HOTS (Higher Order Thinking Skills)

Que 1. If Fig. 6.33, ∠Q> ∠R, PA is the bisector Of ∠QPR and PM ⊥ QR. Prove that ∠APM = $\frac{1}{2}$ (∠Q - ∠R).

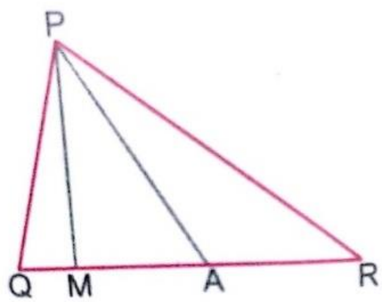

Fig. 6.33

Sol. Since PA is the bisector of ∠QPR

∴ ∠QPA = ∠APR …(i)

In ΔPQM, we have

∠PQM + ∠PMQ + ∠QPM = 180^0 (Angle sum property)

⇒ ∠PQM + 90^0 + ∠QPM = 180^0

⇒ ∠PQM + 90^0 - ∠QPM …(ii)

In ΔPMR, we have

∠PMR + ∠PRM + ∠RPM = 180^0

⇒ 90^0 + ∠PRM + ∠RPM = 180^0

⇒ ∠PRM = 180^0 - 90^0 - ∠RPM

⇒ ∠R = 90^0 - ∠RPM …(iii)

Subtracting (iii) from (ii), we get

∠Q - ∠R = (90^0 - ∠QPM) – (90^0 - ∠RPM)

∠Q - ∠R = ∠RPM - ∠QPM

∠Q - ∠R = (∠RPA + ∠APM) – (∠QPA - ∠APM) …(iv)

∠Q - ∠R = ∠QPA + ∠APM - ∠QPA + ∠APM [Using (i)]

⇒ ∠Q - ∠R = 2 ∠APM

Hence, ∠APM = $\frac{1}{2}$ (∠Q - ∠R).

Que 2. In Fig. 6.34, the side QR of ΔPQR is produced to point S. If the bisector of ∠PQR and ∠PRS meet at point T. then prove that ∠QTR = $\frac{1}{2}$ ∠QPR.

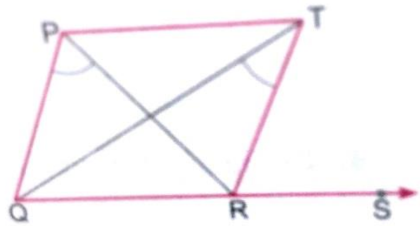

Fig. 6.34

Sol. Given: A Δ PQR, whose side QR is produced to S. The bisectors of ∠PQR and ∠PRS meet at point T.

To prove: ∠PRS = $\frac{1}{2}$ ∠QPR

Proof: Side QR of Δ PQR is produced to S.

∴ ∠PRS = ∠P + ∠Q ⇒ $\frac{1}{2}$ ∠PRS = $\frac{1}{2}$ ∠P + $\frac{1}{2}$ ∠Q

⇒ ∠TRS = $\frac{1}{2}$ ∠P + $\frac{1}{2}$ ∠Q ...(i)

Again, side QR of Δ TQR is produced to S

∴ ∠TRS = ∠QTR + ∠RQT

⇒ ∠TRS = ∠T + $\frac{1}{2}$ ∠Q ...(ii)

From (i) and (ii), we get

$\frac{1}{2}$ ∠P + $\frac{1}{2}$ ∠Q = ∠T + $\frac{1}{2}$ ∠Q

⇒ ∠T = $\frac{1}{2}$ ∠P or ∠QTR = $\frac{1}{2}$ ∠QPR

Que 3. Prove that a triangle must have atleast two acute angles.

Sol. Let us assume a triangle ABC which has only one acute angle (say ∠A)

Then we have the following three cases:

(i) The other two angles (∠B and ∠C) are right angle.

Then ∠A + ∠B + ∠C = ∠A + 90^0 + 90^0 = ∠A + 180^0 > 180^0 which is not possible.

(ii) The other two angles (∠B and ∠C) are obtuse angles.

Then $\angle A + \angle B + \angle C > 180^0$ which is not possible.

(iii) One angle (say $\angle B$) is right and the other angle ($\angle C$) is obtuse.

Then $\angle A + \angle B + \angle C > 180^0$ which is not possible as we know that sum of the three angles of a triangle is 180^0 by angle sum property of a triangle.

Thus, a triangle must have atleast two acute angles.