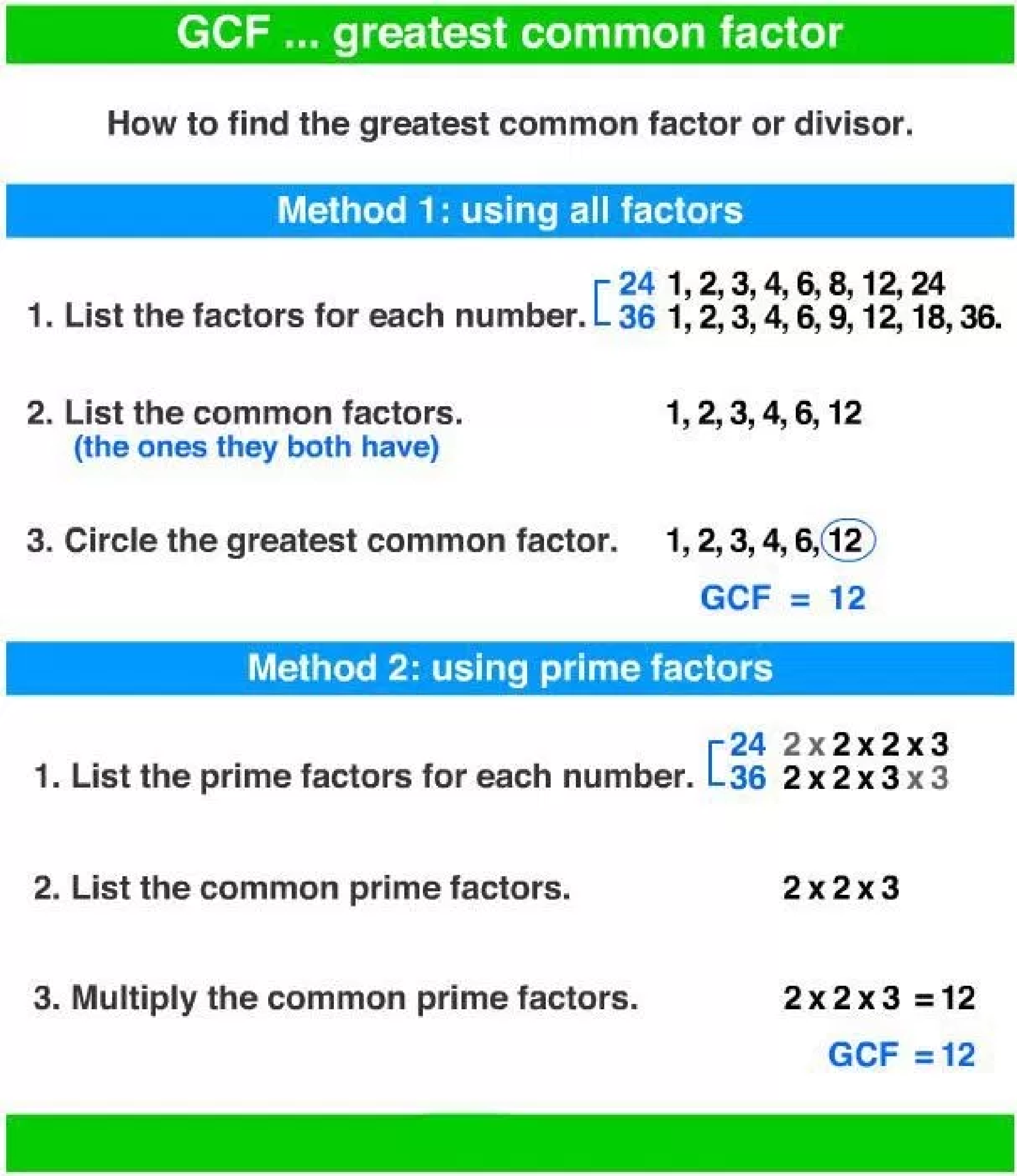
4. Factoring

Common Factors

When two integers are multiplied together, the answer is called a **product.**
The integers that were multiplied together are called the **factors** of the product.
3 • 6 = 18 (3 and **6** are factors of **18)**

The **greatest common factor** of two (or more) integers is the largest integer that is a factor of both (or all) numbers.



Consider the numbers 18, 24, and 36.

The greatest common factor is 6.
(6 is the largest integer that will divide evenly into all three numbers)

The greatest common factor, (GCF), of two (or more) monomials is the product of the greatest common factor of the numerical coefficients (the numbers out in front) and the highest power of every variable that is a factor of each monomial.

Example: Consider $10x^2y^3$ and $15xy^2$
The greatest common factor is $5xy^2$.

The largest factor of 10 and 15 is 5.
The highest power of x that is contained in both terms is x.
The highest power of y that is contained in both terms is y^2 .

When factoring polynomials, first look for the largest monomial which is a factor of each term of the polynomial. Factor out (divide each term by) this largest monomial.

Example 1: Factor: $4x + 8y$
The largest integer that will divide evenly into 4 and 8 is 4. Since the terms do not contain a variable (x or y) in common, we cannot factor any variables.
The greatest common factor is 4. Divide each term by 4.

Answer: $4(x + 2y)$

Example 2: Factor: $15x^2y^3 + 10xy^2$
The largest integer that will divide evenly into 15 and 10 is 5. The largest power of x present in both terms is x.
The largest power of y present in both term is y^2.
The GCF is $5xy^2$. Divide each term by the GCF.

Answer: $5xy^2(3xy + 2)$

Solving A Quadratic Equation By Factoring

(i) $(x+y)^2 = x^2 + 2xy + y^2$

(ii) $(x-y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x-y)(x+y)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(vii) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

(viii) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^3 = 3xyz \quad if \; x+y+z = 0$

Forms of a Quadratic Equation

- **Standard Form of a Quadratic Equation** $ax^2 + bx + c = 0, a \neq 0$

- **Factored Form of a Quadratic Equation** $a(x + p)(x + q) = 0, a \neq 0$

  Factoring means to write the terms in multiplication form (as a product).

- **<u>Zero Product Property</u>**

  If $ab = 0$ then either $a = 0$ or $b = 0$ (or both).

  The expression *must* be set equal to zero to use this property.

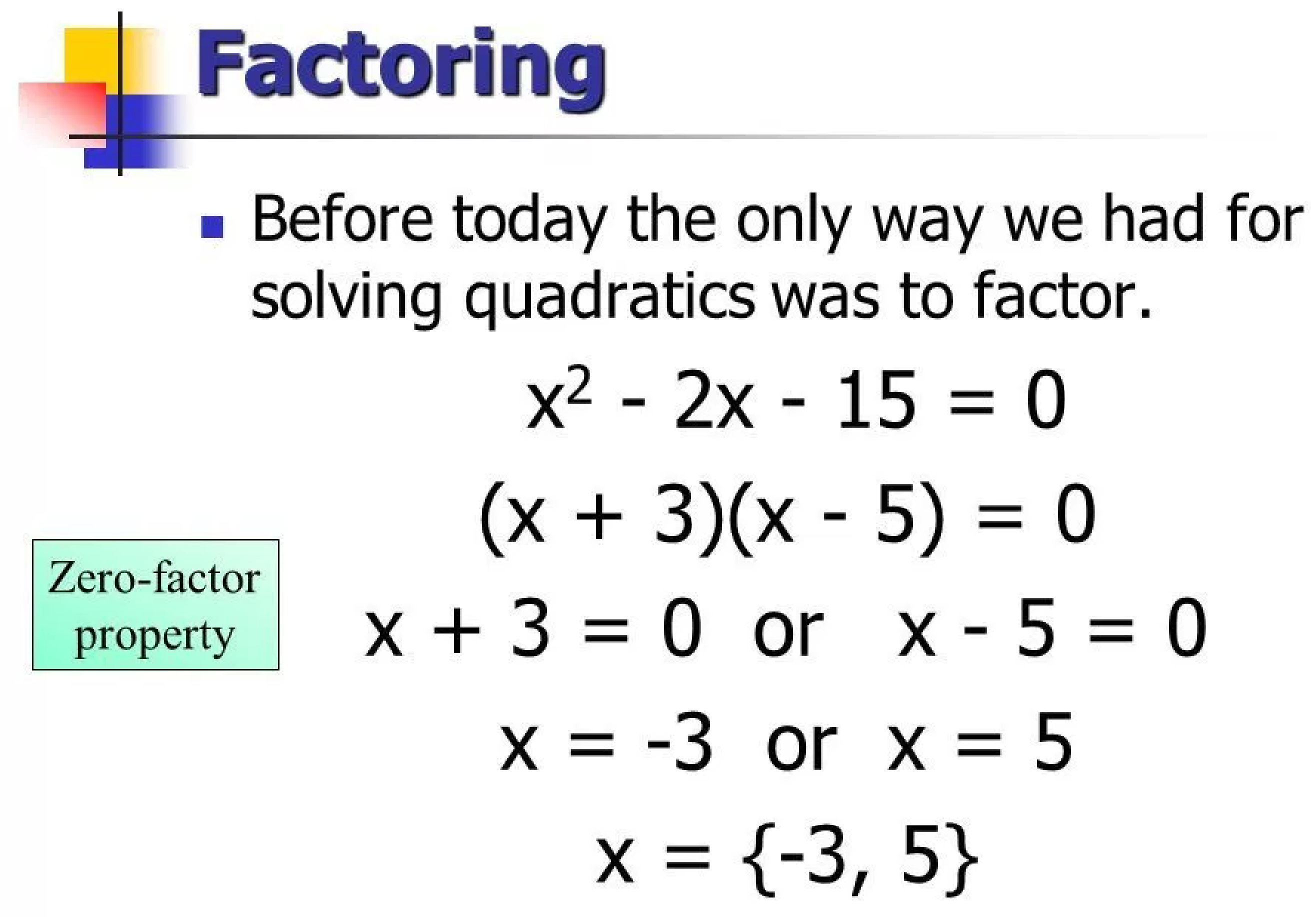
Zero Product Example: Quadratic in Factored Form $(x - 6)(x + 8) = 0$

$x - 6 = 0$ or $x + 8 = 0$

$x = 6$ or $x = 8$

Factoring Help!

Question	Strategy	Answer
$m^2+10m+16$	Both signs are positive, so both signs in answer are positive.	$(m+2)(m+8)$
$n^2-8n-48$	Two negatives, so in our answer, one will be positive (the smaller number) and one will be negative (the larger number)	$(n-12)(n+4)$
$y^2-15y+56$	Second term negative, third term positive; both signs in the answer will be negative	$(y-8)(y-7)$
p^2+p-20	Second term positive, third term negative; one will be positive (the larger number) and one will be negative (the smaller number)	$(p+5)(p-4)$



Since, $3x^2 - 5x + 2$ is a quadratic polynomial;
$3x^2 - 5x + 2 = 0$ is a quadratic equation.
Also,
$3x^2 - 5x + 2 = 3x^2 - 3x - 2x + 2$ [Factorising]
$= 3x(x - 1) - 2(x - 1)$
$= (x - 1)(3x - 2)$
In the same way :
$3x^2 - 5x + 2 = 0 \Rightarrow 3x^2 - 3x - 2x + 2 = 0$ [Factorising L.H.S.]
$\Rightarrow (x - 1)(3x - 2) = 0$
i.e., $x - 1 = 0$ or $3x - 2 = 0$
$\Rightarrow x = 1$ or $x = 2/3$
which is the solution of given quadratic equation.

In order to solve the given Quadratic Equation:

1. Clear the fractions and brackets, if given.
2. By transfering each term to the left hand side; express the given equation as $ax^2 + bx + c = 0$ or $a + bx + cx^2 = 0$
3. Factorise left hand side of the equation obtained (the right hand side being zero).
4. By putting each factor equal to zero; solve it.

Solving A Quadratic Equation By Factoring With Examples

Example 1:

Solve (i) $x^2 + 3x - 18 = 0$ (ii) $(x - 4)(5x + 2) = 0$
(iii) $2x^2 + ax - a2 = 0$; where 'a' is a real number.

Sol.

(i) $x^2 + 3x - 18 = 0$
$\Rightarrow x^2 + 6x - 3x - 18 = 0$
$\Rightarrow x(x + 6) - 3(x + 6) = 0$
i.e., $(x + 6)(x - 3) = 0$
$\Rightarrow x + 6 = 0$ or $x - 3 = 0$
$\Rightarrow x = -6$ or $x = 3$

Roots of the given equation are -6 and 3

(ii) $(x - 4)(5x + 2) = 0$
$\Rightarrow x - 4 = 0$ or $5x + 2 = 0$
$x = 4$ or $x = -2/5$

(iii) $2x^2 + ax - a^2 = 0$
$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$
$\Rightarrow 2x(x + a) - a(x + a) = 0$
i.e., $(x + a)(2x - a) = 0$
$\Rightarrow x + a = 0$ or $2x - a = 0$
$\Rightarrow x = -a$ or $x = a/2$

Example 2:

Solve the following quadratic equations
(i) $x^2 + 5x = 0$ (ii) $x^2 = 3x$ (iii) $x^2 = 4$

Sol.

(i) $x^2 + 5x = 0 \Rightarrow x(x + 5) = 0$
$\Rightarrow x = 0$ or $x + 5 = 0$
$\Rightarrow x = 0$ or $x = -5$

(ii) $x^2 = 3x$
$\Rightarrow x^2 - 3x = 0$
$\Rightarrow x(x - 3) = 0$
$\Rightarrow x = 0$ or $x = 3$

(iii) $x^2 = 4$
$\Rightarrow x = \pm 2$

Example 3:

Solve the following quadratic equations

(i) $7x^2 = 8 - 10x$ (ii) $3(x^2 - 4) = 5x$

(iii) $x(x + 1) + (x + 2)(x + 3) = 42$

Sol.

(i) $7x^2 = 8 - 10x$
$\Rightarrow 7x^2 + 10x - 8 = 0$
$\Rightarrow 7x^2 + 14x - 4x - 8 = 0$
$\Rightarrow 7x(x + 2) - 4(x + 2) = 0$
$\Rightarrow (x + 2)(7x - 4) = 0$
$\Rightarrow x + 2 = 0$ or $7x - 4 = 0$
$\Rightarrow x = -2$ or $x = 4/7$

(ii) $3(x^2 - 4) = 5x$
$\Rightarrow 3x^2 - 5x - 12 = 0$
$\Rightarrow 3x^2 - 9x + 4x -\neg 12 = 0$
$\Rightarrow 3x(x - 3) + 4(x - 3) = 0$
$\Rightarrow (x - 3)(3x + 4) = 0$

⇒ $x - 3 = 0$ or $3x + 4 = 0$
⇒ $x = 3$ or $x = -4/3$

(iii) $x(x + 1) + (x + 2)(x + 3) = 42$
⇒ $x^2 + x + x^2 + 3x + 2x + 6 - 42 = 0$
⇒ $2x^2 + 6x - 36 = 0$
⇒ $x^2 + 3x - 18 = 0$
⇒ $x^2 + 6x - 3x - 18 = 0$
⇒ $x(x + 6) - 3(x + 6) = 0$
⇒ $(x + 6)(x - 3) = 0$
⇒ $x = -6$ or $x = 3$

Example 4:

Solve for x : $12abx^2 - (9a^2 - 8b^2) x - 6ab = 0$
Given equation is $12abx^2 - (9a^2 - 8b^2) x - 6ab = 0$
⇒ $3ax(4bx - 3a) + 2b(4bx - 3a) = 0$
⇒ $(4bx - 3a)(3ax + 2b) = 0$
⇒ $4bx - 3a = 0$ or $3ax + 2b = 0$
⇒ $x = 3a/4b$ or $x = -2b/3a$