Chapter 10. Logarithms

Ex 10.1

Answer 1.

(i) $3^3 = 27$

$\Rightarrow \log_3 27 = 3$

(ii) $5^4 = 625$

$\Rightarrow \log_5 625 = 4$

(iii) $9^0 = 1$

$\Rightarrow \log_9 1 = 0$

(iv) $\frac{1}{8} = 2^{-3}$

$\Rightarrow \log_2 \frac{1}{8} = -3$

(v) $11^2 = 121$

$\Rightarrow \log_{11} 121 = 2$

(vi) $3^{-2} = \frac{1}{9}$

$\Rightarrow \log_3 \frac{1}{9} = -2$

(vii) $10^{-4} = 0.0001$

$\Rightarrow \log_{10} 0.0001 = -4$

(viii) $7^0 = 1$

$\Rightarrow \log_7 1 = 0$

(ix) $\left(\frac{1}{3}\right)^4 = \frac{1}{81}$

$\Rightarrow \log_{\frac{1}{3}} \frac{1}{81} = 4$

(x) $9^{-4} = \frac{1}{6561}$

$\Rightarrow \log_9 \frac{1}{6561} = -4$

Answer 2.

(i) $\log_2 128 = 7$

$\Rightarrow \quad 128 = 2^7$

(ii) $\log_3 81 = 4$

$\Rightarrow \quad 81 = 3^4$

(iii) $\log_{10} 0.001 = -3$

$\Rightarrow \quad 0.001 = 10^{-3}$

(iv) $\log_2 \frac{1}{32} = -5$

$\Rightarrow \quad \frac{1}{32} = 2^{-5}$

(v) $\log_b a = c$

$\Rightarrow \quad a = b^c$

(vi) $\log_2 \frac{1}{2} = -1$

$\Rightarrow \quad \frac{1}{2} = 2^{-1}$

(vii) $\log_5 a = 3$

$\Rightarrow \quad a = 5^3$

(viii) $\log_{\sqrt{3}} 27 = 6$

$\Rightarrow \quad 27 = \left(\sqrt{3}\right)^6$

(ix) $\log_{25} \sqrt{5} = \frac{1}{4}$

$\Rightarrow \quad \sqrt{5} = 25^{1/4}$

(x) $q = \log_a p$

$\Rightarrow \quad p = a^q$

(xi) $\log_{\sqrt{6}} \left(6\sqrt{6}\right) = 3$

$\Rightarrow \quad 6\sqrt{6} = (\sqrt{6})^3$

(xii) $-2 = \log_2 0.25$

$\Rightarrow \quad 2^{-2} = 0.25$

Answer 3.

(i) $\log_x 49 = 2$

$\Rightarrow \quad x^2 = 49$

$\Rightarrow \quad x^2 = 7^2$

$\therefore \quad x = 7$

(ii) $\log_x 125 = 3$

$\Rightarrow \quad x^3 = 125$

$\Rightarrow \quad x^3 = 5^3$

$\therefore \quad x = 5$

(iii) $\log_x 243 = 5$

$\Rightarrow \quad x^5 = 243$

$\Rightarrow \quad x^5 = 3^5$

$\therefore \quad x = 3$

(iv) $\log_8 x = \frac{2}{3}$

$\Rightarrow \quad x = 8^{\frac{2}{3}}$

$\Rightarrow \quad x^3 = 8^2 = 64 = 4^3$

$\therefore \quad x = 4$

(v) $\log_7 x = 3$

$\Rightarrow \quad x = 7^3$

$\Rightarrow \quad x = 343$

(vi) $\log_4 x = -4$

$\Rightarrow \quad x = 4^{-4}$

$\Rightarrow \quad x = \frac{1}{256}$

(vii) $\log_2 0.5 = x$

$\Rightarrow \quad 2^x = 0.5 = \frac{1}{2}$

$\Rightarrow \quad 2^x = 2^{-1}$

$\therefore \quad x = -1$

(viii) $\log_3 243 = x$

$\Rightarrow \quad 243 = 3^x$

$\Rightarrow \quad 3^5 = 3^x$

$\therefore \quad x = 5$

(ix) $\log_{10} 0.0001 = x$

$\Rightarrow \quad 0.0001 = 10^x$

$\Rightarrow \quad 10^x = 10^{-4}$

$\therefore \quad x = -4$

(x) $\log_4 0.0625 = x$

$\Rightarrow \quad 0.0625 = 4^x$

$\Rightarrow \quad 4^x = 4^{-2}$

$\therefore \quad x = -2$

Answer 4.

(i) $\log_{10} 1000$

Let $\log_{10} 1000 = x$

$\Rightarrow 10^x = 1000$

$\Rightarrow 10^x = 10^3$

$\therefore x = 3$

(ii) $\log_3 81$

Let $\log_3 81 = x$

$\Rightarrow 3^x = 81$

$\Rightarrow 3^x = 3^4$

$\therefore x = 4$

(iii) $\log_5 3125$

Let $\log_5 3125 = x$

$\Rightarrow 5^x = 3125$

$\Rightarrow 5^x = 5^5$

$\therefore x = 5$

(iv) $\log_2 128$

Let $\log_2 128 = x$

$\Rightarrow 2^x = 128$

$\Rightarrow 2^x = 2^7$

$\therefore x = 7$

(v) $\log_{\frac{1}{5}} 125$

Let $\log_{\frac{1}{5}} 125 = x$

$\Rightarrow \left(\frac{1}{5}\right)^x = 125$

$\Rightarrow 5^{-x} = 5^3$

$\therefore -x = 3$

$\Rightarrow x = -3$

(vi) $\log_{10} 0.0001$

Let $\log_{10} 0.0001 = x$

$\Rightarrow 0.0001 = 10^x$

$\Rightarrow 10^x = 10^{-4}$

$\therefore x = -4$

(vii) $\log_5 125$

Let $\log_5 125 = x$

$\Rightarrow 125 = 5^x$

$\Rightarrow 5^x = 5^3$

$\therefore x = 3$

(viii) $\log_8 2$

Let $\log_8 2 = x$

$\Rightarrow \quad 2 = 8^x$

$\Rightarrow \quad (2^3)^x = 2$

$\Rightarrow \quad 2^{3x} = 2^1$

$\therefore \ 3x = 1 \quad \Rightarrow \quad x = \frac{1}{3}$

(ix) $\log_{\frac{1}{2}} 16$

Let $\log_{\frac{1}{2}} 16 = x$

$\Rightarrow \quad 16 = \left(\frac{1}{2}\right)^x$

$\Rightarrow \quad 2^{-x} = 2^4$

$\therefore \ -x = 4 \quad \Rightarrow \quad x = -4$

(x) $\log_{0.01} 10$

Let $\log_{0.01} 10 = x$

$\Rightarrow \quad (0.01)^x = 10$

$\Rightarrow \quad (10^{-2})^x = 10^1$

$\Rightarrow \quad 10^{-2x} = 10^1$

$\therefore \ -2x = 1 \quad \Rightarrow \quad x = -\frac{1}{2}$

(xi) $\log_3 81$

Let $\log_3 81 = x$

$\Rightarrow \ 3^x = 81$

$\Rightarrow \ 3^x = 3^4$

$\therefore \quad x = 4$

(xii) $\log_5 \frac{1}{25}$

Let $\log_5 \frac{1}{25} = x$

$\Rightarrow \ 5^x = \frac{1}{25}$

$\Rightarrow \ 5^x = 5^{-2}$

$\therefore \quad x = -2$

(xiii) $\log_2 8$

Let $\log_2 8 = x$

$\Rightarrow \ 2^x = 8$

$\Rightarrow \ 2^x = 2^3$

$\therefore \quad x = 3$

(xiv) $\log_a a^3$

Let $\log_a a^3 = x$

$\Rightarrow a^x = a^3$

$\therefore x = 3$

(xv) $\log_{0.1} 10$

Let $\log_{0.1} 10 = x$

$\Rightarrow 0.1^x = 10$

$\Rightarrow (10^{-1})^x = 10^1$

$\therefore -x = 1 \Rightarrow x = -1$

(xvi) $\log_{\sqrt{3}} (3\sqrt{3})$

Let $\log_{\sqrt{3}} (3\sqrt{3}) = x$

$\Rightarrow (\sqrt{3})^x = 3\sqrt{3}$

$\Rightarrow 3^{\frac{x}{2}} = 3^{1+\frac{1}{2}} = 3^{\frac{3}{2}}$

$\therefore \frac{x}{2} = \frac{3}{2} \Rightarrow x = 3$

Answer 5.

(i) 10^{2a}

$\log_{10} x = a$

$\Rightarrow x = 10^a$

$\therefore 10^{2a} = (10^a)^2 = x^2$

(ii) 10^{a+3}

$\log_{10} x = a$

$\Rightarrow x = 10^a$

$\therefore 10^{a+3} = 10^a . 10^3 = x .1000 = 1000x$

(iii) 10^{-a}

$\log_{10} x = a$

$\Rightarrow x = 10^a$

$\therefore 10^{-a} = x^{-1} = 1 / x$

(iv) 10^{2a-3}

$\log_{10} x = a$

$\Rightarrow x = 10^a$

$\therefore 10^{2a-3} = 10^{2a} . 10^{-3} = (10^a)^2 10^{-3} = \frac{x^2}{1000}$

Answer 6.

(i) 10^{n-1}

$\log_{10} m = n$

$\Rightarrow \quad m = 10^n$

$\therefore \quad 10^{n-1} = 10^n . 10^{-1} = \dfrac{m}{10}$

(ii) 10^{2n+1}

$\log_{10} m = n$

$\Rightarrow \quad m = 10^n$

$\therefore \quad 10^{2n+1} = 10^{2n} . 10^1 = (10^n)^2 . 10 = 10 m^2$

(iii) 10^{-3n}

$\log_{10} m = n$

$\Rightarrow \quad m = 10^n$

$\therefore \quad 10^{-3n} = (10^n)^{-3} = (m)^{-3} = \dfrac{1}{m^3}$

Answer 7.

(i) 10^p

$\log_{10} x = p$

$\Rightarrow \quad x = 10^p$

(ii) 10^{p+1}

$\log_{10} x = p$

$\Rightarrow \quad x = 10^p$

$\therefore \quad 10^{p+1} = 10^p . 10^1 = 10x$

(iii) 10^{2p-3}

$\log_{10} x = p$

$\Rightarrow \quad x = 10^p$

$\therefore \quad 10^{2p-3} = 10^{2p} . 10^{-3} = (10^p)^2 . 10^{-3} = \dfrac{x^2}{1000}$

(iv) 10^{2-p}

$\log_{10} x = p$

$\Rightarrow \quad x = 10^p$

$\therefore \quad 10^{2-p} = 10^2 . 10^{-p} = 100 . x^{-1} = \dfrac{100}{x}$

Answer 8.

$\log_{10} x = a$

$\Rightarrow \quad x = 10^a$

$\log_{10} y = b$

$\Rightarrow \quad y = 10^b$

$\log_{10} z = 2a - 3b$

$\Rightarrow \quad z = 10^{2a-3b}$

$\therefore z = 10^{2a-3b} = (10^a)^2 . (10^b)^{-3} = (x)^2 (y)^{-3} = \dfrac{x^2}{y^3}$

Answer 9.

(i) 10^{2x-3} in terms of a

$\log_{10} a = x$

$\Rightarrow a = 10^x$

$$10^{2x-3} = (10^x)^2 . 10^{-3} = \frac{a^2}{1000}$$

(ii) 10^{3y-1} in terms of b

$\log_{10} b = y$

$\Rightarrow b = 10^y$

$$10^{3y-1} = (10^y)^3 . 10^{-1} = \frac{b^3}{10}$$

(iii) 10^{x-y+z} in terms of a, b and c.

$\log_{10} a = x$

$\Rightarrow a = 10^x$

$\log_{10} b = y$

$\Rightarrow b = 10^y$

$\log_{10} c = z$

$\Rightarrow c = 10^z$

$$10^{x-y+z} = 10^x . 10^{-y} . 10^z = a.b^{-1}.c = \frac{ac}{b}$$

Answer 10.

(i) If $\log_{10} 100 = 2$, then $10^2 = 100$

The statement is TRUE.

(ii) If $\log_{10} p = q$, then $10^p = q$

The statement is FALSE.

$\log_{10} p = q$ implies $10^q = p$

(iii) If $4^3 = 64$, then $\log_3 64 = 4$

The statement is FALSE

$4^3 = 64$ implies $\log_4 64 = 3$

(iv) If $x^y = z$, then $y = \log_x z$

The statement is TRUE.

(v) If $\log_2 8 = 3$, then $\log_8 2 = \frac{1}{3}$

The statement is TRUE

Answer 1.

(i) log 36

$\log 36 = \log(2 \times 2 \times 3 \times 3) = \log(2^2 \times 3^2)$

$= \log 2^2 + \log 3^2 = 2\log 2 + 2\log 3.$

(ii) log 54

$\log 54 = \log(2 \times 3 \times 3 \times 3) = \log(2 \times 3^3)$

$= \log 2 + \log 3^3 = \log 2 + 3\log 3$

(iii) log 144

$\log 144 = \log(2^4 \times 3^2) = \log 2^4 + \log 3^2 = 4\log 2 + 2\log 3$

(iv) log 216

$\log 216 = \log(2^3 \times 3^3) = \log 2^3 + \log 3^3 = 3\log 2 + 3\log 3$

(v) log 648

$\log 648 = \log(2^3 \times 3^4) = \log 2^3 + \log 3^4 = 3\log 2 + 4\log 3$

(vi)

$$\begin{aligned}\log 12^8 &= \log(3 \times 2^2)^8 \\ &= 8\log(3 \times 2^2) \\ &= 8[\log 3 + \log 2^2] \\ &= 8[\log 3 + 2\log 2]\end{aligned}$$

Ex 10.2

Answer 2.

(i) log 20

$\log 20 = \log(2^2 \times 5) = \log 2^2 + \log 5 = 2\log 2 + \log 5$

(ii) log 80

$\log 80 = \log(2^4 \times 5) = \log 2^4 + \log 5 = 4\log 2 + \log 5$

(iii) log 125

$\log 125 = \log 5^3 = 3\log 5$

(iv) log 160

$\log 160 = \log(2^5 \times 5) = \log 2^5 + \log 5 = 5\log 2 + \log 5$

(v) log 500

$\log 500 = \log(2^2 \times 5^3) = \log 2^2 + \log 5^3 = 2\log 2 + 3\log 5$

(vi)

$$\log 250 = \log(5^3 \times 2)$$
$$= \log 5^3 + \log 2$$
$$= 3\log 5 + \log 2$$

Answer 3.

(i) $\log \sqrt[3]{144}$

$$= \log(144)^{1/3}$$
$$= \frac{1}{3}\log 144 = \frac{1}{3}\log(2^4 \times 3^2) = \frac{1}{3}\log 2^4 + \frac{1}{3}\log 3^2 = \frac{4}{3}\log 2 + \frac{2}{3}\log 3$$

(ii) $\log \sqrt[5]{216}$

$$= \log(216)^{1/5}$$
$$= \frac{1}{5}\log 216 = \frac{1}{5}\log(2^3 \times 3^3) = \frac{1}{5}\log 2^3 + \frac{1}{5}\log 3^3 = \frac{3}{5}\log 2 + \frac{3}{5}\log 3$$

(iii) $\log \sqrt[4]{648}$

$$= \log(648)^{1/4}$$
$$= \frac{1}{4}\log 648 = \frac{1}{4}\log(2^3 \times 3^4) = \frac{1}{4}\log 2^3 + \frac{1}{4}\log 3^4 = \frac{3}{4}\log 2 + \frac{4}{4}\log 3 = \frac{3}{4}\log 2 + \log 3$$

(iv) $\log \frac{26}{51} - \log \frac{91}{119}$

$$= \log \frac{2 \times 13}{3 \times 17} - \log \frac{7 \times 13}{7 \times 17}$$
$$= \log \frac{2 \times 13}{3 \times 17} - \log \frac{13}{17} = \log 13 + \log 2 - \log 3 - \log 17 - \log 13 + \log 17 = \log 2 - \log 3$$

(vi)

$$\log\frac{225}{16}-2\log\frac{5}{9}+\log\left(\frac{2}{3}\right)^5$$
$$=\log\frac{225}{16}-2\log\frac{5}{9}+5\log\frac{2}{3}$$
$$=\log 225-\log 16-2[\log 5-\log 9]+5[\log 2-\log 3]$$
$$=\log(5^2\times 3^2)-\log 2^4-2[\log 5-\log 3^2]+5[\log 2-\log 3]$$
$$=\log 5^2+\log 3^2-4\log 2-2[\log 5-2\log 3]+5[\log 2-\log 3]$$
$$=2\log 5+2\log 3-4\log 2-2\log 5+4\log 3+5\log 2-5\log 3$$
$$=\log 2+\log 3$$

Answer 4.

(i) $F=G\frac{m_1 m_2}{d^2}$

Considering log on both the sides, we get

$$\log F=\log\left(G\frac{m_1 m_2}{d^2}\right)=\log(Gm_1m_2)-\log d^2$$
$$=\log G+\log m_1+\log m_2-2\log d$$

(ii) $E=\frac{1}{2}mv^2$

Considering log on both the sides, we get

$$\log E=\log\left(\frac{1}{2}mv^2\right)=\log\frac{1}{2}+\log m+\log v^2=\log 1-\log 2+\log m+2\log v$$
$$=\log m+2\log v-\log 2$$

(iii) $n=\sqrt{\frac{M.g}{m.\ell}}$

$$\Rightarrow n=\left(\frac{M.g}{m.\ell}\right)^{1/2}$$

Considering log on both sides,

$$\log n=\log\left(\frac{M.g}{m.\ell}\right)^{1/2}=\frac{1}{2}\log\left(\frac{M.g}{m.\ell}\right)=\frac{1}{2}[\log(Mg)-\log(m.\ell)]$$
$$=\frac{1}{2}(\log M+\log g-\log m-\log\ell)$$

(iv) $V = \frac{4}{3}\pi r^3$

Considering log on both the sides, we get

$$\log V = \log\left(\frac{4}{3}\pi r^3\right) = \log 4 + \log\pi + \log r^3 - \log 3$$

$$= \log 2^2 + \log\pi + 3\log r - \log 3 = 2\log 2 - \log 3 + \log\pi + 3\log r$$

(v) $V = \frac{1}{D\ell}\sqrt{\frac{T}{\pi r}}$

$$\Rightarrow V = \frac{1}{D\ell}\left(\frac{T}{\pi r}\right)^{1/2}$$

Considering log on both the sides, we get

$$\log V = \log\left[\frac{1}{D\ell}\left(\frac{T}{\pi r}\right)^{1/2}\right] = \log\left(\frac{1}{D\ell}\right) + \log\left(\frac{T}{\pi r}\right)^{1/2} = (\log 1 - \log D - \log\ell) + \frac{1}{2}\log\left(\frac{T}{\pi r}\right)$$

$$= (0 - \log D - \log\ell) + \frac{1}{2}(\log T - \log\pi - \log r)$$

$$= \frac{1}{2}(\log T - \log\pi - \log r) - \log D - \log\ell$$

Answer 5.

(i) $\log 18 + \log 25 - \log 30$

$= \log(2 \times 3^2) + \log 5^2 - \log(2 \times 3 \times 5)$

$= \log 2 + \log 3^2 + 2\log 5 - \{\log 2 + \log 3 + \log 5\}$

$= \log 2 + 2\log 3 + 2\log 5 - \log 2 - \log 3 - \log 5$

$= \log 3 + \log 5 = \log(3 \times 5) = \log 15$

(ii) $\log 144 - \log 72 + \log 150 - \log 50$

$= \log(2^4 \times 3^2) - \log(2^3 \times 3^2) + \log(2 \times 3 \times 5^2) - \log(2 \times 5^2)$

$= \log 2^4 + \log 3^2 - \{\log 2^3 + \log 3^2\} + \log 2 + \log 3 + \log 5^2 - \{\log 2 + \log 5^2\}$

$= 4\log 2 + 2\log 3 - 3\log 2 - 2\log 3 + \log 2 + \log 3 + 2\log 5 - \log 2 - 2\log 5$

$= \log 2 + \log 3 = \log(2 \times 3) = \log 6$

(iii) $2\log 3 - \frac{1}{2}\log 16 + \log 12$

$= 2\log 3 - \frac{1}{2}\log 2^4 + \log(2^2 \times 3)$

$= 2\log 3 - \frac{1}{2}\times 4\log 2 + \log 2^2 + \log 3$

$= 2\log 3 - 2\log 2 + 2\log 2 + \log 3 = 3\log 3 = \log 3^3 = \log 27$

(iv) $2 + \frac{1}{2}\log 9 - 2\log 5$

$= 2 + \frac{1}{2}\log 3^2 - 2\log 5$

$= 2\log 10 + \frac{1}{2}\times 2\log 3 - 2\log 5$

$= \log 10^2 + \log 3 - \log 5^2$

$= \log 100 + \log 3 - \log 25$

$= \log \frac{100\times 3}{25} = \log 12$

(v) $2\log\frac{9}{5} - 3\log\frac{3}{5} + \log\frac{16}{20}$

$= 2\log 9 - 2\log 5 - 3\log 3 + 3\log 5 + \log 16 - \log 20$

$= 2\log(3^2) - 2\log 5 - 3\log 3 + 3\log 5 + \log(4^2) - \log(5\times 4)$

$= 4\log 3 - 2\log 5 - 3\log 3 + 3\log 5 + 2\log 4 - \log 5 - \log 4$

$= (4-3)\log 3 + (-2-1+3)\log 5 + \log 4$

$= \log 3 + \log 4$

$= \log(3\times 4) = \log 12$

(vi) $2\log\frac{15}{18} - \log\frac{25}{162} + \log\frac{4}{9}$

$= 2\log\frac{5}{2\times 3} - \log\frac{5^2}{2\times 3^4} + \log\frac{2^2}{3^2}$

$= 2\log 5 - 2\log 2 - 2\log 3 - \{\log 5^2 - \log 2 - \log 3^4\} + \log 2^2 - \log 3^2$

$= 2\log 5 - 2\log 2 - 2\log 3 - 2\log 5 + \log 2 + 4\log 3 + 2\log 2 - 2\log 3$

$= \log 2$

(vii) $2\log\frac{16}{25} - 3\log\frac{8}{5} + \log 90$

$= 2\log\frac{2^4}{5^2} - 3\log\frac{2^3}{5} + \log(2\times 5\times 3^2)$

$= 2\log 2^4 - 2\log 5^2 - 3\{\log 2^3 - \log 5\} + \log 2 + \log 5 + \log 3^2$

$= 2\times 4\log 2 - 2\times 2\log 5 - 3\times 3\log 2 + 3\log 5 + \log 2 + \log 5 + 2\log 3$

$= 8\log 2 - 4\log 5 - 9\log 2 + 3\log 5 + \log 2 + \log 5 + 2\log 3$

$= 2\log 3 = \log 3^2 = \log 9$

(viii) $\frac{1}{2}\log 25 - 2\log 3 + \log 36$

$= \frac{1}{2}\log 5^2 - 2\log 3 + \log(2^2 \times 3^2)$

$\frac{1}{2} \times 2\log 5 - 2\log 3 + \log 2^2 + \log 3^2$

$= \log 5 + 2\log 2$

$= \log 5 + \log 2^2 = \log 5 + \log 4 = \log(5 \times 4) = \log 20$

(ix) $\log \frac{81}{8} - 2\log\frac{3}{5} + 3\log\frac{2}{5} + \log\frac{25}{9}$

$= \log\frac{3^4}{2^3} - 2\log\frac{3}{5} + 3\log\frac{2}{5} + \log\frac{5^2}{3^2}$

$= \log 3^4 - \log 2^3 - 2\log 3 + 2\log 5 + 3\log 2 - 3\log 5 + \log 5^2 - \log 3^2$

$= 4\log 3 - 3\log 2 - 2\log 3 + 2\log 5 + 3\log 2 - 3\log 5 + 2\log 5 - 2\log 3$

$= \log 5$

(x) $3\log \frac{5}{8} + 2\log\frac{8}{15} - \frac{1}{2}\log\frac{25}{81} + 3$

$= 3\log\frac{5}{2^3} + 2\log\frac{2^3}{3 \times 5} - \frac{1}{2}\log\frac{5^2}{3^4} + 3\log 10$

$= 3\log 5 - 3\log 2^3 + 2\log 2^3 - 2\log 3 - 2\log 5 - \frac{1}{2}\log 5^2 + \frac{1}{2}\log 3^4 + 3\log(2 \times 5)$

$= 3\log 5 - 3 \times 3\log 2 + 2 \times 3\log 2 - 2\log 3 - 2\log 5 - \frac{1}{2} \times 2\log 5 + \frac{1}{2} \times 4\log 3 + 3\log 2 + 3\log 5$

$= 3\log 5 - 9\log 2 + 6\log 2 - 2\log 3 - 2\log 5 - \log 5 + 2\log 3 + 3\log 2 + 3\log 5$

$= 3\log 5 = \log 5^3 = \log 125$

Answer 6.

(i) $2\log 5 + \log 8 - \frac{1}{2}\log 4$

$2\log 5 + \log 8 - \frac{1}{2}\log 4$

$= 2\log 5 + \log 2^3 - \frac{1}{2}\log 2^2$

$= 2\log 5 + 3\log 2 - \frac{1}{2} \times 2\log 2$

$= 2\log 5 + 3\log 2 - \log 2$

$= 2\log 5 + 2\log 2 = 2(\log 5 + \log 2) = 2\log(5 \times 2) = 2\log 10$

$= 2 \times 1 = 2$

(ii) $2\log 7 + 3\log 5 - \log\frac{49}{8}$

$2\log 7 + 3\log 5 - \log\frac{49}{8}$

$= 2\log 7 + 3\log 5 - \log 49 + \log 8$

$= 2\log 7 + 3\log 5 - \log 7^2 + \log 2^3$

$= 2\log 7 + 3\log 5 - 2\log 7 + 3\log 2$

$= 3\log 5 + 3\log 2 = 3(\log 5 + \log 2) = 3\log(5 \times 2) = 3\log 10$

$= 3 \times 1 = 3$

(iii) $3\log\frac{32}{27} + 5\log\frac{125}{24} - 3\log\frac{625}{243} + \log\frac{2}{75}$

$3\log\frac{32}{27} + 5\log\frac{125}{24} - 3\log\frac{625}{243} + \log\frac{2}{75}$

$= 3\log\frac{2^5}{3^3} + 5\log\frac{5^3}{2^3\times 3} - 3\log\frac{5^4}{2\times 3^4} + \log\frac{2}{3\times 5^2}$

$= 3\log 2^5 - 3\log 3^3 + 5\log 5^3 - 5\log 2^3 - 5\log 3 - 3\log 5^4 + 3\log 2 + 3\log 3^4$
$+ \log 2 - \log 3 - \log 5^2$

$= 3\times 5\log 2 - 3\times 3\log 3 + 5\times 3\log 5 - 5\times 3\log 2 - 5\log 3 - 3\times 4\log 5 + 3\log 2$
$+ 3\times 4\log 3 + \log 2 - \log 3 - 2\log 5$

$= 15\log 2 - 9\log 3 + 15\log 5 - 15\log 2 - 5\log 3 - 12\log 5 + 3\log 2 + 12\log 3 + \log 2 - \log 3 - 2\log 5$

$= \log 5 + \log 2 = \log(5\times 2) = \log 10 = 1$

(iv) $12\log\frac{3}{2} + 7\log\frac{125}{27} - 5\log\frac{25}{36} - 7\log 25 + \log\frac{16}{3}$

$12\log\frac{3}{2} + 7\log\frac{125}{27} - 5\log\frac{25}{36} - 7\log 25 + \log\frac{16}{3}$

$= 12\log\frac{3}{2} + 7\log\frac{5^3}{3^3} - 5\log\frac{5^2}{2^2\times 3^2} - 7\log 5^2 + \log\frac{2^4}{3}$

$= 12\log 3 - 12\log 2 + 7\log 5^3 - 7\log 3^3 - 5\log 5^2 + 5\log 2^2 + 5\log 3^2 - 7\log 5^2 + \log 2^4 - \log 3$

$= 12\log 3 - 12\log 2 + 21\log 5 - 21\log 3 - 10\log 5 + 10\log 2 + 10\log 3 - 14\log 5 + 4\log 2 - \log 3$

$= 2\log 2 + 2\log 5$

Answer 7.

(i) $\log(3 - x) - \log(x - 3) = 1$

$\Rightarrow \log\left(\frac{3-x}{x-3}\right) = 1 = \log 10$

$\Rightarrow \left(\frac{3-x}{x-3}\right) = 10$

$\Rightarrow 3 - x = 10(x - 3)$

$\Rightarrow 3 - x = 10x - 30 \quad \Rightarrow 11x = 33 \quad \Rightarrow x = 3$

(ii) $\log(x^2 + 36) - 2\log x = 1$

$\Rightarrow \log(x^2 + 36) - \log x^2 = 1$

$\Rightarrow \log\left(\frac{x^2+36}{x^2}\right) = 1 = \log 10$

$\Rightarrow \left(\frac{x^2+36}{x^2}\right) = 10$

$\Rightarrow x^2 + 36 = 10x^2$

$\Rightarrow 9x^2 = 36$

$\Rightarrow x^2 = 4 \quad \Rightarrow x = 2$

(iii) $\log 7 + \log (3x - 2) = \log (x+3) + 1$

$\Rightarrow \log 7 + \log (3x - 2) - \log (x+3) = 1$

$\Rightarrow \log \frac{7.(3x-2)}{x+3} = \log 10$

$\Rightarrow \frac{7.(3x-2)}{x+3} = 10$

$\Rightarrow 21x - 14 = 10(x + 3)$

$\Rightarrow 21x - 10x = 30 + 14$

$\Rightarrow 11x = 44$

$\Rightarrow x = 44 / 11 = 4$

(iv) $\log (x+1) + \log (x-1) = \log 11 + 2 \log 3$

$\Rightarrow \log [(x+1)(x-1)] = \log 11 + \log 3^2$

$\Rightarrow \log \{ x^2 - 1 \} = \log (11.9)$

$\Rightarrow \log \{ x^2 - 1 \} = \log 99$

$\Rightarrow x^2 - 1 = 99$

$\Rightarrow x^2 = 100$

So, $x = 10$ or -10

Negative value is rejected

So , $x = 10$

(v) $\log_4 x + \log_4 (x-6) = 2$

$\Rightarrow \log_4 \{x(x-6)\} = 2 \log_4 4$

$\Rightarrow \log_4 \{x^2 - 6x\} = \log_4 4^2$

$\Rightarrow x^2 - 6x = 16$

$\Rightarrow x^2 - 6x - 16 = 0$

$\Rightarrow x^2 - 8x + 2x - 16 = 0$

$\Rightarrow x(x-8) + 2(x - 8) = 0$

$\Rightarrow (x-8)(x + 2) = 0$

$\Rightarrow x = 8$ or -2

Negative value is rejected

So , $x = 8$

(vi) $\log_8 (x^2 - 1) - \log_8 (3x + 9) = 0$

$\Rightarrow \log_8 \left(\frac{x^2 - 1}{3x+9}\right) = \log_8 1$

$\Rightarrow \frac{x^2 - 1}{3x+9} = 1$

$\Rightarrow x^2 - 1 = 3x + 9$

$\Rightarrow x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow x = 5$ or $x = -2$

(vii) $\log(x + 1) + \log(x-1) = \log 48$

$\Rightarrow \quad \log\{(x+1)(x-1)\} = \log 48$

$\Rightarrow \quad \log(x^2 - 1) = \log 48$

$\Rightarrow \quad x^2 - 1 = 48$

$\Rightarrow \quad x^2 = 49$

$\Rightarrow \quad x = 7$ (neglecting the negative value)

(viii)

$$\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$$

$$\therefore \frac{1}{\log_x 2} + \frac{1}{\log_x 2^2} + \frac{1}{\log_x 2^4} = \frac{21}{4}$$

$$\therefore \frac{1}{\log_x 2} + \frac{1}{2\log_x 2} + \frac{1}{4\log_x 2} = \frac{21}{4}$$

$$\therefore \frac{1}{\log_x 2}\left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{21}{4}$$

$$\therefore \frac{1}{\log_x 2}\left(\frac{7}{4}\right) = \frac{21}{4}$$

$$\therefore \log_x 2 = \frac{7}{4} \cdot \frac{4}{21}$$

$$\therefore \log_x 2 = \frac{1}{3}$$

$$\therefore x^{\frac{1}{3}} = 2$$

$$\therefore x = 2^3 = 8$$

Answer 8.

(i) $\log(x+5) = 1$

$= \log 10$

$\Rightarrow \quad x + 5 = 10$

$\Rightarrow \quad x = 10 - 5 = 5$

(ii) $\frac{\log 27}{\log 243} = x$

$\Rightarrow \quad \frac{\log 3^3}{\log 3^5} = x$

$\Rightarrow \quad \frac{3\log 3}{5\log 3} = x$

$\Rightarrow \quad x = \frac{3}{5}$

(iii) $\frac{\log 81}{\log 9} = x$

$\Rightarrow \quad \frac{\log 3^4}{\log 3^2} = x$

$\Rightarrow \quad \frac{4\log 3}{2\log 3} = x$

$\Rightarrow \quad x = 2$

(iv) $\frac{\log 121}{\log 11} = \log x$

$\Rightarrow \quad \frac{\log 11^2}{\log 11} = \log x$

$\Rightarrow \quad \frac{2\log 11}{\log 11} = \log x$

$\Rightarrow \quad 2 = \log x$

$\Rightarrow \quad 2\log 10 = \log x \qquad (\text{since } \log 10 = 1)$

$\Rightarrow \quad \log 10^2 = \log x$

$\therefore \; x = 10^2 = 100$

(v) $\frac{\log 125}{\log 5} = \log x$

$\Rightarrow \quad \frac{\log 5^3}{\log 5} = \log x$

$\Rightarrow \quad \frac{3\log 5}{\log 5} = \log x$

$\Rightarrow \quad 3 = \log x$

$\Rightarrow \quad 3\log 10 = \log x \qquad (\text{since } \log 10 = 1)$

$\Rightarrow \quad \log 10^3 = \log x$

$\therefore \; x = 10^3 = 1000$

(vi) $\dfrac{\log 128}{\log 32} = x$

$\Rightarrow \quad \dfrac{\log 2^7}{\log 2^5} = x$

$\Rightarrow \quad \dfrac{7\log 2}{5\log 2} = x$

$\Rightarrow \quad x = \dfrac{7}{5} = 1.4$

(vii) $\dfrac{\log 1331}{\log 11} = \log x$

$\Rightarrow \quad \dfrac{\log 11^3}{\log 11} = \log x$

$\Rightarrow \quad \dfrac{3\log 11}{\log 11} = \log x$

$\Rightarrow \quad 3 = \log x$

$\Rightarrow \quad 3\log 10 = \log x \qquad (\text{since } \log 10 = 1)$

$\Rightarrow \quad \log 10^3 = \log x$

$\therefore\ x = 10^3 = 1000$

(viii) $\dfrac{\log 289}{\log 17} = \log x$

$\Rightarrow \quad \dfrac{\log 17^2}{\log 17} = \log x$

$\Rightarrow \quad \dfrac{2\log 17}{\log 17} = \log x$

$\Rightarrow \quad 2 = \log x$

$\Rightarrow \quad 2\log 10 = \log x \qquad (\text{since } \log 10 = 1)$

$\Rightarrow \quad \log 10^2 = \log x$

$\therefore\ x = 10^2 = 100$

Answer 9.

$$\begin{aligned}\log_{10} 3 + 1 &= \log_{10} 3 + \log_{10} 10 \\ &= \log_{10}(3 \times 10) \\ &= \log_{10} 30\end{aligned}$$

Answer 10A.

False, since $\log xy = \log x + \log y$

Answer 10B.

True, since log 1= 0 and anything multiplied by 0 is 0.

Answer 10C.

False, since $\log_b a = \frac{1}{\log_a b}$.

Answer 10D.

True.

$$\frac{\log 49}{\log 7} = \log y$$
$$\Rightarrow \frac{\log 7^2}{\log 7} = \log y$$
$$\Rightarrow \frac{2\log 7}{\log 7} = \log y$$
$$\Rightarrow 2(1) = \log y$$
$$\Rightarrow 2\log_{10} 10 = \log y$$
$$\Rightarrow \log_{10} 10^2 = \log_{10} y$$
$$\Rightarrow \log_{10} 100 = \log_{10} y$$
$$\Rightarrow y = 100$$

Answer 11A.

$$\log 16 = a,\ \log 9 = b \text{ and } \log 5 = c$$
$$\log 4^2 = a,\ \log 3^2 = b \text{ and } \log 5 = c$$
$$2\log 4 = a,\ 2\log 3 = b \text{ and } \log 5 = c$$
$$\log 4 = \frac{a}{2},\ \log 3 = \frac{b}{2} \text{ and } \log 5 = c$$

Consider,
$$\log 12 = \log(4 \times 3)$$
$$= \log 4 + \log 3$$
$$= \frac{a}{2} + \frac{b}{2}$$
$$= \frac{a+b}{2}$$

Answer 11B.

$$\log 16 = a,\ \log 9 = b \text{ and } \log 5 = c$$
$$\log 4^2 = a,\ \log 3^2 = b \text{ and } \log 5 = c$$
$$2\log 4 = a,\ 2\log 3 = b \text{ and } \log 5 = c$$
$$\log 4 = \frac{a}{2},\ \log 3 = \frac{b}{2} \text{ and } \log 5 = c$$

$$\begin{aligned}\text{Consider, } \log 75 &= \log(5^2 \times 3)\\ &= \log 5^2 + \log 3\\ &= 2\log 5 + \log 3\\ &= 2(c) + \frac{b}{2}\\ &= \frac{4c+b}{2}\end{aligned}$$

Answer 11C.

$$\log 16 = a,\ \log 9 = b \text{ and } \log 5 = c$$
$$\log 4^2 = a,\ \log 3^2 = b \text{ and } \log 5 = c$$
$$2\log 4 = a,\ 2\log 3 = b \text{ and } \log 5 = c$$
$$\log 4 = \frac{a}{2},\ \log 3 = \frac{b}{2} \text{ and } \log 5 = c$$

$$\begin{aligned}\text{Consider, } \log 720 &= \log(4^2 \times 3^2 \times 5)\\ &= \log 4^2 + \log 3^2 + \log 5\\ &= 2\log 4 + 2\log 3 + \log 5\\ &= 2\left(\frac{a}{2}\right) + 2\left(\frac{b}{2}\right) + c\\ &= a + b + c\end{aligned}$$

Answer 11D.

$$\log 16 = a,\ \log 9 = b \text{ and } \log 5 = c$$
$$\log 4^2 = a,\ \log 3^2 = b \text{ and } \log 5 = c$$
$$2\log 4 = a,\ 2\log 3 = b \text{ and } \log 5 = c$$
$$\log 4 = \frac{a}{2},\ \log 3 = \frac{b}{2} \text{ and } \log 5 = c$$

$$\begin{aligned}\text{Consider, } \log 2.25 &= \log\left(\frac{225}{100}\right)\\ &= \log 225 - \log 100\\ &= \log(3^2 \times 5^2) - \log(4 \times 5^2)\\ &= \log 3^2 + \log 5^2 - (\log 4 + \log 5^2)\\ &= \log 3^2 + \log 5^2 - \log 4 - \log 5^2\\ &= 2\log 3 - \log 4\\ &= 2\left(\frac{b}{2}\right) - \frac{a}{2}\\ &= \frac{2b-a}{2}\end{aligned}$$

Answer 11E.

$\log 16 = a,\ \log 9 = b$ and $\log 5 = c$

$\log 4^2 = a,\ \log 3^2 = b$ and $\log 5 = c$

$2\log 4 = a,\ 2\log 3 = b$ and $\log 5 = c$

$\log 4 = \frac{a}{2},\ \log 3 = \frac{b}{2}$ and $\log 5 = c$

Consider, $\log 2\frac{1}{4} = \log\left(\frac{9}{4}\right)$

$= \log 9 - \log 4$

$= \log 3^2 - \log 4$

$= 2\log 3 - \log 4$

$= 2\left(\frac{b}{2}\right) - \frac{a}{2}$

$= \frac{2b - a}{2}$

Answer 12.

$\log x = p + q$ and $\log y = p - q$

$\log \frac{10x}{y^2} = \log 10x - \log y^2$

$\Rightarrow \log \frac{10x}{y^2} = \log 10 + \log x - 2\log y$

$\Rightarrow \log \frac{10x}{y^2} = 1 + p + q - 2(p - q)$

$\Rightarrow \log \frac{10x}{y^2} = 1 - p + 3q$

Answer 13.

$\log \frac{a^3}{b^2} = \log a^3 - \log b^2 = 3\log a - 2\log b$

$= 3p - 2q$

Answer 14.

$$\log\frac{x^2}{10y} = \log x^2 - \log 10y = 2\log x - (\log 10 + \log y)$$

$$= 2\log x - \log y - 1 = 2(A+B) - (A-B) - 1 = A + 3B - 1$$

Answer 15.

(i) 10^{a-1} in terms of x

$$\log x = a \Rightarrow x = 10^a$$

$$\therefore 10^{a-1} = \frac{10^a}{10} = \frac{x}{10}$$

(ii) 10^{2b} in terms of y

$$\log y = b \Rightarrow y = 10^b$$

$$\therefore 10^{2b} = \left(10^b\right)^2 = y^2$$

Answer 16.

(i) 3^{2x-3} in terms of m

$$\log_3 m = x \Rightarrow m = 3^x$$

$$\therefore 3^{2x-3} = \frac{3^{2x}}{3^3} = \frac{\left(3^x\right)^2}{27} = \frac{m^2}{27}$$

(ii) $3^{1-2y+3x}$ in terms of m and n

$$\log_3 m = x \Rightarrow m = 3^x$$

$$\log_3 n = y \Rightarrow n = 3^y$$

$$\therefore 3^{1-2y+3x} = 3.3^{-2y}.3^{3x} = 3.\left(3^y\right)^{-2}.\left(3^x\right)^3$$

$$= 3.n^{-2}.m^3 = \frac{3m^3}{n^2}$$

Answer 17A.

$$2\log x + 1 = 40$$

$$\Rightarrow 2\log x = 39$$

$$\Rightarrow \log x^2 = 39$$

$$\Rightarrow x^2 = 10^{39}$$

$$\Rightarrow x = 10^{\frac{39}{2}}$$

Answer 17B.

$$2\log x + 1 = 40$$
$$\Rightarrow 2\log x + \log 10 = 40$$
$$\Rightarrow 2\log 10x = 40$$
$$\Rightarrow \log 2 \times 5x = 20$$
$$\Rightarrow \log 2 + \log 5x = 20$$
$$\Rightarrow \log 5x = 20 - \log 2$$
$$\Rightarrow \log 5x = 20 - 0.3010 \quad ......(\text{Since} \log 2 = 0.3010)$$
$$\Rightarrow \log 5x = 19.6989$$

Answer 18A.

$$\log_{10}25 = x$$
$$\Rightarrow \log_{10}5^2 = x$$
$$\Rightarrow 2\log_{10}5 = x$$
$$\Rightarrow \log_{10}5 = \frac{x}{2}$$

Answer 18B.

$$\log_{10}27 = y$$
$$\Rightarrow \log_{10}3^3 = y$$
$$\Rightarrow 3\log_{10}3 = y$$
$$\Rightarrow \log_{10}3 = \frac{y}{3}$$

Answer 19.

(i) log 18

$\log 18 = \log (2 \times 3^2) = \log 2 + \log 3^2 = \log 2 + 2 \log 3$
$= 0.3010 + (2 \times 0.4771) = 1.2552$

(ii) log 45

$\log 45 = \log (3^2 \times 5) = \log 3^2 + \log 5 = 2 \log 3 + \log 5$
$= (2 \times 0.4771) + 0.6990 = 1.6532$

(iii) log 540

$\log 540 = \log (2^2 \times 3^3 \times 5) = \log 2^2 + \log 3^3 + \log 5 = 2 \log 2$
$+ 3 \log 3 + \log 5$
$=(2 \times 0.3010) + (3 \times 0.4771) + 0.6990 = 2.7323$

(iv) $\log \sqrt{72}$

$$\log\sqrt{72} = \log(72)^{1/2} = \frac{1}{2}\log 72 = \frac{1}{2}\log\left(2^3 \times 3^2\right)$$
$$= \frac{1}{2}\log 2^3 + \frac{1}{2}\log 3^2 = \frac{3}{2}\log 2 + \frac{2}{2}\log 3 = \frac{3}{2}\log 2 + \log 3$$
$$= \left(\frac{3}{2} \times 0.3010\right) + 0.4771 = 0.9286$$

Answer 20.

$$2\log y - \log x - 3 = 0$$
$$\Rightarrow \log x = 2\log y - 3$$
$$\Rightarrow \log x = \log y^2 - 3\log 10 \qquad [\because \log 10 = 1]$$
$$\Rightarrow \log x = \log y^2 - \log 10^3$$
$$\Rightarrow \log x = \log\left(\frac{y^2}{1000}\right)$$
$$\therefore x = \frac{y^2}{1000}$$

Answer 21.

(i) log 60

$\log 60 = \log(2 \times 3 \times 10) = \log 2 + \log 3 + \log 10 = x + y + 1$

(ii) log 1.2

$\log 1.2 = \log\left(\frac{12}{10}\right) = \log 12 - \log 10 = \log(2^2 \times 3) - 1$

$= \log 2^2 + \log 3 - 1$

$= 2\log 2 + \log 3 - 1 = 2x + y - 1$

Answer 22.

(i) log 8

$\log 4 = 0.6020 \Rightarrow \log 2^2 = 0.6020 \Rightarrow 2\log 2 = 0.6020 \Rightarrow \log 2 = \frac{0.6020}{2} = 0.3010$

$\therefore \quad \log 8 = \log 2^3 = 3\log 2 = 3 \times 0.3010 = 0.9030$

(ii) log 2.5

$\log 2.5 = \log\left(\frac{10}{4}\right) = \log 10 - \log 4 = 1 - 0.6020 = 0.3980$

Answer 23.

(i) log 4

$\log 8 = \log 2^3 = 3\log 2 = 0.90 \Rightarrow \log 2 = 0.90 / 3 = 0.3$

$\therefore \quad \log 4 = \log 2^2 = 2\log 2 = 2 \times 0.30 = 0.60$

(ii) $\log\sqrt{32}$

$\log 8 = \log 2^3 = 3\log 2 = 0.90 \Rightarrow \log 2 = 0.90 / 3 = 0.3$

$\log\sqrt{32} = \frac{1}{2}\log 32 = \frac{1}{2}\log 2^5 = \frac{5}{2}\log 2 = \frac{5}{2} \times 0.30 = 0.75$

Answer 24.

$\log 27 = \log 3^3 = 3 \log 3 = 1.431 \Rightarrow \log 3 = \frac{1.431}{3} = 0.477$

$\therefore \log 9 = \log 3^2 = 2 \log 3 = 2 \times 0.477 = 0.954$

$\log 300 = \log(3 \times 100) = \log(3 \times 10^2) = \log 3 + 2\log 10 = 0.477 + 2$

$= 2.477$

Answer 25.

$$x^2 + y^2 = 6xy$$
$$\Rightarrow x^2 + y^2 - 2xy = 6xy - 2xy$$
$$\Rightarrow (x-y)^2 = 4xy$$
$$\Rightarrow \left(\frac{x-y}{2}\right)^2 = xy$$
$$\Rightarrow \left(\frac{x-y}{2}\right) = \sqrt{xy}$$

Considering log both sides, we get

$$\log\left(\frac{x-y}{2}\right) = \log(xy)^{1/2}$$
$$\Rightarrow \log\left(\frac{x-y}{2}\right) = \frac{1}{2}\log(xy)$$
$$\Rightarrow \log\left(\frac{x-y}{2}\right) = \frac{1}{2}[\log x + \log y]$$

Answer 26.

$$x^2 + y^2 = 7xy$$
$$\Rightarrow x^2 + y^2 + 2xy = 7xy + 2xy$$
$$\Rightarrow (x+y)^2 = 9xy$$
$$\Rightarrow \left(\frac{x+y}{3}\right)^2 = xy$$
$$\Rightarrow \left(\frac{x+y}{3}\right) = \sqrt{xy}$$

Considering log both sides, we get

$$\log\left(\frac{x+y}{3}\right) = \log(xy)^{1/2}$$
$$\Rightarrow \log\left(\frac{x+y}{3}\right) = \frac{1}{2}\log(xy)$$
$$\Rightarrow \log\left(\frac{x+y}{3}\right) = \frac{1}{2}[\log x + \log y]$$

Answer 27.

$$\frac{\log x}{\log 5} = \frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$$

Considering the first equality

$$\frac{\log x}{\log 5} = \frac{\log 36}{\log 6}$$

$$\Rightarrow \quad \frac{\log x}{\log 5} = \frac{\log 6^2}{\log 6} = \frac{2\log 6}{\log 6} = 2$$

$$\Rightarrow \quad \log x = 2\log 5 = \log 5^2 = \log 25$$

$$\therefore \quad x = 25$$

Considering the second equality

$$\frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$$

$$\Rightarrow \quad \frac{\log 6^2}{\log 6} = \frac{2\log 6}{\log 6} = 2 = \frac{\log 64}{\log y}$$

$$\Rightarrow \quad \log y = \frac{\log 64}{2} = \frac{\log 8^2}{2} = \frac{2\log 8}{2} = \log 8$$

$$\therefore \quad y = 8$$

Answer 28.

$$\log x^2 - \log \sqrt{y} = 1$$

$$\Rightarrow \quad \log\left(\frac{x^2}{\sqrt{y}}\right) = \log 10$$

$$\Rightarrow \quad \frac{x^2}{\sqrt{y}} = 10$$

$$\Rightarrow \quad \sqrt{y} = \frac{x^2}{10}$$

Squaring both sides, we get

$$y = \left(\frac{x^2}{10}\right)^2 = \frac{x^4}{100}$$

Now, when $x = 2$, $y = \frac{2^4}{100} = \frac{16}{100} = \frac{4}{25}$

Answer 29.

(i) x

$$2\log x + 1 = \log 360$$

$\Rightarrow \quad \log x^2 + \log 10 = \log 360$

$\Rightarrow \quad \log(10x^2) = \log 360$

$\Rightarrow \quad 10x^2 = 360$

$\Rightarrow \quad x^2 = \frac{360}{10} = 36$

$\Rightarrow \quad x = \sqrt{36} = \pm 6$

As negative value is rejected,

$\therefore \quad x = 6$

(ii) log (2x – 2)

$$2\log x + 1 = \log 360$$

$\Rightarrow \quad \log x^2 + \log 10 = \log 360$

$\Rightarrow \quad \log(10x^2) = \log 360$

$\Rightarrow \quad 10x^2 = 360$

$\Rightarrow \quad x^2 = \frac{360}{10} = 36$

$\Rightarrow \quad x = \sqrt{36} = \pm 6$

As negative value is rejected,

$\therefore \quad x = 6$

$\therefore \log(2x - 2) = \log(2.6 - 2) = \log 10 = 1$

(iii) log (3x² – 8)

$$2\log x + 1 = \log 360$$

$\Rightarrow \quad \log x^2 + \log 10 = \log 360$

$\Rightarrow \quad \log(10x^2) = \log 360$

$\Rightarrow \quad 10x^2 = 360$

$\Rightarrow \quad x^2 = \frac{360}{10} = 36$

$\Rightarrow \quad x = \sqrt{36} = \pm 6$

As negative value is rejected,

$\therefore \quad x = 6$

$\therefore \log(3x^2 - 8) = \log\{3(6)^2 - 8\} = \log(108 - 8) = \log 100 = \log 10^2$

$= 2\log 10 = 2 \times 1 = 2$

Answer 30.

$$x + \log 4 + 2\log 5 + 3\log 3 + 2\log 2 = \log 108$$

$$\Rightarrow x = \log 108 - \log 4 - 2\log 5 - 3\log 3 - 2\log 2$$

$$= \log(2^2.3^3) - \log 2^2 - \log 5^2 - \log 3^3 - \log 2^2$$

$$= \log\left(\frac{2^2.3^3}{2^2.5^2.3^3.2^2}\right) = \log\left(\frac{1}{100}\right)$$

$$\Rightarrow x = \log 1 - \log 100 = 0 - 2 = -2$$

$$\therefore x = -2$$

Answer 21A.

$$\log a^2 + \log a^{-1}$$

$$= 2\log a + (-1)\log a$$

$$= 2\log a - \log a$$

$$= \log a\,(2 - 1)$$

$$= \log a$$

Answer 31B.

$$\log b \div \log b^2$$

$$= \log b \div 2\log b$$

$$= \frac{\log b}{2\log b}$$

$$= \frac{1}{2}$$

Answer 32A.

$$\frac{\log\sqrt{8}}{8} = \frac{\log 2\sqrt{2}}{8}$$

$$= \frac{\log 2\sqrt{2}}{8}$$

$$= \frac{1}{8}\left(\log 2 + \log\sqrt{2}\right)$$

$$= \frac{1}{8}\left(\log 2 + \log 2^{\frac{1}{2}}\right)$$

$$= \frac{1}{8}\log 2 + \frac{1}{8}\log 2^{\frac{1}{2}}$$

$$= \frac{1}{8}\log 2 + \frac{1}{2}\frac{1}{8}\log 2$$

$$= \frac{1}{8}\log 2 + \frac{1}{16}\log 2$$

$$= \frac{2}{16}\log 2 + \frac{1}{16}\log 2$$

$$= \frac{3}{16}\log 2$$

Answer 32B.

$$\frac{\log\sqrt{27}+\log 8+\log\sqrt{1000}}{\log 120}$$

$$=\frac{\log(27)^{\frac{1}{2}}+\log 2^3+\log 1000^{\frac{1}{2}}}{\log(3\times 2^2\times 10)}$$

$$=\frac{\log(3)^{3\times\frac{1}{2}}+\log 2^3+\log(10)^{3\times\frac{1}{2}}}{\log 3+\log 2^2+\log 10}$$

$$=\frac{\frac{3}{2}\log 3+3\log 2+\frac{3}{2}\log(10)}{\log 3+2\log 2+\log 10}$$

$$=\frac{\frac{3}{2}\log 3+\frac{3}{2}(2\log 2)+\frac{3}{2}(1)}{\log 3+2\log 2+1}$$

$$=\frac{\frac{3}{2}[\log 3+2\log 2+1]}{\log 3+2\log 2+1}$$

$$=\frac{3}{2}$$

Answer 32C.

$$\frac{\log\sqrt{125}-\log\sqrt{27}-\log\sqrt{8}}{\log 6-\log 5}$$

$$=\frac{\log(125)^{\frac{1}{2}}-\log(27)^{\frac{1}{2}}-\log(8)^{\frac{1}{2}}}{\log 6-\log 5}$$

$$=\frac{\log(5)^{3\times\frac{1}{2}}-\log(3)^{3\times\frac{1}{2}}-\log(2)^{3\times\frac{1}{2}}}{\log 6-\log 5}$$

$$=\frac{\frac{3}{2}\log(5)-\frac{3}{2}\log(3)-\frac{3}{2}\log(2)}{\log(2\times 3)-\log 5}$$

$$=\frac{\frac{3}{2}[\log(5)-\log(3)-\log(2)]}{\log 2+\log 3-\log 5}$$

$$=\frac{\frac{3}{2}[\log(5)-\log(3)-\log(2)]}{-[\log 5-\log 3-\log 2]}$$

$$=-\frac{3}{2}$$

Answer 33.

Consider $\log\left(5^{a+b-c}\right)$

$$= (a+b-c)\log 5 = \left(\log\frac{3}{5}+\log\frac{5}{4}-2\log\sqrt{\frac{3}{4}}\right)\log 5$$

$$= \left(\log\frac{3}{5}+\log\frac{5}{4}-\log\left[\sqrt{\frac{3}{4}}\right]^2\right)\log 5 = \left(\log\frac{3}{5}+\log\frac{5}{4}-\log\frac{3}{4}\right)\log 5$$

$$= \log\left(\frac{\frac{3}{5}\times\frac{5}{4}}{\frac{3}{4}}\right)\log 5 = \log 1\times\log 5 = 0 \qquad [\because \log 1 = 0]$$

$$\therefore \quad 5^{a+b-c} = 10^0 = 1$$

Answer 34.

(i) $3\log x - 2\log y = 2$

$\Rightarrow \quad \log x^3 - \log y^2 = 2\log 10$

$\Rightarrow \quad \log\left(\frac{x^3}{y^2}\right) = \log 10^2 = \log 100$

$\Rightarrow \quad \left(\frac{x^3}{y^2}\right) = 100$

$\Rightarrow \quad x^3 = 100\,y^2$

(ii) $2\log x + 3\log y = \log a$

$\Rightarrow \quad \log x^2 + \log y^3 = \log a$

$\Rightarrow \quad \log(x^2 . y^3) = \log a$

$\Rightarrow \quad x^2y^3 = a$

(iii) $m\log x - n\log y = 2\log 5$

$\Rightarrow \quad \log x^m - \log y^n = \log 5^2$

$\Rightarrow \quad \log\left(\frac{x^m}{y^n}\right) = \log 5^2$

$\Rightarrow \quad \left(\frac{x^m}{y^n}\right) = 5^2 = 25$

$\Rightarrow \quad x^m = 25\,y^n$

(iv) $2\log x + \frac{1}{2}\log y = 1$

$\Rightarrow \quad \log x^2 + \log \sqrt{y} = \log 10$

$\Rightarrow \quad \log (x^2\sqrt{y}) = \log 10$

$\Rightarrow \quad x^2 \sqrt{y} = 10$

(v) $5 \log m - 1 = 3 \log n$

$\Rightarrow \quad \log m^5 - \log 10 = \log n^3$

$\Rightarrow \quad \log\left(\frac{m^5}{10}\right) = \log n^3$

$\Rightarrow \quad \left(\frac{m^5}{10}\right) = n^3$

$\Rightarrow \quad m^5 = 10 n^3$

Answer 35.

$\log (1+2+3) = \log 6$

$= \log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$

No, this property is not true for any numbers x, y, z

For example, $\log (1+3+5) = \log 9$

$\log 1 + \log 3 + \log 5 = \log (1 \times 3 \times 5) = \log 15$

$\log (1+3+5) \neq \log 1 + \log 3 + \log 5$

Answer 36.

L.H.S. = $(\log a)^2 - (\log b)^2$

$= (\log a + \log b)(\log a - \log b) \quad \{\text{using identity } m^2 - n^2 = (m+n)(m-n)\}$

$= \log(ab)\log\left(\frac{a}{b}\right) = \log\left(\frac{a}{b}\right).\log(ab) = \text{R.H.S}$

Answer 37.

Given $a\log b + b\log a - 1 = 0$

$\Rightarrow \quad \log(b)^a + \log(a)^b - \log 10 = 0$

$\Rightarrow \quad \log(b^a . a^b) = \log 10$

$\therefore \quad b^a . a^b = 10$

Answer 38.

$$\log(a+1) = \log(4a-3) - \log 3$$
$$\Rightarrow \log(a+1) + \log 3 = \log(4a-3)$$
$$\Rightarrow \log\{3(a+1)\} = \log(4a-3)$$
$$\Rightarrow 3(a+1) = 4a - 3$$
$$\Rightarrow 3a + 3 = 4a - 3$$
$$\Rightarrow 4a - 3a = 3 + 3$$
$$\therefore a = 6$$

Answer 39.

$$\text{L.H.S} = \log_{10} 125$$
$$= \log_{10}\left(\frac{1000}{8}\right)$$
$$= \log_{10} 1000 - \log_{10} 8$$
$$= \log_{10} (10)^3 - \log_{10} (2)^3$$
$$= 3\log_{10} 10 - 3\log_{10} 2$$
$$= 3 \times 1 - 3\log_{10} 2$$
$$= 3(1 - \log_{10} 2) = \text{R.H.S.}$$

Answer 40.

$$\text{LHS} = \frac{\log_p x}{\log_{pq} x}$$
$$= \frac{\left(\frac{\log x}{\log p}\right)}{\left(\frac{\log x}{\log pq}\right)}$$
$$= \frac{\log x}{\log p} \times \frac{\log pq}{\log x}$$
$$= \frac{\log pq}{\log p}$$
$$= \frac{\log p + \log q}{\log p}$$
$$= 1 + \frac{\log q}{\log p}$$
$$= 1 + \log_p q$$
$$= \text{RHS}$$

Hence proved.

Answer 41A.

$$\text{LHS} = \frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30}$$

$$= \frac{1}{\frac{\log 30}{\log 2}} + \frac{1}{\frac{\log 30}{\log 3}} + \frac{1}{\frac{\log 30}{\log 5}}$$

$$= \frac{\log 2}{\log 30} + \frac{\log 3}{\log 30} + \frac{\log 5}{\log 30}$$

$$= \frac{1}{\log 30}(\log 2 + \log 3 + \log 5)$$

$$= \frac{1}{\log(2 \times 3 \times 5)}(\log 2 + \log 3 + \log 5)$$

$$= \frac{(\log 2 + \log 3 + \log 5)}{(\log 2 + \log 3 + \log 5)}$$

$$= 1$$

$$= \text{LHS}$$

Hence proved.

Answer 41B.

$$\text{LHS} = \frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36}$$

$$= \log_{36} 8 + \log_{36} 9 + \log_{36} 18$$

$$= \frac{\log 8}{\log 36} + \frac{\log 9}{\log 36} + \frac{\log 18}{\log 36}$$

$$= \frac{1}{\log 36}(\log 8 + \log 9 + \log 18)$$

$$= \frac{1}{\log 36}(\log 2^3 + \log 3^2 + \log(2 \times 3^2))$$

$$= \frac{1}{\log(2^2 \times 3^2)}(\log 2^3 + \log 3^2 + \log 2 + \log 3^2)$$

$$= \frac{1}{\log(2^2 \times 3^2)}(3\log 2 + 2\log 3 + \log 2 + 2\log 3)$$

$$= \frac{1}{2\log 2 + 2\log 3}(4\log 2 + 4\log 3)$$

$$= \frac{4}{2(\log 2 + \log 3)}(\log 2 + \log 3)$$

$$= 2$$

$$= \text{RHS}$$

Hence proved.

Answer 42.

$$a = \log\frac{p^2}{qr}, b = \log\frac{q^2}{rp}, c = \log\frac{r^2}{pq}$$

Consider,

$a + b + c$

$= \log\frac{p^2}{qr} + \log\frac{q^2}{rp} + \log\frac{r^2}{pq}$

$= \log p^2 - \log qr + \log q^2 - \log rp + \log r^2 - \log pq$

$= 2\log p - (\log q + \log r) + 2\log q - (\log r + \log p) + 2\log r - (\log p + \log q)$

$= 2\log p - \log q - \log r + 2\log q - \log r - \log p + 2\log r - \log p - \log q$

$= 0$

Answer 43.

$a = \log 20, \quad b = \log 25$ and

$2\log(p-4) = 2a - b$

$\Rightarrow 2\log(p-4) = 2\log 20 - \log 25$

$\Rightarrow \log(p-4)^2 = \log 20^2 - \log 25$

$\Rightarrow \log(p-4)^2 = \log\left(\frac{400}{25}\right)$

$\Rightarrow (p-4)^2 = \frac{400}{25}$

$\Rightarrow p^2 - 8p + 16 = 16$

$\Rightarrow p^2 - 8p = 0$

$\Rightarrow p(p-8) = 0$

$\Rightarrow p = 0$ or $p = 8$