PRACTICE PAPER 11

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: *As given in Practice Paper – 1.*

Section–A

Choose the correct option:

1. If $A = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$ then A^{20} is

(*a*) $\begin{bmatrix} 0 & 0 \\ 2^{20} & 2^{20} \end{bmatrix}$ (*b*) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 0 \\ 40 & 40 \end{bmatrix}$ (*d*) $\begin{bmatrix} 40 & 0 \\ 0 & 40 \end{bmatrix}$

2. The value of the determinant $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$ is

(*a*) $9x^2(x+y)$ (*b*) $9y^2(x+y)$ (*c*) $3y^2(x+y)$ (*d*) $7x^2(x+y)$

3. If A is a matrix of order 3 × 3, then $(A^2)^{-1}$ is

(*a*) $(A^{-1})^3$ (*b*) $(A^{-1})^9$ (*c*) $(A^{-1})^2$ (*d*) None of these

4. If $f(x) = \frac{a}{x}$ then $f''(x)$ is equal to

(*a*) $\frac{-1}{x^3}$ (*b*) $\frac{a}{x^3}$ (*c*) $\frac{a}{x^2}$ (*d*) $\frac{2a}{x^3}$

5. If m be the slope of a tangent to the curve $e^y = 1 + x^2$ then

(*a*) $|m| > 1$ (*b*) $m < 1$ (*c*) $|m| < 1$ (*d*) $|m| \le 1$

6. $\int \left(\frac{\log x - 1}{1 + (\log x)^2} \right)^2 dx$ is equal to

(*a*) $\frac{\log x + 1}{1 + (\log x)^2} + C$ (*b*) $\frac{xe^x}{1 + x^3} + C$ (*c*) $\frac{x}{1 + (\log x)^2} + C$ (*d*) $\frac{\log x}{1 + (\log x)^2} + C$

7. If $\int \frac{\sin x \, dx}{\sin (x - \alpha)} = Ax + B \log \sin (x - \alpha) + C,$ the value of (A, B) is

(*a*) $(\sin \alpha, \cos \alpha)$ (*b*) $(\sin \alpha, -\sin \alpha)$ (*c*) $(-\cos \alpha, \sin \alpha)$ (*d*) $(\cos \alpha, \sin \alpha)$

8. If $\int_0^{\pi/2} \log \sin x \, dx = k$, then $\int_0^{\pi} \log (1 + \cos x) \, dx$ is equal to

(*a*) $\log 9 + 4k$ (*b*) $\pi \log 2 + 4k$ (*c*) $\pi \log 2 + 4k^2$ (*d*) none of these

9. If $f(x) = \int_0^1 \frac{dt}{1+|x-t|}$, then $f'(1/2)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) 0 (d) none of these

10. The area bounded by the curve $y = 2\cos x$ and the x-axis from $x = 0$ to $x = 2\pi$ is

(a) 8 sq. units (b) 18 sq. units (c) 6 sq. units (d) 4 sq. units

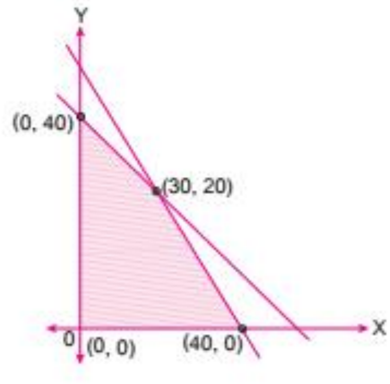
11. The number of arbitrary constants in the particular solution of a differential equation of third order are

(a) 3 (b) 2 (c) 1 (d) 0

12. Which of the following differential equations has $y = x$ as one of its particular solution?

(a) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$ (b) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$ (c) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$ (d) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

13. The maximum value of $Z = 0.7x + y$ for feasible region given below is



(a) 45 (b) 40 (c) 50 (d) 41

14. A discrete random variable X has the following probability distribution

X	1	2	3	4	5	6	7
$P(X)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{1}{50}$	$\frac{17}{100}$

Mean of the distribution is

(a) 3.66 (b) 2.66 (c) 1.66 (d) 4.66

15. If X follows the binomial distribution with parameters $n = 5$, $p = \frac{1}{5}$ then the mean of the distribution is

(a) 2 (b) 1 (c) 5 (d) None of these

Section–B (B1)

16. Let $A = \{1, 2, 3\}$. Then number of equivalence relation containing (1, 2) is

(a) 1 (b) 2 (c) 3 (d) 4

17. Let $f:[0, 1] \to [0, 1]$ be defined by $f(x) = \begin{cases} x, \text{ if } x \text{ is rational} \\ 1-x, \text{ if } x \text{ is irrational} \end{cases}$. Then $(fof)(x)$ is

(a) $1 - x$ (b) x (c) $1 + x$ (d) constant

18. Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$ then for all x. $f(g(x))$ *i.e.*, $fog\ (x)$ is

19. **Let * be a binary operation on ℝ (set of reals) as $a * b = a + b - \sqrt{2}$ then the value of $(\sqrt{2} * \sqrt{3}) * \sqrt{11}$ is**

(a) $\sqrt{3}+\sqrt{11}$ (b) $\sqrt{3}+\sqrt{11}-\sqrt{2}$ (c) $\sqrt{3}+\sqrt{11}+\sqrt{2}$ (d) None of these

20. ***Read the following statements.***

Statement I : Let $f : R \longrightarrow R$ is defined by $f(x) = 3x - 4$ then $f^{-1}(x) = \dfrac{x+4}{3}$.

Statement II : Let f be a function defined by $f(x) = \dfrac{2x+1}{1-3x}$, then $f^{-1}(x) = \dfrac{x-1}{3x+2}$.

Choose the correct option:

(a) Statement I is correct but statement II is not correct.

(b) Statement II is correct but statement I is not correct.

(c) Both statements I and II are correct.

(d) None of these

21. **The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ is**

(a) $\frac{25}{24}$ (b) $\frac{25}{7}$ (c) $\frac{24}{25}$ (d) $\frac{7}{24}$

22. **The principal value of $\text{cosec}^{-1}(-1)$ is**

(a) $\frac{-\pi}{2}$ (b) 0 (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

23. **The value of $\tan^{-1}(\sqrt{3}) + \cot^{-1}(-1) + \sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is**

(a) $\frac{-\pi}{12}$ (b) $\frac{11\pi}{12}$ (c) $\frac{5\pi}{4}$ (d) $\frac{23\pi}{12}$

24. **The value of $\sin[\cot^{-1}\{\tan(\cos^{-1} x)\}]$ is**

(a) $\sqrt{1-x^2}$ (b) 1 (c) x (d) x^2

25. **If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, then the value of x and y such that $A^2 + xI_2 = yA$ is**

(a) (8, 8) (b) (–8, 8) (c) (0, 0) (d) (8, –8)

26. **If $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ then what is the value of $x - y + z$?**

(a) 1 (b) 0 (c) 4 (d) 2

27. **Let A be a symmetric matrix such the $|A| = 5$ then $|A'|$ is**

(a) 5 (b) 5^2 (c) $\frac{1}{5}$ (d) none of these

28. **If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $adj\ A$ is**

(a) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$ (b) $\begin{bmatrix} d & a \\ b & c \end{bmatrix}$ (c) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (d) $\begin{bmatrix} a & b \\ d & c \end{bmatrix}$

29. **Let $f(x) = \begin{cases} \cos[x], & x \geq 0 \\ |x| + a, & x < 0 \end{cases}$ where $[x]$ denotes the greatest integer $\leq x$. If $\lim_{x \to 0} f(x)$ exists then a is equal to**

(a) 1 (b) 3 (c) 0 (d) none of these

30. If $3\sin(xy) + 4\cos(xy) = 5$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{3\sin(xy) + 4\cos(xy)}{3\cos(xy) - 4\sin(xy)}$ (b) $-\frac{y}{x}$ (c) $\frac{3\sin(xy)}{4\cos(xy)}$ (d) none of these

31. The value of c in Rolle's Theorem for the function $f(x) = e^x \sin x$ such that $x \in [0, \pi]$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{3\pi}{4}$ (d) none of these

32. $f : [-2a, 2a] \to R$ is an odd function such that the left hand derivative at $x = a$ is zero and $f(x) = f(2a - x)$ $\forall\, x \in [a, 2a]$. Then its left hand derivative at $x = -a$ is

(a) 0 (b) a (c) 1 (d) does not exist.

33. The least value of the function $f(x) = ax + \frac{b}{x} (a > 0, b > 0, x > 0)$ is

(a) $\frac{a}{b}$ (b) $2\sqrt{ab}$ (c) 0 (d) none of these

34. *Read the following statements.*

Statement I : The anti-derivative of $\sqrt{x} + \frac{1}{\sqrt{x}}$ is $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$

Statement II : $\int \sin x\, dx = \cos x + C$

Choose the correct option:

(a) Statement I is correct but statement II is not correct.
(b) Statement II is correct but statement I is not correct.
(c) Both statements I and II are correct.
(d) None of these

35. The value of $\int \tan(x - \alpha)\tan(x + \alpha)\tan(2x)\, dx$ is

(a) $\log\left|\frac{\sqrt{\sec 2x}\sec(x+\alpha)}{\sec(x-\alpha)}\right| + C$ (b) $\log\left|\tan^{-1}(\sec x + \cos x)\right| + C$

(c) $\log\left|\frac{\sqrt{\sec 2x}\sec(x-\alpha)}{\sec(x+\alpha)}\right| + C$ (d) $\log\left|\frac{\sqrt{\sec 2x}}{\sec(x-\alpha)\sec(x+\alpha)}\right| + C$

36. $\int \tan^{-1}(\sqrt{x})\, dx$ is equal to

(a) $(x + 1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$ (b) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(c) $\sqrt{x} - x\tan^{-1}\sqrt{x} + C$ (d) $\sqrt{x} - (x + 1)\tan^{-1}\sqrt{x} + C$

37. The area of the region bounded by the line $2y = -x + 8$, the x-axis and the lines $x = 2$ and $x = 4$ is

(a) 4 sq. units (b) 2 sq. units (c) 5 sq. units (d) 3 sq. units

38. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(a) $e^x + e^{-y} = C$ (b) $e^x + e^y = C$ (c) $e^{-x} + e^y = C$ (d) $e^{-x} + e^{-y} = C$

39. A homogeneous differential equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution

(*a*) $y = vx$ (*b*) $v = yx$ (*c*) $x = vy$ (*d*) $x = v$

40. **The unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$ is**

(*a*) $\frac{1}{\sqrt{26}}\hat{i} + \frac{5}{\sqrt{26}}\hat{k}$ (*b*) $\frac{1}{\sqrt{3}}\hat{i} - \frac{5}{\sqrt{26}}\hat{k}$ (*c*) $\frac{1}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{20}}\hat{k}$ (*d*) $\frac{1}{\sqrt{32}}\hat{i} - \frac{5}{\sqrt{3}}\hat{k}$

41. **The vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with *y* and *z*-axis, respectively is**

(*a*) $\vec{r} = \pm 4\hat{i} - 5\hat{j}$ (*b*) $\vec{r} = \pm 3\hat{i} + 3\hat{j}$ (*c*) $\vec{r} = \pm 5\hat{i} + 5\hat{j}$ (*d*) $\vec{r} = \pm 4\hat{i} + 5\hat{j}$

42. **The magnitude of vector $6\hat{i} + 2\hat{j} + 3\hat{k}$ is**

(*a*) 5 (*b*) 7 (*c*) 12 (*d*) 1

43. **The value of λ for which the two vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \lambda\hat{j} + \hat{k}$ are perpendicular is**

(*a*) 2 (*b*) 4 (*c*) 6 (*d*) 8

44. **If a line makes angles $\frac{\pi}{2}, \frac{3\pi}{4}$, and $\frac{\pi}{4}$ with *x, y, z* axes, respectively, then the direction cosines are**

(*a*) $\pm(1, 1, 1)$ (*b*) $\pm\left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (*c*) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (*d*) $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

45. **If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}\alpha$ with *x*-axis, then the value of α is**

(*a*) $\frac{\sqrt{3}}{2}$ (*b*) $\frac{\sqrt{2}}{3}$ (*c*) $\frac{2}{7}$ (*d*) $\frac{3}{7}$

46. **The angle between the planes $\vec{r}.(2\hat{i} - 3\hat{j} + \hat{k}) = 1$ and $\vec{r}.(\hat{i} - \hat{j}) = 4$ is**

(*a*) $\cos^{-1}\left(\frac{5}{2\sqrt{7}}\right)$ (*b*) $\cos^{-1}\left(\frac{7}{2\sqrt{3}}\right)$ (*c*) 0 (*d*) $\cos^{-1}\left(\frac{8}{3\sqrt{7}}\right)$

47. **The coordinates of the point where the line through (3, – 4, – 5) and (2, –3, 1) crosses the plane passing through three points (2, 2, 1), (3, 0, 1) and (4, – 1, 0) are**

(*a*) (0, – 2, 7) (*b*) (3, – 2, 5) (*c*) (1, – 2, – 7) (*d*) (1, – 2, 7)

48. ***Read the following statements.***

Statement I : If *A* and *B* are two events such that $P(B) = \frac{3}{5}, P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$ then $P(B/A') = \frac{3}{5}$.

Statement II : $P(A/B) = \frac{P(A)}{P(B)}$

Choose the correct option:

(*a*) Statement I is correct but statement II is not correct.

(*b*) Statement II is correct but statement I is not correct.

(*c*) Both statements I and II are correct.

(*d*) None of these

49. **If *A* and *B* are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ equals to**

(*a*) $\frac{14}{15}$ (*b*) $\frac{18}{45}$ (*c*) $\frac{11}{3}$ (*d*) $\frac{2}{9}$

50. A box contains 6 bolts and 10 nuts. Half of the bolts and nuts are rusted. If one of them is chosen at random, the probability that it is rusted or is a bolt is

(*a*) 3/16 (*b*) 5/16 (*c*) 11/16 (*d*) 7/8