

Soil Compaction

5.1 Introduction

Construction of many structures requires "stabilisation" of the soil mass i.e. an artificial improvement of its engineering properties. There are various methods of soil stabilisation, the most common being the mechanical stabilisation and the simplest technique of mechanical stabilisation is compaction.

Compaction is a process by which the soil particles are artificially rearranged and packed together into a closer state of contact by mechanical means in order to reduce the volume of air voids of the soil and thus increase its dry density.

A soil mass can be compacted by either a dynamic process or a static one. In the dynamic method the soil mass is compacted by repeated applications of a dead load, while in static method, compaction is done by a steady increase of static load.

Do you know? The dynamic method gives better result in coarse grained soils and static method is suitable for less permeable fine grained soils.

5.2 Principles of Compaction

Usually when soil is excavated from one place, transported and dumped in another place it is loosened up to a great extent. In an uncompacted fill, the void ratio is relatively high and the corresponding density is low. Consequently, the shearing strength of uncompacted soil is low while its permeability is high. A structure on such a fill may crack extensively due to non uniformity of settlement. Therefore, it is necessary to artificially compact the fill.

Soil compaction is the process where soil particles are forced to pack more closely by reducing air voids. This can be achieved by applying some mechanical energy (static or dynamic) on soil fill.

The degree of compaction of a soil is measured in terms of dry unit weight i.e. the amount of soil solids that can be packed in a unit volume of the soil.

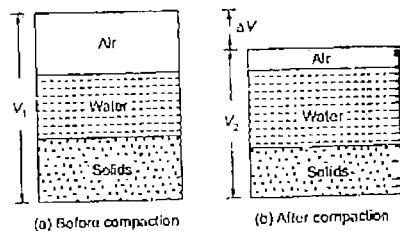


Fig. 5.1

Remember: Compaction is somewhat different from consolidation. In consolidation volume reduction takes place due to expulsion of pore water from saturated voids.

5.3 Difference between Compaction and Consolidation

5.3.1 Compaction

- Almost an instantaneous phenomenon.
- Soil is always unsaturated.
- Densification is due to a reduction in the volume of air voids at a given water content.
- Specified compaction techniques are used in this process.

5.3.2 Consolidation

- It is a time dependent phenomenon.
- Soil is completely saturated.
- Volume reduction is due to expulsion of pore water from voids.
- Consolidation occurs on account of a load placed on the soil.

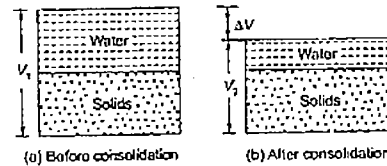


Fig. 5.2

5.4 Advantages of Compaction

Proper compaction of a soil mass lead to

- Increase in shear strength of soil.
- Improved stability and bearing capacity of soil.
- Reduction in compressibility and permeability of soil.
- Increase in the load carrying capacity of soil subgrade.
- Prevention of detrimental settlements and undesirable volume changes through swelling and shrinkage.



When a heavily compacted soil mass (near to maximum dry unit weight) is sheared, it tends to expand and gets loosened. Therefore, heavily compacted soils tend to show decrease in strength when sheared.

5.5 Laboratory Compaction

Laboratory compaction tests are designed to estimate the dry density of soils. These tests are based on any one of the following methods or type of compaction; dynamic or impact, kneading, static and vibration. The tests are performed by compacting a wet soil sample in a mould in a specified number of layers. Each layer is compacted at specified number of blows. After compaction of final layer, the bulk density of soil and corresponding moisture content are determined. Test are repeated on fresh samples with changed moisture content. With known values of bulk density and moisture content, the dry density may be calculated as given below.

$$\text{Dry density, } \rho_d = \frac{\rho_t}{1 + w}$$

Now a graph of dry density Vs moisture content is plotted and the maximum dry density and optimum moisture content are read from the graph.

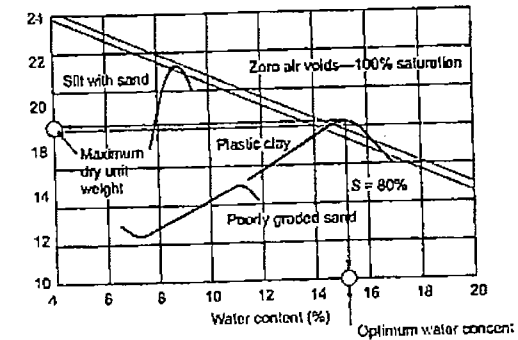


Fig. 5.3

5.5.1 Standard Proctor Test

- Standard volume mould (944 cc or 1/30 cubic feet).
- Filled up with soil in three layers.
- Each layer is compacted by 25 blows of standard hammer (weight 2.495 kg or 5.5 lbs) falling through 304.8 mm (12 inch).
- Dry weight is calculated by knowing the wet weight of compacted soil and its water content.

Dry unit weight,

$$\gamma_d = \frac{\gamma}{1 + w}$$

where, γ = bulk unit weight

$$= \frac{\text{weight of compacted soil}}{\text{volume of the mould}}$$

w = water content

- Test is repeated at different water contents.
- Compaction curve is plotted between moisture content and dry unit weight.
- The peak of compaction curve corresponding to the maximum dry unit weight is referred as $\gamma_{d(max)}$.
- The moisture content corresponding to $\gamma_{d(max)}$ is known as optimum moisture content (OMC) at a given compactive effort.

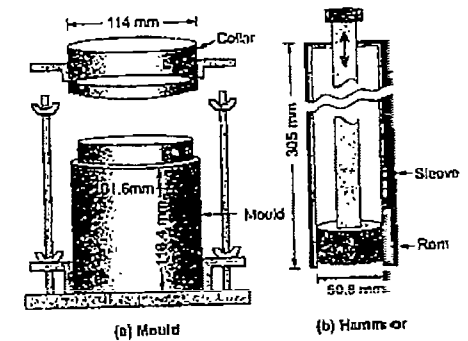


Fig. 5.4 Standard Proctor

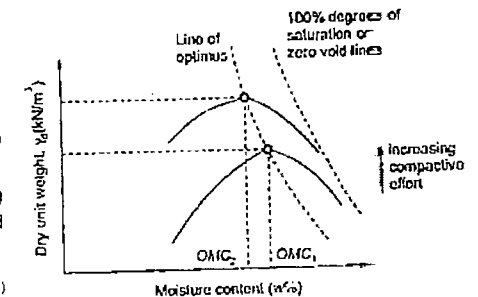


Fig. 5.5 Compaction Curve

NOTE

- Typical values of maximum dry unit weights range from 16 to 20 kN/m³ with the widest range being 13 to 24 kN/m³.
 - Typical optimum moisture content values range from 10 to 20% with a maximum range of 5 to 30%.
 - Maximum dry unit weight so obtained is only for a given amount of compaction effort and method of compaction. It is also not necessary that the maximum dry unit weight can be obtained in the field.
- The specifications for compaction of fills in the field are usually based on maximum dry unit weight but sometimes on both the maximum dry unit weight and the OMC.

5.5.2 Modified Proctor Test

- Developed during World War II, to simulate the compaction required for air fields to support heavier aircrafts.
- Standard volume mould (944 cc or 1/30 cubic feet).
- Filled up with soil in five layers.
- This test employs heavier hammer (4.54 kg or 10 lbs).
- Height of fall is 457.2 mm (18 inches) and each layer is tamped 25 times into a standard proctor mould.

5.5.3 Indian Standard Equivalent of Standard Proctor Test (Light Compaction Test)

- Volume of mould is 1000 cc.
- Weight of hammer is 2.6 kg and drop is 310 mm.
- Mould is filled in three layers and each layer tamped by 25 blows.

5.5.4 Indian Standard Equivalent of Modified Proctor Test (Heavy Compaction Test)

- Volume of mould is 1000 cc.
- Weight of hammer is 4.9 kg and height of drop is 450 mm.
- Soil is compacted in 5 layers and each layer tamped by 25 blows.

Do you know? Under heavier compaction the moisture-density curve is shifted upward and simultaneously towards the left, resulting in a lower OMC but a greater $\gamma_{d(max)}$.

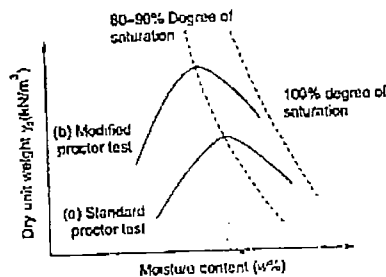
5.6 Comparison of Standard and Modified Proctor Test

Fig. 5.6

Remember

- Compactive energy applied in proctor test per unit volume is given by

$$E = \frac{nNWh}{V}$$

where,

- n = Number of layer
- N = Number of blows per layer
- W = Weight of hammer
- h = Height of free fall of hammer
- V = Volume of mould

- Compactive energy per unit volume in Standard Proctor Test

$$E = \frac{3 \times 25 \times 2.495 \times 0.3048 \times 9.81}{944 \times 10^{-6}} = 592.71 \text{ kJ/m}^3$$

- Compactive energy per unit volume in Modified Proctor Test

$$E = \frac{5 \times 25 \times 4.54 \times 0.4572 \times 9.81}{944 \times 10^{-6}} = 2696.30 \text{ kJ/m}^3$$

- Compactive energy per unit volume in Light Compaction Test

$$E = \frac{3 \times 25 \times 2.6 \times 0.310 \times 9.81}{1000 \times 10^{-6}} = 593.01 \text{ kJ/m}^3$$

- Compactive energy per unit volume in Heavy Compaction Test

$$E = \frac{5 \times 25 \times 4.9 \times 0.45 \times 9.81}{1000 \times 10^{-6}} = 2703.88 \text{ kJ/m}^3$$

Example 5.1

The compaction of a floor area is carried out in 250 mm thick layers. The hammer used for compaction has the foot area of 0.05 m². The energy developed per drop of the rammer is 50 kg-m. Assuming 40% more energy in each pass over the compacted area due to overlap, calculate the number of passes required to develop compactive energy equivalent to IS heavy compaction for each layer.

Solution:

Compactive energy as per IS heavy compaction test

$$= \frac{4.9(\text{kg}) \times 0.45(\text{m}) \times 5(\text{layers}) \times 25(\text{blows/layer})}{10^3 \times 10^{-6}(\text{m}^3)} = 27562.5 \text{ kg/m}^3$$

Compactive energy per drop provided by the rammer per m³ of the soil

$$= \frac{50}{0.05 \times 250 \times 10^{-3}} = 4000 \text{ kg/m}^3$$

However, in each pass over a layer, the energy supplied will be 1.4 times this value on account of overlap of hammer foot print. Let n be the number of passes required to develop compactive energy equivalent to IS heavy compaction.

$$\therefore n \times 1.4 \times 4000 = 27562.5$$

$$n = 4.92 \text{ say } 5$$

5.7 Zero Air Void Line

At constant moisture content, the dry unit weight reaches its theoretical maximum when all the air is expelled from the void spaces, i.e. when degree of saturation is 100%. Therefore the zero air void line is a line joining points having dry unit weights corresponding to 100% saturation at different moisture contents. Therefore, it is also called the saturation line.

Zero air void lines can be defined as "The lines showing the dry density as a function of water content for soil containing no air voids."

We can derive its equation as follows:

$$\gamma_d = \frac{G_s \gamma_w}{1 + e} \quad \dots(i)$$

For any degree of saturation,

$$e = \frac{w G_s}{S}$$

Therefore we can write the expression for dry unit weight corresponding to any degree of saturation as

$$\gamma_{ds} = \frac{G_s \gamma_w}{1 + \frac{w G_s}{S}} = \frac{\gamma_w}{\left(\frac{1}{G_s}\right) + \left(\frac{w}{S}\right)} \quad \dots(ii)$$

where, γ_{ds} = dry unit weight at degree of saturation

γ_w = unit weight of water, e = void ratio

w = water content, G_s = sp. gravity of solids

For zero air voids, degree of saturation becomes 100%. Therefore equation (ii) becomes

$$\gamma_{d0} = \frac{\gamma_w}{\left(\frac{1}{G_s}\right) + w} \quad \dots(iii)$$

where, γ_{d0} = dry unit weight at zero air void

Remember: Zero air voids line or saturation line is always a steadily decreasing line.

5.8 Constant Percentage Air Void Lines

These are lines which show the water content, dry density relation for the compacted soil containing a constant percentage air void is known as an air voids line.

By the definition of percentage air voids, we have

$$\frac{n_a}{100} = \frac{V_a}{V_v} = \frac{V_v - V_w}{V_v} = 1 - S$$

$$\text{or} \quad S = 1 - \frac{n_a}{100} \quad \dots(iv)$$

Substituting value of S into equation (ii), we have

$$\gamma_{da} = \frac{G_s \gamma_w \left(1 - \frac{n_a}{100}\right)}{\left(1 - \frac{n_a}{100}\right) + w G_s}$$

where, γ_{da} = dry unit weight at constant percentage air voids

of soil

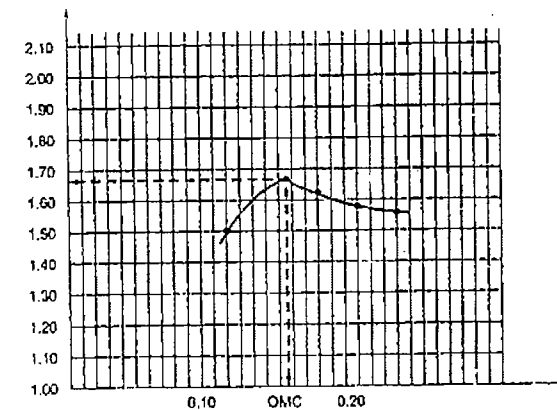
Example 5.2 The following results were obtained from a IS Light Compaction Test on sample

Water content (%)	0.12	0.15	0.18	0.21	0.24
Mass of Wet Soil (kg)	1.65	1.95	1.93	1.90	1.86

Plot the compaction curve. Hence obtain the maximum dry density and the optimum moisture content. Also calculate the void ratio the degree of saturation and theoretical maximum dry density ($G = 2.68$).

Solution:

Water Content (w)	0.12	0.15	0.18	0.21	0.24
Mass of wet soil M (kg)	1.65	1.95	1.93	1.90	1.86
Bulk density $\rho = \frac{M}{V} = \frac{M}{1.0} \text{ kg/m}^3$	1.65	1.95	1.93	1.90	1.86
Dry density $\rho_d = \frac{\rho}{1+w}$	1.47	1.69	1.67	1.57	1.5
Void ratio $e = \frac{G \rho_s}{\rho_d} - 1$	0.62	0.58	0.60	0.70	0.79
Degree of saturation $S = \frac{w G}{e}$	0.39	0.69	0.60	0.604	0.61
Theoretical max dry density $\rho_{d(max)} = \frac{G \rho_s}{1 + w G}$	2.03	1.91	1.81	1.71	1.63



From the compaction curve,
Optimum moisture content,

$$\rho_{d(max)} = 1.66 \text{ g/cc}$$

$$\text{OMC} = 0.163 \text{ or } 16.3\%$$

Example 5.3 The results of a standard compaction test are shown in the table below. Determine the maximum dry unit weight and optimum water content.

Water Content (%)	6.2	8.1	9.8	11.5	12.3	13.2
Bulk unit weight (kN/m ³)	16.9	18.7	19.5	20.5	20.4	20.1

- (a) What is the dry unit weight and water content at 95% standard compaction?
 (b) Determine the degree of saturation at the maximum dry density.
 (c) Plot the zero air voids line.
 Compute γ_d and then plot the results of γ_d versus w (%). Then extract the required information.

Solution:

Water Content (%)	6.2	8.1	9.8	11.5	12.3	13.2
Bulk unit weight (kN/m ³)	16.9	18.7	19.5	20.5	20.4	20.1
Dry unit weight (kN/m ³) $\gamma_d = \frac{\gamma}{1+w}$	15.9	17.3	17.8	18.4	18.2	17.8

$$\gamma_{d(max)} = 18.4 \text{ kN/m}^3$$

and optimum moisture content,

$$w_{opt} = 11.5\%$$

From Graph below,

$$\therefore \text{At 95\% compaction, } \gamma_d = 18.4 \times 0.95 = 17.5 \text{ kN/m}^3$$

$$\text{Water content, } w = 9.2\%$$

- (b) Degree of saturation at maximum dry unit weight

We know,

$$\gamma_d = \frac{G_s \gamma_w}{1 + \frac{wG_s}{S}}$$

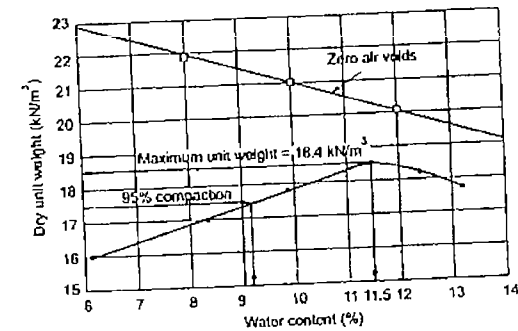
$$S = \frac{wG_s \gamma_{d(max)} / \gamma_w}{G_s - \gamma_{d(max)} / \gamma_w}$$

$$= \frac{0.115 \times 2.7 \times (18.4/9.8)}{2.7 - (18.4/9.8)}$$

$$= 0.71 \text{ or } 71\%$$

- (c) Zero air void lines

Water Content (%)	6	8	10	12	14
Dry unit weight (kN/m ³) $\gamma_d = \frac{G_s \gamma_w}{1 + \frac{wG_s}{S}}$ at $S = 1$	22.8	21.8	20.8	20.0	19.2



Example 5.4 The maximum dry density of a sample by the standard compaction test is 1.76 g/cc at an optimum moisture content of 14.5%. Find the air voids and the degree of saturation ($G = 2.68$).

What would be the corresponding value of dry density on the zero air void line at optimum moisture content?

Solution:

- (i) We know,

$$\rho_d = \frac{\left(1 - \frac{n_a}{100}\right) G p_w}{1 + wG}$$

$$1.76 = \frac{\left(1 - \frac{n_a}{100}\right) \times 2.68 \times 1}{1 + 0.145 \times 2.68}$$

$$n_a = 0.088 \text{ or } 8.80\%$$

- (ii)

$$\rho_d = \frac{G p_w}{1 + e} = \frac{G p_w}{1 + \frac{wG}{S}}$$

$$1.76 = \frac{2.68 \times 1}{1 + \left(\frac{0.145 \times 2.68}{S}\right)}$$

$$1.76 + \frac{0.683}{S} = 2.68$$

$$S = 0.7423 \text{ or } 74.23\%$$

- (iii) At zero air void line, $S = 100\%$

$$\rho_{d(100\%, \text{max})} = \frac{G p_w}{1 + \frac{wG}{S}} = \frac{G p_w}{1 + wG} = \frac{2.68 \times 1}{1 + 0.145 \times 2.68} = 1.93 \text{ g/cc}$$

5.9 Factors Affecting Compaction

The major factors which affect compaction are:

- (i) water content
- (ii) compactive effort
- (iii) type of soil
- (iv) method of compaction

(i) **Water Content:** At lower water contents, the soil particles offer more resistance to compaction and soil behaves like a stiff material. Increasing the moisture content helps the particles to move closer because of the lubrication effect. On further increasing the moisture content beyond a certain limit, the water starts to replace the soil particles. Thus the dry unit weight continuous to increase till the Optimum Moisture Content (OMC) is reached and with further increase in moisture content beyond OMC, unit weight starts decreasing.

Do You Know?

Using,

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{G\gamma_w}{1 + \frac{wG}{S}}$$

we conclude $\gamma_d \propto S$ i.e. degree of saturation.

Therefore for a given water content, the theoretical maximum value of dry unit weight for a compacted soil is obtained corresponding to the situation when no air voids are left i.e. when degree of saturation becomes 100%.

(ii) **Compactive Effort:** For a given type of compaction, an increase in the amount of compaction will initially result in closer packing of the soil particles and maximum dry unit weight increase while the optimum moisture content at which it is attained decreases.

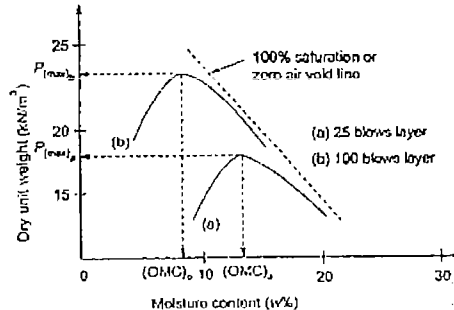


Fig. 5.7 Effect of compactive effort

(iii) **Type of Soil**

- Coarse grained soils, well graded can be compacted to high dry unit weight, especially if they contains some fines.
- However, if quantity of fines is excessive the maximum dry unit weight decreases.
- Poorly graded or uniform sands lead to lowest dry unit weight value.
- In clayey soils, maximum dry unit weight tends to decrease as plasticity increases.
- Cohesive soils have generally high value of OMC.

- Heavy clays with high plasticity have very low value of maximum dry unit weight and very high value of OMC.

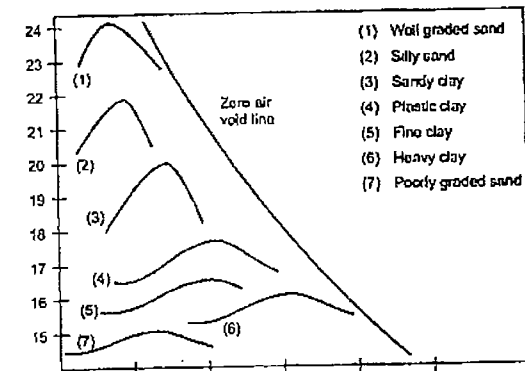


Fig. 5.8 Typical proctor compaction curve for different soils

(iv) **Method of Compaction:** Since the field compaction is essentially a kneading or rolling type compaction whereas the laboratory tests are dynamic-impact type compaction, therefore, laboratory compaction tests have more value of maximum dry unit weight.

5.10 Compaction Behaviour of Sand

The moisture-dry density relationship, as obtained from a laboratory test on a cohesionless sand is shown in figure below:

- Initially there is decrease in dry unit weight with the increase in water content. This is due to bulking of sand i.e. capillary tension in pore water prevents soil particles coming closer. The maximum bulking occurs at 4-5% water content.
- The maximum dry unit weight occurs when soil is either completely saturated or dry.
- When water content is increased further, there is fall in dry unit weight again.

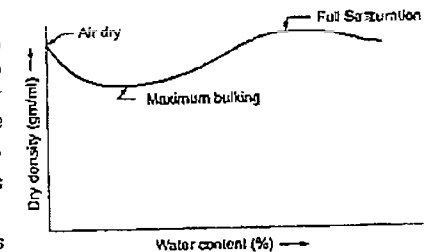


Fig. 5.9 Compaction Behaviour of Sand

Remember: Coarse grained soils do not absorb water, but fine grained soils do and hence the Lambe's double layer theory is not applicable to them.

5.11 Effect of Compaction on Properties of Soils

5.11.1 Soil Structure

- The water content at which the soils is compacted plays an important role in the engineering properties of the soils.

- At low water content, attractive forces between the particles are stronger than repulsive forces. Hence, soils compacted at a water content less than the optimum water content generally have a flocculated structure.
- Increasing the water content, increases the repulsive forces. Hence, soils compacted at water content more than the optimum water content usually have a dispersed structure.
- If the compactive effort is increased there is a corresponding increase in the orientation of the particles and higher dry densities are obtained, as shown by the upper curve.

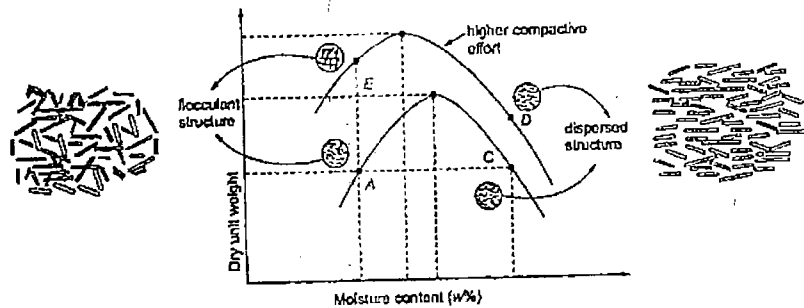


Fig. 5.10

5.11.2 Permeability

- The permeability of a soil depends upon the size of voids. Due to increase in water content for a given compactive effort there is an improved orientation of the particles and a corresponding reduction in the size of voids which cause a decrease in permeability.
- For a given compactive effort, the permeability decreases sharply with increase in water content on the dry side of optimum. The minimum permeability occurs at or slightly above the OMC. Beyond OMC, the permeability may show a slight increase, but always remain much smaller than on the dry side of optimum.
- If the compactive effort is increased, the permeability of the soil decrease due to increased dry density and better orientation of particle.

5.11.3 Compressibility

- At relatively low stress levels, a soil compacted wet side (dispersed soil) of optimum is more compressible than the soil compacted dry side (flocculated soil) of optimum. Therefore, for construction of embankment, the soil is compacted on the dry side of optimum.
- At high stress levels, the compressibility increase due to breakdown of the structure, now the flocculated structures with larger void volume, can undergo a large volume decrease.

5.11.4 Swelling

- A soil on the dry side of optimum has a higher water deficiency and a more random particle arrangement. It can therefore, imbibe more water than a soil on the wet of optimum, therefore more swelling occur on dry side of optimum.

5.11.5 Shrinkage

- Soils compacted on the wet of optimum, have nearly parallel orientation of particles which allows the particles to pack more efficiently as compared to the randomly oriented particles on the dry side of optimum.
- Therefore, soils compacted on the wet of optimum tend to exhibit more shrinkage upon drying than those compacted on the dry of optimum.

5.11.6 Shear Strength

- The shear strength of the compacted soils depends upon the soil type, the moulded water content, drainage conditions, the method of compaction etc.
- In general, at a given water content, the shear strength of the soil increases with an increase in the compactive effort till a critical degree of saturation is reached. With further increase in the compactive effort, the shear strength decreases.

5.12 Field Compaction and Equipment

In the construction of highway embankments and earth dams, soil is first dumped in the form of loose fills, and then compacted to improve the density and the strength characteristics. In the field, soil is compacted by applying energy through mechanical equipment. The energy is transmitted to the soil by applying pressure in any one of the following three ways

- Static pressure (using rollers)
- Impact (using rammers)
- Vibration (using vibrator)

The choice of equipment will depend on the type of soil and economic consideration. The main compaction equipment along with suitability of soil and nature of project are summarized below:

S.No.	Type of Equipment	Suitability for Soil type	Nature of Project
1	Rammer or Tampers	All soils	In confined areas such as fills behind retaining walls, basement walls, trench fills etc.
2	Smooth wheeled rollers	Crushed rocks, gravels, sands	Road construction
3	Pneumatic tyred rollers	Sand, gravels, silts, clayey soils	Base, sub-base and embankment compaction for highways, air field and earth dams
4	Sheep foot rollers	Clayey soils	Core of earth dams
5	Vibratory rollers	Sands	Embankments for oil storage tanks etc.

5.13 Evaluation of Compaction

The method adopted to assess the degree of compaction obtained in the field is the relative compaction. It is the ratio of the dry unit weight attained in the field to specified standard unit weight expressed as a percentage. The specified standard unit weight may be proctor unit weight, Indian standard light or heavy unit weight.

$$\text{Relative compaction} = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\text{max})}} \times 100$$

5.14 Compaction Quality Control

The compacted dry unit weights attained in the field should be checked occasionally for quality control. Three methods are generally used to check the in-situ density.

- Sand cone method
 - Rubber-balloon method
 - Nuclear density meters
- (i) Sand Cone Method: A sand cone apparatus is shown in figure.

Procedure:

- Fill the jar with a standard sand of known density.
- Determine the weight of the sand cone apparatus with the jar filled with sand (say W_1).
- Determine the weight of sand to fill the cone (say W_2).
- Excavate the small hole in the soil and determine the weight of the excavated soil (say W_3).
- Fill the hole with the standard sand by inverting the sand cone apparatus over the hole and opening the valve.
- Determine the weight of the sand cone apparatus with the remaining sand in the jar (say W_4).
- Calculate the unit weight of the soil as follows:

Weight of sand to fill hole,

$$W_s = W_1 - (W_2 + W_4)$$

Volume of hole, $V = \frac{W_s}{\gamma_{\text{standard}}}$

Weight of dry soil, $W_d = \frac{W_3}{1 + w}$

Dry unit weight, $\gamma_d = \frac{W_d}{V}$

- (ii) Rubber-balloon Method: Figure shows balloon test apparatus

Procedure:

- Fill the cylinder with water and record its volume, V_1 .
- Excavate a small hole in the soil and determine the weight of the excavated soil (W).
- Determine the water content of the excavated soil (w).
- Use the pump to invert the balloon to fill the hole.
- Record the volume of water remaining in the cylinder, V_2 .
- Calculate the unit weight of soil as follows:

$$\gamma = \frac{W}{V_1 - V_2}; \quad \gamma_d = \frac{\gamma}{1 + w}$$

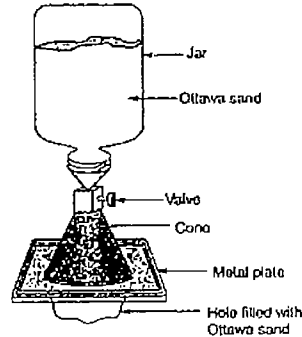


Fig. 5.11 Sand Cone Apparatus

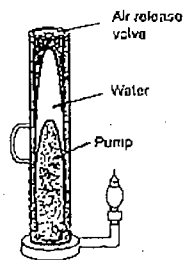


Fig. 5.12 Balloon Test Method

(iii) Nuclear Density Meter

- Nuclear density meter are used for direct determination of compacted unit weights and water content. Therefore, nuclear field density tests are much faster than conventional tests.
- Nuclear density meter works on the principle of scattering. Soil particles cause radiation to scatter to a detector tube and the amount of scatter is counted.
- The scatter count rate is inversely proportional to the unit weight of soil.
- If water is present in the soil, the hydrogen in water scatters the neutrons and the amount of scatter is proportional to the water content.

$$\gamma_d = \frac{\gamma}{1 + w}$$

5.15 Settlement During Compaction

Let e_0 be the initial void ratio before compaction has been started. After compaction, the void ratio of soil be e_r .

We know,

Void ratio, $e = \frac{V_v}{V_s} = \frac{H_v}{H_s} \quad (\because V \propto H)$

$$\therefore e_0 = \frac{H_{v1}}{H_s}$$

$$\Rightarrow H_{v1} = e_0 H_s$$

Similarly, $H_{v2} = e_r H_s$

\therefore change in the thickness of soil layer

$$\Delta H = H_{v1} - H_{v2} = (e_0 - e_r) H_s$$

$$\therefore \frac{\Delta H}{H} = \frac{(e_0 - e_r) H_s}{H_s + e_0 H_s} = \frac{e_0 - e_r}{1 + e_0}$$

$$\Delta H = \left(\frac{e_0 - e_r}{1 + e_0} \right) H$$

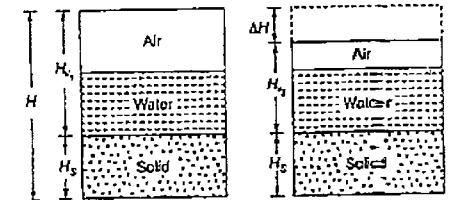


Fig. 5.13 Settling compaction

Example 5.5

The unit weight of a compacted sand backfill was determined by field measurement to be 1738 kg/m^3 . The water content and void ratio of the laboratory compacted soil was 10.2% and 0.607 respectively. What was the degree of compaction achieved in field? Assume water content remain constant ($G = 2.7$).

Solution:

Given,

$$\gamma_{\text{field}} = 1738 \text{ kg/m}^3$$

$$w = 10.2\%$$

For field,

$$\gamma_d = \frac{\gamma_{\text{field}}}{1 + w} = \frac{1738}{1 + 0.102} = 1577.13 \text{ kg/m}^3$$

For laboratory sample,

$$w = 10.2\%$$

$$e = 0.607$$

We know,

$$\gamma_{(w)} = \left(\frac{G + S_o}{1 + e} \right) \gamma_w = \frac{G(1+w)}{1+e} \gamma_w$$

$$= \frac{2.7(1+0.102)}{1+0.607} \times 1000 = 1851.5 \text{ kg/m}^3$$

$$\gamma_{d(\text{lab})} = \frac{\gamma_{(w)}}{1+w} = \frac{1851.5}{1+0.102} = 1680.12 \text{ kg/m}^3$$

$$\therefore \text{Degree of compaction} = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\text{lab})}} \times 100 = \frac{1577.13}{1680.12} \times 100 = 93.87\%$$

Illustrative Examples:

Example 5.6 Soil has been compacted in an embankment at a bulk density of 2.15 g/cc and the water content of 12%. The specific gravity of soil solids is 2.7. The water table is well below the foundation level. Estimate

- (i) void ratio (ii) the dry density
(iii) degree of saturation (iv) air content of the compacted soil

Solution:

Given, $\rho = 2.15 \text{ g/cc}$, $w = 12\%$ or 0.12, $G = 2.7$

(i) Using, $\rho = \frac{G(1+w)}{1+e} \rho_w$

$$2.15 = \frac{2.7(1+0.12)}{1+e} \times 1$$

$$1+e = \frac{2.7(1+0.12)}{2.15} = 1.406$$

$$\therefore e = 0.406$$

(ii) $\rho_d = \frac{\rho}{1+w} = \frac{2.15}{1+0.12} = 1.92 \text{ g/cc}$

(iii) Using, $Se = wG$

$$S = \frac{wG}{e} = \frac{0.12 \times 2.7}{0.406} = 0.798 \text{ or } 79.80\%$$

(iv) Air content = $\frac{V_a}{V_v} = \frac{V_v - V_h}{V_v} = 1 - S$

$$= 1 - 0.798$$

$$= 0.202 \text{ or } 20.2\%$$

Example 5.7 During a compaction test, a soil attains the maximum dry unit weight of 18.6 kN/m³ at a water content of 15 percent. The specific gravity of soil is 2.7. Determine the degree of saturation, air content at the maximum dry unit weight. What would be the maximum theoretical dry density corresponding to zero air voids at optimum water content?

Solution:

Given,

$$\gamma_{d(\text{max})} = 18.6 \text{ kN/m}^3$$

$$w = 15\% \text{ or } 0.15$$

OMC,

using,

$$\gamma_d = \frac{G\gamma_w}{1 + \frac{w}{S}}$$

$$18.6 = \frac{2.7 \times 9.81}{1 + \frac{0.15 \times 2.7}{S}}$$

$$1 + \frac{0.405}{S} = 1.424$$

$$S = 0.955 \text{ or } 95.5\%$$

$$\text{Air content} = \frac{V_a}{V_v} = 1 - S$$

$$= 1 - 0.955 = 0.045 \text{ or } 4.5\%$$

At OMC of 15%, the theoretical maximum dry density will occur when air voids reduce to zero i.e. soil is completely saturated ($S = 1$).

$$\gamma_{d(\text{max})} = \frac{G \cdot \gamma_w}{1 + \frac{w \cdot G}{S}} = \frac{2.7 \times 9.81}{1 + \frac{0.15 \times 2.7}{1}} = 18.85 \text{ kN/m}^3$$

Example 5.8 As per the compaction specification, a highway fill has to be compacted to 90% of Indian standard light compaction test dry density. A borrow pit available near the project site has a dry density of 1.78 g/cc at 100% compaction and a void ratio of 0.63. Compute the volume of borrow material needed to construct a highway fill of height 5 m and length 1.5 m with side slope of 1 : 2. The top width of the fill is 10.5 m. $G = 2.7$.

Solution:

given

$$\rho_{d(\text{borrow})} = 1.78 \text{ g/cc}$$

$$e_{(\text{borrow})} = e_b = 0.63$$

$$G = 2.7$$

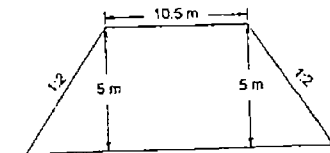
$$\text{Base width of highway fill} = 2(2 \times 5) + 10.5 = 30.5 \text{ m}$$

$$\text{Volume of highway fill} = \frac{1}{2} (10.5 + 30.5) \times 5 \times 1500 = 153750 \text{ m}^3$$

$$\text{Dry density of highway fill (90\%)} = 0.90 \times 1.78 = 1.602 \text{ g/cc}$$

To find void ratio of fill:

We know $\rho_{d(\text{fill})} = \frac{G\rho_w}{1+e_f}$



$$1.602 = \frac{2.7 \times 1}{1 + e_f}$$

$$e_f = 0.685$$

Since volume of soil solid remain constant in both borrow pit and fill, Hence

$$\frac{V_b}{(1 + e_b)} = \frac{V_f}{(1 + e_f)}$$

$$V_b = \frac{(1 + e_b)}{(1 + e_f)} \times V_f = \frac{(1 + 0.63)}{(1 + 0.685)} \times 573750 = 57486 \text{ m}^3$$

Example 5.9 The in-situ void ratio of a granular soil deposit is 0.59. The void ratios in loosest and densest state were found to be 0.81 and 0.37 respectively. Determine the relative density and relative compaction of the soil deposit. $G = 2.7$.

Solution:

Given:

$$e = 0.59, e_{max} = 0.81, e_{min} = 0.37$$

$$\text{Relative density (\% } I_D) = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100\% = \frac{0.81 - 0.59}{0.81 - 0.37} \times 100 = 50\%$$

$$\gamma_{d(max)} = \frac{G\gamma_w}{1 + e_{min}} = \frac{2.7 \times 9.81}{1 + 0.37} = 19.33 \text{ kN/m}^3$$

$$\gamma_{d(in-situ)} = \frac{G\gamma_w}{1 + e} = \frac{2.7 \times 9.81}{1 + 0.59} = 16.65 \text{ kN/m}^3$$

$$\text{Relative compaction} = \frac{\gamma_{d(in-situ)}}{\gamma_{d(max)}} \times 100 = \frac{16.65}{19.33} \times 100 = 86.13\%$$

Example 5.10 In Indian standard light compaction test, the bulk unit weight of soil sample was found to be 18 kN/m^3 at a moisture content of 13%. Determine

- degree of saturation of sample
 - additional moisture content required for complete saturation
 - maximum theoretical dry unit weight
- $G = 2.68$

Solution:

(i) Given:

$$\gamma_t = \frac{G(1 + w)}{1 + \frac{wG}{S}} \gamma_w$$

$$18 = \frac{2.68(1 + 0.13) \times 9.81}{1 + \frac{(0.13 \times 2.68)}{S}}$$

$$1 + \frac{0.3484}{S} = 1.65$$

$$S = 0.5356 \text{ or } 53.56\%$$

(ii) Void ratio at 53.56% saturation

$$e = \frac{w \cdot G}{S} = \frac{0.13 \times 2.68}{0.5356} = 0.65$$

At a given compactive effort, void ratio remain constant. The water content at full saturation may be given as

$$w = \frac{1 \times e}{G} = \frac{0.65}{2.68} = 0.242 \text{ or } 24.20\%$$

$$\therefore \text{Additional water content required for complete saturation} = 24.20 - 13 = 11.2\%$$

Example 5.11 In the construction of a levee by compacting soil excavated from a borrow area to dry density of 1.78 g/cc , to make the compaction process more workable, an optimum water content of 15% is necessary. However the natural moisture content and bulk density of the soil were 9% and 1.83 g/cc respectively. Find out the quantity of water to be added for every 100 m^3 of finished embankment.

Solution:

For embankment Given:

$$\gamma_{d(max)} = 1.78 \text{ g/cc} = 1.78 \text{ t/m}^3$$

$$W_s = 1.78 \times 100 = 178 \text{ tonnes}$$

The weight of water needed in embankment

$$W_w = wW_s = 0.15 \times 178 = 26.7 \text{ ton}$$

For borrow area given:

$$\gamma_t = 1.83 \text{ g/cc} = 1.83 \text{ t/m}^3$$

$$w = 9\% \text{ or } 0.09$$

$$\gamma_t = \frac{\text{weight of soil}}{\text{volume of soil}} = \frac{W_s + W_w}{V_b} = \frac{W_s(1 + w)}{V_b}$$

$$1.83 = \frac{178(1 + 0.09)}{V_b}$$

$$V_b = 106.02 \text{ m}^3$$

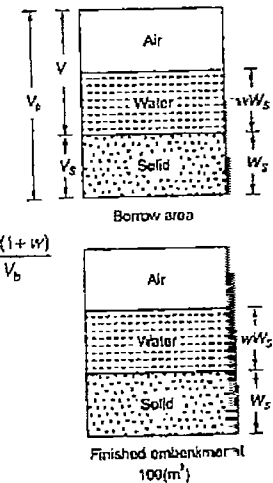
Weight of water present in the soil taken from borrow area,

$$W_w = wW_s = 0.09 \times 178 = 16.02 \text{ ton}$$

Therefore quantity of water to be added

$$= 26.70 - 16.02 = 10.68 \text{ ton}$$

$$\therefore \text{Volume of water to be added} = \frac{W_w}{\rho_w} = \frac{10.68}{10^{-3}} = 10680 \text{ litres}$$



Summary



- Soil compaction is the process of increasing the density of the soil by applying some mechanical energy and thereby reducing air voids.
- Factors affecting the compaction of a soil are moisture content, compactive effort and method of compaction.
- Higher compactive effort increases the maximum dry unit weight and reduces the optimum water content.
- Comparison of properties of cohesive soil on dry and wet sides of OMC

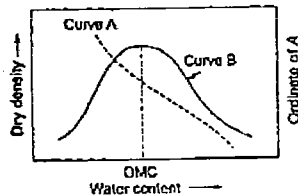
Table : Comparison of dry of Optimum with Wet of Optimum Compaction

Property	Dry optimum	Wet of optimum
Structure after compaction	Flocculated (random)	Dispersed (oriented)
Water deficiency	More	Less
Permeability	More, isotropic	Less isotropic ($k_v > k_h$)
Compressibility at low stress at high stress	Low High	High Low
Swollability	High	Low
Shrinkage	Low	High
Stress strain behaviour	Brittle; high peak higher elastic modulus	Ductile; no peak lower elastic modulus
Construction pore water pressures	Low	High
Strength (undrained) as moulded after saturation	High somewhat higher if swelling prevent	Much lower low



Objective Brain Teasers

- Q.1 Curve B shows the typical compaction curve of a soil in the Proctor test. Dotted curve A is shown superimposed on the same graph. Which one of the following expressions corresponds to the ordinate axis of curve A?



- (a) Zero air voids
(b) Wet density
(c) Penetration resistance of Proctor needle
(d) 95% saturation
- Q.2 At what value of saturation does the zero air voids curve in a compaction test represent the dry density?
- (a) 0% (b) 80%
(c) 100% (d) 50%

- Q.3 Which one of the following is the correct statements?

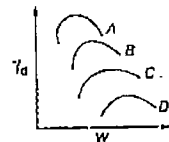
Vibratory rollers are suitable for compacting

(a) organic soil
(b) clays
(c) sands and gravels
(d) clayey silts

- Q.4 Why are sheep foot rollers more effective in compacting clayey soils?

(a) There is differential expulsion of water under the roller
(b) Contact pressure is high
(c) Roller speed is high
(d) Drum width is large

- Q.5 The results (curves A, B, C and D) of four compaction tests on different soils are shown in the graph:



- Silty sand, modified test
 - Silty sand, standard test
 - Fat clay, modified test
 - Fat clay, standard test
- Curves A, B, C and D correspond respectively to tests:

(a) 1, 3, 2 and 4 (b) 1, 2, 3 and 4
(c) 2, 1, 3 and 4 (d) 2, 1, 4 and 3

- Q.6 Given below are methods of compaction:

- Vibration technique
- Flooding the soil
- Sheep foot roller
- Tandem roller
- Heavy weights dropped from a height

The methods suitable for cohesionless soils include:

(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 1, 2 and 5 (d) 3, 4 and 5

- Q.7 Match List-I (Type of soils) List-II (Compaction parameters) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Sand	1. OMC = 18%, $\gamma_{dry, max} = 17 \text{ kN/m}^3$
B. Sandy clay	2. OMC = 14%, $\gamma_{dry, max} = 18.9 \text{ kN/m}^3$
C. Silty clay	3. OMC = 15%, $\gamma_{dry, max} = 17.4 \text{ kN/m}^3$
D. Heavy clay	4. OMC = 10%, $\gamma_{dry, max} = 20.5 \text{ kN/m}^3$

Codes:

	A	B	C	D
(a)	2	4	3	1
(b)	2	4	1	3
(c)	4	2	3	1
(d)	4	2	1	3

- Q.8 Consider the following statements regarding the flow nets:

- Flow lines and equipotential lines always intersect one-another at right angles irrespective of the permeability characteristics.

- For an isotropic soil, the spacing of lines is inversely proportional to the hydraulic gradient.
- For an anisotropic soil having greater horizontal coefficient of permeability, a flow net of approximate squares can be constructed by reducing the vertical dimensions of the flow domain to a certain scale.
- For an isotropic soil having greater horizontal coefficient of permeability, a flow net of approximate squares can be constructed by reducing the vertical dimensions of the flow domain to a certain scale.

Which of these statements are correct?

(a) 2, 3 and 4 (b) 2 and 3
(c) 1, 2 and 4 (d) 1 and 3

- Q.9 A clear dry sand sample is tested in a direct shear test. The normal stress and the shear stress at failure are both equal to 120 kN/m^2 . The angle of shearing resistance of the sand will be:

(a) 25° (b) 35°
(c) 45° (d) 55°

- Q.10 Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I	List-II
A. Optimum moisture content	1. Compaction of cohesive soil
B. Vibratory rollers	2. Compaction of granular soil
C. Zero air void line	3. Maximum dry density
	4. Relative density
	5. 100% saturation

Codes:

	A	B	C
(a)	4	1	3
(b)	3	2	5
(c)	4	1	5
(d)	3	2	4

- Q.11 If specific gravity of soil solids of a soil having optimum moisture content 16.5% and maximum

dry density 1.57 g/cc is 2.65, void ratio at optimum moisture content is

- (a) 0.588 (b) 0.688
(c) 0.788 (d) 0.888

- Q.12 The maximum dry density upto which any soil can be compacted depends upon
(a) amount of compaction energy alone
(b) moisture content alone
(c) both compaction energy and moisture content
(d) none of these

- Q.13 At optimum moisture content of soil mass, the soil saturation will be around
(a) 100% (b) 90 - 95%
(c) 0% (d) none of these

- Q.14 The percentage of voids filled with air in a soil compacted under OMC, shall be around
(a) 0% (b) 5 - 10%
(c) 100% (d) none of these

Direction: The following items consists of two statements, one labelled as 'Assertion A' and the other as 'Reason R'. Select your answer to these items using the codes given below.

- (a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false
(d) A is false but R is true

- Q.15 Assertion (A): The process of compaction is accompanied by the expulsion of air.
Reason (R): The degree of compaction of a soil is characterized by its dry density.

- Q.16 Assertion (A): For a given soil, the optimum moisture content increases with the increase in compactive effort.

Reason (R): Higher the compactive effort, higher is the dry density at the same moisture content.

- Q.17 Consider the following:

- Increase in shear strength and bearing capacity
- Increase in slope stability

- Decrease in settlement of soil
- Decrease in permeability

Which of the above with respect to compaction of soil is/are correct.

- (a) 1 only (b) 1 and 2 only
(c) 2 and 3 only (d) 1, 2, 3 and 4

- Q.18 The insitu void ratio of a granular soil deposit is 0.50. The maximum and minimum void ratios of the soil were determined to be 0.75 and 0.35. $G_s = 2.67$ the relative density and relative compaction of the soil are respectively ($\gamma_w = 9.8$ kN/m³)
(a) 62.5%, 89.9% (b) 89.9%, 62.5%
(c) 62.5%, 96.6% (d) 89.9%, 96.6%

- Q.19 Compaction of an embankment is carried out in 400 mm thick layers. The rammer used for compaction has a foot area of 0.1 m² and the energy imparted in every drop of rammer is 350 N-m. Assuming 40% more energy in each pass over the compacted area due to overlap, the number of passes required to develop compactive energy equivalent to Indian Standard light compaction for each layer would be
(a) 26 (b) 33
(c) 49 (d) 54

- Q.20 Assertion (A): Road built on black cotton soils show crack after some time.
Reason (R): Black cotton soils settle and this results in deformation.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Answers

1. (c) 2. (c) 3. (c) 4. (b) 5. (b)
6. (c) 7. (c) 8. (d) 9. (c) 10. (b)
11. (b) 12. (c) 13. (b) 14. (b) 15. (b)
16. (d) 17. (d) 18. (a) 19. (d) 20. (a)

Hints and Explanations:

4. (b)
Under sheep foot rollers the contact pressure is very high and this results in kneading action which is responsible for effective compaction of clayey soils
5. (b)
With the increase of fine particles in sand, the OMC decreases and maximum dry density increases upto certain extent. Thereafter any further increase in fine particles will lead to increase of OMC and decrease of maximum dry density. Further in modified test, the OMC will be less and maximum dry density will be more compared to that in standard test. Therefore combining both:
OMC $1 < 2 < 3 < 4$
Maximum dry density
 $1 < 2 < 3 < 4$
So A-1, B-2, C-3, D-4
6. (c)
Cohesionless soils can be compacted by vibration; flooding and dropping heavy weight from height. Kneading and tamping actions provided by sheep foot roller and tandem roller are not effective for cohesionless soils.
7. (c)
Heavy (fat) clay means pure clay.
As the fine particle content is increased in a cohesionless soil, the dry density achieved is very high compared to that in cohesionless soil. However when fine particles increase beyond a limit, the dry density decrease and OMC increases.
Thus OMC of $D > C > B > A$
So D - 1, C - 3, B - 2, A - 4
Maximum dry density of $A > B > C > D$
So A - 4, B - 2, C - 3, D - 1
8. (d)
The flow net is governed by velocity potential function and stream function.
 $\frac{\partial \phi}{\partial x} = k_x; \frac{\partial h}{\partial x} = v_x$ & $\frac{\partial \phi}{\partial z} = k_z; \frac{\partial h}{\partial z} = v_z$

$$\text{and } \frac{\partial \psi}{\partial z} = v_x, \frac{\partial \psi}{\partial x} = -v_z$$

The flow lines and equipotential lines are thus always orthogonal to each other.

The line will be spaced proportional to the hydraulic gradient.

For square mesh potential drop is same throughout the field.

For anisotropic soil, the horizontal dimension of flow domain is reduced by $\sqrt{k_x/k_z}$ factor.

Thus statements (1) and (3) are correct.

9. (c)
 $\tau = \sigma_n \tan \phi$ given $\tau = \sigma_n = 120 \text{ kN/m}^2$
 $\therefore \tan \phi = 1$ $\phi = 45^\circ$

10. (b)
A - 3; B - 2; C - 5

11. (b)
given
 $\gamma_d = 1.57$
 $w = 0.165$
 $G = 2.65$

$$\text{using, } \gamma_d = \frac{G \cdot \gamma_w}{1 + e}$$

$$1.57 = \frac{2.65 \times 1}{1 + e}$$

$$1 + e = 1.688$$

$$\therefore e = 0.688$$

14. (b)
%Air voids = $1 - S = 5 - 10\%$

18. (a)
Relative density

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100\%$$

$$= \frac{0.75 - 0.50}{0.75 - 0.35} \times 100 = 62.5\%$$

$$\gamma_{d(max)} = \frac{G \cdot \gamma_w}{1 + e_{min}}$$

$$= \frac{2.67 \times 9.8}{1 + 0.35} = 19.38 \text{ kN/m}^3$$

$$\gamma_{d(\max)} = \frac{G_s \gamma_w}{1 + e_{\max}}$$

$$= \frac{2.67 \times 9.8}{1 + 0.75} = 14.95 \text{ kN/m}^3$$

$$\gamma_{d(\text{resu})} = \frac{G_s \gamma_w}{1 + e_{\text{resu}}}$$

$$= \frac{2.67 \times 9.8}{1 + 0.50} = 17.44 \text{ kN/m}^3$$

∴ Relative compaction,

$$\frac{\gamma_{d(\text{resu})}}{\gamma_{d(\max)}} = \frac{17.44}{19.38} \times 100 = 89.9\%$$

19. (d)

As per Indian Standard light compaction test, a hammer of 2.6 kg is allowed to fall from a height of 310 mm and 3 layers are tamped 25 times in a mould of volume 1000 cc.

∴ Energy imparted

$$= 2.6 \times 25 \times 3 \times 9.81 \times \frac{310}{1000}$$

$$= 593.01 \text{ N-m}$$

For embankment compaction, the volume of soil covered by rammer

$$= 0.1 \times 10^4 \times \frac{400}{10} = 40000 \text{ cc}$$

∴ Energy imparted in every drop of hammer

$$= \left(1 + \frac{40}{100}\right) \times 350 = 490 \text{ N-m}$$

If 'n' number of passes are required to develop equivalent energy to Indian Standard light compaction test, then

$$n \times \frac{490}{40000} = \frac{593.01}{1000}$$

$$\Rightarrow n = 48.41 \approx 49 \text{ (approx.)}$$

Q.4 A sample of soil was prepared by mixing a quantity of dry soil with 10% by mass of water. Find the mass of this wet mixture required to produce a cylindrical, compacted specimen of 15 cm diameter and 12.5 cm high, having 6% air content. Find also the void ratio and the dry density of the specimen if $G = 2.68$.

$$\text{Ans. } M = 460.65 \text{ g, } e = 0.285,$$

$$\rho_d = 2.085 \text{ g/cc}$$

Q.5 The soil from a borrow area having an average in-situ unit weight of 15.5 kN/m^3 and water content of 10%, was used for the construction of an embankment of total finished volume 6000 m^3 . In half of embankment, due to improper control during rolling, the dry unit weight achieved was slightly lower. If the dry unit weight in the two parts is 16.5 kN/m^3 and 16.0 kN/m^3 , find the volume of borrow area soil used in each part.

$$\text{Ans. } 3512.9 \text{ m}^3 \text{ and } 3.406.45 \text{ m}^3$$



Student's Assignments

Q.1 The total unit weight (γ_t) of soil is 18.8 kN/m^3 , the specific gravity (G) of the solid particles of the soil is 2.67 and the water content (w) of the soil is 12%. Calculate the dry unit weight (γ_d), void ratio (e) and the degree of saturation.

$$\text{Ans. } \gamma_d = 16.79 \text{ kN/m}^3, e = 0.56 \text{ and } S = 57.2\%$$

Q.2 Embankment for a highway 30 m wide and 1.5 m is to be constructed from a sandy soil trucked from a borrow pit. The water content of the sandy soil in the borrow pit is 15% and its void ratio is 0.69. The specification requires the embankment be compacted to a dry unit weight of 18 kN/m^3 . Determine, for 1 km length of embankment, the following:

- The volume of sandy soil from the borrow pit required to construct the embankment.
- The number of trucks taking 10 m^3 of volume of soil required for the construction.

(iii) The weight of water per truck load of sandy soil.

(iv) The degree of saturation of the sandy soil in-situ.

$$\text{Ans. } V_b = 51.6 \times 10^3 \text{ m}^3, N = 5160 \text{ trucks}$$

$$W_w/\text{Truck} = 23.6 \text{ kN, } S = 59\%$$

Q.3 Some soil has been dumped loosely from a scraper. It has a unit weight of 16 kN/m^3 , a water content of 10.5% and a specific gravity of solids of 2.68. Find the void ratio, porosity and unit weight of the soil in the loose condition.

To make the compaction process more workable, an optimum water content of 15% is necessary. How much of water should be added in liters per cubic meter of soil to raise the water content to the optimum?

$$\text{Ans. Loose condition: } \rho_d = 1.468 \text{ g/cc}$$

$$\text{and } \gamma_c = 14.40 \text{ kN/m}^3$$

$$\text{Water to be added} = 51.4 \text{ kg}$$