Exercise 12.R

Answer 1CC.

A **vector** requires both magnitude and direction for complete definition. A Euclidean vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point A with a terminal point B and denoted by $\overline{AB}$.

A **scalar** is defined as a simple physical quantity that is not changed by coordinate system rotations or translations. In other words, we can say that a scalar require only magnitude for definition.

Answer 1E.

(a)

Find the equation of the sphere that passes through the point $(6,-2,3)$ and has center $(-1,2,1)$

Recollect that:

An equation of a sphere with center $C(h,k,l)$ and radius r is,

$$(x-h)^2+(y-k)^2+(z-l)^2=r^2$$

So, the required sphere equation at the center $(-1,2,1)$.

$$(x+1)^2+(y-2)^2+(z-1)^2=r^2$$

And passes through the point the $(6,-2,3)$ then the sphere equation,

$$\begin{aligned}(6+1)^2+(-2-2)^2+(3-1)^2&=r^2\\(7)^2+(-4)^2+(2)^2&=r^2\\49+16+4&=r^2\\69&=r^2\end{aligned}$$

Therefore the required sphere equation is, $\boxed{(x+1)^2+(y-2)^2+(z-1)^2=69}$

(b)

The curve this sphere intersects the $yz-$plane.

The $yz-$plane is $x=0$

The sphere equation is, $(x+1)^2+(y-2)^2+(z-1)^2=69$

$$(0+1)^2+(y-2)^2+(z-1)^2=69$$
$$1+(y-2)^2+(z-1)^2=69$$
$$1+y^2+4-4y+z^2+1-2z=69$$
$$y^2-4y+z^2-2z=69-6$$

Therefore the required sphere equation intersects the $yz-$plane is,

$$\boxed{y^2-4y+z^2-2z=63}$$

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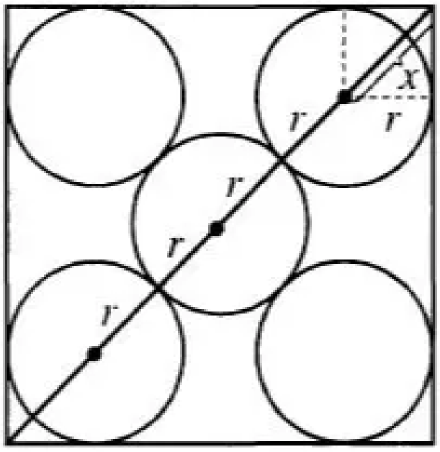
The equation in the form of an equation of a sphere and the complete squares:

$$x^2+y^2+z^2-8x+2y+6z+1=0$$
$$(x^2-8x+16-16)+(y^2+2y+1-1)+(z^2+6z+9-9)+1=0$$
$$(x^2-8x+16)+(y^2+2y+1)+(z^2+6z+9)+1-16-1-9=0$$
$$(x-4)^2+(y+1)^2+(z+3)^2-25=0$$
$$(x-4)^2+(y+1)^2+(z+3)^2=25$$

Comparing this equation with the above standard form.

Therefore the sphere with center $(4,-1,-3)$ and radius 5

Answer 1P.



Since three-dimensional situations are often difficult to visualize and work with, let us first try to find an analogous problem in two dimensions. The analogue of a cube is a square and the analogue of a sphere is a circle. Thus a similar problem in two dimensions is the following: if five circles with the same radius r are contained in a square of side 1 m so that the circles touch each other and four of the circles touch two sides of the square, find r.

The diagonal of the square is $\sqrt{2}$. The diagonal is also $4r + 2x$. But x is the diagonal of a smaller square of side r.

Therefore x =√2 r $\Rightarrow$ √2= 4r + 2x = 4r + 2 √2r = (4 + 2 √2)r

r = √2 /(4+2√2)

Let us use these ideas to solve the original three-dimensional problem. The diagonal of the cube is $\sqrt{1^2+1^2+1^2}$ = √3. The diagonal of the cube is also 4r + 2x where x is the diagonal of a smaller cube with edge r. Therefore x = $\sqrt{r^2+r^2+r^2}$ = √3 r

√3 = 4r + 2x == 4r + 2 √3 r = (4 + 2 √3)r. Thus

r = √3 /(4+2√3) = (2√3-3)/2 . The radius of each ball is (√3 -3/2)m.

Answer 1TFQ.

The given statement is **false**.

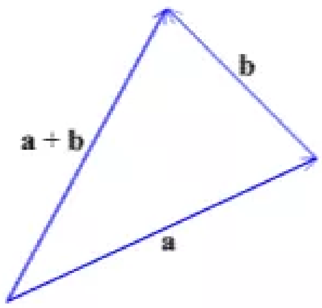
If $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ are two vectors, then the dot product of the vectors is given by the number $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2$.

Therefore, we can say that $\mathbf{u} \cdot \mathbf{v} \neq \langle u_1 v_1, u_2 v_2 \rangle$.

Answer 2CC.

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$. Thus, we can say that the sum of two vectors is obtained by adding the corresponding component vectors.

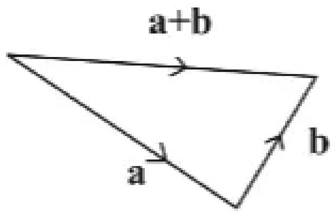
If **a** and **b** are vectors positioned so that the initial point of **v** is at terminal point of **u**, then the sum **a** + **b** is the vector from the initial point of **a** to the terminal point of **b**.



Answer 2E.

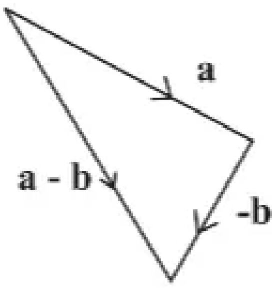
(A)

$\vec{a}+\vec{b}$



(B)

$\vec{a}-\vec{b}$

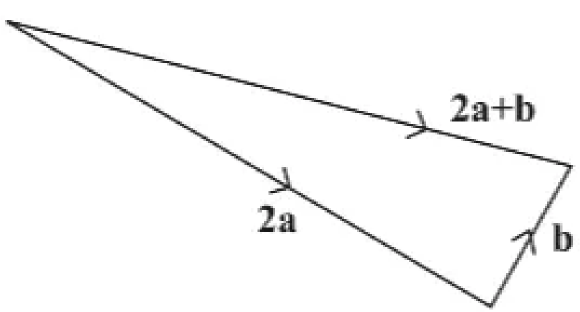


(C)

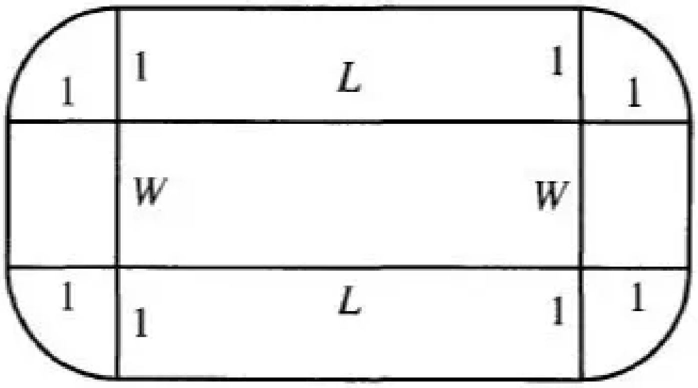
$-\frac{1}{2}\vec{a}$

(D)

$2\vec{a}+\vec{b}$



Answer 2P.



Try an analogous problem in two dimensions. Consider a rectangle with

length L and width W and find the area of S in terms of L and W. Since

S contains B, it has area

A(S) = LW + the area of two L x 1 rectangles

+ the area of two 1 x W rectangles

+ the area of four quarter-circles of radius 1 as seen in the diagram.

So A(S) = LW + 2L + 2W + π (12)

Now in three dimensions. the volume of S is

LWH + 2(L x W x 1) + 2(1 x W x H) + 2(L x 1 x H)

+ the volume of 4 quarter-cylinders with radius 1 and height W

+ the volume of 4 quarter-cylinders with radius 1 and height L

+ the volume of 4 quarter-cylinders with radius 1 and height H

+ the volume of 8 eighths of a sphere of radius 1

So

V(S) = LWH + 2LW + 2WH + 2LH + π (1)2 W + π (1)2 L + π(1)2 H

+ 4/3π (1)3

V(S) = LWH + 2(LW +WH + LH) + π(L + W + H) + 4/3 π

Answer 2TFQ.

The given statement is **false**.

Let $\mathbf{u}$ be $2\mathbf{i}+3\mathbf{j}$ and $\mathbf{v}$ be $\mathbf{i}+\mathbf{j}$. We know that $\mathbf{u}+\mathbf{v}=(u_1+v_1)\mathbf{i}+(u_2+v_2)\mathbf{j}$.

Find $\mathbf{u}+\mathbf{v}$.

$$\begin{aligned}\mathbf{u}+\mathbf{v} &= (2+1)\mathbf{i}+(3+1)\mathbf{j}\\ &= 3\mathbf{i}+4\mathbf{j}\end{aligned}$$

Determine $|\mathbf{u}+\mathbf{v}|$.

$$\begin{aligned}|\mathbf{u}+\mathbf{v}| &= \sqrt{3^2+4^2}\\ &= \sqrt{9+16}\\ &= 5\end{aligned}$$

Now, find $|\mathbf{u}|$ and $|\mathbf{v}|$.

$$\begin{aligned}|\mathbf{u}| &= \sqrt{2^2+3^2} & |\mathbf{v}| &= \sqrt{1^2+1^2}\\ &= \sqrt{4+9} & &= \sqrt{2}\\ &= \sqrt{13} & &= \sqrt{2}\end{aligned}$$

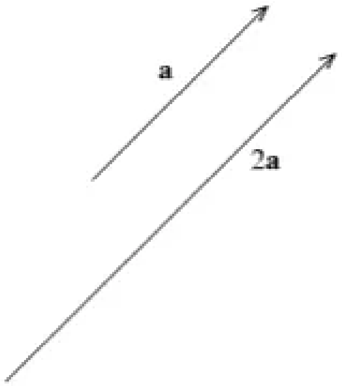
We thus get $|\mathbf{u}|$ and $|\mathbf{v}|$ as $\sqrt{13}+\sqrt{2}$.

Therefore, we can say that $|\mathbf{u}|+|\mathbf{v}| \neq |\mathbf{u}+\mathbf{v}|$.

Answer 3CC.

If c is a vector and **a** is a vector, then the vector $c\mathbf{a}$ is the vector whose length is $|c|$ times the length of **a** and whose direction is the same as **a** if $c > 0$ and in the opposite direction if $c < 0$. In other words, we can say that c works like a scaling factor.

For example, let $c = 2$ and sketch $2\mathbf{a}$.



We can find $c\mathbf{a}$ algebraically by multiplying each of the component vectors of **a**. If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, then $c\mathbf{a} = ca_1\mathbf{i} + ca_2\mathbf{j} + ca_3\mathbf{k}$.

Answer 3E.

$$|\vec{u}| = 2,\ |\vec{v}| = 3,\ \theta = 45^\circ$$

Then
$$\begin{aligned} \vec{u} \cdot \vec{v} &= |\vec{u}||\vec{v}|\cos\theta \\ &= (2)(3)\cos 45^\circ \\ &= 2 \times 3 \times \frac{1}{\sqrt{2}} \\ &= 3\sqrt{2} \end{aligned}$$

And
$$\begin{aligned} |\vec{u} \times \vec{v}| &= |\vec{u}||\vec{v}|\sin\theta \\ &= (2)(3)\sin 45^\circ \\ &= 2 \times 3 \times \frac{1}{\sqrt{2}} \\ &= 3\sqrt{2} \end{aligned}$$

Also $\vec{u} \times \vec{v}$ is directed out of the page.

Answer 3P.

(a) We find the line of intersection L . Observe that the point (-1, c, c) lies on both planes. Now since L lies in both planes, it is perpendicular to both of the normal vectors n1 and n2, and thus parallel to their cross product

$$n_1 \times n_2 = \begin{vmatrix} i & j & k \\ c & 1 & 1 \\ 1 & -c & c \end{vmatrix} = \left\langle 2c,\ -c^2+1,\ -c^2-1 \right\rangle$$

So symmetric equations of L can be written as

$$\frac{x+1}{-2c} = \frac{y-c}{c^2-1} = \frac{z-c}{c^2+1}$$ provided that $c \neq 0, \pm 1$.

If c = 0, then the two planes are given by y + z = 0 and x = -1. so symmetric equations of L are x = -1, y = - z.

If c = -1, then the two planes are given by -x + y + z = -1 and x + y + z = -1, and they intersect in the line x = 0, y = -z - 1. If c = 1, then the two planes are given by x + y + z = 1 and x - y + z = 1, and they intersect in the line y = 0, x = 1 - z.

(b)

If we set z = t in the symmetric equations and solve for x and y separately, we get x + 1 = $\frac{(t-c)(2c)}{c^2+1}$, $y-c=\frac{(t-c)(c^2-1)}{c^2+1}$

$$x=\frac{-2ct+(c^2-1)}{c^2+1}$$

$$y=\frac{(c^2-1)t+2c}{c^2+1}$$

Eliminating c from these equations, we have x 2 + y2 = t2 + 1.

So the curve traced out by L in the plane z = t is a circle with center at (0,0, t) and radius $\sqrt{t^2+1}$

(c) The area of a horizontal cross-section of the solid is

A(z) = π(z2 + 1), so

$$V=\int_0^1 A(z)\,dz=\pi\left[\frac{1}{3}z^3+z\right]_0^1=\frac{4\pi}{3}$$

Answer 3TFQ.

The given statement is **false**.

Let **u** be $2\mathbf{i}+3\mathbf{j}$ and **v** be $\mathbf{i}+\mathbf{j}$. We know that $\mathbf{u}\cdot\mathbf{v}=u_1v_1+u_2v_2$.

Find $\mathbf{u}\cdot\mathbf{v}$.

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v}&=(2)(1)+(3)(1)\\&=2+3\\&=5\end{aligned}$$

Also, we get $|\mathbf{u}\cdot\mathbf{v}|$ as 5.

Now, find $|\mathbf{u}|$ and $|\mathbf{v}|$.

$$|\mathbf{u}| = \sqrt{2^2+3^2} \qquad |\mathbf{v}| = \sqrt{1^2+1^2}$$
$$= \sqrt{4+9} \qquad = \sqrt{2}$$
$$= \sqrt{13} \qquad = \sqrt{2}$$

We thus get $|\mathbf{u}||\mathbf{v}|$ as $\left(\sqrt{13}\right)\left(\sqrt{2}\right)$ or $\sqrt{26}$.

Therefore, we can say that $|\mathbf{u}\cdot\mathbf{v}| \neq |\mathbf{u}|\cdot|\mathbf{v}|$.

Answer 4CC.

The vector from a starting point $P(x_1, y_1, z_1)$ to a terminal point $R(x_2, y_2, z_2)$ is obtained by subtracting the coordinates of the first point from the coordinates of the second. Then, the vector $\overrightarrow{PR}$ is given by $(x_2 - x_1, y_2 - , y_1, z_2 - z_1)$.

Answer 4E.

Given that $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}, \vec{c} = \hat{j} - 5\hat{k}$

(A) $$2\vec{a} + 3\vec{b} = 2\left(\hat{i} + \hat{j} - 2\hat{k}\right) + 3\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$$
$$= 2\hat{i} + 2\hat{j} - 4\hat{k} + 9\hat{i} - 6\hat{j} + 3\hat{k}$$
$$= 11\hat{i} - 4\hat{j} - \hat{k}$$

(B) $$\left|\vec{b}\right| = \sqrt{3^2 + (-2)^2 + 1^2}$$
$$= \sqrt{9+4+1}$$
$$= \sqrt{14}$$

(C) $$\vec{a}.\vec{b} = \left(\hat{i} + \hat{j} - 2\hat{k}\right).\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$$
$$= 3 + (1)(-2) + (-2)(1)$$
$$= 3 - 2 - 2$$
$$= -1$$

(D) $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 3 & -2 & 1 \end{vmatrix}$$
$$= (1-4)\hat{i} - (1+6)\hat{j} + (-2-3)\hat{k}$$
$$= -3\hat{i} - 7\hat{j} - 5\hat{k}$$

(E) $\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -2 & 1\\ 0 & 1 & -5\end{vmatrix}$

$$=(10-1)\hat{i}-(-15-0)\hat{j}+(3-0)\hat{k}$$

$$=9\hat{i}+15\hat{j}+3\hat{k}$$

Then $\left|\vec{b}\times\vec{c}\right|=\sqrt{9^2+(15)^2+3^2}$

$$=\sqrt{81+225+9}$$

$$=\sqrt{315}$$

(F) $\vec{a}.\left(\vec{b}\times\vec{c}\right)=\left(\hat{i}+\hat{j}-2\hat{k}\right).\left(9\hat{i}+15\hat{j}+3\hat{k}\right)$

$$=9+15-6$$

$$=18$$

(G) $\vec{c}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & -5\\ 0 & 1 & -5\end{vmatrix}$

$$=i(-5+5)-j(0-0)+k(0-0)$$

$\vec{c}\times\vec{c}=0$

(H) $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -2\\ 9 & 15 & 3\end{vmatrix}$

$$=i(3+30)-j(3+18)+k(15-9)$$

$$=33\hat{i}-21\hat{j}+6\hat{k}$$

(I) $\text{Comp}_{\vec{a}}\vec{b}=\dfrac{\vec{a}.\vec{b}}{\left|\vec{a}\right|}$

$$=\frac{-1}{\sqrt{1^2+1^2+(-2)^2}}$$

$$=-\frac{1}{\sqrt{6}}$$

(J) $\text{Proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}$

$$= \frac{-1}{\sqrt{6}} \times \frac{1}{\sqrt{6}}\left(\hat{i}+\hat{j}-2\hat{k}\right)$$

$$= -\frac{1}{6}\left(\hat{i}+\hat{j}-2\hat{k}\right)$$

(K) If θ is the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{-1}{\sqrt{6}\sqrt{14}}$$

$$= \frac{-1}{2\sqrt{21}}$$

$$\text{or, } \theta = \cos^{-1}\left(-\frac{1}{2\sqrt{21}}\right) = 96^\circ$$

Answer 4P.

A plane is capable of flying at a speed of 180 km/ h in still air.

| **vi** |= 180 km/h, where **vi** is the initial velocity.

The pilot takes off from an airfield and heads due north according to the plane's compass.

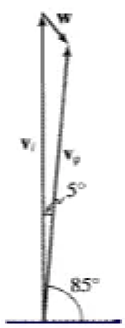
After 30 minutes of flight time, the pilot notices that, due to the wind, the plane has actually traveled 80 km at an angle 5° east of north.

t= 30 min = 0.5 hr

| **vg** |= 80km/0.5hr = 160 km/hr, where **vg** is the velocity relative to the ground.
vg = 160cos(90°-5°) **i**+ 160sin(90°-5°)**j**

Let be **W** the win velocity

The figure for these datas is given by



a)

From the figure we can see that

vg = **vi**+**W**

W = **vg**-**vi**

W= (160 cos85° **i**+ 160 sin85° **j**)-180**j**
W= -160(0.0871557) **i**+ 160(0.996195)**j**-180**j**

W= 13.945 **i**+ 159.391**j**-180**j**
W= 13.945 **i** +(159.391-180)**j**
W= 13.945 **i** -20.6091**j**

The wind velocity is ~ 13.945 **i** -20.6091**j**

The wind speed is ~ $\sqrt{(13.945)^2 + (20.609)^2}$ ~ 24.9 km/h and

b)

Let **v** be the velocity that the pilot need to take.

The actual velocity is **v**+**W**.

We need that **vi** = **v**+**W**

v = **vi**-**W**

v ~180**j** - (13.945 **i** -20.6091**j**)
v ~180**j** - 13.945 **i** +20.6091**j**

v ~ - 13.945 **i** +(20.6091+180)**j**

v ~ - 13.945 **i** +200.6091**j**

The angle for this vector is given by

tanθ ~ 200.6091/(- 13.945)

θ ~tan -1 (-14.385)

θ ~ 93.9766° or 3.9766° west of north.

Answer 4TFQ.

The given statement is **false**.

If θ is the angle between the vectors **u** and **v**, then $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}|\sin\theta$.

Therefore, we can say that $|\mathbf{u} \times \mathbf{v}| \neq |\mathbf{u}||\mathbf{v}|$.

Answer 5CC.

We know that the dot product of two vectors **a** and **b** is given by $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where $|\mathbf{a}|$ and $|\mathbf{b}|$ represent the length of the vectors and θ is the angle between them. Thus, the dot product of two vectors can be obtained by multiplying the length of the vectors to the cosine of the angle between them.

Let $\mathbf{a}\langle a_1, a_2, a_3\rangle$ and $\mathbf{b}\langle b_1, b_2, b_3\rangle$ be two vectors. Then, the dot product of the vectors is obtained by multiplying the corresponding components and adding. Mathematically, we can represent the dot product as $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$

Answer 5E.

If the vectors <3, 2, x> and <2x, 4, x> are orthogonal then their dot product is zero.

i.e. $<3, 2, x>. <2x, 4, x> = 0$

i.e. $6x + 8 + x^2 = 0$

i.e. $x^2 + 6x + 8 = 0$

i.e. $x^2 + 2x + 4x + 8 = 0$

i.e. $(x+2)(x+4) = 0$

i.e. $\boxed{x = -2 \text{ or } -4}$

Answer 5P.

It is given that $|\mathbf{v}_1| = 2$, $|\mathbf{v}_2| = 3$, and $\mathbf{v}_1 \cdot \mathbf{v}_2 = 5$.

Find $\mathbf{v}_3$.

$|\mathbf{v}_3| = |\text{Proj}_{\mathbf{v}_1}\mathbf{v}_2|$ $\quad\quad \left(\because \text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|}\right)$

$= \left(\frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{|\mathbf{v}_1|}\right)\frac{\mathbf{v}_1}{|\mathbf{v}_1|}$

$= \frac{5}{2}\frac{\mathbf{v}_1}{|\mathbf{v}_1|}$

Now, find $\mathbf{v}_4$ given by $\left|\text{Proj}_{\mathbf{v}_2}\mathbf{v}_3\right|$.

$$|\mathbf{v}_4| = \left|\text{Proj}_{\mathbf{v}_2}\mathbf{v}_3\right|$$

$$= \left(\frac{\mathbf{v}_2 \cdot \mathbf{v}_3}{|\mathbf{v}_2|}\right)\frac{\mathbf{v}_2}{|\mathbf{v}_2|}$$

$$|\mathbf{v}_4| = \left(\frac{\mathbf{v}_2 \cdot \frac{5}{2}\frac{\mathbf{v}_1}{|\mathbf{v}_1|}}{|\mathbf{v}_2|}\right)\frac{\mathbf{v}_2}{|\mathbf{v}_2|}$$

$$= \left(\frac{\frac{5}{2}\cdot\frac{5}{2}}{3}\right)\frac{\mathbf{v}_2}{|\mathbf{v}_2|}$$

$$= \frac{25}{12}\frac{\mathbf{v}_2}{|\mathbf{v}_2|}$$

Similarly, find $\mathbf{v}_5$ given by $\mathbf{v}_5 = \left|\text{Proj}_{\mathbf{v}_3}\mathbf{v}_4\right|$

$$|\mathbf{v}_5| = \frac{\frac{5}{2}\frac{\mathbf{v}_1}{|\mathbf{v}_1|}\cdot\frac{25}{12}\frac{\mathbf{v}_2}{|\mathbf{v}_2|}}{|\mathbf{v}_3|}$$

$$= \frac{\frac{5}{2}\left(\frac{25}{12}\right)\left(\frac{1}{6}\right)(5)}{\frac{5}{2}}$$

$$= \frac{125}{72}$$

Then, $\sum_{n=1}^{\infty}|\mathbf{v}_n| = 2+3+\frac{5}{2}+\frac{25}{12}+\frac{125}{72}+\ldots$

$$\sum_{n=1}^{\infty}|\mathbf{v}_n| = 5+\frac{5}{2}+\left(\frac{5}{2}\right)\left(\frac{5}{6}\right)+\left(\frac{5}{2}\right)\left(\frac{5}{6}\right)^2+$$

$$= 5+\frac{5/2}{1-5/6}$$

$$= 20$$

Thus, we get $\boxed{\sum_{n=1}^{\infty}|\mathbf{v}_n| = 20}$

Answer 5TFQ.

The given statement is **true**.

We know that the dot product of two vectors is commutative.

Therefore, we can say that $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$.

Answer 6CC.

We know that the dot product of two vectors can be used to determine the angle between two vectors. The angle between two vectors is given by $\theta = \cos^{-1}\left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)$. Also, we know that two vectors are perpendicular if their dot product is zero. Dot product can be used to determine the direction angle and direction cosine of a non-zero vector.

Answer 6E.

Consider the following vectors:

$\mathbf{j}+2\mathbf{k}$ and $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$

Assume,

$a=\mathbf{j}+2\mathbf{k}$ and $b=\mathbf{i}-2\mathbf{j}+3\mathbf{k}$

The objective is to find two unit vectors that are orthogonal to both $\mathbf{j}+2\mathbf{k}$ and $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$.

The cross product $a \times b$ is orthogonal to both a and b.

$$\begin{aligned} a \times b &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ 1 & -2 & 3 \end{vmatrix} \\ &= (3+4)\mathbf{i}-(0-2)\mathbf{j}+(0-1)\mathbf{k} \\ &= 7\mathbf{i}+2\mathbf{j}-\mathbf{k} \end{aligned}$$

This vector has length $\sqrt{(7)^2+(2)^2+(-1)^2} = \sqrt{54}$.

Therefore, the unit vector is,

$$\begin{aligned} \frac{a \times b}{\|a \times b\|} &= \frac{7\mathbf{i}+2\mathbf{j}-\mathbf{k}}{\sqrt{(7)^2+(2)^2+(-1)^2}} \\ &= \frac{7\mathbf{i}+2\mathbf{j}-\mathbf{k}}{\sqrt{54}} \\ &= \frac{7}{\sqrt{54}}\mathbf{i}+\frac{2}{\sqrt{54}}\mathbf{j}-\frac{1}{\sqrt{54}}\mathbf{k} \\ &= \left\langle \frac{7}{\sqrt{54}}, \frac{2}{\sqrt{54}}, -\frac{1}{\sqrt{54}} \right\rangle \end{aligned}$$

Thus, the unit vector orthogonal to both a and b is $\boxed{\left\langle \frac{7}{\sqrt{54}}, \frac{2}{\sqrt{54}}, -\frac{1}{\sqrt{54}} \right\rangle}$.

Since, the unit vector in the opposite direction is also orthogonal to both a and b. Hence the opposite vector is $-\frac{a\times b}{\|a\times b\|}$.

That implies,

$$\begin{aligned}-\frac{a\times b}{\|a\times b\|} &= -\left\langle \frac{7}{\sqrt{54}},\frac{2}{\sqrt{54}},-\frac{1}{\sqrt{54}}\right\rangle \\ &= \left\langle -\frac{7}{\sqrt{54}},-\frac{2}{\sqrt{54}},\frac{1}{\sqrt{54}}\right\rangle\end{aligned}$$

Hence, the two unit vectors orthogonal to both a and b are $\boxed{\left\langle \frac{7}{\sqrt{54}},\frac{2}{\sqrt{54}},-\frac{1}{\sqrt{54}}\right\rangle}$ and $\boxed{\left\langle -\frac{7}{\sqrt{54}},-\frac{2}{\sqrt{54}},\frac{1}{\sqrt{54}}\right\rangle}$.

Answer 6P.

The given condition can be rewritten as $x^2+y^2+z^2-2(x+2y+3z)<136$ or $(x-1)^2+(y-2)^2+(z-3)^2<150$. We have to find the equation of the largest sphere that passes through (-1, 1, 4) and lies inside the sphere $(x-1)^2+(y-2)^2+(z-3)^2<150$.

Now, the distance between (1, 2, 3) and (−1, 1, 4) is given as $\sqrt{4+1+1}$ or $\sqrt{6}$.

The largest sphere that can lie in this sphere has the diameter passing through (−1, 1, 4), (1, 2, 3) and has the length as $\sqrt{150}+\sqrt{6}$.

The line joining (−1, 1, 4) and (1, 2, 3) is given by $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-4}{-1}=t$. Thus, any point on this line is given by $x=2t-1, y=t+1$, and $z=4-t$.

The co-ordinate of the point that lies at a distance $\frac{\sqrt{150}+\sqrt{6}}{2}$ or $3\sqrt{6}$ is obtained by replacing t with 3. Then, $x=5, y=4$, and $z=1$.

Thus, the largest sphere has center at (5, 4, 1) and radius $3\sqrt{6}$.

The equation of the sphere is obtained as $(x-5)^2+(y-4)^2+(z-1)^2<54$.

Answer 6TFQ.

The given statement is **false**.

Let $\mathbf{u}$ be $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{v}$ be $\mathbf{i}+\mathbf{j}+\mathbf{k}$.

We know that $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$.

Substitute the known values and find $\mathbf{u}\times\mathbf{v}$.

$$\begin{aligned} \mathbf{u}\times\mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} \\ &= \mathbf{i}(2-3)-\mathbf{j}(1-3)+\mathbf{k}(1-2) \\ &= -\mathbf{i}+2\mathbf{j}-\mathbf{k} \end{aligned}$$

We thus get $\mathbf{u}\times\mathbf{v} = -\mathbf{i}+2\mathbf{j}-\mathbf{k}$.

Now, find $\mathbf{v}\times\mathbf{u}$.

$$\begin{aligned} \mathbf{v}\times\mathbf{u} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} \\ &= \mathbf{i}(3-2)-\mathbf{j}(3-1)+\mathbf{k}(2-1) \\ &= \mathbf{i}-2\mathbf{j}+\mathbf{k} \end{aligned}$$

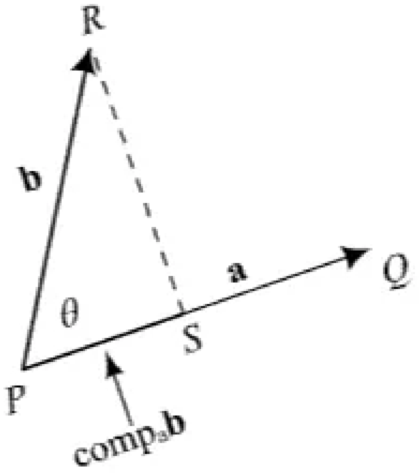
Thus, $\mathbf{v}\times\mathbf{u} = \mathbf{i}-2\mathbf{j}+\mathbf{k}$.

Therefore, we can say that $\mathbf{u}\times\mathbf{v} \neq \mathbf{v}\times\mathbf{u}$.

Answer 7CC.

(a). An ellipse is the set of points in a plane the sum of whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are called the foci.

The scalar projection of **b** onto **a** is defined to be the signed magnitude of eth vector projection, which is the number $|\mathbf{b}|\cos\theta$, where θ is the angle between **a** and **b**. The scalar projection is denoted by $\text{comp}_{\mathbf{a}}\mathbf{b}$ and $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$.



Answer 7E.

properties:

If **a**, **b**, **c** are vectors and c is a scalar then

1) $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$

2) $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$

It is given that $\mathbf{u}.(\mathbf{v}\times\mathbf{w}) = 2$

(a)

We know that $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = (\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}$

Then $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w} = 2$

(b)

We know $\mathbf{v}\times\mathbf{w} = -\mathbf{w}\times\mathbf{v}$

Then $\mathbf{u}\cdot(\mathbf{w}\times\mathbf{v}) = -\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})$

$= -2$

(c)

As $\mathbf{v}\cdot(\mathbf{u}\times\mathbf{w})=(\mathbf{u}\times\mathbf{w})\cdot\mathbf{v}$ $\{\text{As } \mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}\}$

$=\mathbf{u}\cdot(\mathbf{w}\times\mathbf{v})$ $(\because \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c})$

$=-2$

(d)

Now $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{v}=\mathbf{u}\cdot(\mathbf{v}\times\mathbf{v})$

$=\mathbf{u}\cdot 0$

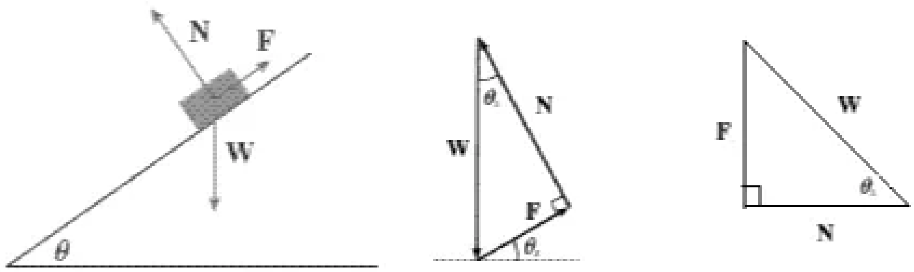
$=0$

i.e. $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{v}=0$

Answer 7P.

a) Observe that **N** + **F** + **W** = **0**

From the problem we know that if θ is not too large, friction will prevent that block from movig at all. When θ = θs then | **F** |= μsn, and the block is not moving, so the sum of the forces on the block must be **0**, thus **N** + **F** + **W** = **0**.



From the figure, we can find the trigonometric function that we need.

$$\tan\theta_s = \frac{|F|}{|N|}$$

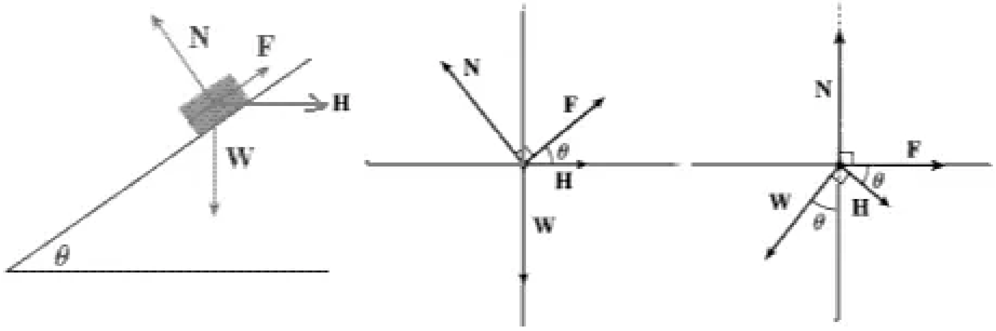
Since |**N**|=n and | **F** |= μsn we get

$$\tan\theta_s = \frac{\mu_s n}{n} = \mu_s$$

So

$$\mu_s = \tan\theta_s$$

b) For θ > θs , and the force **H,** where |**H**| = h and is applied to the block, horizontally from the left. If | **F** | is maximal we know that | **F** |= μsn



The vectors can be expressed in terms of components parallel and perpendicular to the inclined plane as:

N= n **j** ,   **F**= μsn **i** , **W**= -mg sinθ **i** - mg cosθ **j** , **H**= h min cosθ **i** - hmin sinθ **j**

Using that **N+F+W+H=0**

Equating components:

(μsn - mg sinθ **+** h min cosθ) **i** + (n - mg cosθ **-** hmin sinθ) **j = 0i +0 j**

$(\mu_s n - mg \sin\theta + h_{min} \cos\theta) = 0$ **and** $(n - mg \cos\theta - h_{min} \sin\theta) = 0$

We can rewrite as

$\mu_s n + h_{min} \cos\theta = mg \sin\theta$ **and** $mg \cos\theta + h_{min} \sin\theta = n$

---

c)

$\mu_s n + h_{min} \cos\theta = mg \sin\theta$ (1)

$mg \cos\theta + h_{min} \sin\theta = n$ (2)

Substituting (2) into (1)

$\mu_s (mg \cos\theta + h_{min} \sin\theta) + h_{min} \cos\theta = mg \sin\theta$

$\mu_s mg \cos\theta + \mu_s h_{min} \sin\theta + h_{min} \cos\theta = mg \sin\theta$

μs hmin sinθ + h min cosθ = mg sinθ - μs mg cosθ

hmin(μs sinθ + cosθ) = mg (sinθ - μs cosθ)

hmin = mg (sinθ - μs cosθ) / (μs sinθ + cosθ)

$$h_{\min} = \mathrm{mg}\left[\frac{\frac{\sin\theta - \mu_s \cos\theta}{\cos\theta}}{\frac{\mu_s \sin\theta + \cos\theta}{\cos\theta}} \right]$$

$$h_{\min} = \mathrm{mg}\left[\frac{\frac{\sin\theta}{\cos\theta} - \frac{\mu_s \cos\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\mu_s \sin\theta}{\cos\theta}} \right]$$

hmin = mg (tanθ - μs) / (1+ μs tanθ)

From part (a) we know that $\mu_s = \tan\theta_s$

hmin = mg (tanθ - tanθs) / (1+ tanθs tanθ)

Using a trigonometric identity

hmin = mg tan (θ - θs)

Since the block is at rest for $\theta = \theta s \longrightarrow$ hmin= tan 0= 0, so no additional force is necessary to prevent it form moving, which makes sense.

As θ increases, the factor tan ($\theta - \theta s$) $\neq 0$ and hmin increases, this seems reasonable, because we need a horizontal force to keep the block motionless.

For $\theta \longrightarrow 90°$, the plane is vertically and we need a great amount of horizontal force to keep an object from moving vertically.

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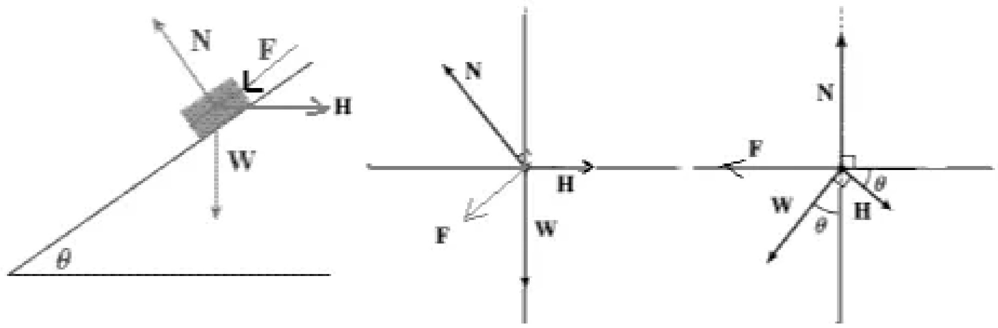
d)



The direction of **F** change 180°.

The vectors can be expressed in terms of components parallel and perpendicular to the inclined plane as:

N= n **j** , **F**= -μsn **i** , **W**= -mg sinθ **i** - mg cosθ **j** , **H**=h maxcosθ **i** - h max sinθ **j**

Using that **N+F+W+H=0**

Equating components:

(-μsn - mg sinθ **+** h max cosθ) **i** + (n - mg cosθ -h maxsinθ) **j** = 0**i** +0 **j**

(-μsn - mg sinθ **+** h maxcosθ) = 0 **and** (n - mg cosθ - h max sinθ) = 0

We can rewrite as

h maxcosθ-μsn = mg sinθ **and** mg cosθ + h max sinθ = n

h maxcosθ-μsn = mg sinθ (1)
mg cosθ +h maxsinθ = n (2)

Substituting (2) into (1)

h max cosθ-μs (mg cosθ + h max sinθ) = mg sinθ

-μs mg cosθ + μs h max sinθ - h max cosθ = mg sinθ

-μs h max sinθ +h max cosθ = mg sinθ + μs mg cosθ

h max(-μs sinθ + cosθ) = mg (sinθ + μs cosθ)

h max= mg (sinθ + μs cosθ) / (-μs sinθ + cosθ)

$$h_{\max} = \mathrm{mg}\left[\frac{\frac{\sin\theta + \mu_S \cos\theta}{\cos\theta}}{\frac{-\mu_S \sin\theta + \cos\theta}{\cos\theta}}\right]$$

$$h_{\max} = \mathrm{mg}\left[\frac{\frac{\sin\theta}{\cos\theta} + \frac{\mu_S \cos\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} - \frac{\mu_S \sin\theta}{\cos\theta}}\right]$$

h max = mg (tanθ + μs) / (1- μs tanθ)

From part (a) we know that $\mu_s = \tan\theta_s$

h max = mg (tanθ + tanθs) / (1- tanθs tanθ)

Using a trigonometric identity

h max = mg tan (θ + θs)

As θ increases we have that h max increases, and h max has values always largen than h min

As h max increases, the normal force increases as well.

If (90°- θs) $\leq$θ $\leq$90°, the horizontal force is completely counteracted by the sum of the normal and frictional force, then the horizontal force does not contributes to moving the block.

Answer 7TFQ.

The given statement is **true**.

We know that $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$.

Then, $|\mathbf{u} \times \mathbf{v}| = |-(\mathbf{v} \times \mathbf{u})|$ or $|\mathbf{u} \times \mathbf{v}| = |\mathbf{v} \times \mathbf{u}|$.

Therefore, we can say that $|\mathbf{u} \times \mathbf{v}| = |\mathbf{v} \times \mathbf{u}|$.

Answer 8CC.

We know that a vector is completely determined by its magnitude and direction. Then, we can say that $\mathbf{a} \times \mathbf{b}$ is the vector perpendicular to the both **a** and **b**, whose orientation is determined by the right hand rule and whose length is given by $|\mathbf{a}||\mathbf{b}|\sin\theta$.

Let $\mathbf{a}\langle a_1, a_2, a_3\rangle$ and $\mathbf{b}\langle b_1, b_2, b_3\rangle$ be two vectors. Then, the cross product of the vectors is obtained by evaluating the determinant given by

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

Answer 8E.

Consider $\left(\vec{a}\times\vec{b}\right).\left[\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\right]$

$$=\left(\vec{a}\times\vec{b}\right).\left[\left\{\left(\vec{b}\times\vec{c}\right).\vec{a}\right\}\vec{c}-\left\{\left(\vec{b}\times\vec{c}\right).\vec{c}\right\}\vec{a}\right]$$

(As $\vec{x}\times\left(\vec{y}\times\vec{z}\right)=\left(\vec{x}\times\vec{z}\right)\vec{y}-\left(\vec{x}\times\vec{y}\right)\vec{z}$)

$$=\left(\vec{a}\times\vec{b}\right).\left[\left\{\left(\vec{b}\times\vec{c}\right).\vec{a}\right\}\vec{c}-\left\{\vec{b}.\left(\vec{c}\times\vec{c}\right)\vec{a}\right\}\right]$$

(As $\left(\vec{x}\times\vec{y}\right).\vec{z}=\vec{x}.\left(\vec{y}\times\vec{z}\right)$)

$$=\left(\vec{a}\times\vec{b}\right).\left[\left\{\left(\vec{b}\times\vec{c}\right).\vec{a}\right\}\vec{c}-0\right]$$

(As $\vec{c}\times\vec{c}=0$)

$$=\left[\left(\vec{a}\times\vec{b}\right).\vec{c}\right]\left[\left(\vec{b}\times\vec{c}\right).\vec{a}\right]$$

$$=\left[\vec{a}.\left(\vec{b}\times\vec{c}\right)\right]\left[\vec{a}.\left(\vec{b}\times\vec{c}\right)\right]$$

$$=\left[\vec{a}.\left(\vec{b}\times\vec{c}\right)\right]^2$$

Hence proved

Answer 8TFQ.

The given statement is **true**.

If $\mathbf{u} = \langle u_1, u_2\rangle$ and $\mathbf{v} = \langle v_1, v_2\rangle$ are two vectors, then the dot product of the vectors is given by the number $\mathbf{u}\cdot\mathbf{v} = u_1v_1 + u_2v_2$. Then, $k(\mathbf{u}\cdot\mathbf{v}) = ku_1v_1 + ku_2v_2$.

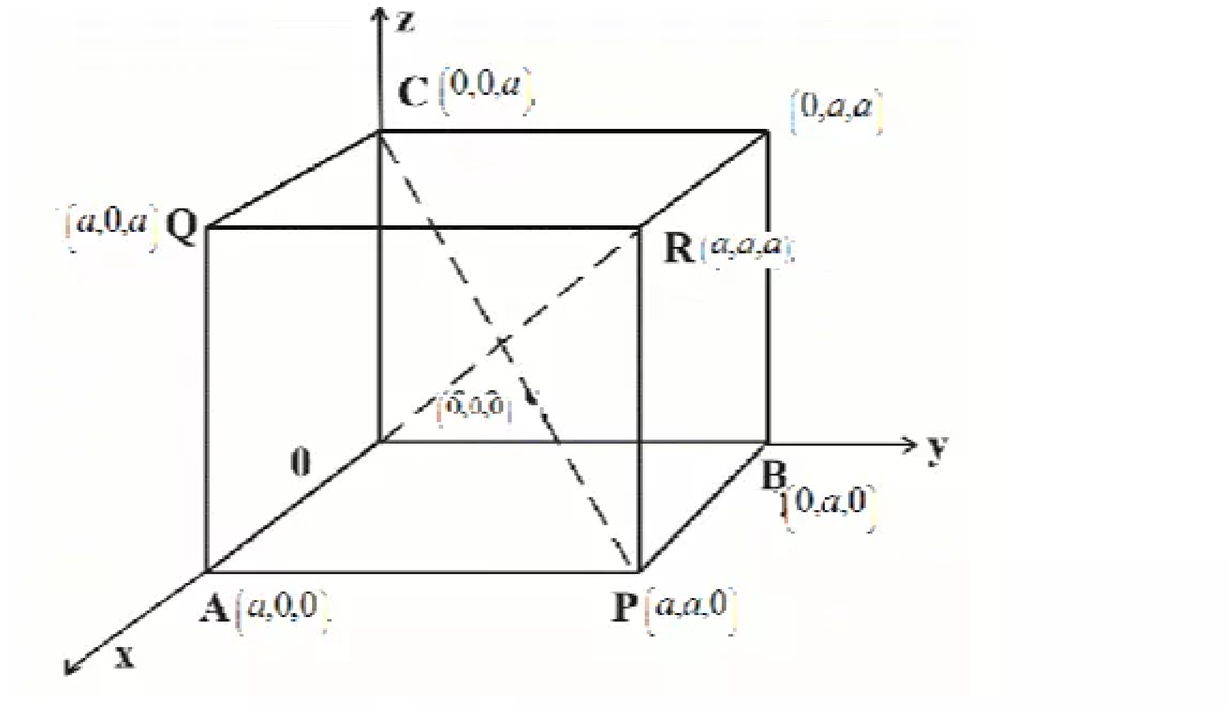
Now, we have $k\mathbf{u} = \langle ku_1, ku_2\rangle$.

Find $(k\mathbf{u})\cdot\mathbf{v}$.

$$\begin{aligned}(k\mathbf{u})\cdot\mathbf{v} &= \langle ku_1, ku_2\rangle\cdot\langle v_1, v_2\rangle\\ &= ku_1v_1 + ku_2v_2\end{aligned}$$

Therefore, we can say that $k(\mathbf{u}\cdot\mathbf{v}) = (k\mathbf{u})\cdot\mathbf{v}$.

Answer 9E.



Let the each side of cube is a unit. Then the co – ordinates of each side are shown as above. The diagonals of the cube are OR and CP

Then $\overrightarrow{OR} = \langle a, a, a \rangle$

And $\overrightarrow{CP} = \langle a, a, -a \rangle$

Then $\left|\overrightarrow{OR}\right| = \sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$

And $\left|\overrightarrow{CP}\right| = \sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$

Let θ be the angle between them.

As we know $\overrightarrow{OR}.\overrightarrow{CP} = \left|\overrightarrow{OR}\right|\left|\overrightarrow{CP}\right|\cos\theta$

Then $\cos\theta = \dfrac{\overrightarrow{OR}\overrightarrow{CP}}{\left|\overrightarrow{OR}\right|\left|\overrightarrow{CP}\right|}$

$= \dfrac{a^2 + a^2 - a^2}{\left(a\sqrt{3}\right)\left(a\sqrt{3}\right)}$

$= \dfrac{a^2}{a^2 3} = \dfrac{1}{3}$

Then $\theta = \cos^{-1}\left(\dfrac{1}{3}\right) = 71^{\circ}$

Hence the angle between two diagonals of a cube is $\boxed{71^{\circ}}$

Answer 9TFQ.

The given statement is **true**.

We know that $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$.

Therefore, we can say that $k(\mathbf{u} \times \mathbf{v}) = (k\mathbf{u}) \times \mathbf{v}$.

Answer 10CC.

(a)We have the cross product of vectors **a** and **b** as $\mathbf{a} \times \mathbf{b}$. The length of the cross product of vectors gives the area of a parallelogram determined by **a** and **b**.

(b)The volume of the parallelepiped determine by the vectors **a**, **b**, and **c** is the magnitude of their scalar triple product $V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$.

Answer 10E.

The given points are

$$A(1, 0, 1),\ B(2, 3, 0),\ C(-1, 1, 4),\ D(0, 3, 2)$$

Then adjacent edges of parallelepiped are:

$$\overrightarrow{AB} = <1, 3, -1>$$

$$\overrightarrow{AC} = <-2, 1, 3>$$

$$\overrightarrow{AD} = <-1, 3, 1>$$

Then volume of parallelepiped is

$$v = \left|\overrightarrow{AB}.\left(\overrightarrow{AC} \times \overrightarrow{AD}\right)\right|$$

$$= \begin{vmatrix} 1 & 3 & -1 \\ -2 & 1 & 3 \\ -1 & 3 & 1 \end{vmatrix}$$

$$= |1(1-9) - 3(-2+3) + (-1)(-6+1)|$$

$$= |-8-3+5|$$

$$= 6$$

Hence volume of parallelepiped is $\boxed{6}$ cubic units.

Answer 10TFQ.

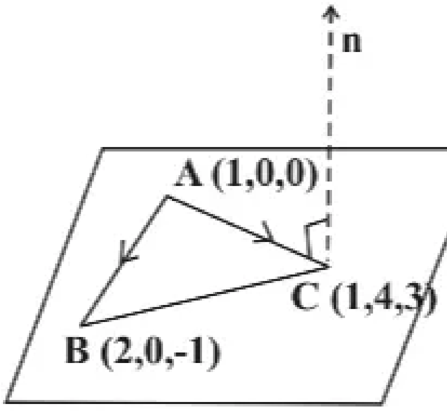
The given statement is **true**.

We know that vector multiplication is distributive over addition. Then, we can say that $(\mathbf{u} + \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times \mathbf{w} + \mathbf{v} \times \mathbf{w}$.

Answer 11CC.

We have the general equation of a plane as $ax+by+cz+d=0$, where $d=-(ax_0+by_0+cz_0)$. Then, a vector perpendicular to the plane is given by $\mathbf{n}=\langle a,b,c\rangle$.

Answer 11E.



(A)

$$A(1,0,0),\ B(2,0,-1),\ C(1,4,3)$$

Then $\overrightarrow{AB}=<1,\ 0,\ -1>$

$\overrightarrow{AC}=\ <0,\ 4,\ 3>$

Let $\vec{n}$ be the vector normal to plane contain.

Then $\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & -1\\ 0 & 4 & 3\end{vmatrix}$$

$$=\hat{i}(0+4)-\hat{j}(3-0)+\hat{k}(4-0)$$

$$=4\hat{i}-3\hat{j}+4\hat{k}$$

Or $\vec{n}=\ <4,-3,4>$

(B)

Area of triangle ABC = 1/2 area of parallelogram with AB and AC as adjacent sides

$$=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$$

$$=\frac{1}{2}\sqrt{4^2+3^2+4^2}$$

$$=\frac{1}{2}\sqrt{41}$$

Hence are of $\Delta ABC=\boxed{\frac{1}{2}\sqrt{41}}$

Answer 11TFQ

The given statement is **true**.

Let $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$, $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, and $\mathbf{w} = \langle w_1, w_2, w_3 \rangle$.

Evaluate $\mathbf{v} \times \mathbf{w}$.

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$
$$= \mathbf{i}(v_2 w_3 - w_2 v_3) - \mathbf{j}(v_1 w_3 - w_1 v_3) + \mathbf{k}(v_1 w_2 - w_1 v_2)$$

We thus get $\mathbf{v} \times \mathbf{w}$ as $\langle v_2 w_3 - v_3 w_2, v_3 w_1 - v_1 w_3, v_1 w_2 - v_2 w_1 \rangle$.

Now, find $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

$$\begin{aligned} \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) &= \langle u_1, u_2, u_3 \rangle \cdot \langle v_2 w_3 - v_3 w_2, v_3 w_1 - v_1 w_3, v_1 w_2 - v_2 w_1 \rangle \\ &= u_1(v_2 w_3 - v_3 w_2) + u_2(v_3 w_1 - v_1 w_3) + u_3(v_1 w_2 - v_2 w_1) \\ &= u_1 v_2 w_3 - u_1 v_3 w_2 + u_2 v_3 w_1 - u_2 v_1 w_3 + u_3 v_1 w_2 - u_3 v_2 w_1 \end{aligned}$$

Rearrange the terms in the equation.

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (u_2 v_3 - u_3 v_2) w_1 + (u_3 v_1 - u_1 v_3) w_2 + (u_1 v_2 - u_2 v_1) w_3$$

On evaluating $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$, we get $(u_2 v_3 - u_3 v_2) w_1 + (u_3 v_1 - u_1 v_3) w_2 + (u_1 v_2 - u_2 v_1) w_3$.

Therefore, we can say that $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$

Answer 12CC.

The angle between two intersecting planes is obtained by finding the angle between the normal vectors of the plane.

Answer 12E.

The object is moved from (1, 0, 2) to (5, 3, 8)
Then the displacement of the object is:

$$\vec{s} = <4, 3, 6> = 4i + 3j + 6k$$

The force is $\vec{F} = 3\hat{i} + 5\hat{j} + 10\hat{k}$

Then the work done is

$$W = \vec{F}\,\vec{s}$$
$$= (3\cdot 4) + (5\cdot 3) + (10\cdot 6)$$
$$= 12 + 15 + 60$$
$$= \boxed{87\text{ J}}$$

Answer 12TFQ.

The given statement is **false**.

Let $\mathbf{u} = \langle 1, 0, 1\rangle$, $\mathbf{v} = \langle -1, 2, 3\rangle$, and $\mathbf{w} = \langle 2, 1, 0\rangle$.

Evaluate $\mathbf{v} \times \mathbf{w}$.

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 3 \\ 2 & 1 & 0 \end{vmatrix}$$
$$= -3\mathbf{i} + 6\mathbf{j} - 5\mathbf{k}$$

We thus get $\mathbf{v} \times \mathbf{w}$ as $\langle -3, 6, -5\rangle$. On using a CAS, we get $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ as $\langle -6, 2, 6\rangle$.

Now, find $\mathbf{u} \times \mathbf{v}$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ -1 & 2 & 3 \end{vmatrix}$$
$$= -2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$$

Evaluate $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$.

$$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -4 & 2 \\ 2 & 1 & 0 \end{vmatrix}$$
$$= 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$$

We note that $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) \neq (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$.

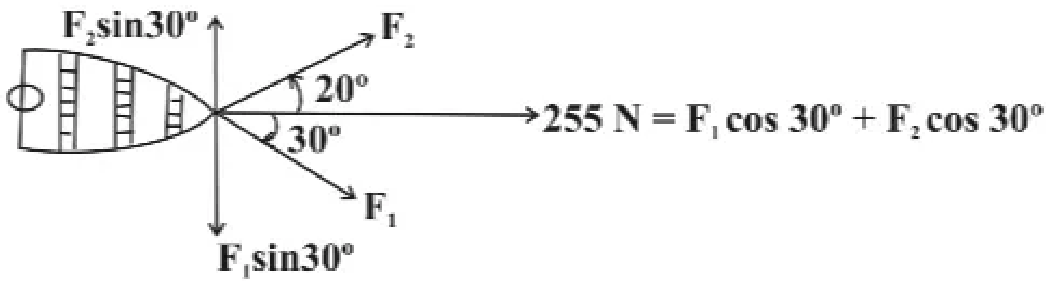
Answer 13CC.

We know that $P_0(x_0, y_0, z_0)$ is a point on the line L. Now, let $P(x, y, z)$ be an arbitrary point on L, $\mathbf{r}_0$ and $\mathbf{r}$ be the position vectors of P_0 and P. Then, the vector equation of the line is given by $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$.

The parametric equations of a line in space parallel to a nonzero vector $\mathbf{v} = \langle a, b, c \rangle$ and passing through the point $P(x_1, y_1, z_1)$ are $x = x_1 + at$, $y = y_1 + bt$, and $z = z_1 + ct$. The numbers a, b, and c are called direction numbers.

The symmetric equations of the line is obtained by eliminating the parameter t from the parametric equations and is given by $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$.

Answer 13E.



Let $\vec{F_1}$ and $\vec{F_2}$ be the forces in two ropes respectively and let F1 and F2 be the respective magnitudes of these forces.

Then resolving these forces horizontally and vertically we find

$F_1 \cos 30° + F_2 \cos 20° = 255$ ---------------- (1)

And $F_1 \sin 30° = F_2 \sin 20°$ ---------------- (2)

From equation (2),

$F_1 = F_2 \sin 20° / \sin 30°$

i.e. $F_1 = 0.684\, F_2$

Using this value in equation (1)

$F_1 \cos 30° + F_2 \cos 20° = 255$

or, $(0.684\, F_2)(0.866) + F_2(0.939) = 255$

or, $(0.5923 + 0.939) F_2 = 255$

Then $F_2 = \dfrac{255}{(0.5923)(0.939)}$

$= 166 \text{ N}$

Therefore $F_1 = (0.684)(166)$

$= 114 \text{ N}$

Hence the forces required in two ropes are

$\boxed{166 \text{ N}}$ and $\boxed{114 \text{ N}}$

Answer 13TFQ.

The given statement is **true**.

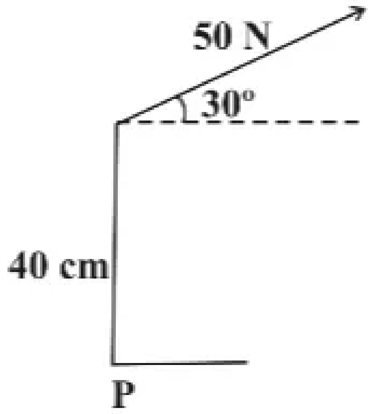
We know that the vector $\mathbf{u} \times \mathbf{v}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$. It is also known that two vectors are perpendicular if their dot product is zero. Thus, we get $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \mathbf{0}$ and $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = 0$.

Answer 14CC.

We know that a plane in space is determined by a point $P_0(x_0, y_0, z_0)$ in the plane and a vector $\mathbf{n}$ that is orthogonal to the plane. Now, let $P(x, y, z)$ be an arbitrary point in the plane, $\mathbf{r}_0$ and $\mathbf{r}$ be the position vectors of P_0 and P. Then, the vector equation of the plane is given by $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$.

In order to obtain the scalar equation for the plane, we write $\mathbf{n} = \langle a, b, c \rangle$, $\mathbf{r} = \langle x, y, z \rangle$, and $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$. Then, the scalar equation of the plane is obtained as $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

Answer 14E.



The magnitude of force is $|\vec{F}| = 50\text{ N}$

And magnitude of position vector is $|\vec{r}| = 40\text{ cm}$

$$= 0.4\text{ m}$$

Then magnitude of torque about P is:

$$|\vec{\tau}| = |\vec{F}||\vec{r}|\sin\theta$$

Here $\theta = 90° - 30° = 60°$

Then $|\vec{\tau}| = (50)(0.4)\sin 60°$

$$= \boxed{17.3\text{ J}}$$

Answer 14TFQ.

The given statement is **true**.

Start by expanding the left hand side of the given equation.

$$(\mathbf{u}+\mathbf{v})\times\mathbf{v} = (\mathbf{u}\times\mathbf{v})+(\mathbf{v}\times\mathbf{v})$$

We know that $\mathbf{v}\times\mathbf{v}$ is 0.

$$\begin{aligned}(\mathbf{u}+\mathbf{v})\times\mathbf{v} &= (\mathbf{u}\times\mathbf{v})+0\\ &= \mathbf{u}\times\mathbf{v}\end{aligned}$$

Therefore, we can say that $(\mathbf{u}+\mathbf{v})\times\mathbf{v} = \mathbf{u}\times\mathbf{v}$.

Answer 15CC.

(a)We know that two vectors are parallel if the angle between them is zero. In other words, we can say that two non-zero vectors **a** and **b** are parallel if and only if their cross product is zero, $\mathbf{a}\times\mathbf{b} = 0$.

(b)We know that two vectors are perpendicular if the angle between them is $90°$. In other words, we can say that two vectors **a** and **b** are perpendicular if and only if their dot product is zero, $\mathbf{a}\cdot\mathbf{b} = 0$.

(c)Two planes are parallel if their normal vectors are parallel. Thus, we can say that two planes with normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are parallel if $\mathbf{n}_1\cdot\mathbf{n}_2 = 0$.

Answer 15E.

We have been given that the line passes through the points $A(4,-1,2)$, and $B(1,1,5)$

The direction numbers are $<1-4,\ 1+1,\ 5-2>$

i.e. $<-3, 2, 3>$

The parametric equations of the line with point (4, -1, 2) and direction numbers <-3, 2, 3> are $x=4-3t$, $y=-1+2t$, $x=2+3t$

Answer 15TFQ.

The given statement is **false**.

Let $\mathbf{v}\langle 3,-1,2\rangle$ be the given vector.

The normal to the plane is given by $\mathbf{n} = \langle 6,-2,4\rangle$. We know that two vectors are parallel if their cross product is zero.

Find $\mathbf{v} \times \mathbf{n}$.

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 2 \\ 6 & -2 & 4 \end{vmatrix}$$

$$= \mathbf{i}(-4+4) + \mathbf{j}(12-12) + \mathbf{k}(-6+6)$$

$$= 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$

$$= 0$$

We note that the given vector is parallel to the normal to the plane. Since the normal is perpendicular to the plane, we can say that the given vector is also perpendicular to the plane.

Therefore, we can say that the given vector is not parallel to the plane.

Answer 16CC.

(a) If three points lie on the same line, then we call them collinear points. Let P, Q, and R be the three points. The points are collinear if the cross product of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are zero. Mathematically, we can say that P, Q, and R lie on the same line if $\overrightarrow{PQ} \times \overrightarrow{PR} = 0$.

(b) Four points that lie on the same plane are called co planar points.

We know the scalar triple product is used to establish whether three vectors lie in the same plane. If the scalar triple product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$, then the three vectors do lie in the same plane, they are co-planar.

The three vectors formed by the given points are $\overrightarrow{PQ}$, $\overrightarrow{PR}$, and $\overrightarrow{PS}$. Thus, we can say that P, Q, R, and S lie on the same plane if $\overrightarrow{PQ} \cdot (\overrightarrow{PR} \times \overrightarrow{PS}) = 0$.

Answer 16E.

The parametric equations of the line through the point (1, 0, -1) and parallel to the line $\frac{x-4}{3} = \frac{y}{2} = z+2$ means parallel to the vector <3, 2, 1> are

$$x = 1+3t,\ y = 0+2t,\ z = -1+t$$

$$x = 1+3t,\ y = 2t,\ z = -1+t$$

Answer 16TFQ.

The given statement is **false**.

The equation $Ax + By + Cz + d = 0$ represents a plane in space, where $d = -(Ax_0 + By_0 + Cz_0)$. The equation is also known as the linear equation in x, y, and z. The normal vector to the plane is given by $\mathbf{n}\langle a, b, c\rangle$.

Answer 17CC.

(a)Let L_1 be a line in three dimensions be specified by two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ lying on it. Then, the distance D between L_1 and a $P_0(x_0, y_0, z_0)$ is given by

$$D = \frac{|(P_0 - P_1) \times (P_0 - P_2)|}{|P_2 - P_1|}.$$

(b)The distance D from a point $P_1(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is given by

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

(c)Consider two lines in space L_1 and L_2 such that L_1 passes through point P_1 and is parallel to vector $\mathbf{v}_1$ and L_2 passes through P_2 and is parallel to $\mathbf{v}_2$. The smallest distance D between the two lines depend on whether the lines are intersecting, parallel, or skew.

If the two lines intersect, then it is clear that the distance between them is 0. If they do not intersect and are parallel, then D corresponds to the distance between point P_2 and line L_1 and is given by

$$D = \frac{\left\|\overrightarrow{P_1P_2} \times \mathbf{v}_1\right\|}{\|\mathbf{v}_1\|}.$$

Now, if the lines are skew. Let $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2$ be a vector perpendicular to both the lines. Then, the distance between the lines is given by

$$D = \frac{\left|\overrightarrow{P_1P_2} \cdot \mathbf{n}\right|}{|\mathbf{n}|}.$$

Answer 17E.

We know that the parametric equations of the lines are:

$x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$ where t is the parameter.

a,b, c are the direction numbers.

The equation of the plane is $2x - y + 5z = 12$

The normal vector of this plane is

$\mathbf{n} = \langle 2, -1, 5 \rangle$

Normal vector is a vector perpendicular to the plane.

Since the given line is perpendicular to the plane, so normal vector and and the lines are parallel.

Thus $\langle a, b, c \rangle = \langle 2, -1, 5 \rangle$

The parametric equations of the line through the point (-2, 2, 4) and perpendicular to the plane $2x - y + 5z = 12$ are

$$x = -2 + 2t,\ y = 2 + (-1)t,\ z = 4 + 5t$$

$$x = -2 + 2t,\ y = 2 - t,\ z = 4 + 5t$$

Therefore required parametric equations of the line are

$$\boxed{x = -2 + 2t \quad y = 2 - t \quad z = 4 + 5t}$$

Answer 17TFQ.

The given statement is **false**.

We note that z is missing in $x^2 + y^2 = 1$.

Since the equations $x^2 + y^2 = 1$ and $z = k$ represents a circle with radius 1 in the plane $z = k$, we can say that the surface $x^2 + y^2 = 1$ is a circular cylinder whose axis is the z-axis.

Answer 18CC.

The curves of intersection of a surface with planes parallel to the coordinate planes are called traces of the surface. The traces are used to sketch the graph of cylinders and quadric surfaces. The trace of a surface along the xy-plane is determined by setting $z = 0$. Similarly, the trace along xz-plane is obtained by setting $y = 0$ and the trace along yz-plane is obtained by setting $x = 0$.

Answer 18E.

An equation of the plane through the point (2, 1, 0) and parallel to $x + 4y - 3z = 1$ means the normal vector is parallel to the normal vector $< 1, 4, -3 >$ is

$$1(x-2) + 4(y-1) - 3(z-0) = 0$$

or, $x - 2 + 4y - 4 - 3z = 0$

or, $x + 4y - 3z = 6$

Answer 18TFQ.

The given statement is **false**.

We note that z is missing in the equation. This means that any horizontal plane with $z = k$ intersects the graph in a curve with equation $y = x^2$. So, we can say that these horizontal traces are parabolas. Thus, we can conclude that the equation represents a parabolic cylinder.

Answer 19CC.

We know that a quadric surface is the graph of a second–degree equation in three variables $x, y,$ and z. The most general equation is
$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$.

From the general equation, we can derive six types of quadric surfaces.

The standard equation of an ellipsoid is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. All the traces of the surface represent ellipses.

The second form of quadric surface is the elliptic paraboloid. The horizontal traces are ellipses and the vertical traces are parabolas. The standard equation is given by
$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

The equation of the form $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ represent a hyperbolic paraboloid. The horizontal traces are hyperbolas and the vertical traces are parabolas.

A quadric surface with ellipse as horizontal traces and planes as vertical traces represents a cone and the standard equation is of the form $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

The quadric surfaces with equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ represent a hyperboloid of one sheet. The surface has ellipses as horizontal traces and hyperbolas as vertical traces.

A quadric surface with standard equation $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ represents a hyperboloid of two sheets.

Answer 19E.

$P(3,-1,1)$, $Q(4,0,2)$, $R=(6,3,1)$

The vector $\vec{a}$ and $\vec{b}$ corresponding to $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are

$\vec{a} = <1,1,1> \quad \vec{b} = <3,4,0>$

With the point $P(3,-1,1)$ and the normal vector $\vec{n}$, an equation of the plane is

$$-4(x-3)+3(y+1)+1(z-1)=0$$

or, $-4x+12+3y+3+z-1=0$

or, $-4x+3y+z=-14$

or, $4x-3y-z=14$

Answer 19TFQ.

The given statement is **false**.

Let $\mathbf{u}\langle 1,-1,2\rangle$ and $\mathbf{v}\langle 2,2,0\rangle$ be two non-zero vectors.

Find $\mathbf{u}\cdot\mathbf{v}$.

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \langle 1,-1,2\rangle\cdot\langle 2,2,0\rangle\\ &= (1)(2)+(-1)(2)+(2)(0)\\ &= 2-2+0\\ &= 0\end{aligned}$$

We note that $\mathbf{u}\cdot\mathbf{v} = 0$, when both $\mathbf{u}$ and $\mathbf{v}$ are non-zero vectors.

Answer 20E.

The plane passing through the point, $\mathbf{r_o}=(x_o,y_o,z_o)=(1,2,-2)$.

The plane contains the line $x=2t, y=3-t, z=1+3t$.

The objective is to find equation of the plane.

First, write the equation of line in symmetric form.

$$\frac{x-0}{2}=\frac{y-3}{-1}=\frac{z-1}{3}.$$

This line contains the point $(0,3,1)$ which lies on the plane.

Substitute $t=1$ in $x=2t, y=3-t, z=1+3t$.

Then $x=2, y=2, z=4$

So, another point on the plane is $(2,2,4)$.

Let $\mathbf{a}$ be the vector from $(2,2,4)$ to $(0,3,1)$.

That is $\mathbf{a}=\langle -2,1,-3\rangle$

Let $\mathbf{b}$ be the vector from $(2,2,4)$ to $(1,2,-2)$.

That is $\mathbf{b}=\langle -1,0,-6\rangle$

The normal of two vectors is,

$$\begin{aligned}\mathbf{n} &= \mathbf{a}\times\mathbf{b}\\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -2 & 1 & -3\\ -1 & 0 & -6\end{vmatrix}\\ &= \mathbf{i}(-6-0)-\mathbf{j}(12-3)+\mathbf{k}(0+1)\\ &= -6\mathbf{i}-9\mathbf{j}+\mathbf{k}\end{aligned}$$

The equation of the plane is $\mathbf{n}\cdot(\mathbf{r}-\mathbf{r}_o)=0$.

$$\begin{aligned}\langle -6,-9,1\rangle\cdot\langle x-1, y-2, z+2\rangle &= 0\\ -6(x-1)-9(y-2)+1(z+2) &= 0\\ -6x+6-9y+18+z+2 &= 0\\ 6x+9y-z &= 26\end{aligned}$$

Therefore, the equation of the plane is $\boxed{6x+9y-z=26}$.

Answer 20TFQ.

The given statement is **false**.

Let $\mathbf{u}\langle 3, -1, 2\rangle$ and $\mathbf{v}\langle 6, -2, 4\rangle$ be two non-zero vectors.

Find $\mathbf{u} \times \mathbf{v}$.

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 2 \\ 6 & -2 & 4 \end{vmatrix}$$
$$= \mathbf{i}(-4+4) + \mathbf{j}(12-12) + \mathbf{k}(-6+6)$$
$$= 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$
$$= 0$$

We note that $\mathbf{u} \times \mathbf{v} = 0$, when both $\mathbf{u}$ and $\mathbf{v}$ are non-zero vectors.

Answer 21E.

Substitute the expressions $x = 2 - t$, $y = 1 + 3t$, $z = 4t$ from parametric equations into the plane $2x - y + z = 2$

$$2(2-t)-(1+3t)+4t = 2$$
or, $4-2t-1-3t+4t = 2$
or, $3-t = 2$
or, $3-2 = t$
i.e. $t = 1$

Therefore the point of intersection is

$$x = 2-1 = 1$$
$$y = 1+3(1) = 4$$
$$z = 4(1) = 4 \quad \text{i.e.} \quad \boxed{(1, 4, 4)}$$

Answer 21TFQ.

The given statement is **true**.

We know that the dot product of two vectors $\mathbf{u}$ and $\mathbf{v}$ is zero if the vectors are perpendicular to each other. Also, the cross product of two vectors is zero if they are parallel to each other.

Now, it is known that no two vectors can be both parallel and perpendicular to each other. Thus, we can say that $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ is possible if and only if $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

Answer 22E.

Consider the line equation:

$x = 1 + t$, $y = 2 - t$ and $z = -1 + 2t$

The objective is to find the distance from the origin to the given line.

The parametric representation of the line is $\mathbf{r}(t) = \langle 1+t, 2-t, -1+2t \rangle$.

The direction vector is $\mathbf{v} = \langle 1, -1, 2 \rangle$.

To find the distance from the origin to the line, find the value of t by setting $\mathbf{r}(t) \cdot \mathbf{v} = 0$ and then find the modulus of $\mathbf{r}(t)$ i.e. $D = |\mathbf{r}(t)|$ by substituting t value in $\mathbf{r}(t)$.

Now find the value of t by setting $\mathbf{r}(t) \cdot \mathbf{v} = 0$

$$\begin{aligned} \mathbf{r}(t) \cdot \mathbf{v} &= 0 \\ \langle 1+t, 2-t, -1+2t \rangle \cdot \langle 1, -1, 2 \rangle &= 0 \\ (1+t)(1) + (2-t)(-1) + (-1+2t)(2) &= 0 \quad \text{(dot product rule)} \\ 1 + t - 2 + t - 2 + 4t &= 0 \end{aligned}$$

$$\begin{aligned} -3 + 6t &= 0 \quad \text{(Simplify)} \\ t &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

The parametric representation of the line at $t = \frac{1}{2}$ is,

$$\begin{aligned} \mathbf{r}(t) &= \langle 1+t, 2-t, -1+2t \rangle \\ \mathbf{r}\left(\frac{1}{2}\right) &= \left\langle 1+\frac{1}{2}, 2-\frac{1}{2}, -1+2\cdot\frac{1}{2} \right\rangle \\ &= \left\langle \frac{3}{2}, \frac{3}{2}, 0 \right\rangle \end{aligned}$$

The position vector is orthogonal to the direction vector of the line, its length is the minimum distance between the line and the origin.

Thus, the distance from the origin to the line is,

$$D = |\mathbf{r}(t)|$$
$$= \left|\mathbf{r}\left(\frac{1}{2}\right)\right|$$
$$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + (0)^2} \quad \left(\text{Since, } \mathbf{r}\left(\frac{1}{2}\right) = \left\langle \frac{3}{2}, \frac{3}{2}, 0 \right\rangle\right)$$
$$= \sqrt{\frac{9}{4} + \frac{9}{4} + 0}$$
$$= \sqrt{\frac{18}{4}}$$
$$= \frac{3}{\sqrt{2}}$$
$$= \frac{3\sqrt{2}}{2}$$

Hence, the distance from the origin to the line is $\boxed{D = \frac{3\sqrt{2}}{2}}$.

Answer 22TFQ.

Consider the statement

"If $\mathbf{u}$ and $\mathbf{v}$ are in V_3, then $|\mathbf{u} \cdot \mathbf{v}| \le |\mathbf{u}||\mathbf{v}|$."

The given statement is true according to Cauchy-Schwarz Inequality.

Cauchy-Schwarz Inequality:

For any vectors $\mathbf{u}$ and $\mathbf{v}$ are in V_2 or V_3, then

$$|\mathbf{a} \cdot \mathbf{b}| \le |\mathbf{a}||\mathbf{b}|.$$

Answer 23E.

The given lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = t \ (\text{say}) \qquad \text{-------------- (1)}$$

And $$\frac{x+1}{6} = \frac{y-3}{-1} = \frac{z+5}{2} = s \ (\text{say}) \qquad \text{-------------- (2)}$$

The direction numbers of line (1) are <2, 3, 4> and direction numbers of line (2) are <6, -1, 2>

Since the direction numbers of both the lines are neither equal nor proportional then we say that these two lines are not parallel.

Now any point on line (1) is $(2t+1,\ 3t+2,\ 4t+3)$

And any point on line (2) is $(6s-1,\ -s+3,\ 2s-5)$

If these two lines are intersecting, then for some values of t and s,

$$2t+1=6s-1$$
$$3t+2=-s+3$$
$$4t+3=2s-5$$

If we solve first two of these equations, we find t = 1/5 and s = 2/5

But these values of t and s do not satisfy the third equation. Then we say that the given lines do not intersect.

And hence the given lines are skew.

Answer 24E.

(A)

The normal vectors of two planes x + y – z = 1 and 2x – 3y + 4z = 5 are

$$\overrightarrow{n_1} = <1, 1, -1>,\ \overrightarrow{n_2} = <2, -3, 4>$$

Because $\overrightarrow{n_1} \neq \overrightarrow{n_2}$, therefore planes are not parallel and

$$\overrightarrow{n_1}.\overrightarrow{n_2} = (1)(2)+1(-3)+(-1)4 = -5 \neq 0$$

Therefore planes are not perpendicular. Because the normal vectors $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ are not perpendicular.

(B)

If θ is the angle between these two planes then θ is also the angle between their normal vector $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$

$$\cos\theta = \frac{\overrightarrow{n_1}.\overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|} = \frac{-5}{\sqrt{1^2+1^2+(-1)^2}\sqrt{2^2+(-3)^2+4^2}} = \frac{-5}{\sqrt{3}\sqrt{29}}$$

or, $\theta = \cos^{-1}(-0.5361) = 122°$

Answer 25E.

First pull the normals from the first two equations and you'll get:

n1=<1,0,-1>

n2=<0,1,2>

Then set z=0 and solve for a point on the intersection line:

x-z=1

x=1

y+2z=3

y=3

so a point on the line is (1,3,0)

Also the direction of the line is equal to the cross product of the 2 normal vectors.

L1= n1 X n2 = <1,0,-1>X<0,1,2>= $\begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = i - 2j + k = \; <1, \; -2, 1>$

L1=<1,-2,1>

Now, pull the normal from x+y-2z=1

and you get N=<1,1,-2>

Now in order to find the normal of the desired plane we have to take the cross product of L1and N

L1 X N = <1,-2,1>X<1,1,-2> $\begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = \; 3i + 3j + 3k = 3(1, \, 1, \, 1)$

So now we have the normal of our desired plane, and point on our plane

so, our plane is:

ANSWER: (x-1)+(y-3)+z=0 OR x+y+z=4

Answer 27E.

Consider the following planes:

$$3x + y - 4z = 2$$
$$3x + y - 4z = 24$$

The objective is to find the distance between the above planes.

Note that, the planes are parallel because their normal vectors $\langle 3,1,-4\rangle$ and $\langle 3,1,-4\rangle$ are parallel.

To find the distance *D* between the planes by choosing any point on one plane and calculate its distance to the other plane.

In particular, if $y = z = 0$ is substituted in the equation of the first plane, it will result in

$3x = 2 \Rightarrow x = \frac{2}{3}$ and so $\left(\frac{2}{3},0,0\right)$ is a point in this plane.

The distance *D* from a point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is,

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

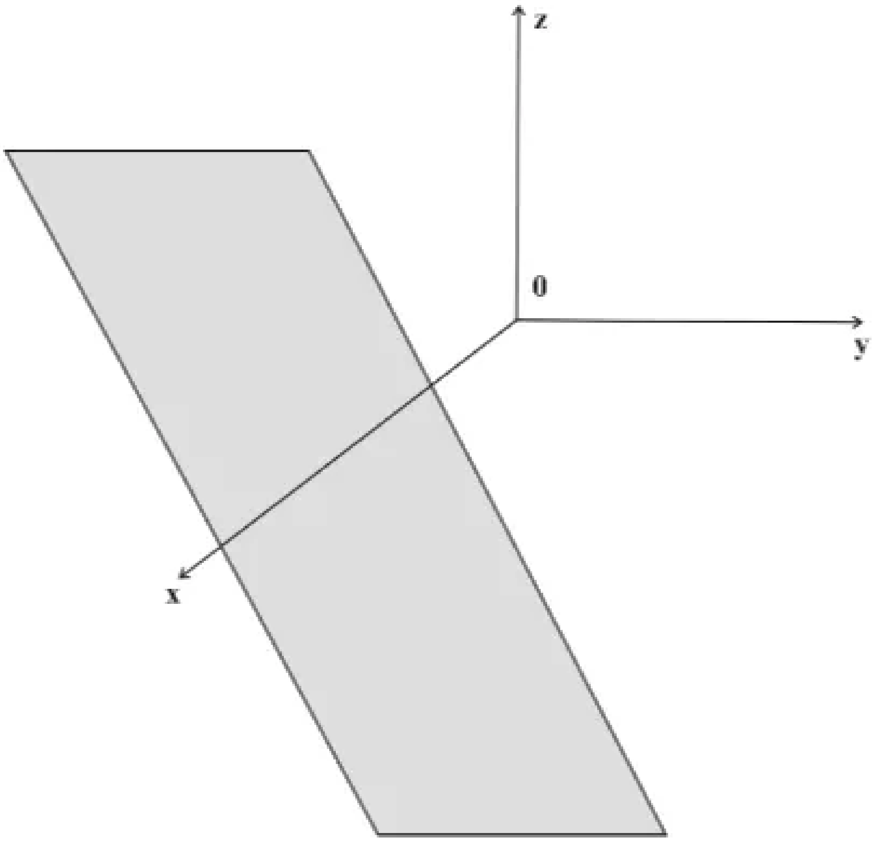
Therefore, the distance *D* from a point $P\left(\frac{2}{3},0,0\right)$ to the plane $3x + y - 4z - 24 = 0$ is,

$$\begin{aligned} D &= \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} \\ &= \frac{\left|3\left(\frac{2}{3}\right) + 1(0) + (-4)(0) - 24\right|}{\sqrt{3^2 + 1^2 + (-4)^2}} \\ &= \frac{|2 + 0 + 0 - 24|}{\sqrt{9 + 1 + 16}} \\ &= \frac{|-22|}{\sqrt{26}} \\ &= \frac{22}{\sqrt{26}} \end{aligned}$$

Thus, the distance between the planes is $\boxed{\frac{22}{\sqrt{26}}}$.

Answer 28E.

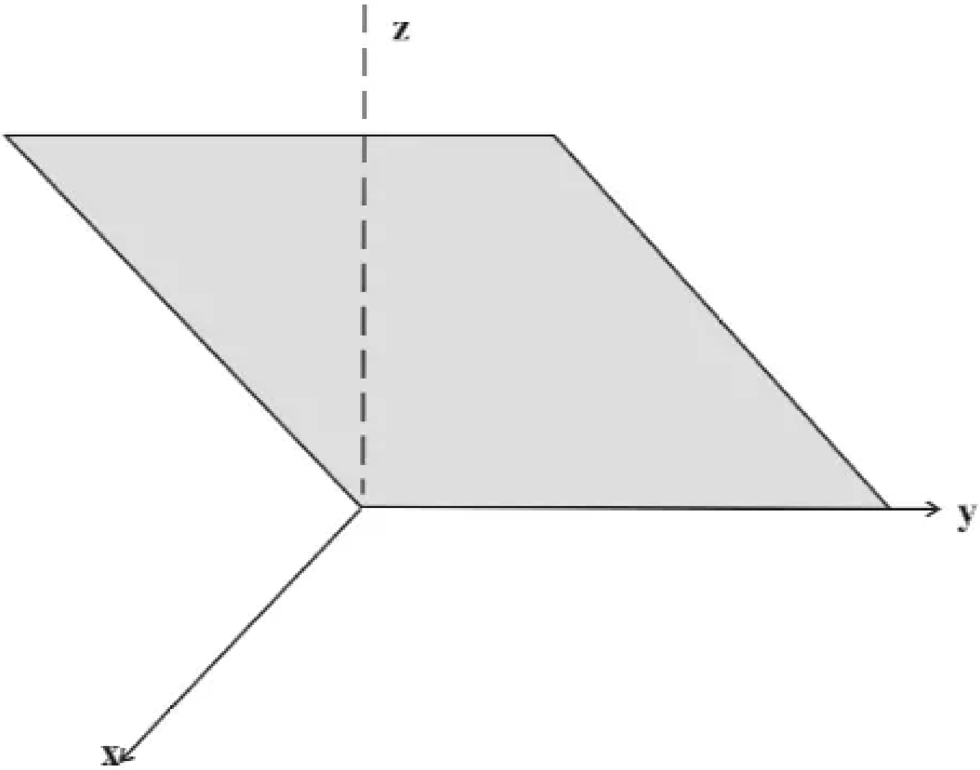
The given surface $x = 3$ is a vertical plane that is parallel to yz- plane and three units from it in the positive direction of x



The given surface is $x = z$

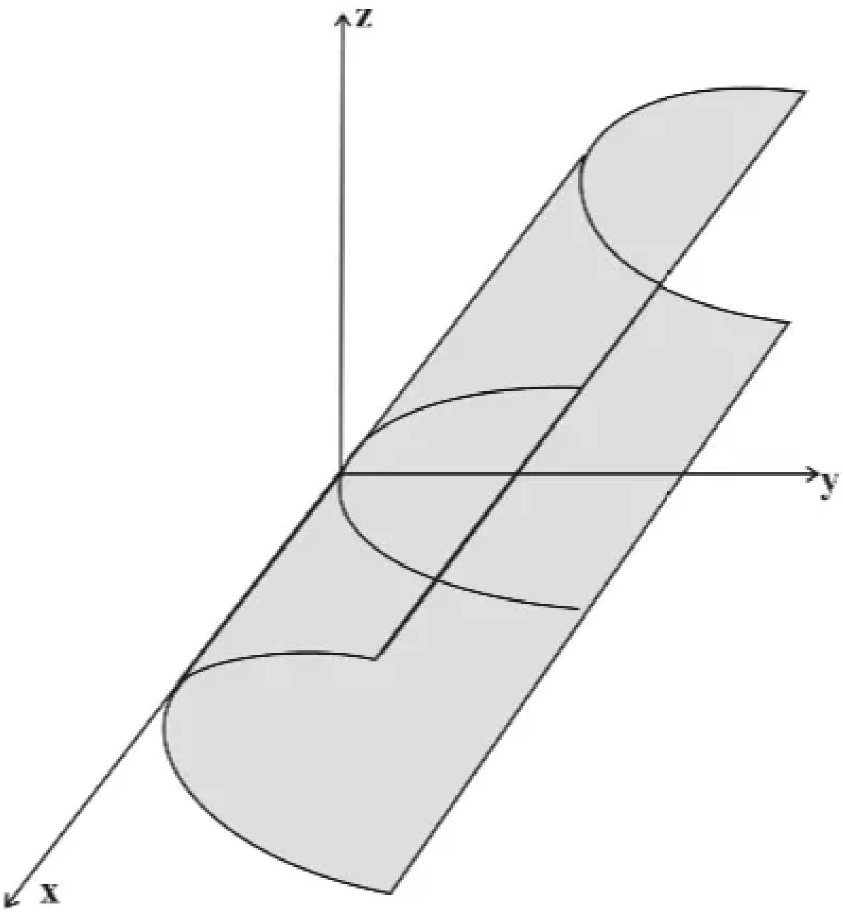
This surface represents a plane in which the x- co- ordinate is equal to z- co-ordinate.

Answer 29E.



Answer 30E.

The given surface $y = z^2$ does not involve x.
This means any vertical plane with equation $x = k$ (parallel to yz- plane) intersects the graph in a curve with equation $y = z^2$. So these vertical traces are parabolas.
This surface is called a parabolic cylinder, made up of infinitely many shifted copies of the same parabola. Here the rulings of the cylinder are parallel to x –



Answer 31E.

Consider the surface.

$$x^2 = y^2 + 4z^2.$$

The objective is to identify and sketch the given surface.

$$x^2 = y^2 + 4z^2$$

Divide by 4 on both sides,

$$\frac{x^2}{4} = \frac{y^2}{4} + \frac{z^2}{1}$$

$$\frac{x^2}{2^2} = \frac{y^2}{2^2} + \frac{z^2}{1^2}$$

This surface is a cone with axis along *x*-axis.

Horizontal traces are ellipses. If $k \neq 0$, then vertical traces in the planes $y = k$ and $z = k$ are hyperbolas and if $k = 0$, then vertical traces are pair of lines

Use Maple software to sketch the graph of the cone, $x^2 = y^2 + 4z^2$ as shown below:

Maple input:

with(plots);

Maple output:

```
> with(plots);
```

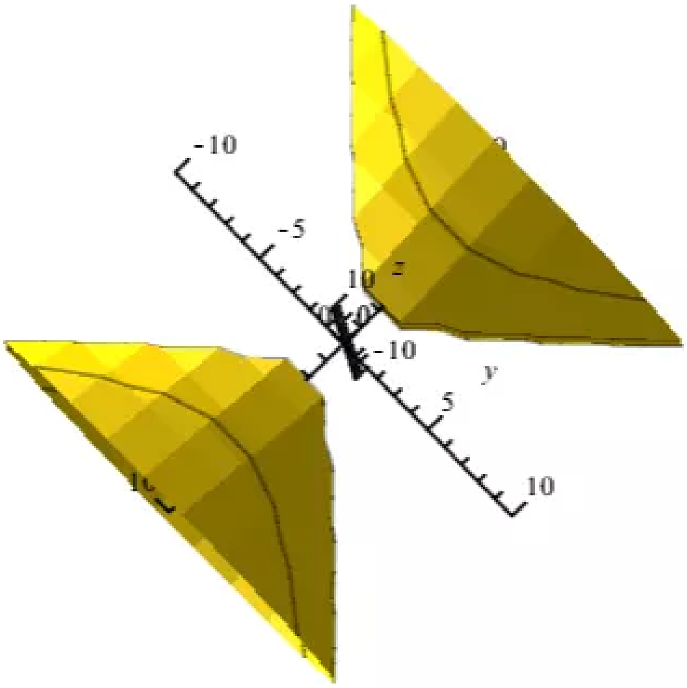
[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

Maple input:

implicitplot3d(x^2=y^2+4*z^2,x=-10..10,y=-10..10,z=-10..10);

Maple output:

$implicitplot3d(x^2 = y^2 + 4 \cdot z^2, x = -10..10, y = -10..10, z = -10..10);$



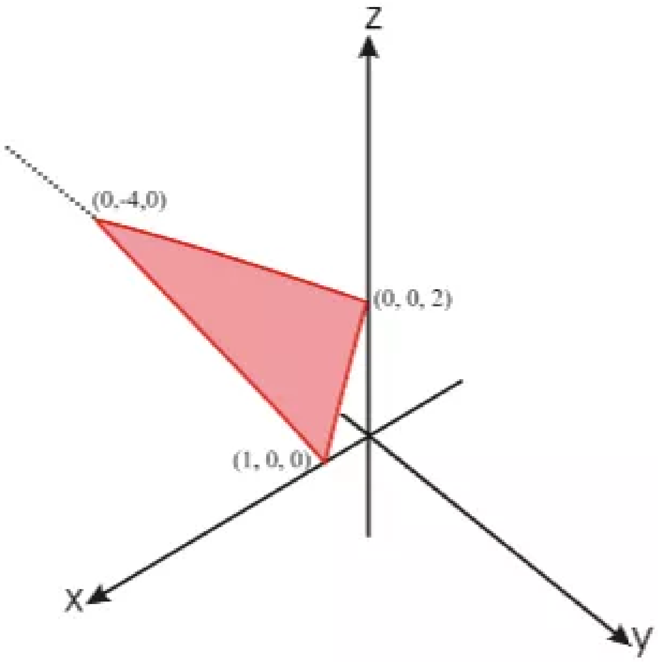
Answer 32E.

The given surface is

$$4x - y + 2z = 4$$

The equation is of the form $Ax + By + Cz = D$, which is the equation of a plane

Hence $4x - y + 2z = 4$, represents a plane whose intercepts are $(1, 0, 0)$, $(0, 0, 2)$ and $(0, -4, 0)$



Answer 33E.

Consider the equation, $-4x^2 + y^2 - 4z^2 = 4$

It represents a hyperboloid of two sheets, the only difference being that in this case the axis of the hyperboloid is the $y-$axis.

The traces in the $xy-$and $yz-$planes are the hyperbolas.

$-4x^2 + y^2 = 4$, $z = 0$ and $y^2 - 4z^2 = 4,\ x = 0$

The surface has no trace in the $xz-$plane, but traces in the plane $y = k$ for $|k| > 1$ are the ellipses.

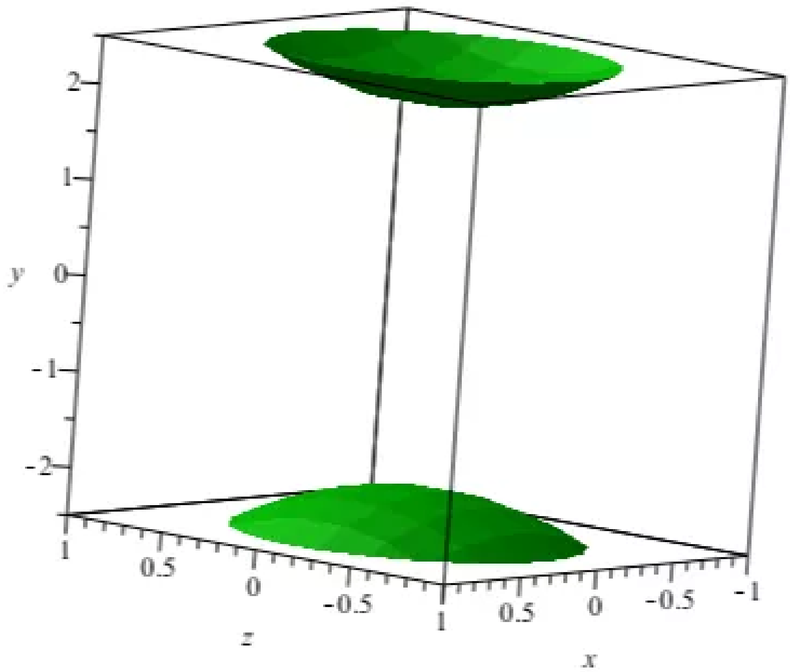
$-4x^2 - 4z^2 = 4 - k^2,\quad y = k$

Which can be written as,

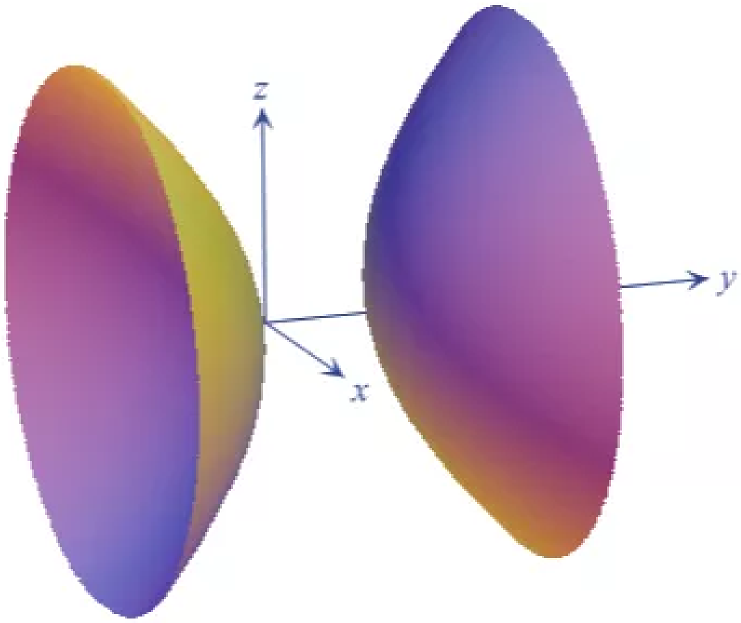
$$-4x^2 - 4z^2 = 4 - k^2,\quad y = k$$
$$4x^2 + 4z^2 = k^2 - 4,\quad y = k$$
$$\frac{4x^2 + 4z^2}{k^2 - 4} = \frac{k^2 - 4}{k^2 - 4},\quad y = k$$
$$\frac{4x^2}{k^2 - 4} + \frac{4z^2}{k^2 - 4} = 1,\quad y = k$$

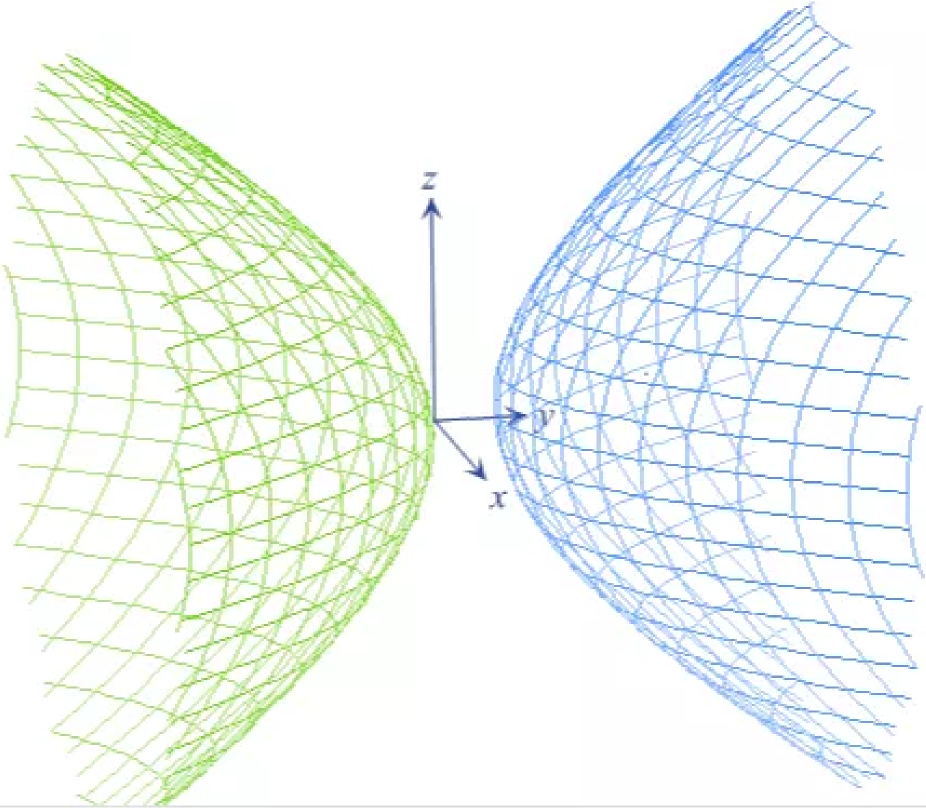
These surface are used to make the sketch in the below.

> *plots*[:-*implicitplot3d*]($-4*x^2 + y^2 - 4*z^2 - 4 = 0, x = -1 .. 1, y = -2.5 .. 2.5, z = -1 .. 1$)



These traces are used to make the sketch in the below.





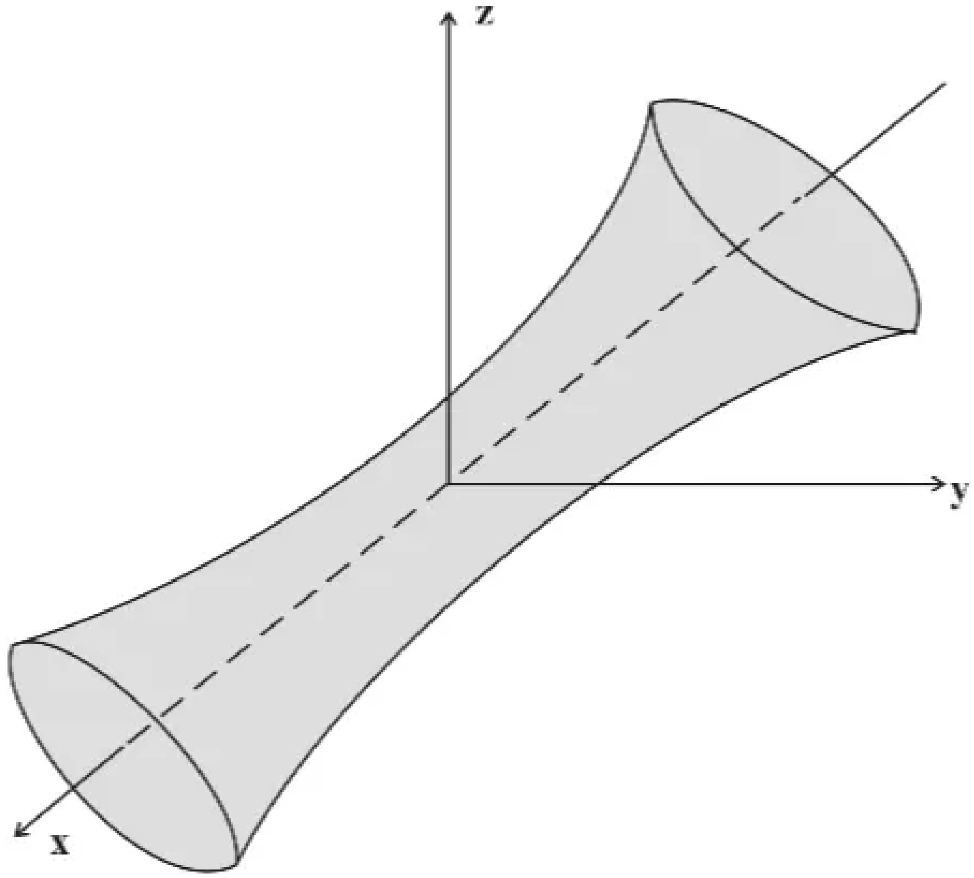
Answer 34E.

The given surface is

$$y^2 + z^2 = 1 + x^2$$

Or $$-x^2 + y^2 + z^2 = 1$$

This surface is a hyperboloid of one sheet with the axis of symmetry along x- axis



Answer 35E.

The given surface is

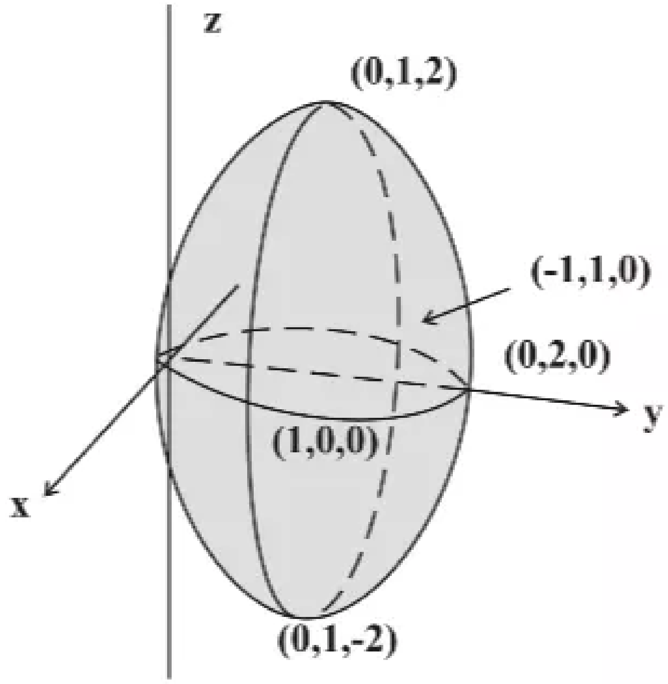
$$4x^2+4y^2-8y+z^2=0$$

Or $$4x^2+\left(4y^2-8y+4\right)+z^2=4$$

Or $$4x^2+\left(2y-2\right)^2+z^2=4$$

Or $$x^2+\left(y-1\right)^2+\frac{z^2}{4}=1$$

This surface is an ellipsoid



Answer 36E.

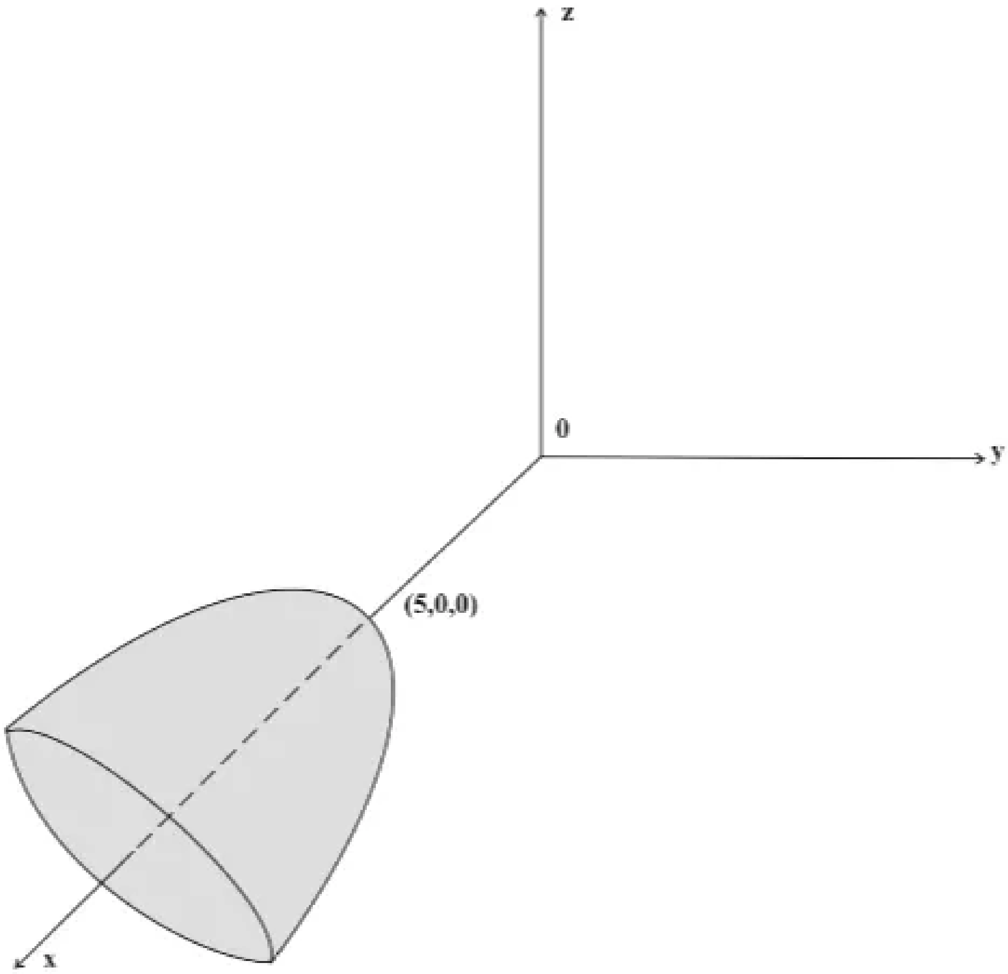
The given surface is

$$x = y^2 + z^2 - 2y - 4z + 5$$

Or $\quad x = (y^2 - 2y + 1) + (z^2 - 4z + 4) + 5 - 1 - 4$

Or $\quad x = (y-1)^2 + (z-2)^2$

Which is an elliptic parabolic with axis along x- axis



Answer 37E.

The equation of given ellipse is $4x^2+y^2=16$
When this ellipse is rotated about x- axis in space then z- coefficient varies and we obtain an ellipsoid whose equation is given by

$$4x^2+y^2+z^2=16$$

Answer 38E.

Let point P has co- ordinates (x,y,z)
The equation of plane is $y=1$ or $y-1=0$

The distance of P from above plane is

$$d=\frac{|y-1|}{\sqrt{1^2}}$$
$$=|y-1|$$

And distance of P from point (0, -1, 0) is

$$\sqrt{(x-0)^2+(y+1)^2+(z-0)^2}$$
$$=\sqrt{x^2+y^2+z^2+2y+1}$$

According to given condition

$$|y-1|=2\sqrt{x^2+y^2+z^2+2y+1}$$

Squaring both sides

$$(y-1)^2=4\left(x^2+y^2+z^2+2y+1\right)$$
$$y^2+1-2y=4x^2+4y^2+4z^2+8y+4$$

Or $\quad 4x^2+3y^2+4z^2+10y+3=0$

Or $\quad 4x^2+\left(3y^2+10y+\frac{25}{3}\right)+4z^2+3-\frac{25}{3}=0$

Or $\quad 4x^2+\left(\sqrt{3}y+\frac{5}{\sqrt{3}}\right)^2+4z^2=\frac{16}{3}$

Or $\quad 4x^2+3\left(y+\frac{5}{3}\right)^2+4z^2=\frac{16}{3}$

Or, $\quad \frac{x^2}{4/3}+\frac{(y+5/3)^2}{16/9}+\frac{z^2}{4/3}=1$

This is the equation of the ellipsoid

The points on this ellipsoid are obtained by taking intersection with three planes viz. $x = 0, z = 0$ and y = -5/3

These are

$$\left(0, \frac{-5}{3}, \frac{2}{\sqrt{3}}\right), \text{ and } \left(0, \frac{-5}{3}, \frac{-2}{\sqrt{3}}\right)$$

$$\left(\frac{2}{\sqrt{3}}, \frac{-5}{3}, 0\right), \text{ and } \left(\frac{-2}{\sqrt{3}}, \frac{-5}{3}, 0\right)$$

And $(0, -3, 0)$, and $\left(0, \frac{-1}{3}, (\ )\right)$

The centre of the ellipsoid is at the point $\left(0, \frac{-4}{3}, 0\right)$

The sketch is