Chapter - 10

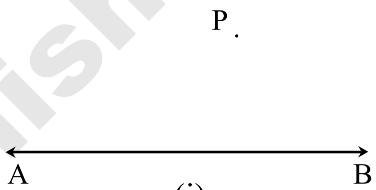
Practical Geometry

10.1 You have already learnt the preliminary concepts of geometrical construction in Class VI. In the previous class you have learnt about drawing of perpendicular line, perpendicular bisector of side, angles, bisector of angles and the angles of specific measurment. In this chapter we are going to discuss about construction of parallel lines and Triangles.

10.2 Construction of a line Parallel to a Given Line, through a point not on a line.

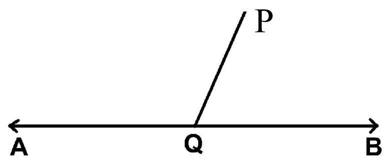
Let AB be a straight line.

P is a point outside AB. We have to draw a line through point 'P' parallel to AB.



(i)

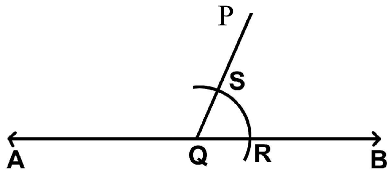
Step 1 ꞉ Take a point 'Q' at AB. Join P and Q.



(ii)

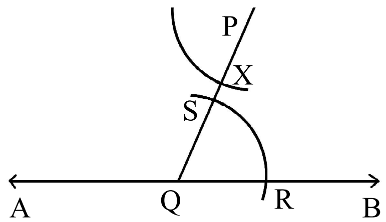
Step 2 ꞉ With Q as the centre, draw an arc which intersect QB and QP.

The arc cuts at R and at S of QB and QP respectively.



(iii)

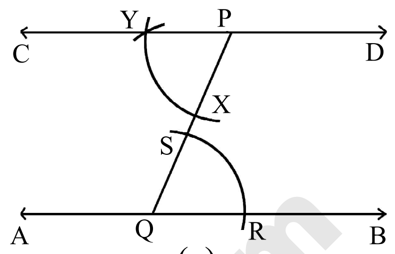
Step 3 ꞉ Again taking P as centre another arc having same radius to the previous arc is drawn In such a way, that the arc passes through PQ.The arc cuts at point 'X'.



(iv)

Step 4 : Again taking X as centre and taking same radius equal to RS another arc is drawn. This arc intersect the 2nd arc at point 'Y'.

Step 5 : The line $\overleftrightarrow{CD}$ is drawn through P and Y . CD is the line which is to be drawn parallel to AB.



(v)

Figure - 10.1

Here CD is parallel to AB, do you know why? You already know that when a transversal cuts two parallel lines, then each pair of interior alternate angles formed are equal. On the other hand, if each pair of interior alternate angles are equal when a transveral cuts a pair of lines, then the lines are parallel to each other. Here $\angle PQB$ and $\angle CPQ$ are a pair of interior internal angle which are equal. So $AB || CD$.

Exercise -10.1

1. Draw a line *m.* Take a point 'A' which is not in the line *m.* Draw a line through 'A' which is parallel to *m.*
2. Draw a line segment AB of length 10 cm. Draw another ray AX at point A such that $\angle BAX = 60^0$. Take a point D on AX so that AD = 4 cm. Draw a line through point D which is parallel to the AB.
3. Draw a line PQ. Draw a perpendicular line at any point of PQ. Take a point R on the perpendicular line which is 5·6 cm away from PQ. Draw a line through R which is parallel to the PQ.
4. Draw a line segment XY of 7·5 cm. Draw two perpendicular lines of X and Y. Identify two points A and B in the perpendicular line which are 5 cm away from the XY. Through point A draw a line parallel to XY. Will this line pass through the point B?

10.3 Construction of Triangles :

You have already learnt about Triangle and some of its properties. In this chapter we will discuss the construction of triangle.

In the previous lesson the congruency of triangle was also discussed. Congruency of triangles can be shown by using postulates Side-Side-Side (S-S-S). Side-Angle-Side (S-A-S), Angle-Side-Angle (A-S-A) and Right -Angle Hypotenuse Side (R-H-S). From these postulates of triangle, it is undestood that a specific triangle can be drawn, if anyone of the following measurements are given–

(i) Measurement of three sides of a triangle.

(ii) Measurement of two sides of a triangle and the measurements of angle formed by two sides.

(iii) Measurement of line and the measurement of two angles formed at two ends of the line.

(iv) Measurement of Hypotenuse and one side of a right angle triangle.

Let us discuss such constructions.

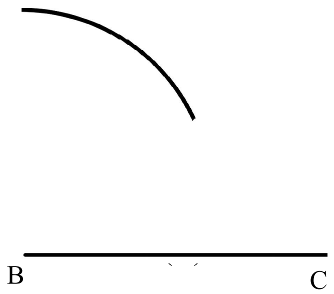
10.3.1 Construction of a triangle if three sides are given (Side-Side-Side)

Let ABC be a triangle and its three sides are AB = 4 cm, BC = 5 cm, AC = 6 cm. We have to construct the triangle.

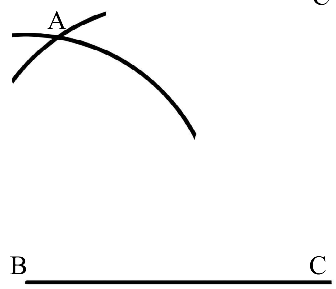
Step 1 : Anyone side of the triangle for example BC = 5 cm is drawn.

(i)

Step 2 : Taking B as the centre draw an arc of radius 4 cm.



Step 3 : Taking 'C' as centre another arc of radius 6 cm is drawn so that it cuts the previous arc, at point 'A'.



(iii)

Step 4 : AB, BC and AC are joined.

Here ΔABC is constructed which has 3 sides where AB = 4 cm, BC = 5 cm and AC = 6 cm.

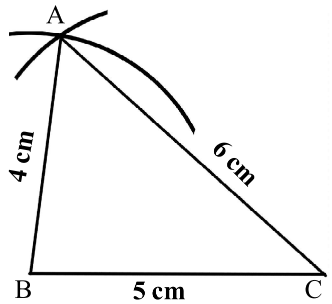


Figure - 10.2 (iv)

Do yourself :

As discussed in 10.2 try to construct a triangle PQR on a paper so that the sides are PQ= 4 cm, QR = 5 cm, PR = 6 cm. Cut out the triangle with the help of a scissor. Place the cut out triangle on the previous triangle and fit them. Do the triangles coincide exactly ? Therefore, we can say that if the corresponeding sides of two triangles are same then the triangles are congruent. This is known as Side-Side-Side congruency postulate which was discussed earlier.

Think :

Can you always construct a triangle when three sides of any length are given? For example : Can you construct triangle ABC when AB = 2 cm, BC = 7 cm and AC = 3 cm? First draw BC = 7 cm. Now draw an arc AB = 2 cm, taking B as centre as earlier method. Now draw an arc AC = 3 cm taking C as centre. What happened? The two arcs never intersect. So, ABC can not be completed. Why? (Remember that sum of the measurement of any two sides of a triangle is greater than third side. Here sum of measurments of AB & AC = (2+3) cm = 5cm. But it is not greater than the measurement of third side (BC = 7 cm).

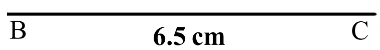
Exercise - 10.2

1. Construct triangle ABC so that
   (i) AB = 3 cm BC = 4 cm AC = 2.5 cm
   (ii) AB = 6 cm BC = 4 cm AC = 7 cm
2. Construct an isosceles triangle base of which is 5 cm and two equal sides are of 4 cm.
3. Construct an equilateral triangle DEF of which DE = 6.5 cm.
4. Construct the triangle XYZ of which XY = YZ = 5.5 cm and XZ = 4 cm.

10.3.2 Constructing a triangle when the lengths of two sides and the measure of the angle between them are given.

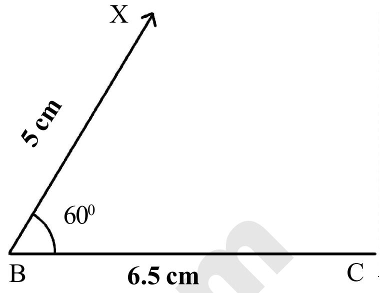
Let ABC be a triangle of which AB = 5 cm, BC = 6·5 cm and $\angle B = 60^0$. The triangle to be constructed

Step 1 : Draw a line BC of length 6.5 cm.



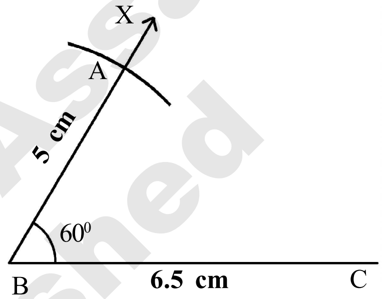
(i)

Step 2 : Construct angle $\angle XBC = 60^0$ at point B.



(ii)

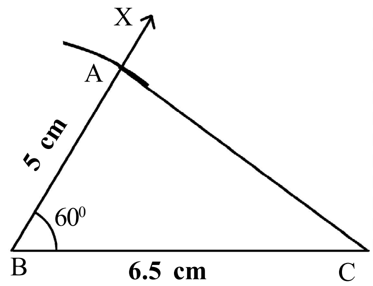
Step 3 : Considering B as centre draw an arc of radius AB 5 cm. Which will cut BX at A.



(iii)

Step 4 : A and C arc joined.

In this way the triangle ABC is constructed of which AB = 5 cm, BC = 6.5 cm, $\angle B = 60^0$.



(iv)

Figure - 10.3

Do yourself :

According to the method discussed above construct a triangle PQR on a paper so that PQ = 5 cm, QR = 6˙5 cm, $\angle Q = 60^0$.

Now cut out the triangle the help of a Scissor. Place the cut out triangle on the previous triangle. Does the triangle coincide exactly ? From this we can say that if two sides and the interior angle between the two given sides are equal to the corresponding sides and angle of an another triangle then the two triangles are congruent. This is known as Side-Angle-Side Congruency postulate of triangle which we learnt in our earlier chapter.

Think :

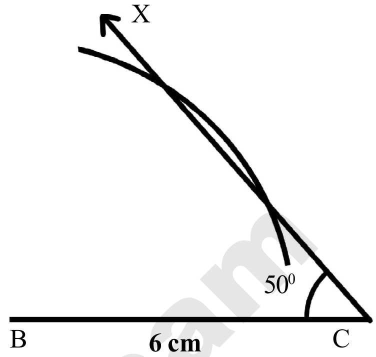


Figure - 10.4

1. Suppose in triangle ABC, AB = 5 cm, BC = 6 cm and $\angle C = 50^0$. To construct the triangle first draw a line segment BC = 6 cm. Construct $\angle BCX = 50^0$ at C. Now taking B as centre draw an arc of radius 5 cm. This arc does not cut CX in definite points. (Observe that the arc cuts CX in two anywhere and if the radius of the arc is less than 5 cm, than the arc may not cut CX points). So, here we do not get a definite triangle).

2. Suppose in triangle ABC, AB = 5 cm, BC = 6 cm and $\angle A = 50^0$. In this case also to construct ABC. We can draw AC = 5cm & $\angle A = 50^0$. But 'C' can not be determine exactly.

3. We can always constuct a triangle if two sides and the measure of the angle between them are given.

Exercise - 10.3

1. Construct a triangle ABC of which AB = 6 cm, BC = 7 cm and $\angle B = 75^0$.
2. Construct a triangle PQR of which QR = PR= 8 cm and $\angle R = 60^0$.
3. Construct a triangle DEF of which DE = 5˙5 cm, DF = 7˙5 cm and $\angle D = 80^0$.
4. Construct right- angle triangle, the two perpendicular line of which is 4˙5 cm.
5. Construct a triangle of which two sides are 5˙5 cm and 6˙5 cm and the angle between two sides is 120^0.

10.3.3 Constructing a triangle when the measure of two of its angles and the length of the side included between them is given (ASA criterion)

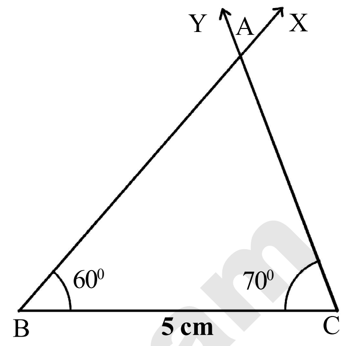
Suppose in ΔABC, $\angle B = 60^0$, $\angle C = 70^0$ and BC = 5 cm. We have to construct the triangle.

Step 1 : BC a line is drawn of length 5cm.

(i)

Step 2 : Construct $\angle CBX = 60^0$ at B and $\angle BCY = 70^0$ at C. Ray BX and CY cut at A.

Here in ΔABC, $\angle B = 60^0$, $\angle C = 70^0$ and BC = 5 cm. Hence ΔABC is required triangle.



(ii)

Think :

1. Suppose in ΔABC, $\angle B = 50^0$, $\angle C = 70^0$ and AB = 6 cm. AB is not the interior to $\angle B$ and $\angle C$. Can you construct ΔABC ?

You already know that sum of measure of three angles of a triangle is 180^0. By using this you can find $\angle A$ &$\angle B$ at the end point A & B determined. Hence you can construct triangle by using above method.

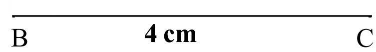
Exercise - 10.4

1. Construct a ΔABC of which $\angle B = 65^0$, $\angle C = 55^0$ and BC = 7 cm.
2. Construct a triangle of which one side is 6˙5 cm and the angles formed at two ends of the side are 45^0 and 75^0.
3. Construct a triangle PQR of which $\angle P = 60^0$, $\angle Q = 50^0$ and QR = 6 cm.
4. Construct a ΔXYZ of which XZ = 5˙5 cm, $\angle X = 100^0$ and $\angle Z = 30^0$.

10.3.4 Constructing a right-angled triangle when the length of a side and its hypotenuse are given (RHS criterion)

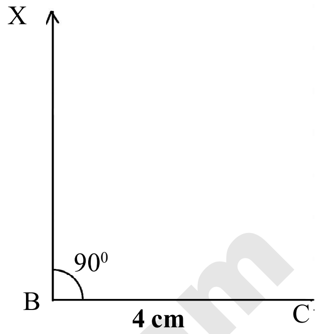
Let ABC is a right-angle triangle and $\angle B = 90^0$. Hypotenuse AC = 5cm and BC = 4 cm. The triangle to be constructed.

Step 1 : Draw a line BC = 4 cm.



(i)

Step 2 : Construct an angle $\angle CBX = 90^0$ at point B.

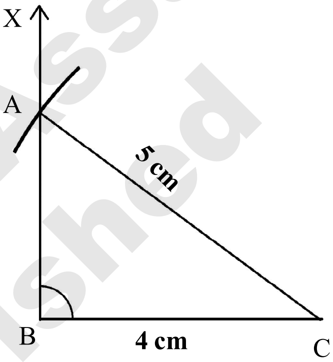


(ii)

Step 3 : Considering the point C as centre draw an arc of radius 5 cm which will cut BX at point A .

Step 4 : Join A and C. AC = 5 cm

In this way ΔABC is drawn of which $\angle B = 90^0$ AC = 5 cm and CB = 4 cm. This is the triangle to be constructed.



(iii)

Figure - 10.6

Exercise - 10.5

1. Construct ΔABC of which $\angle A = 90^0$, BC = 8 cm and AB = 5 cm.
2. The length of hypotenuse of a right-angled triangle is 10 cm and another side is 6 cm. Construct the triangle.
3. Construct ΔPQR of which $\angle Q = 90^0$, PR = 7·5 cm and PQ = 5 cm.

What we have learnt

1. If a line and a point which is not on the line are given we can use the idea of 'equal alternate angle' to draw a parallel line through the point parallel to the given line.
2. To construct a triangle, the concept of congruence of triangle is used indirectly. We can construct a trianlge when the following are given –
   (i) The length of three sides of a triangle. (SSS)
   (ii) The length of any two sides and the measure of the angle between them. (SAS)
   (iii) The measures of two angles and the length of side included between them. (ASA)
   (iv) The length of hypotenuse of a right angled triangle and the length of one of its leg.