1. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\frac{dy}{dx}$ શોધો. : $x = 2at^2$, $y = at^4$**

➥ $x = 2at^2 \Rightarrow \frac{dx}{dt} = 4at$

$y = at^4 \Rightarrow \frac{dy}{dt} = 4at^3$

હવે $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4at^3}{4at} = t^2$

2. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\frac{dy}{dx}$ શોધો. : $x = a \cos\theta$, $y = b \cos\theta$**

➥ $x = a \cos\theta \Rightarrow \frac{dx}{d\theta} = -a\sin\theta$

$y = b \cos\theta \Rightarrow \frac{dy}{d\theta} = -b\sin\theta$

હવે $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-b\sin\theta}{-a\sin\theta} = \frac{b}{a}$

3. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\frac{dy}{dx}$ શોધો. : $x = \sin t$, $y = \cos 2t$**

➥ $x = \sin t \Rightarrow \frac{dx}{dt} = \cos t$

$y = \cos 2t \Rightarrow \frac{dy}{dt} = -2\sin 2t$

હવે $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{\cos t}$

$= \frac{-4\sin t\ \cos t}{\cos t}$

$= -4\ \sin t$

4. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\frac{dy}{dx}$ શોધો. : $x = 4t$, $y = \frac{4}{t}$**

➥ $x = 4t \Rightarrow \frac{dx}{dt} = 4$

$y = \frac{4}{t} \Rightarrow \frac{dy}{dt} = -\frac{4}{t^2}$

હવે $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-4}{t^2}}{4} = -\frac{1}{t^2}$

5. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\frac{dy}{dx}$ શોધો. : $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$**

➥ $x = \cos\theta - \cos 2\theta \Rightarrow \frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$

$y = \sin\theta - \sin 2\theta \Rightarrow \dfrac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$

હવે $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$

$= \dfrac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$

6. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\dfrac{dy}{dx}$ શોધો. : $x = a\ \sec\theta,\quad y = b\tan\theta$**

➡ $x = a\ \sec\theta \Rightarrow \dfrac{dx}{d\theta} = a\ \sec\theta\ \tan\theta$

$y = b\tan\theta \Rightarrow \dfrac{dy}{d\theta} = b\ \sec^2\theta$

હવે $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{b\sec^2\theta}{a\sec\theta \cdot \tan\theta}$

$= \dfrac{b}{a}\operatorname{cosec}\theta$

7. $\dfrac{dy}{dx}$ **મેળવો** : $x = \cos^2\theta$ અને $y = \sin^2\theta$

➡ **−1**

8. $\dfrac{dy}{dx}$ **મેળવો** : $x = t + \dfrac{1}{t}$ અને $y = t - \dfrac{1}{t}$

➡ $\dfrac{t^2 + 1}{t^2 - 1}$

9. $\dfrac{dy}{dx}$ **મેળવો** : $x = te^t,\ y = 1 + \log t$

➡ $\dfrac{1}{te^t(1 + t)}$

10. $\dfrac{dy}{dx}$ **મેળવો** : $x = a\ \sec^3\theta,\ y = a\ \tan^3\theta$

➡ **$\sin\theta$**

11. $\dfrac{dy}{dx}$ **મેળવો** : $x = a\ \sin^2\theta\ \cos\theta,\ y = 2b\ \cos^2\theta\ (-\sin\theta)$

➡ $-\dfrac{b^{\frac{2}{3}}\ y^{\frac{1}{3}}}{a^{\frac{2}{3}}\ x^{\frac{1}{3}}}$

12. $\dfrac{dy}{dx}$ **મેળવો** : $x = 2\cos\theta - \cos^2\theta$ અને $y = 2\sin\theta - \sin 2\theta$ માટે સાબિત કરો કે $\dfrac{dy}{dx} = -1$ જ્યારે $\theta = \dfrac{\pi}{2}$.

➡ સ્વપ્રયત્ને

13. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\dfrac{dy}{dx}$ શોધો. : $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$**

➡ $x = a(\theta - \sin\theta) \Rightarrow \dfrac{dx}{d\theta} = \dfrac{d}{d\theta}[a(\theta - \sin\theta)]$

$= a[1 - \cos\theta]$

$$y = a(1 + \cos\theta) \Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta}[a(1+\cos\theta)]$$
$$= a(-\sin\theta)$$

હવે $$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$
$$= -\cot\frac{\theta}{2}$$

14. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\frac{dy}{dx}$ શોધો. :**

$$x = a\left(\cos t + \log\tan\frac{t}{2}\right),\ y = a\sin t$$

➡ $$x = a\left(\cos t + \log\tan\frac{t}{2}\right)$$
$$\therefore \frac{dx}{dt} = a\left(-\sin t + \frac{1}{\tan\frac{t}{2}}\cdot\sec^2\frac{t}{2}\cdot\frac{1}{2}\right)$$
$$= a\left(-\sin t + \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}\cdot\frac{1}{2\cos^2\frac{t}{2}}\right)$$
$$= a\left(-\sin t + \frac{1}{2\cos\frac{t}{2}\sin\frac{t}{2}}\right)$$
$$= a\left(-\sin t + \frac{1}{\sin t}\right)$$
$$= a\left(-\frac{\sin^2 t + 1}{\sin t}\right)$$
$$= \frac{a\cos^2 t}{\sin t} \qquad \text{..........(i)}$$
$$y = a\sin t$$
$$\therefore \frac{dy}{dt} = a\cos t \qquad \text{..........(ii)}$$

હવે $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\cos^2 t}\sin t$$ ($\because$ (i) અને (ii) ઉપરથી)
$$= \tan t$$

15. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\frac{dy}{dx}$ શોધો. : $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$**

➡ $$x = a(\cos\theta + \theta\sin\theta)$$
$$\therefore \frac{dx}{d\theta} = a\cdot\frac{d}{d\theta}(\cos\theta + \theta\sin\theta)$$
$$\therefore \frac{dx}{d\theta} = a[-\sin\theta + \theta\cos\theta + \sin\theta]$$
$$= a\theta\cos\theta \qquad \text{.........(i)}$$
$$y = a(\sin\theta - \theta\cos\theta)$$
$$\therefore \frac{dy}{d\theta} = a\cdot\frac{d}{d\theta}(\sin\theta - \theta\cos\theta)$$
$$= a[\cos\theta - \cos\theta + \theta\sin\theta]$$
$$= a\theta\sin\theta \qquad \text{.........(ii)}$$

હવે $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$= \dfrac{a\,\theta\,\sin\theta}{a\,\theta\,\cos\theta}$ ($\because$ પરિણામ (i) અને (ii) ઉપરથી)

$= \tan\theta$

16. $\dfrac{dy}{dx}$ **મેળવો :** $x = e^{\cos 2t}$ અને $y = e^{\sin 2t}$ હોય તો સાબિત કરો કે, $\dfrac{dy}{dx} = \dfrac{-y\log x}{x\log y}$.

➥ સ્વપ્રયત્ને

17. $\dfrac{dy}{dx}$ **મેળવો :** $x = a\sin 2t\,(1 + \cos 2t)$ અને $y = b\cos 2t\,(1 - \cos 2t)$ હોય તો સાબિત કરો કે,

$\left(\dfrac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \dfrac{b}{a}$.

➥ સ્વપ્રયત્ને

18. **x અને y પ્રચલ સમીકરણ સ્વરૂપે આપેલ હોય તો પ્રચલનો લોપ કર્યા વગર $\dfrac{dy}{dx}$ શોધો. :**

$$x = \frac{\sin^3 t}{\sqrt{\cos 2t}},\ y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$$

➥ $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$

$$\therefore \frac{dx}{dt} = \frac{\sqrt{\cos 2t}\cdot\frac{d}{dt}(\sin^3 t) - \sin^3 t\cdot\frac{d}{dt}\sqrt{\cos 2t}}{\left(\sqrt{\cos 2t}\right)^2}$$

$$= \frac{\sqrt{\cos 2t}\cdot 3\sin^2 t\cos t - \sin^3 t\cdot\left(\frac{1}{2\sqrt{\cos 2t}}\cdot(-2\sin 2t)\right)}{\cos 2t}$$

$$= \frac{3\sin^2 t\cdot\cos t\cdot\sqrt{\cos 2t} + \frac{\sin^3 t\cdot\sin 2t}{\sqrt{\cos 2t}}}{\cos 2t}$$

$$= \frac{3\sin^2 t\cdot\cos t\cdot\cos 2t + \sin^3 t\cdot\sin 2t}{(\cos 2t)^{\frac{3}{2}}} \quad ..(i)$$

$y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\therefore \frac{dy}{dt} = \frac{\sqrt{\cos 2t}\cdot\frac{d}{dt}(\cos^3 t) - \cos^3 t\frac{d}{dt}(\sqrt{\cos 2t})}{(\sqrt{\cos 2t})^2}$$

$$= \frac{-\sqrt{\cos 2t}\cdot 3\cos^2 t\sin t + \cos^3 t\cdot\frac{1}{2\sqrt{\cos 2t}}2\sin 2t}{(\sqrt{\cos 2t})^2}$$

$$= \frac{-3\cos^2 t\cdot\sin t\cos 2t + \cos^3 t\cdot\sin 2t}{(\cos 2t)^{\frac{3}{2}}} \quad .(ii)$$

હવે $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$= \dfrac{\cos^3 t\cdot\sin 2t - 3\cos^2 t\,\sin t\,\cos 2t}{3\sin^2 t\cdot\cos t\cdot\cos 2t + \sin^3 t\cdot\sin 2t}$ ((i) અને (ii) ઉપરથી)

$$= \frac{\sin 2t - 3\tan t \cdot \cos 2t}{3\tan^2 t \cdot \cos 2t + \tan^3 t \cdot \sin 2t}$$ (અંશ તથા છેદને $\cos^3 t$ વડે ભાગતાં)

$$= \frac{2\tan t - 3\tan t\,(1 - \tan^2 t)}{3\tan^2 t\,(1 - \tan^2 t) + \tan^3 t \cdot 2\tan t}$$

➡ $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$

$$\therefore \frac{dx}{dt} = \frac{\sqrt{\cos 2t} \cdot \frac{d}{dt}(\sin^3 t) - \sin^3 t \cdot \frac{d}{dt}\sqrt{\cos 2t}}{\left(\sqrt{\cos 2t}\right)^2}$$

$$= \frac{\sqrt{\cos 2t} \cdot 3\sin^2 t \cos t - \sin^3 t \cdot \left(\frac{1}{2\sqrt{\cos 2t}} \cdot (-2\sin 2t)\right)}{\cos 2t}$$

$$= \frac{3\sin^2 t \cdot \cos t \cdot \sqrt{\cos 2t} + \frac{\sin^3 t \cdot \sin 2t}{\sqrt{\cos 2t}}}{\cos 2t}$$

$$= \frac{3\sin^2 t \cdot \cos t \cdot \cos 2t + \sin^3 t \cdot \sin 2t}{(\cos 2t)^{\frac{3}{2}}} \quad ..(i)$$

$$y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$$

$$\therefore \frac{dy}{dt} = \frac{\sqrt{\cos 2t} \cdot \frac{d}{dt}(\cos^3 t) - \cos^3 t \frac{d}{dt}(\sqrt{\cos 2t})}{(\sqrt{\cos 2t})^2}$$

$$= \frac{-\sqrt{\cos 2t} \cdot 3\cos^2 t \sin t + \cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}} 2\sin 2t}{(\sqrt{\cos 2t})^2}$$

$$= \frac{-3\cos^2 t \cdot \sin t \cos 2t + \cos^3 t \cdot \sin 2t}{(\cos 2t)^{\frac{3}{2}}} \quad .(ii)$$

હવે $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$$= \frac{\cos^3 t \cdot \sin 2t - 3\cos^2 t \sin t \cos 2t}{3\sin^2 t \cdot \cos t \cdot \cos 2t + \sin^3 t \cdot \sin 2t}$$ ((i) અને (ii) ઉપરથી)

$$= \frac{\sin 2t - 3\tan t \cdot \cos 2t}{3\tan^2 t \cdot \cos 2t + \tan^3 t \cdot \sin 2t}$$ (અંશ તથા છેદને $\cos^3 t$ વડે ભાગતાં)

$$= \frac{2\tan t - 3\tan t\,(1 - \tan^2 t)}{3\tan^2 t\,(1 - \tan^2 t) + \tan^3 t \cdot 2\tan t}$$

19. **જો $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$ હોય, તો સાબિત કરો કે, $\dfrac{dy}{dx} = \dfrac{-y}{x}$.**

➡ $x = \sqrt{a^{\sin^{-1} t}}$

$$\therefore \frac{dx}{dt} = \frac{d}{dt}\left(a^{\sin^{-1} t}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(a^{\sin^{-1} t}\right)^{\frac{1}{2}-1} \cdot \frac{d}{dt}\left(a^{\sin^{-1} t}\right)$$

$$= \frac{1}{2\sqrt{a^{\sin^{-1} t}}} \cdot a^{\sin^{-1} t} \cdot \log a \cdot \frac{d}{dt}(\sin^{-1} t)$$

$$= \frac{\sqrt{a^{\sin^{-1} t}} \cdot \log a}{2} \times \frac{1}{\sqrt{1 - t^2}} \quad ....(i)$$

$$y = \sqrt{a^{\cot^{-1} t}}$$

$$\therefore \frac{dy}{dt} = \frac{d}{dt}\left(a^{\cos^{-1} t}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(a^{\cos^{-1} t}\right)^{\frac{1}{2}-1} \cdot \frac{d}{dt}\left(a^{\cos^{-1} t}\right)$$

$$= \frac{1}{2\sqrt{a^{\cos^{-1} t}}} \cdot a^{\cos^{-1} t} \cdot \log a \cdot \frac{d}{dt}(\cos^{-1} t)$$

$$= \frac{\sqrt{a^{\cos^{-1} t}} \cdot \log a}{2} \times \frac{-1}{\sqrt{1-t^2}} \quad \text{..........(ii)}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{\frac{\sqrt{a^{\cos^{-1} t}} \cdot \log a}{2} \cdot \left(-\frac{1}{\sqrt{1-t^2}}\right)}{\frac{\sqrt{a^{\sin^{-1} t}} \cdot \log a}{2} \cdot \left(\frac{1}{\sqrt{1-t^2}}\right)}$$

➥ $$x = \sqrt{a^{\sin^{-1} t}}$$

$$\therefore \frac{dx}{dt} = \frac{d}{dt}\left(a^{\sin^{-1} t}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(a^{\sin^{-1} t}\right)^{\frac{1}{2}-1} \cdot \frac{d}{dt}\left(a^{\sin^{-1} t}\right)$$

$$= \frac{1}{2\sqrt{a^{\sin^{-1} t}}} \cdot a^{\sin^{-1} t} \cdot \log a \cdot \frac{d}{dt}(\sin^{-1} t)$$

$$= \frac{\sqrt{a^{\sin^{-1} t}} \cdot \log a}{2} \times \frac{1}{\sqrt{1-t^2}} \quad \text{..........(i)}$$

$$y = \sqrt{a^{\cot^{-1} t}}$$

$$\therefore \frac{dy}{dt} = \frac{d}{dt}\left(a^{\cos^{-1} t}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(a^{\cos^{-1} t}\right)^{\frac{1}{2}-1} \cdot \frac{d}{dt}\left(a^{\cos^{-1} t}\right)$$

$$= \frac{1}{2\sqrt{a^{\cos^{-1} t}}} \cdot a^{\cos^{-1} t} \cdot \log a \cdot \frac{d}{dt}(\cos^{-1} t)$$

$$= \frac{\sqrt{a^{\cos^{-1} t}} \cdot \log a}{2} \times \frac{-1}{\sqrt{1-t^2}} \quad \text{..........(ii)}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{\frac{\sqrt{a^{\cos^{-1} t}} \cdot \log a}{2} \cdot \left(-\frac{1}{\sqrt{1-t^2}}\right)}{\frac{\sqrt{a^{\sin^{-1} t}} \cdot \log a}{2} \cdot \left(\frac{1}{\sqrt{1-t^2}}\right)}$$