

MATHEMATICS PAPER IIA - MAY 2008

ALGEBRA & PROBABILITY

TIME : 3hrs

Max. Marks.75

Note: This question paper consists of three sections A,B and C.

SECTION A

Very Short Answer Type Questions.

10X2 =20

Note : Attempt all questions. Each question carries 2 marks.

1. If the equation $x^2 - 15 - m(2x - 8) = 0$ has equal roots find the value of m.
2. If the product of two of its roots of $4x^3 + 16x^2 - 9x - a = 0$ is 3, find a.
3. If $A = \begin{bmatrix} -2 & 1 & 0 \\ 3 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \\ -1 & 5 \end{bmatrix}$ then find $A+B^T$.
4. Show that $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & c-a \\ c-a & a-b & b-c \end{vmatrix} = 0$
5. If ${}^nP_3 = 1320$ then find 'n'
6. Find the number of ways of arranging all the letters of the word "MISSAMMA".
7. Find the term independent of 'x' in the expansion of $\left(\sqrt{\frac{x}{7}} - \frac{\sqrt{5}}{x^2}\right)^{10}$
8. Find the sum of the series $\frac{1}{5} + \frac{1}{2.5^2} + \frac{1}{3.5^3} + \frac{1}{4.5^4} + \dots$
9. Two fair dice are rolled. What is the probability that the sum on the faces of the two dice is 7?
10. If X is poisson variate such that $P(X = 1) = 3P(X = 2)$ then find the variance of X.

SECTION B

Short Answer Type Questions.

5X4 =20

Note : Answer any FIVE questions. Each question carries 4 marks.

11. If x is real, find the range of $\frac{x+2}{2x^2+3x+6}$
12. If n is a positive integer and $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ then show that $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$
13. Find the number of ways of arranging 5 different Mathematics books, 4 different Physics and 3 different chemistry books such that the books of the same subject are together.
14. Show that $\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1.3.5.....(4n-1)}{\{1.3.5.....(2n-1)\}^2}$
15. Resole $\frac{x^3}{(x-a)(x-b)(x-c)}$ into partial fractions.
16. Find the sum of the series $1 + \frac{2x}{1} + \frac{2x^2}{2} + \frac{4x^3}{3} + \dots$
17. If E_1, E_2 are any two events of a random experiment and P is a probability function, then $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.

SECTION C

Long Answer Type Questions.

5X7 =35

Note: Answer any Five of the following. Each question carries 7 marks.

18. If the roots of $x^3 + 3px^2 + 3qx + r = 0$ are in A.P show that $2p^3 - 3qp + r = 0$.
19. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$
20. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ then show that the adjoint of A is $3A^T$. Find A^{-1} .

21. If n is a positive integer and x is any nonzero real number, then prove that

$$C_0 + C_1 \cdot \frac{x}{2} + C_2 \cdot \frac{x^2}{3} + C_3 \cdot \frac{x^3}{4} + \dots + C_n \cdot \frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

22. If $x = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + \dots \infty$ then find the value of $3x^2 + 6x$

23. A, B, C are aiming to shoot a balloon. A will succeed 4 times out of 5 attempts. The chance of B to shoot the balloon is 3 out of 4 and that of C is 2 out of 3. If the three aim the balloon simultaneously, then find the probability that at least two of them hit the balloon.

24. A random variable X has the following probability distribution.

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) then mean and (iii) $P(0 < X < 5)$
