Polynomials

Case Study Based Questions

Case Study 1

A reputed school of Meerut decided to conduct different types of Tours for the students to educate them. So in class X, $\frac{1}{6}$th times the square of the total number of students planned to visit historical monuments, $\frac{5}{6}$th times the total number of students planned to visit old age homes while 10 students decided to teach poor children.

On the basis of the above information, solve the following questions:

Q 1. Using above information, express the total number of students as a polynomial in terms of x:

a. $\frac{x^2}{6}+\frac{5}{6}x+10$ b. $\frac{x^2}{4}+\frac{7}{4}x+10$

c. $\frac{7x^2}{12}+\frac{1}{12}x+10$ d. $\frac{x^2}{4}+\frac{7}{4}x+15$

Q 2. Write the coefficient of x in polynomial.

a. $\frac{9}{13}$ b. $\frac{5}{6}$ c. $\frac{11}{12}$ d. $\frac{13}{12}$

Q 3. Write the coefficient of x^2 in polynomial.

a. $\frac{1}{13}$ b. $\frac{1}{10}$ c. $\frac{1}{6}$ d. 15

Q 4. Value of $p(x)$ at $x = 2$ is:

a. $\frac{37}{3}$ b. $\frac{11}{2}$ c. $\frac{22}{3}$ d. $\frac{14}{3}$

Q 5. When $p(x)$ is divided by x, then quotient is:

a. $\frac{x}{6}+\frac{5}{6}-\frac{10}{x}$ b. $\frac{x}{6}+\frac{5}{6}+\frac{10}{x}$

c. $\frac{x^3}{6}+\frac{5}{6}x^2+10x$ d. $\frac{x^3}{6}+\frac{5}{6}x+10$

Solutions

1. (a) Let the total number of students be x then $\frac{1}{6}$ th times the square of total students $=\frac{x^2}{6}$ and $\frac{5}{6}$ th times the number of students $=\frac{5x}{6}$.

Therefore the total students $=\frac{x^2}{6}+\frac{5x}{6}+10$.

Hence, the polynomial will be $p(x)=\frac{x^2}{6}+\frac{5x}{6}+10$.

So, option (a) is correct.

2. (b) $\frac{5}{6}$ is the coefficient of x.

So, option (b) is correct.

3. (c) $\frac{1}{6}$ is the coefficient of x^2.

So, option (c) is correct.

4. (a) Let $p(x) = \frac{x^2}{6} + \frac{5x}{6} + 10$

At $x = 2$,

$$p(2) = \frac{(2)^2}{6} + \frac{5 \times 2}{6} + 10$$

$$= \frac{4}{6} + \frac{10}{6} + 10 = \frac{4 + 10 + 60}{6}$$

$$p(2) = \frac{74}{6} = \frac{37}{3}$$

So, option (a) is correct.

5. (b) When $p(x)$ is divided by x, then quotient is:

$$\frac{\frac{x^2}{6} + \frac{5}{6}x + 10}{x} = \frac{x}{6} + \frac{5}{6} + \frac{10}{x}$$

So, option (b) is correct.

Case Study 2

Amit along with his four friends visited the house of Rohit, who was a common friend. There they meet his father, who was having keen interest in mathematics. Rohit's father wanted to test the practical knowledge of all his friends, so he showed some objects like a cuboid shaped geometry box, a rectangular photo frame, a circular cardboard, square shaped files and a cube. He started asking the following questions one by one.

(Cuboid Geometry Box)

(Rectangular Photo Frame)

(Circular Cardboard)

On the basis of the above information, solve the following questions:

Q 1. If the area of circular cardboard is $49\pi x^2 + 70\pi x + 25\pi$, what is the radius of this object?

a. $(7x + 5)$ b. $\pi(7x + 5)$

c. $-5/7$ d. $7/5$

Q 2. If the volume of geometry box is $x^3 - 2x^2 - x + 2$, then the possible dimensions of this box are:

a. $(x + 1), (x + 1), (x + 2)$ b. $(x + 1), (x - 1), (x + 2)$

c. $(x - 1), (x + 1), (x - 2)$ d. $(x - 1), (x - 1), (x + 2)$

Q 3. If the area of a file is $4x^2 + 4x + 1$, what is the perimeter of this file?

a. $2x + 1$ b. $4x + 1$

c. $4(2x + 1)$ d. $(8x + 2)$

Q 4. If the area of rectangular photo frame is $12x^2 - 7x + 1$, what are the possible dimensions of photo frame?

a. $(3x - 1), (4x - 1)$ b. $(3x + 1), (4x + 1)$

c. $(3x - 1), (4x + 1)$ d. $(3x + 1), (4x - 1)$

Q 5. If the volume of cube is $8a^3 - b^3 - 12a^2b + 6ab^2$, what is the side of cube?

a. $(2a + b)$ b. $(2a - b)$

c. $(2a + 3b)$ d. $(3a - 2b)$

Solutions

1. (a) Given,

Area of circular cardboard $= 49\pi x^2 + 70\pi x + 25\pi$

We know that,

Area of circle $= \pi r^2$

$\therefore \quad \pi r^2 = \pi(49x^2 + 70x + 25)$

$\Rightarrow \quad r^2 = (49x^2 + 35x + 35x + 25)$

$= [7x(7x + 5) + 5(7x + 5)]$

$= [(7x + 5)^2] = (7x + 5)\,(7x + 5)$

$\therefore \quad r = 7x + 5$

Hence, the radius of the circle is $(7x + 5)$.

So, option (a) is correct.

2. (c) Volume $= x^3 - 2x^2 - x + 2$

$= x^3 - x^2 - x^2 + x - 2x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x^2 - x - 2)\ (x - 1)$

$= (x - 1)\ (x^2 - 2x + x - 2)$

$= (x - 1)\ [x(x - 2) + 1\ (x - 2)]$

$= (x - 1)\ (x - 2)\ (x + 1)$

Hence, possible dimensions are $(x - 1)$, $(x + 1)$, $(x - 2)$.

So, option (c) is correct.

3. (c) We have, area of a square shape file $= 4x^2 + 4x + 1$

We know that,

Area of square $= (\text{Side})^2$

$\therefore \quad (\text{Side})^2 = (4x^2 + 4x + 1) = (2x + 1)^2$

$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$\therefore \quad$ Perimeter $= 4 \times (2x + 1) = 4(2x + 1)$

So, option (c) is correct.

4. (a) Area of rectangle $= l \times b = 12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (4x - 1)\ (3x - 1)$

Hence, possible dimensions are $(4x - 1)$, $(3x - 1)$.

So, option (a) is correct.

5. (b) Volume of cube $= 8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 6ab(2a - b)$

$= (2a - b)\ [4a^2 + 2ab + b^2] - 6ab\ [2a - b]$

$= (2a - b)\ [4a^2 - 4ab + b^2]$

$= (2a - b)\ (2a - b)^2$

$= (2a - b)^3$ [$\because$ volume of cube $= (\text{side})^3$]

Hence, the side of cube is $(2a - b)$.

So, option (b) is correct.

Case Study 3

Nari Niketan is an organisation to help the women and child having distress. Swati donated some amount to this organisation for betterment. The amount of donation is represented by the expression ₹ $4x^2 + \frac{1}{4x^2}$. She also discussed her friends about this organisation. Some of her friends wanted to know the amount of donation, but she did not disclose this amount to anyone. Some how her friend got to know that she gave amount having expression $\left(2x + \frac{1}{2x}\right)$, whose value is ₹ 90.

On the basis of the above information, solve the following questions:

Q 1. The amount donated by Swati in the expression form is:

a. linear equation b. quadratic equation
c. algebraic expression d. polynomial

Q 2. If $x = \sqrt{2}$, then the amount donated by Swati is:

a. ₹ 8 b. ₹ 8.125 c. ₹ 8.75 d. ₹ 9

Q 3. The value of $\left(2x + \frac{1}{2x}\right)^2$ is:

a. 8000 b. 8100 c. 8200 d. 8300

Q 4. The amount donated by Swati (in ₹) is:

a. ₹ 9020 b. ₹ 8096 c. ₹ 8090 d. ₹ 9000

Q 5. If $x = 5$, then the value of donated expression is:

a. $\frac{10001}{100}$ b. $\frac{1003}{100}$ c. $\frac{999}{100}$ d. $\frac{10005}{100}$

Solutions

1. (c) The amount donated by Swati in the expression form is algebraic expression.

So, option (c) is correct.

2. (b) Since, amount donated by Swati is $4x^2 + \frac{1}{4x^2}$.

At $x = \sqrt{2}$, then

$$4x^2 + \frac{1}{4x^2} = 4(\sqrt{2})^2 + \frac{1}{4(\sqrt{2})^2} = 8 + \frac{1}{8}$$

$$= 8 + 0.125 = ₹\ 8.125$$

So, option (b) is correct.

3. (b) Given, $\left(2x + \frac{1}{2x}\right) = 90$

Squaring both sides, we get

$$\left(2x + \frac{1}{2x}\right)^2 = (90)^2$$

$$= 8100$$

So, option (b) is correct.

4. (b) Now, $4x^2 + \frac{1}{4x^2} = (2x)^2 + \frac{1}{(2x)^2} + 4 - 4$

$$= \left(2x + \frac{1}{2x}\right)^2 - 4$$

$$= (90)^2 - 4 = 8100 - 4 = ₹\ 8096$$

Hence, amount donated by Swati is ₹ 8096.

So, option (b) is correct.

5. (a) Given, $x = 5$

$$\therefore \left(4x^2 + \frac{1}{4x^2}\right) = 4(5)^2 + \frac{1}{4(5)^2}$$

$$= 4 \times 25 + \frac{1}{4 \times 25}$$

$$= 100 + \frac{1}{100} = \frac{10001}{100}$$

So, option (a) is correct.

Case Study 4

In the current scenario, people use such door whose top half part is made of glass and bottom half part is wooden.

The glass portion of the door is having length and width in the ratio of 5: 3. The wooden frame around the glass portion adds 11 inches to the total width and 14 inches to the total length. Consider the length of the glass portion as 5x inches:

On the basis of the above information, solve the following questions:

Q1. Find the total length of the glass portion of the door (in inches) is represented in terms of x.

Q 2. Find the total width of the glass portion of the door (in inches).

Q3. Write the polynomial representation of the area top half part of the door.

Q4. Find the zeroes of the polynomial representing the area.

Solutions

1. The total length of the glass portion in the door is represented by (5x + 14) inches.

2. The total width of the glass portion in the door is (3x + 11) inches.

3. The area of top half part of the door

$$= \text{length} \times \text{width}$$
$$= (5x + 14)\ (3x + 11)$$
$$= 15x^2 + 55x + 42x + 154$$
$$= 15x^2 + 97x + 154$$

4. We have area, p(x) = (5x + 14) (3x + 11)

For finding zeroes, put p(x) = 0

$\therefore (5x+14)(3x+11)=0$

$\Rightarrow \quad (5x+14)=0 \text{ or } (3x+11)=0$

$\Rightarrow \quad x=\frac{-14}{5} \text{ or } x=\frac{-11}{3}$

Case Study 5

A teacher told 10 students to write a polynomial on the blackboard.

The students wrote the following polynomials:

1. $\sqrt{5}x^3+1$	2. $20x^3+3x+8$
3. $x-2$	4. $x^2+\frac{12x}{35}+\frac{1}{35}$
5. $3x^3-4x^2-12x+16$	6. $-2x-5$
7. $\frac{\pi}{2}x^2+x$	8. $9x^2-361$
9. $\frac{12}{x}-64-3x^2+24x$	10. $8x^3$

On the basis of the above information, solve the following questions:

Q1. How many students wrote quadratic polynomial?

Q2. How many students wrote a binomial?

Q3. Find the zeroes of the polynomial p(x) = −2x – 5.

Q4. Factorise: $9x^2 – 361$.

Solutions

1. There are 3 students to write a quadratic polynomial on the blackboard.

   i.e., $x^2+\frac{12x}{35}+\frac{1}{35}, \frac{\pi}{2}x^2+x, 9x^2-361$

2. There are 5 students to write a binomial on the blackboard.

   i.e., $\sqrt{5}x^3+1,\ x-2,\ -2x-5,\ \frac{\pi}{2}x^2+x,\ 9x^2-361$

3. Consider $p(x)=0$

   $\Rightarrow \quad -2x-5=0$

   $\Rightarrow \quad x=\frac{-5}{2}$

So, $\frac{-5}{2}$ is a zero of the polynomial $(-2x - 5)$.

4. $9x^2 - 361 = (3x)^2 - (19)^2$ [$\because a^2 - b^2 = (a - b)(a + b)$]

$= (3x - 19)(3x + 19)$