.2}{40.2} = 1 = \tan 45°$

$\Rightarrow$ $\theta = 45°$

Hence, required angle of elevation is 45°.

OR

Let AB be tower, CD be the pole of height h m and distance between foot of pole and tower is x m.



$\therefore$ $CD = h$ m $\Rightarrow$ $PB = h$ m $[\because CD = PB]$

$AP = AB - PB = (50 - h)$ m

In ΔABD, $\tan 60° = \dfrac{AB}{BD}$

$\Rightarrow \sqrt{3} = \dfrac{50}{x} \Rightarrow x = \dfrac{50}{\sqrt{3}}$ ...(i)

In ΔAPC, $\tan 45° = \dfrac{AP}{PC}$

$\Rightarrow 1 = \dfrac{50 - h}{x} \Rightarrow x = 50 - h$ or $h = 50 - x$

$\Rightarrow h = 50 - \dfrac{50}{\sqrt{3}}$ [Using (i)]

$= 50 - 28.86 = 21.14$ (approx)

Hence, height of the pole is 21.14 m.

10. The cumulative frequency distribution table for the given data can be drawn as :

Class-interval	**Frequency (f_i)**	**Cumulative frequency ($c.f.$)**
0-10	5	5
10-20	x	$5 + x$
20-30	20	$25 + x$
30-40	14	$39 + x$
40-50	y	$39 + x + y$
50-60	8	$47 + x + y$
Total	$\sum f_i = 68$	

Given, median is 27, it lies in the interval 20-30.

$\therefore$ Median class is 20-30.

So, $l = 20, h = 10, f = 20, c.f. = 5 + x, \dfrac{n}{2} = \dfrac{68}{2} = 34$

$\therefore$ Median $= l + \left(\dfrac{\frac{n}{2} - c.f.}{f}\right) \times h$

$\Rightarrow$ $27 = 20 + \left(\dfrac{34 - (5 + x)}{20}\right) \times 10 \Rightarrow 7 = \dfrac{34 - 5 - x}{2}$

$\Rightarrow$ $14 = 29 - x \Rightarrow x = 15$ ...(i)

Also, $47 + x + y = 68$

$\Rightarrow$ $y = 68 - 47 - 15$ (From (i))

$\Rightarrow$ $y = 6$

11. Given : In ΔPQR, $PQ = 10$ cm, $QR = 8$ cm and $PR = 12$ cm.

We know that, the lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ $QM = QL = x$(say), $RM = RN = y$ (say),

$PL = PN = z$ (say)

$QR = QM + MR = x + y = 8$ ...(1)

$PQ = PL + LQ = z + x = 10$ ...(2)

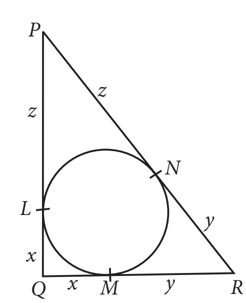
$PR = PN + NR = z + y = 12$ ...(3)

Adding (1), (2) and (3) , we have

$(x + y) + (z + x) + (z + y) = 8 + 10 + 12$

$\Rightarrow$ $2(x + y + z) = 30$

$\Rightarrow$ $x + y + z = 15$ ...(4)



From (2) and (4), we get

$10 + y = 15 \Rightarrow y = 5$

From (3) and (4), we get

$12 + x = 15 \Rightarrow x = 3$

From (1) and (4), we get

$z + 8 = 15 \Rightarrow z = 7$

$RN = y = 5$ cm, $PL = z = 7$ cm

$\therefore$ $QM = x = 3$ cm

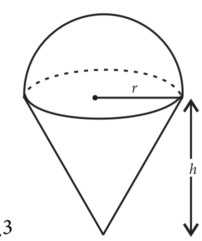
12. Volume of cylindrical tub $= \pi r^2 h$

$= \pi\left(\frac{12}{2}\right)^2 \times 15 = 540\pi \text{ cm}^3$

Quantity of ice-cream, given to one child

$= \frac{540\pi}{10} = 54\pi \text{ cm}^3$ ... (i)

Let r be the radius of conical base and h be the height of conical portion.



$\therefore \quad h = 2(2r) = 4r$

Capacity of one ice-cream cone

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2(4r) + \frac{2}{3}\pi r^3$

$= 2\pi r^3$ ... (ii)

From (i) and (ii), $2\pi r^3 = 54\pi$

$\Rightarrow r^3 = 27 \quad \Rightarrow \quad r = 3$ cm

$\therefore$ Diameter of conical part of ice-cream cone $= 2r = 6$ cm

OR

We have, radius of cone (r) = 5 cm

Height of cone (h) = 24 cm

Radius of cylinder (R) = 10 cm

Let height of water in cylinder be h_1.

$\therefore$ According to question

$\frac{3}{4}$ of volume of cone = Volume of water in cylinder

$\Rightarrow \frac{3}{4} \times \frac{1}{3} \times \pi \times r^2 \times h = \pi(R)^2 h_1$

$\Rightarrow \frac{1}{4}(5)^2 \times (24) = (10)^2 \times h_1$

$\Rightarrow \quad h_1 = \frac{150}{100} = 1.5$ cm

13. We have, 3rd term = 4 and 9th term = –8

i.e., $a + 2d = 4$ ...(1)

and $a + 8d = -8$ ...(2)

Solving (1) and (2), we get

$d = -2, a = 8$

(i) Let $t_n = -160 \Rightarrow a + (n-1)d = -160$

$\Rightarrow 8 + (n-1)(-2) = -160 \Rightarrow (n-1)(-2) = -168$

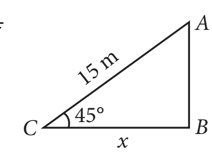
$\Rightarrow n - 1 = 84 \Rightarrow n = 85$

So, $t_{85} = -160$

(ii) $t_{75} = a + 74d = 8 + 74(-2) = -140$

14. (i) Let h be the height of the pole.

In ΔABC, $\frac{h}{15} = \sin 45° \Rightarrow \frac{h}{15} = \frac{1}{\sqrt{2}}$



$\Rightarrow h = \frac{15}{\sqrt{2}}$ m

(ii) Let x be the required distance.

In ΔABC,

$\frac{x}{15} = \cos 45° = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad x = \frac{15}{\sqrt{2}}$ m