Class 11

Important Formulas

Binomial Theorem

1. (*Binomial theorem*) If x and a are real numbers, then for all $n \in N$, we have

$$(x+a)^n = {}^nC_0\, x^n a^0 + {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 + \ldots\ldots\ldots + {}^nC_r\, x^{n-r} a^r + \ldots + {}^nC_{n-1}\, x^1 a^{n-1} + {}^nC_n\, x^0 a^n$$

i.e., $(x+a)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} a^r$

This expansion has the following properties:

(i) It has $(n+1)$ terms.

(ii) The sum of the indices of x and a in each term is n.

(iii) The coefficients of terms equidistant from the beginning and the end are equal.

(vi) General term is given by $T_{r+1} = {}^nC_r\, x^{n-r} a^r$

(v) $(x+a)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} a^r$ can also be expressed as $(x+a)^n = \sum_{r+s=n} \frac{n!}{r!\,s!} x^r a^s$

(vi) Replacing a by $-a$ in the expansion of $(x+a)^n$, we get

$$(x-a)^n = {}^nC_0\, x^n a^0 - {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 - {}^nC_3\, x^{n-3} a^3 + \ldots + (-1)^r\, {}^nC_r\, x^{n-r} a^r + \ldots + (-1)^n\, {}^nC_n\, x^0 a^n$$

The general term in the expansion of $(x-a)^n$ is $T_{r+1} = (-1)^r\, {}^nC_r\, x^{n-r} a^r$

(vii) Putting $x = 1$ and replacing a by x in the expansion of $(x+a)^n$, we get

$$(1+x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \ldots + {}^nC_n\, x^n = \sum_{r=0}^{n} {}^nC_r\, x^r$$

This is expansion of $(1+x)^n$ is ascending powers of x. In this case, $T_{r+1} = {}^nC_r\, x^r$

(viii) Putting $a = 1$ in the expansion of $(x+a)^n$, we get

$$(1+x)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1} + {}^nC_2\, x^{n-2} + \ldots + {}^nC_n\, x^0 = \sum_{r=0}^{n} {}^nC_r\, x^{n-r}$$

This is the expansion of $(1+x)^n$ in descending powers of x.. In this case, $T_{r+1} = {}^nC_r\, x^{n-r}$

(ix) $(x+a)^n + (x-a)^n = 2\left\{{}^nC_0\, x^n a^0 + {}^nC_2\, x^{n-2} a^2 + \ldots\right\}$

$= 2$ {Sum of the odd terms in the expansion of $(x+a)^n$}

$(x+a)^n - (x-a)^n = 2\left\{{}^nC_1\, x^{n-1} a^1 + {}^nC_3\, x^{n-3} a^3 + \ldots\right\}$

$= 2$ {Sum of the even terms in the expansion of $(x+a)^n$}

If n is odd, then $\left\{(x+a)^n + (x-a)^n\right\}$ and $\left\{(x+a)^n - (x-a)^n\right\}$ both have $\left(\frac{n+1}{2}\right)$ terms.

If n is even, then $\left\{(x+a)^n + (x-a)^n\right\}$ has $\left(\frac{n}{2}+1\right)$ terms whereas $\left\{(x+a)^n - (x-a)^n\right\}$ has $\left(\frac{n}{2}\right)$ terms.

(x) If O and E denote respectively the sums of odd terms and even terms in the expansion of $(x+a)^n$, then

(a) $(x+a)^n = O+E$ and $(x-a)^n = O-E$ (b) $(x^2-a^2)^n = O^2-E^2$

(c) $4\,OE = (x-a)^{2n} - (n-a)^{2n}$ (d) $(x+a)^{2n} + (x-a)^{2n} = 2\,(O^2+E^2)$

(xi) If n is even, then $\left(\frac{n}{2}+1\right)^{\text{th}}$ term is the middle term.

If n is odd, then $\left(\frac{n+1}{2}\right)$ and $\left(\frac{n+3}{2}\right)$ are middle terms.