CBSE Board
Class XII Mathematics
Sample Paper 2

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. **All** the questions are **compulsory**.
2. The question paper consists of **37** questions divided into **three parts** A, B, and C.
3. **Part A** comprises of **20** questions of **1 mark** each. **Part B** comprises of **11** questions of **4 marks** each. **Part C** comprises of **6** questions of **6 marks** each.
4. There is no overall choice. However, an internal choice has been provided in **three questions of 4 marks** each, **four questions of 6 marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is **not** permitted.

Section A

Q 1 – Q 20 are multiple choice type questions. Select the correct option.

1. If A and B are two events associated to a random experiment such that $P(A \cap B) = \frac{7}{10}$ and $P(B) = 17/20$, then $P(A/B) =$

A. $\frac{14}{17}$

B. $\frac{17}{14}$

C. $\frac{4}{7}$

D. $\frac{7}{4}$

2. A parallelepiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes. The length of a diagonal of the parallelepiped is

A. 40

B. $\sqrt{38}$

C. $\sqrt{155}$

D. 7

3. The value of $\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ is equal to

A. $\frac{\pi}{4}$

B. -1

C. 1

D. $-\frac{\pi}{4}$

4. The distance of a point P(a, b, c) from the x-axis is

A. $\sqrt{b^2 + c^2}$

B. $\sqrt{a^2 + c^2}$

C. $\sqrt{a^2 + b^2}$

D. none of these

5. If the function $f(x) = 2x^2 - kx + 5$ is increasing on [1, 2], then k lies in the interval:

A. $[0, 2]$

B. $(-\infty, 4)$

C. $(4, \infty)$

D. $(-4, 4)$

6. Find the number of all possible matrices of order 3 × 3 with each entry 0 or 1.

A. 64

B. 128

C. 256

D. 512

7. If $x < 0$, $y < 0$ such that $xy = 1$, then $\tan^{-1}x + \tan^{-1}y$ equals

A. 0

B. $\frac{\pi}{2}$

C. $-\frac{\pi}{2}$

D. $\frac{3\pi}{2}$

8. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then find the position vector of their mid-point.

A. $3\hat{i} + \hat{j} - 4\hat{k}$

B. $3\hat{i} - \hat{j} + 4\hat{k}$

C. $\hat{i} + 3\hat{j} + 4\hat{k}$

D. $-\hat{i} - \hat{j} - \hat{k}$

9. Which of the following is the integrating factor of the differential equation

$\cos^2 x \frac{dy}{dx} + y = \tan x$?

A. tan x

B. sec x tan x

C. $e^{\tan x}$

D. $e^{\sec x}$

10. The area bounded by the curve $x^2 = 4y$ and $y = 4$ in the first quadrant is

A. $\frac{32}{3}$ sq. units

B. $\frac{4}{3}$ sq. units

C. $\frac{16}{3}$ sq. units

D. $\frac{2}{3}$ sq. units

11. If $y = 10^{10^x}$ then $\frac{dy}{dx}$ is equal to

A. $10^{10^x} \cdot 10^x (\log 10)^2$

B. $10^x (\log 10)^2$

C. $10^{10^x} \cdot 10^x (\log 10)$

D. $10^{10^x} (\log 10)^2$

12. Two vectors $\vec{a}$ and $\vec{b}$ have the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$. Then their magnitude is

A. $\sqrt{3}$

B. 3

C. 1

D. $3\sqrt{3}$

13. The function $f(x) = \frac{-x}{2} + \sin x$ defined on $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ is increasing on

A. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

C. $\left[-\frac{\pi}{3},\frac{\pi}{3}\right]$

D. $\left[0,\frac{\pi}{2}\right]$

14. The number of binary operations that can be defined on a set of 2 elements is

A. 8
B. 4
C. 1
D. 16

15. If $f(x) = (x + 1)^{\cot x}$ be continuous at $x = 0$, then $f(0)$ is equal to

A. 1
B. ∞
C. e
D. log e

16. Find the general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$.

A. $y=\frac{k}{x}$
B. $y = kx$
C. $k = 0$
D. $y = e^{x}k$

17. Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$. Then, the value of $5a + 4b + 3c + 2d + e$ is equal to:

A. 0
B. 11
C. −11
D. 1

18. Find $\int\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$.

A. $-2 \tan x \log(\cos x) + c$
B. $\tan x - x + c$
C. $\tan x + c$
D. $x + c$

19. Let R = {(a, a^3): a is a prime number less than 5} be a relation. Then the range of R is

A. {2, 3}
B. {8, 27}
C. R
D. {4, 9}

20. Evaluate $\int_{1}^{\sqrt{3}} \frac{1}{1+x^2} dx$.

A. $-\frac{\pi}{6}$

B. $\frac{\pi}{6}$

C. $-\frac{\pi}{12}$

D. $\frac{\pi}{12}$

Section B

21. Find the family of curves passing through the point (x, y) for which the slope of the tangent is equal to the sum of y-coordinate and exponential raise to the power of x-coordinate.

OR

Form the differential equation of the family of curves y = A cos 2x + B sin 2x, where A and B are constants.

22. Evaluate: $\int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx$

OR

Evaluate $\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$.

23. Show that the function, $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ 2, & x = 0 \end{cases}$

is continuous at x = 0.

OR

If sin y = x sin(a + y), prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

24. Let f: N → N be defined by $f(n) = \begin{cases} \dfrac{n+1}{2} & \text{if } n \text{ is odd} \\ \dfrac{n}{2} & \text{if } n \text{ is even} \end{cases}$

Find whether the function f is bijective or not.

25. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum numbers of times must he fire so that the probability of hitting the target at least once is more than 0.99?

26. If $\tan^{-1}\left(\dfrac{2x-4}{2x-3}\right) + \tan^{-1}\left(\dfrac{2x+4}{2x+3}\right) = \dfrac{\pi}{4}$, then find the value of x.

27. Without expanding the determinant prove that $\begin{vmatrix} 3x+y & 2x & x \\ 4x+3y & 3x & 3x \\ 5x+6y & 4x & 6x \end{vmatrix} = x^3$.

28. If $x = \dfrac{a}{1+t^3}$ and $y = \dfrac{at}{1+t^3}$, then find $\dfrac{dy}{dx}$.

29. Find the value of $\int_0^4 (x-3)(x-1)\,dx$.

30. If the vectors $\vec{a} \neq \vec{0}, \vec{b}$ and $\vec{c}$ have magnitude 1, 1 and 4 respectively such that $\left|\vec{b} \times \vec{c}\right| = \sqrt{15}$ and $\vec{c} - 2\vec{b} = \lambda\vec{a}$. Then find the value(s) of λ.

31. A line passes through the point (−1, 3, −4) and it is also perpendicular to the plane x + 2y − 5z + 9 = 0. Find the equation of the line.

Section C

32. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4, and y = 0 into three equal parts.

OR

Find the area of the region $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$.

33. Find the volume of the largest cylinder which can be inscribed in a sphere of radius r.

OR

Find the equations of the normals to the curve $3x^2 - y^2 = 8$, parallel to the line x + 3y = 4.

34. Find the inverse of $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ by elementary row transformation.

OR

Given two matrices, $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, verify that BA = 6I, use the result to solve the system x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7

35. A variable plane which remains at a constant distance 3k from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of ΔABC is $x^{-2} + y^{-2} + z^{-2} = k^{-2}$.

36. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model A and Rs. 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

37. A bag contains 25 balls of which 10 are purple and the remaining are pink. A ball is drawn at random, its colour is noted and it is replaced. 6 balls are drawn in this way, find the probability that

i. All balls were purple
ii. Not more than 2 were pink
iii. An equal number of purple and pink balls were drawn.
iv. Atleast one ball was pink

OR

A doctor is to visit a patient. Form past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively. But if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that the doctor came by train?