

Reg. No. : .....

Code No. 8018

Name : .....

For Scheme-I Candidates only

**Second Year – 2015  
SAY / IMPROVEMENT**

Time : 2½ Hours  
Cool-off time : 15 Minutes

Part – III

**MATHEMATICS (SCIENCE)**

Maximum : 80 Scores

**General Instructions to Candidates :**

- There is a 'cool-off time' of 15 minutes in addition to the writing time of 2½ hrs.
- You are not allowed to discuss anything with others during the 'cool-off time'.
- Use the 'cool-off time' to plan your answers.
- Read question carefully.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

**നിർദ്ദേശങ്ങൾ :**

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും. ഈ സമയത്ത് ചോദ്യങ്ങൾക്ക് ഉത്തരം എഴുതാനോ, മറ്റുള്ളവരുമായി ആശയവിനിമയം നടത്താനോ പാടില്ല.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- എല്ലാ ചോദ്യങ്ങൾക്കും ഉത്തരം എഴുതണം.
- ഒരു ചോദ്യനമ്പർ ഉത്തരമെഴുതാൻ തെരഞ്ഞെടുത്തു കഴിഞ്ഞാൽ ഉപചോദ്യങ്ങളും അതേ ചോദ്യനമ്പറിൽ നിന്ന് തന്നെ തെരഞ്ഞെടുക്കേണ്ടതാണ്.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നൽകിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

1. (a) When a relation  $R$  on a set  $A$  is said to be reflexive ? (Score : 1)
- (b) Find  $\text{gof}$  and  $\text{fog}$ , if  $f(x) = 8x^3$  and  $g(x) = x^{\frac{1}{3}}$ . (Scores : 2)
- (c) Show that  $f : [-1, 1] \longrightarrow \mathbb{R}$  given by  $f(x) = \frac{x}{x+2}$  is one-one. (Scores : 2)
2. (a) Write the principal value of  $\cos^{-1}\left(\frac{-1}{2}\right)$ . (Score : 1)
- (b) Express  $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ ,  $x \neq 0$  in the simplest form. (Scores : 3)
3. Consider a  $2 \times 2$  matrix  $A = [a_{ij}]$  with  $a_{ij} = 2^i + j$ .
- (a) Construct  $A$  (Score : 1)
- (b) Find  $A + A'$  and  $A - A'$  (Score : 1)
- (c) Express  $A$  as sum of a symmetric and skew-symmetric matrix. (Score : 1)
4. (a) Find the relationship between 'a' and 'b' if the function  $f$  defined by
- $$f(x) = ax + 1, \quad x \leq 3$$
- $$= bx + 3, \quad x > 3$$
- is continuous (Scores : 2)
- (b) Find  $\frac{dy}{dx}$ , if  $y^x = x^y$ . (Scores : 2)
- (c) If  $e^y(x+1) = 1$ , show that  $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ . (Scores : 2)

5. Consider the matrix  $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ .

(a) Find  $A^2$

(Scores : 2)

(b) Find  $k$  so that  $A^2 = kA - 2I$

(Score : 1)

6. (a) If  $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = 15$ , then find the values of  $x$ .

(Score : 1)

(b) Solve the following system of equations :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

(Scores : 4)

7. (a) Find the equation of the tangent to the curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$  at  $(1, 1)$ .

(Scores : 2)

(b) Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

(Scores : 3)

8. (a)  $\int \frac{2x}{1+x^2} dx = \underline{\hspace{2cm}}$

(Score : 1)

(b) Find  $\int \frac{1}{x^2 - 6x + 13} dx$

(Scores : 2)

(c) Find  $\int x^2 e^{x^3} dx$

(Scores : 2)

9. Consider the vectors  $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$  and  $\vec{b} = -\hat{i} + 7\hat{k}$

(a) Find  $\vec{a} + \vec{b}$

(Score : 1)

(b) Find a unit vector in the direction of  $\vec{a} + \vec{b}$ .

(Scores : 2)

10. (a) Evaluate  $\int_1^2 \frac{x}{(x+1)(x+2)} dx$ . (Scores : 2)

(b) Evaluate  $\int_0^{\pi} \frac{x}{1 + \sin x} dx$ . (Scores : 3)

**OR**

Evaluate  $\int_0^2 e^x dx$  as the limit of a s. (Scores : 5)

11. Consider the triangle ABC with vertices A(1, 2, 3), B(-1, 0, 4) and C(0, 1, 2)

(a) Find  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ . (Score : 1)

(b) Find  $\angle A$ . (Scores : 2)

(c) Find the area of triangle ABC. (Scores : 2)

12. (a) Find the area of the region enclosed by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (Scores : 3)

(b) Find the area of the region bounded by the parabolas  $y = x^2$  and  $y^2 = x$ . (Scores : 3)

13. (a) Write the order and degree of the differential equation

$$xy \left( \frac{d^2y}{dx^2} \right)^2 + x \left( \frac{dy}{dx} \right)^3 - y \frac{dy}{dx} = 0$$
 (Score : 1)

(b) Find the general solution of the differential equation  $y \log y dx - x dy = 0$ . (Scores : 3)

(c) Find the integrating factor of the differential equation  $x \frac{dy}{dx} - y = 2x^2$ . (Scores : 2)

14. (a) Find the value of 'p' if the lines

$$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1} \text{ and } \frac{x}{1} = \frac{y}{p} = \frac{z}{3} \text{ are perpendicular.}$$

(Score : 1)

- (b) Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}) \text{ and}$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

(Scores : 3)

15. Consider the linear programming problem :

$$\text{Maximize } z = 4x + y$$

Subject to the constraints :

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x \geq 0, y \geq 0$$

- (a) Draw its feasible region

(Scores : 3)

- (b) Find the corner points of the feasible region.

(Scores : 2)

- (c) Find the corner at which z attains its maximum

(Score : 1)

16. Consider a plane which passes through the point (5, 2, -4) and perpendicular to the line

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$$

- (a) Write its equation in Cartesian form.

(Score : 1)

- (b) Find its distance from the point (1, 2, -1)

(Score : 1)

- (c) Find the angle made by it with the line

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$$

(Scores : 2)



17. (a) A die is thrown thrice. Find the probability of getting an odd number atleast once.

(Scores : 2)

- (b) Two cards are drawn successively with replacement from a pack of 52 cards. Find the probability distribution of the number of aces.

(Scores : 3)

**OR**

- (a) If  $P(A) = 0.8$ ,  $P(B) = 0.5$  and  $P\left(\frac{B}{A}\right) = 0.4$ , then find  $P(A \cup B)$ .

(Scores : 2)

- (b) If a fair coin is tossed 10 times, then find the probability of getting exactly 6 heads.

(Scores : 3)