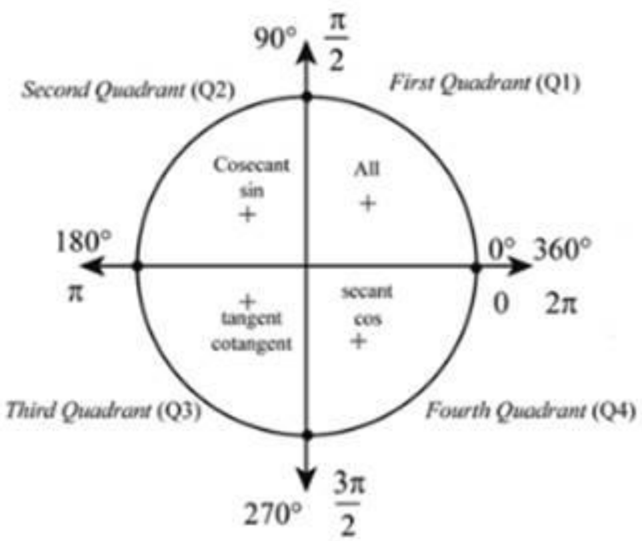
Trigonometric, Or Circular, Functions

Exercise 15A

Q. 1. If $\cos\theta = \frac{-\sqrt{3}}{2}$ and θ lies in Quadrant III, find the value of all the other five trigonometric functions.

Answer : Given: $\cos\theta = \frac{-\sqrt{3}}{2}$



Since, θ is in III[rd] Quadrant. So, sin and cos will be negative but tan will be positive.

We know that,

$\cos^2\theta + \sin^2\theta = 1$

Putting the values, we get

$$\left(-\frac{\sqrt{3}}{2}\right)^2 + \sin^2\theta = 1 \text{ [given]}$$

$$\Rightarrow \frac{3}{4} + \sin^2\theta = 1$$

$$\Rightarrow \sin^2\theta = 1 - \frac{3}{4}$$

$$\Rightarrow \sin^2\theta = \frac{4-3}{4}$$

$$\Rightarrow \sin^2\theta = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \sin\theta = \pm\frac{1}{2}$$

Since, θ in III[rd] quadrant and sinθ is negative in III[rd] quadrant

$$\therefore \sin\theta = -\frac{1}{2}$$

Now,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Putting the values, we get

$$\tan\theta = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$$

$$= -\frac{1}{2} \times \left(-\frac{2}{\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{3}}$$

Now,

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

Putting the values, we get

$$\operatorname{cosec}\theta = \frac{1}{-\frac{1}{2}}$$

= -2

Now,

$$\sec\theta = \frac{1}{\cos\theta}$$

Putting the values, we get

$$\sec\theta = \frac{1}{-\frac{\sqrt{3}}{2}}$$

$$= -\frac{2}{\sqrt{3}}$$

Now,

$$\cot\theta = \frac{1}{\tan\theta}$$

Putting the values, we get

$$\cot\theta = \frac{1}{\frac{1}{\sqrt{3}}}$$

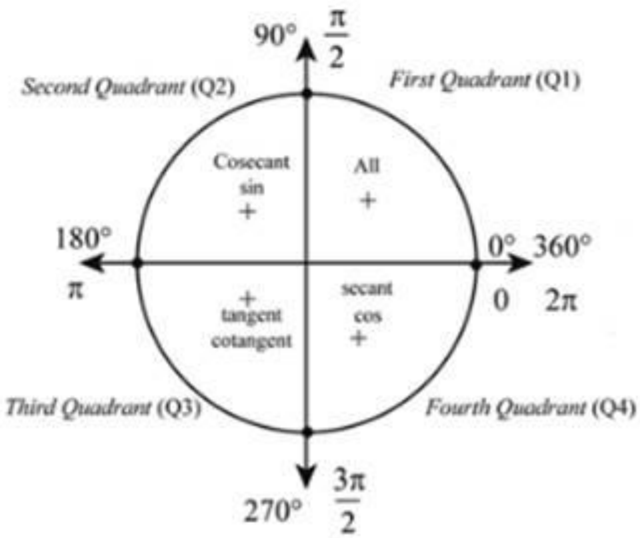
$= \sqrt{3}$

Hence, the values of other trigonometric Functions are:

Cos θ	Sin θ	Tan θ	Cosec θ	Sec θ	Cot θ
$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	-2	$-\frac{2}{\sqrt{3}}$	$\sqrt{3}$

Q. 2. If $\sin\theta = \frac{-1}{2}$ and θ lies in Quadrant IV, find the values of all the other five trigonometric functions.

Answer : Given: $\sin\theta = \frac{-1}{2}$



Since, θ is in IVth Quadrant. So, sin and tan will be negative but cos will be positive.

We know that,

$\sin^2 \theta + \cos^2 \theta = 1$

Putting the values, we get

$\left(-\frac{1}{2}\right)^2 + \cos^2\theta = 1$ [given]

$\Rightarrow \frac{1}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{1}{4}$

$\Rightarrow \cos^2\theta = \frac{4-1}{4}$

$\Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos\theta = \sqrt{\frac{3}{4}}$

$\Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$

Since, θ in IVth quadrant and cosθ is positive in IVth quadrant

$\therefore \cos\theta = \frac{\sqrt{3}}{2}$

Now,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Putting the values, we get

$\tan\theta = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= -\frac{1}{2} \times \left(\frac{2}{\sqrt{3}}\right)$

$= -\frac{1}{\sqrt{3}}$

Now,

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Putting the values, we get

$\operatorname{cosec}\theta = \frac{1}{-\frac{1}{2}}$

= -2

Now,

$\sec\theta = \frac{1}{\cos\theta}$

Putting the values, we get

$\sec\theta = \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}}$

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{-\frac{1}{\sqrt{3}}}$

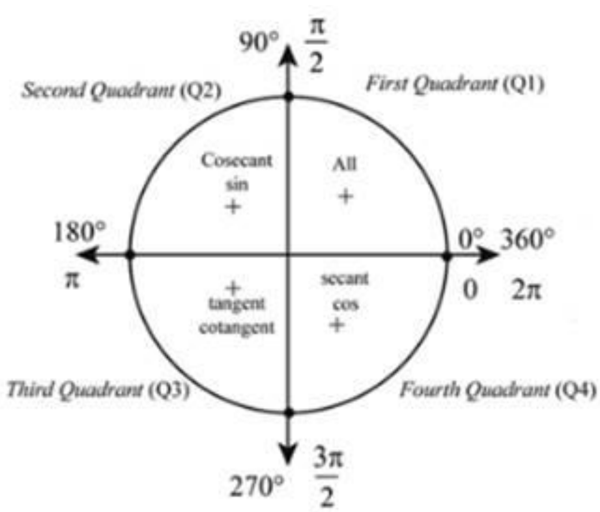
$= -\sqrt{3}$

Hence, the values of other trigonometric Functions are:

Cos θ	Sin θ	Tan θ	Cosec θ	Sec θ	Cot θ
$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$	$-\sqrt{3}$

Q. 3. If $cosec\ \theta = \frac{5}{3}$ and θ lies in Quadrant II, find the values of all the other five trigonometric functions.

Answer : Given: $cosec\ \theta = \frac{5}{3}$



Since, θ is in II[nd] Quadrant. So, cos and tan will be negative but sin will be positive.

Now, we know that

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

Putting the values, we get

$\sin\theta = \frac{1}{\frac{5}{3}}$

$\sin\theta = \frac{3}{5}$ …(i)

We know that,

$\sin^2\theta + \cos^2\theta = 1$

Putting the values, we get

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$ [from (i)]

$\Rightarrow \frac{9}{25} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{9}{25}$

$\Rightarrow \cos^2\theta = \frac{25-9}{25}$

$\Rightarrow \cos^2\theta = \frac{16}{25}$

$\Rightarrow \cos\theta = \sqrt{\frac{16}{25}}$

$\Rightarrow \cos\theta = \pm\frac{4}{5}$

Since, θ in II^{nd} quadrant and cosθ is negative in II^{nd} quadrant

$\therefore \cos\theta = -\frac{4}{5}$

Now,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Putting the values, we get

$$\tan\theta = \frac{\frac{3}{5}}{-\frac{4}{5}}$$

$$= \frac{3}{5} \times \left(-\frac{5}{4}\right)$$

$$= -\frac{3}{4}$$

Now,

$$\sec\theta = \frac{1}{\cos\theta}$$

Putting the values, we get

$$\sec\theta = \frac{1}{-\frac{4}{5}}$$

$$= -\frac{5}{4}$$

Now,

$$\cot\theta = \frac{1}{\tan\theta}$$

Putting the values, we get

$$\cot\theta = \frac{1}{-\frac{3}{4}}$$

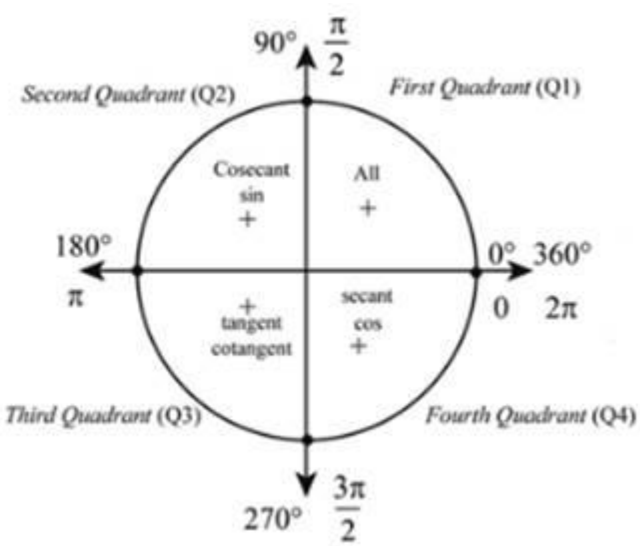
$$= -\frac{4}{3}$$

Hence, the values of other trigonometric Functions are:

Cos θ	Sin θ	Tan θ	Cosec θ	Sec θ	Cot θ
$-\frac{4}{5}$	$\frac{3}{5}$	$-\frac{3}{4}$	$\frac{5}{3}$	$-\frac{5}{4}$	$-\frac{4}{3}$

Q. 4. If sec θ $\sqrt{2}$ and θ lies in Quadrant IV, find the values of all the other five trigonometric functions.

Answer : Given: sec θ = $\sqrt{2}$



Since, θ is in IVth Quadrant. So, sin and tan will be negative but cos will be positive.

Now, we know that

$$\cos\theta = \frac{1}{\sec\theta}$$

Putting the values, we get

$$\cos\theta = \frac{1}{\sqrt{2}} \quad \ldots(i)$$

We know that,

$\cos^2\theta + \sin^2\theta = 1$

Putting the values, we get

$\left(\frac{1}{\sqrt{2}}\right)^2 + \sin^2\theta = 1$ [Given]

$\Rightarrow \frac{1}{2} + \sin^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \frac{1}{2}$

$\Rightarrow \sin^2\theta = \frac{2-1}{2}$

$\Rightarrow \sin^2\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sqrt{\frac{1}{2}}$

$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}$

Since, θ in IVth quadrant and sinθ is negative in IVth quadrant

$\therefore \sin\theta = -\frac{1}{\sqrt{2}}$

Now,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Putting the values, we get

$\tan\theta = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$= -\frac{1}{\sqrt{2}} \times \left(\sqrt{2}\right)$

= – 1

Now,

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Putting the values, we get

$\operatorname{cosec}\theta = \frac{1}{-\frac{1}{\sqrt{2}}}$

= $-\sqrt{2}$

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{-1}$

= – 1

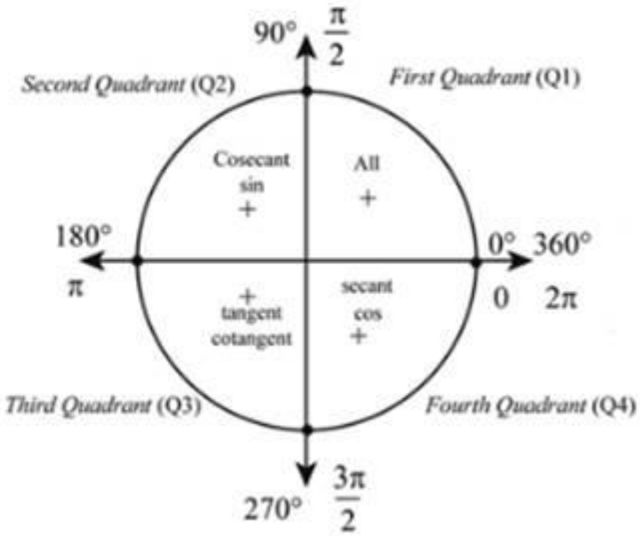
Hence, the values of other trigonometric Functions are:

Cos θ	Sin θ	Tan θ	Cosec θ	Sec θ	Cot θ
$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1	-√2	√2	-1

Q. 5. If $\sin x = -\frac{2\sqrt{6}}{5}$ and x lies in Quadrant III, find the values of cos x and cot x.

Answer : Given: $\sin x = -\frac{2\sqrt{6}}{5}$

To find: cos x and cot x



Since, x is in III[rd] Quadrant. So, sin and cos will be negative but tan will be positive.

We know that,

$\sin^2 x + \cos^2 x = 1$

Putting the values, we get

$\left(-\frac{2\sqrt{6}}{5}\right)^2 + \cos^2 x = 1$ [Given]

$\Rightarrow \frac{24}{25} + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \frac{24}{25}$

$\Rightarrow \cos^2 x = \frac{25-24}{25}$

$\Rightarrow \cos^2 x = \frac{1}{25}$

$\Rightarrow \cos x = \sqrt{\frac{1}{25}}$

$\Rightarrow \cos x = \pm\frac{1}{5}$

Since, x in III^rd quadrant and cos x is negative in III^rd quadrant

$\therefore \cos x = -\frac{1}{5}$

Now,

$\tan x = \frac{\sin x}{\cos x}$

Putting the values, we get

$\tan x = \frac{-\frac{2\sqrt{6}}{5}}{-\frac{1}{5}}$

$= -\frac{2\sqrt{6}}{5} \times (-5)$

= 2√6

Now,

$\cot x = \frac{1}{\tan x}$

Putting the values, we get

$\cot x = \frac{1}{2\sqrt{6}}$

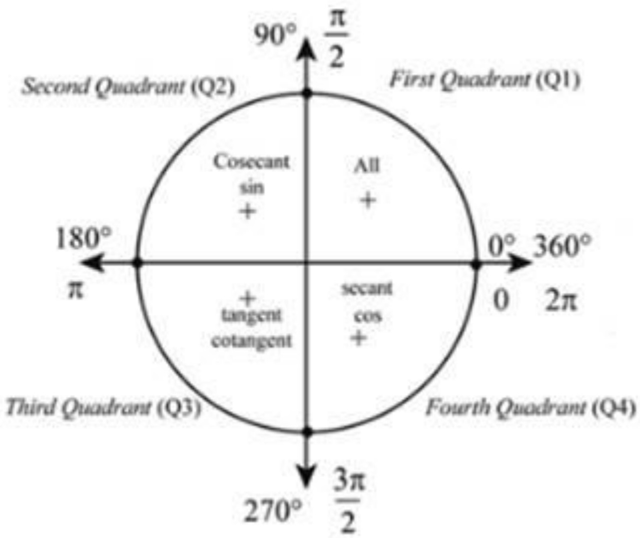
Hence, the values of other trigonometric Functions are:

Cos x	Sin x	Cot x
$-\frac{1}{5}$	$-\frac{2\sqrt{6}}{5}$	$\frac{1}{2\sqrt{6}}$

Q. 6. If $\cos x = \frac{-\sqrt{15}}{4}$ and $\frac{\pi}{2} < x < \pi$**, find the value of sin x.**

Answer : Given: $\cos x = -\frac{\sqrt{15}}{4}$

To find: value of sinx



Given that: $\frac{\pi}{2} < x < \pi$

So, x lies in II[nd] quadrant and sin will be positive.

We know that,

$\cos^2 \theta + \sin^2 \theta = 1$

Putting the values, we get

$\left(-\frac{\sqrt{15}}{4}\right)^2 + \sin^2 \theta = 1$ [Given]

$\Rightarrow \frac{15}{16} + \sin^2 \theta = 1$

$\Rightarrow \sin^2 \theta = 1 - \frac{15}{16}$

$\Rightarrow \sin^2\theta = \frac{16-15}{16}$

$\Rightarrow \sin^2\theta = \frac{1}{16}$

$\Rightarrow \sin\theta = \sqrt{\frac{1}{16}}$

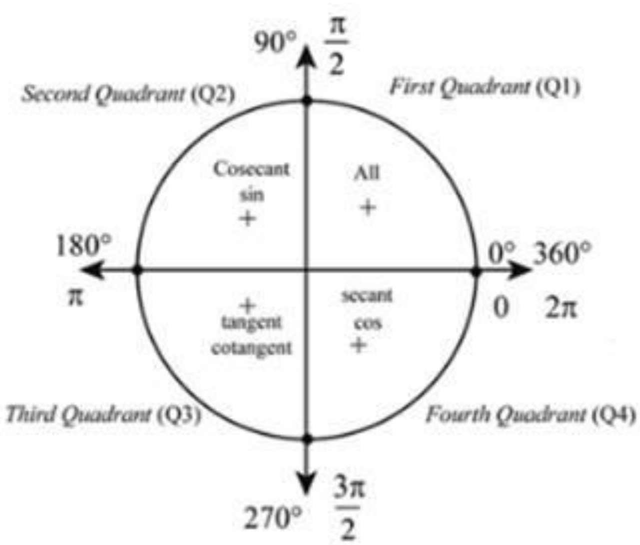
$\Rightarrow \sin\theta = \pm\frac{1}{4}$

Since, x in IInd quadrant and sinθ is positive in IInd quadrant

$\therefore \sin\theta = \frac{1}{4}$

Q. 7. If $\sec x = -2 \text{ and } \pi < x < \frac{3\pi}{2}$ **, find the values of all the other five trigonometric functions.**

Answer : Given: sec x = -2



Given that: $\pi < x < \frac{3\pi}{2}$

So, x lies in IIIrd Quadrant. So, sin and cos will be negative but tan will be positive.

Now, we know that

$\cos x = \frac{1}{\sec x}$

Putting the values, we get

$\cos x = \frac{1}{-2}$ …(i)

We know that,

$\cos^2 x + \sin^2 x = 1$

Putting the values, we get

$\left(-\frac{1}{2}\right)^2 + \sin^2 x = 1$ [Given]

$\Rightarrow \frac{1}{4} + \sin^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \frac{1}{4}$

$\Rightarrow \sin^2 x = \frac{4-1}{4}$

$\Rightarrow \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \sqrt{\frac{3}{4}}$

$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$

Since, x in IIIrd quadrant and sinx is negative in IIIrd quadrant

$\therefore \sin x = -\frac{\sqrt{3}}{2}$

Now,

$\tan x = \frac{\sin x}{\cos x}$

Putting the values, we get

$$\tan x = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$= -\frac{\sqrt{3}}{2} \times (-2\ )$$

= √3

Now,

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

Putting the values, we get

$$\operatorname{cosec} x = \frac{1}{-\frac{\sqrt{3}}{2}}$$

$$= \ -\frac{2}{\sqrt{3}}$$

Now,

$$\cot x = \frac{1}{\tan x}$$

Putting the values, we get

$$\cot x = \frac{1}{\sqrt{3}}$$

Hence, the values of other trigonometric Functions are:

Cos x	Sin x	Tan x	Cosec x	Sec x	Cot x
$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	√3	$-\frac{2}{\sqrt{3}}$	-2	$\frac{1}{\sqrt{3}}$

Q. 8. A. Find the value of

$$\sin\left(\frac{31\pi}{3}\right)$$

Answer :

$$\begin{array}{r} 10 \\ 3\overline{)31} \\ 30 \\ \hline 1 \end{array}$$

To find: Value of sin $\frac{31\pi}{3}$

$$\sin\frac{31\pi}{3} = \sin\left(10\pi + \frac{1}{3}\pi\right)$$

$$= \sin\left(5 \times (2\pi) + \frac{1}{3}\pi\right)$$

Value of sin x repeats after an interval of 2π, hence ignoring 5 × (2π)

$$= \sin\left(\frac{1}{3}\pi\right)$$

$$= \sin\left(\frac{1}{3} \times 180°\right)$$

= sin 60°

$$= \frac{\sqrt{3}}{2}\left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$$

Q. 8. B. Find the value of

$$\cos\left(\frac{17\pi}{2}\right)$$

Answer :

$$
\begin{array}{r|l}
 & 8 \\
\hline
2 & 17 \\
 & 16 \\
\hline
 & 1
\end{array}
$$

To find: Value of cos $\frac{17\,n}{2}$

$\cos\frac{17\pi}{2} = \cos\left(8\pi + \frac{1}{2}\pi\right)$

$= \cos\left(4 \times (2\pi) + \frac{1}{2}\pi\right)$

Value of cos x repeats after an interval of 2π, hence ignoring 4 × (2π)

$= \cos\left(\frac{1}{2}\pi\right)$

$= \cos\left(\frac{1}{2} \times 180°\right)$

= cos 90°

= 0 [∵ cos 90° = 1]

Q. 8. C. Find the value of

$$\tan\left(\frac{-25\pi}{3}\right)$$

Answer :

$$
\begin{array}{r|l}
 & 8 \\
\hline
3 & 25 \\
 & 24 \\
\hline
 & 1
\end{array}
$$

To find: Value of tan $\frac{-25\pi}{3}$

We know that,

tan(-θ) = - tan θ

$\therefore \tan\left(-\frac{25\pi}{3}\right) = -\tan\left(\frac{25\pi}{3}\right)$

$\tan\left(-\frac{25\pi}{3}\right) = -\tan\left(\frac{25\pi}{3}\right) = -\tan\left(8\pi + \frac{1}{3}\pi\right)$

$= -\tan\left(4 \times (2\pi) + \frac{1}{3}\pi\right)$

Value of tan x repeats after an interval of 2π, hence ignoring 4 × (2π)

$= -\tan\left(\frac{1}{3}\pi\right)$

$= -\tan\left(\frac{1}{3} \times 180°\right)$

= - tan 60°

= -√3

[∵ tan 60° = √3]

Q. 8. D. Find the value of

$$\cot\left(\frac{13\pi}{4}\right)$$

Answer : To find: Value of cot $\frac{13\pi}{4}$

We have,

$\cot\frac{13\pi}{4}$

Putting π = 180°

$= \cot\left(\frac{13 \times 180°}{4}\right)$

= cot (13 × 45°)

= cot (585°)

= cot [90° × 6 + 45°]

= cot 45°

[Clearly, 585° is in IIIrd Quadrant and the multiple of 90° is even]

= 1 [∵ cot 45° = 1]

Q. 8. E. Find the value of

$$\sec\left(\frac{-25\pi}{3}\right)$$

Answer : To find: Value of $\sec\left(-\frac{25\pi}{3}\right)$

We have,

$$\sec\left(-\frac{25\pi}{3}\right) = \sec\frac{25\pi}{3}$$

[∵ sec(-θ) = sec θ]

Putting π = 180°

$$= \sec\frac{25\times180}{3}$$

= sec[25 × 60°]

= sec[1500°]

= sec [90° × 16 + 60°]

Clearly, 1500° is in Ist Quadrant and the multiple of 90° is even

= sec 60°

= 2 $[\because\ \sec 60° = 2]$

Q. 8. F. Find the value of

$$\text{cosec}\left(\frac{-41\pi}{4}\right)$$

Answer : To find: Value of $\text{cosec}\left(-\frac{41\pi}{4}\right)$

We have,

$\text{cosec}\left(-\frac{41\pi}{4}\right) = -\text{cosec}\frac{41\pi}{4}$

[∵ cosec(-θ) = -cosec θ]

Putting π = 180°

$= -\text{cosec}\frac{41\times180}{4}$

= -cosec[41 × 45°]

= -cosec[1845°]

= -cosec [90° × 20 + 45°]

Clearly, 1845° is in I[st] Quadrant and the multiple of 90° is even

= -cosec 45°

= -√2 $[\because \text{cosec}\,45° = \sqrt{2}]$

Q. 9. A. Find the value of

sin 405°

Answer : To find: Value of sin 405°

We have,

sin 405° = sin [90° × 4 + 45°]

= sin 45°

[Clearly, 405° is in I^{st} Quadrant and the multiple of 90° is even]

$= \frac{1}{\sqrt{2}}$ [$\because$ $\sin 45° = \frac{1}{\sqrt{2}}$]

Q. 9. B. Find the value of

sec (-1470^0)

Answer : To find: Value of sec (-1470°)

We have,

sec (-1470°) = sec (1470°)

[$\because$ sec(-θ) = sec θ]

= sec [90° × 16 + 30°]

Clearly, 1470° is in I^{st} Quadrant and the multiple of 90° is even

= sec 30°

$= \frac{2}{\sqrt{3}}$ [$\because$ $\sec 30° = \frac{2}{\sqrt{3}}$]

Q. 9. C. Find the value of

tan (-300^0)

Answer : To find: Value of tan (-300°)

We have,

tan (-300°) = - tan (300°)

[$\because$ tan(-θ) = -tan θ]

= - tan [90° × 3 + 30°]

Clearly, 300° is in IV^{th} Quadrant and the multiple of 90° is odd

= - cot 30°

= -√3 [$\because$ $\cot 30° = \sqrt{3}$]

Q. 9. D. Find the value of

cot (585^0)

Answer : To find: Value of cot $\frac{13\pi}{4}$

We have,

cot (585°) = cot [90° × 6 + 45°]

= cot 45°

[Clearly, 585° is in IIIrd Quadrant and the multiple of 90° is even]

= 1 [∵ cot 45° = 1]

Q. 9. E. Find the value of

cosec (-750^0)

Answer : To find: Value of cosec (-750°)

We have,

cosec (-750°) = - cosec(750°)

[∵ cosec(-θ) = -cosec θ]

= - cosec [90° × 8 + 30°]

Clearly, 405° is in Ist Quadrant and the multiple of 90° is even

= - cosec 30°

= -2 [∵ cosec 30° = 2]

Q. 9. F. Find the value of

cos (-2220^0)

Answer : To find: Value of cos 2220°

We have,

cos (-2220˚) = cos 2220°

[∵ cos(-θ) = cos θ]

= cos [2160 + 60°]

= cos [360° × 6 + 60°]

= cos 60°

[Clearly, 2220° is in Ist Quadrant and the multiple of 360° is even]

$= \frac{1}{2}$ [∵ $\cos 60° = \frac{1}{2}$]

Q. 10. A. Prove that

$$\tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2} = 8$$

Answer :

To prove: $\tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2} = 8$

Taking LHS,

$= \tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2}$

Putting π = 180°

$= \tan^2\frac{180}{3} + 2\cos^2\frac{180}{4} + 3\sec^2\frac{180}{6} + 4\cos^2\frac{180}{2}$

= $\tan^2$ 60° + 2 $\cos^2$ 45° + 3 $\sec^2$ 30° + 4 $\cos^2$ 90°

Now, we know that,

$\tan 60° = \sqrt{3}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$\sec 30° = \frac{2}{\sqrt{3}}$

$\cos 90° = 0$

Putting the values, we get

$$= \left(\sqrt{3}\right)^2 + 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 3 \times \left(\frac{2}{\sqrt{3}}\right)^2 + 4(0)^2$$

$$= 3 + 2 \times \frac{1}{2} + 3 \times \frac{4}{3}$$

= 3 + 1 + 4

= 8

= RHS

∴ LHS = RHS

Hence Proved

Q. 10. B. Prove that

$$\sin\frac{\pi}{6}\cos 0 + \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cos\frac{\pi}{6} = \frac{7}{4}$$

Answer :

To prove: $\sin\frac{\pi}{6}\cos 0 + \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cos\frac{\pi}{6} = \frac{7}{4}$

Taking LHS,

$$= \sin\frac{\pi}{6}\cos 0 + \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cos\frac{\pi}{6}$$

Putting π = 180°

$$= \sin\frac{180}{6}\cos 0 + \sin\frac{180}{4}\cos\frac{180}{4} + \sin\frac{180}{3}\cos\frac{180}{6}$$

= sin 30° cos 0° + sin 45° cos 45° + sin 60° cos 30°

Now, we know that,

$\sin 30° = \frac{1}{2}$

$\cos 0° = 1$

$\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

Putting the values, we get

$= \frac{1}{2} \times 1 + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2} + \frac{1}{2} + \frac{3}{4}$

$= \frac{2+2+3}{4}$

$= \frac{7}{4}$

= RHS

∴ LHS = RHS

Hence Proved

Q. 10. C. Prove that

$$4\sin\frac{\pi}{6}\sin^2\frac{\pi}{3} + 3\cos\frac{\pi}{3}\tan\frac{\pi}{4} + \operatorname{cos ec}^2\frac{\pi}{2} = 4$$

Answer : To prove: $4\sin\frac{\pi}{6}\sin^2\frac{\pi}{3} + 3\cos\frac{\pi}{3}\tan\frac{\pi}{4} + \operatorname{cosec}^2\frac{\pi}{2} = 4$

Taking LHS,

$= 4\sin\frac{\pi}{6}\sin^2\frac{\pi}{3} + 3\cos\frac{\pi}{3}\tan\frac{\pi}{4} + \operatorname{cosec}^2\frac{\pi}{2}$

Putting π = 180°

$= 4\sin\frac{180}{6}\sin^2\frac{180}{3} + 3\cos\frac{180}{3}\tan\frac{180}{4} + \operatorname{cosec}^2\frac{180}{2}$

= 4 sin 30° $\sin^2$ 60° + 3 cos 60° tan 45° + cosec^2 90°

Now, we know that,

$\sin 30° = \frac{1}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

tan 45° = 1

cosec 90° = 1

Putting the values, we get

$= 4 \times \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 + 3 \times \frac{1}{2} \times 1 + (1)^2$

$= 2 \times \frac{3}{4} + \frac{3}{2} + 1$

$= \frac{3}{2} + \frac{3}{2} + 1$

$= \frac{3+3+2}{2}$

= 4

= RHS

∴ LHS = RHS

Hence Proved

Exercise 15B

Q. 1. Find the value of

(i) cos 840^0
(ii) sin 870^0
(iii) tan (- 120^0)
(iv) sec (- 420^0)
(v) cosec (- 690^0)
(vi) tan (225^0)
(vii) cot (- 315^0)
(viii) sin (- 1230^0)
(ix) cos (495^0)

Answer : (i) Cos840° = Cos(2.360° + 120°) …………(using Cos(2ϖ + x) = Cosx)

= Cos(120°)

= Cos(180° - 60°)

= - Cos60° ……………(using Cos(ϖ - x) = - Cosx)

$$= -\frac{1}{2}$$

(ii) sin870° = sin(2.360° + 150°) …………(using sin(2ϖ + x) = sinx)

= sin150°

= sin(180° - 30°) ………(using sin(ϖ - x) = sinx)

= sin30°

$$= \frac{1}{2}$$

(iii) tan(- 120°) = - tan12 …….(tan(- x) = tanx)

= - tan(180° - 60°) ……. (in II quadrant tanx is negative)

= - (- tan60°)

= tan60°

$= \sqrt{3}$

(iv) $\sec(-420°) = \dfrac{1}{\cos(-420°)}$

$= \dfrac{1}{-\cos 420°}$ ……….(using cos(- x) = - cosx)

$= \dfrac{-1}{-\cos(360° + 60)}$ ………..(using cos(2ϖ + x) = cosx)

$= \dfrac{-1}{\cos 60°} \Rightarrow \dfrac{-1}{1/2} = -2$

(v) $\operatorname{cosec}(690°) = \dfrac{1}{\sin(-690°)} \Rightarrow \dfrac{1}{-\sin(690°)} = \dfrac{1}{-\sin(2.360 - 30°)}$

……..(IV quadrant sinx is negative)

$= \dfrac{1}{-(-\sin 30°)} \Rightarrow \dfrac{1}{1/2} = 2$

(vi) tan225° = tan(180° + 45°) …………(in III quadrant tanx is positive)

$\Rightarrow \tan 45° = 1$

(vii) $\cot(-315°) = \dfrac{1}{\tan(-315)°} \Rightarrow \dfrac{1}{-\tan(315°)} = \dfrac{1}{-\tan(360° - 45°)}$

…..(tan(- x) = - tanx)

$= \dfrac{1}{-(-\tan 45°)} \Rightarrow 1$ …..(in IV quadrant tanx is negative)

(viii) sin(- 1230°) = sin1230° …………(using sin(- x) = sinx)

= sin(3.360° + 150°)

= sin150°

= sin(180° - 30°) …………(using sin(180° - x) = sinx)

= sin30°

$$= \frac{1}{2}$$

(ix) cos495° = cos(360° + 135°) …………(using cos(360° + x) = cosx)

= cos135°

= cos(180° - 45°) …………(using cos(180° - x) = - cosx)

= - cos45°

$$= -\frac{1}{\sqrt{2}}$$

Q. 2. Find the values of all trigonometric functions of 135^0

Answer : Sin135° = sin(180° - 45°) ……....(using sin(180° - x) = sinx)

$$= \sin 45° \Rightarrow \frac{1}{\sqrt{2}}$$

Cos135° = cos(180° - 45°) ……...(using cos(180° - x) = - cosx)

$$= \cos 45° \Rightarrow -\frac{1}{\sqrt{2}}$$

$$\text{Tan}135° = \frac{\sin 135°}{\cos 135°} \Rightarrow \frac{1/\sqrt{2}}{-1/\sqrt{2}} = -1$$

$$\text{Cosec}135° = \frac{1}{\sin 135°} \Rightarrow \sqrt{2}$$

$$\text{Sec}135° = \frac{1}{\cos 135°} \Rightarrow -\sqrt{2}$$

$$\text{Cot}135° = \frac{1}{\tan 135°} \Rightarrow -1$$

Q. 3. Prove that

(i) $\sin 80^0 \cos 20^0 - \cos 80^0 \sin 20^0 = \frac{\sqrt{3}}{2}$

(ii) $\cos 45^0 \cos 15^0 - \sin 45^0 \sin 15^0 = \frac{1}{2}$

(iii) $\cos 75^0 \cos 15^0 + \sin 75^0 \sin 15^0 = \frac{1}{2}$

(iv) $\sin 40^0 \cos 20^0 + \cos 40^0 \sin 20^0 = \frac{\sqrt{3}}{2}$

(v) $\cos 130^0 \cos 40^0 + \sin 130^0 \sin 40^0 = 0$

Answer : (i) sin80°cos20° - cos80°sin20° = sin(80° - 20°)

(using sin(A - B) = sinAcosB - cosAsinB)

= sin60°

$$= \frac{\sqrt{3}}{2}$$

(ii) cos45°cos15° - sin45°sin15° = cos(45° + 15°)

(Using cos(A + B) = cosAcosB - sinAsinB)

= cos60°

$$= \frac{1}{2}$$

(iii) cos75°cos15° + sin75°sin15° = cos(75° - 15°)

(using cos(A - B) = cosAcosB + sinAsinB)

= cos60°

$$= \frac{1}{2}$$

(iv) sin40°cos20° + cos40°sin20° = sin(40° + 20°)

(using sin(A + B) = sinAcosB + cosAsinB)

= sin60°

$$= \frac{\sqrt{3}}{2}$$

(v) cos130°cos40° + sin130°sin40° = cos(130° - 40°)

(using cos(A - B) = cosAcosB + sinAsinB)

= cos90°

= 0

Q. 4. Prove that

(i) $\sin(50^0+\theta)\cos(20^0+\theta)-\cos(50^0+\theta)\sin(20^0+\theta)=\frac{1}{2}$

(ii) $\cos(70^0+\theta)\cos(10^0+\theta)+\sin(70^0+\theta)\sin(10^0+\theta)=\frac{1}{2}$

Answer : (i) sin(50° + θ)cos(20° + θ) - cos(50° + θ)sin(20° + θ)

= sin(50° + θ - (20° + θ))(using sin(A - B) = sinAcosB - cosAsinB)

= sin(50° + θ - 20° - θ)

= sin30°

$$=\frac{1}{2}$$

(ii) cos(70° + θ)cos(10° + θ) + sin(70° + θ)sin(10° + θ)

= cos(70° + θ - (10° + θ))(using cos(A - B) = cosAcosB + sinAsinB)

= cos(70° + θ - 10° - θ)

= cos60°

$$=\frac{1}{2}$$

Q. 5. Prove that

(i) $\cos(n+2)x\cos(n+1)x+\sin(n+2)x\sin(n+1)x=\cos x$

(ii) $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Answer : (i)cos(n + 2)x.cos(n + 1)x + sin(n + 2)x.sin(n + 1)x

= sin((n + 2)x + (n + 1)x)(using cos(A - B) = cosAcosB + sinAsinB)

= cos(nx + 2x - (nx + x))

= cos(nx + 2x - nx - x)

= cosx

(ii) $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

$= \cos\left(\frac{\pi}{4}-x+\frac{\pi}{4}-y\right)$(using cos(A + B) = cosAcosB - sinAsinB)

$= \cos\left(\frac{2\pi}{4}-x-y\right)$

$= \cos\left(\frac{\pi}{2}-(x+y)\right)\left(\text{using}\cos\left(\frac{\pi}{2}-x\right) = \sin x\right)$

= sin(x + y)

Q. 6.

Prove that $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$

Answer :

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\frac{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}.\tan x}}{\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}.\tan x}}$$

$$\Rightarrow\frac{\dfrac{1+\tan x}{1-1.\tan x}}{\dfrac{1-\tan x}{1+1.\tan x}}=\frac{1+\tan x}{1-\tan x}.\frac{1+\tan x}{1-\tan x}$$

$$\Rightarrow\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Hence, Proved.

Q. 7. Prove that

(i) $\sin 75^0=\frac{(\sqrt{6}+\sqrt{2})}{4}$

(ii) $\frac{\cos 135^0-\cos 120^0}{\cos 135^0+\cos 120^0}=(3-2\sqrt{2})$

(iii) $\tan 15^0+\cot 15^0=4$

Answer : (i) sin75° = sin(90° - 15°) ……..(using sin(A - B) = sinAcosB - cosAsinB)

= sin90°cos15° - cos90°sin15°

= 1.cos15° - 0.sin15°

= cos15°

Cos15° = cos(45° - 30°) …………(using cos(A - B) = cosAcosB + sinAsinB)

= cos45°.cos30° + sin45°.sin30°

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}.1 \Rightarrow \frac{\sqrt{3}+1}{2\sqrt{2}}.\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin 75° = \cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$$

(ii) $\frac{\cos 135° - \cos 120°}{\cos 135° + \cos 120°} = \frac{\cos(180° - 45°) - \cos(180° - 60°)}{\cos(180° - 45°) + \cos(180° - 60°)}$ (using sin(180° - x) = sinx)

(using cos(180° - x) = - cosx)

$$= \frac{-\cos 45° - (-\cos 60°)}{-\cos 45° + (-\cos 60°)}$$

$$= \frac{\cos 60° - \cos 45°}{-(\cos 60° + \cos 45°)}$$

$$= -\frac{\frac{1}{2} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + \frac{1}{2}} \Rightarrow -\frac{\frac{1-\sqrt{2}}{2}}{\frac{\sqrt{2}+1}{2}} = -\frac{1-\sqrt{2}}{\sqrt{2}+1}.\frac{(-\sqrt{2}+1)}{(-\sqrt{2}+1)}$$

$$= -\frac{-\sqrt{2}+1+2-\sqrt{2}}{-2+\sqrt{2}-\sqrt{2}+1} \Rightarrow -\frac{-2\sqrt{2}+3}{-1} = 3-2\sqrt{2}$$

(iii) tan15° + cot15° =

First, we will calculate tan15°,

$$\tan 15^\circ = \frac{\sin 15^\circ}{\cos 15^\circ}$$ ....(1)

$$[\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}, \sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ.\cos 30^\circ - \cos 45^\circ.\sin 30^\circ = \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\tan 15^\circ = \frac{\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} \Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} \text{and } \cot 15^\circ = \frac{1}{\tan 15^\circ} \frac{1}{\frac{\sqrt{3}-1}{\sqrt{3}+1}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Putting in eq(1),

$$\tan 15^\circ + \cot 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{\left(\sqrt{3}-1\right)^2 + \left(\sqrt{3}+1\right)^2}{3-1} \frac{3+1-2\sqrt{3}+3+1+2\sqrt{3}}{2}$$

$$= \frac{8}{2} = 4$$

Q. 8. Prove that

(i) $\cos 15^0 - \sin 15^0 = \frac{1}{\sqrt{2}}$

(ii) $\cot 105^0 - \tan 105^0 = 2\sqrt{3}$

(iii) $\frac{\tan 69^0 + \tan 66^0}{1 - \tan 69^0 \tan 66^0} = -1$

Answer :

(i) $\cos 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\text{Sin} 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\text{Cos} 15^0 - \sin 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{2}{2\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

(ii) cot105° - tan105° = cot(180° - 75°) - tan(180° - 75°)

(II quadrant tanx is negative and cotx as well)

= - cot75° - (- tan75°)

= tan75° - cot75°

Tan75° = $\frac{\sin 75°}{\cos 75°} \Rightarrow \frac{\sin(90° - 15°)}{\cos(90° - 15°)} = \frac{-\cos 15°}{\sin 15°}$

(using sin(90° - x) = - cosx and cos(90° - x) = sinx)

$$= -\frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} \Rightarrow \frac{-\sqrt{3}-1}{\sqrt{3}-1}$$

Cot75° = $\frac{1}{\tan 75°} \Rightarrow \frac{\sqrt{3}-1}{-\sqrt{3}-1}$

Cot105° - tan105°

= $\frac{\sqrt{3}-1}{-\sqrt{3}-1} - \frac{-\sqrt{3}-1}{\sqrt{3}-1} \Rightarrow \frac{(\sqrt{3}-1)-(-\sqrt{3}-1)}{(-\sqrt{3}-1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}-(3+1+2\sqrt{3})}{(-3+1-\sqrt{3}+\sqrt{3})}$

$$= \frac{-4\sqrt{3}}{-2} \Rightarrow 2\sqrt{3}$$

(iii) $\frac{\tan 69° + \tan 66°}{1 - \tan 69°.\tan 66°} = \tan(69° + 66°) \Rightarrow \tan 135° = \tan(180° - 45°)$

(II quadrant tanx negative)

$\Rightarrow$ - tan45° = - 1

Q. 9. Prove that $\frac{\cos 9^0 + \sin 9^0}{\cos 9^0 - \sin 9^0} = \tan 54^0$

Answer : First we will take out cos9°common from both numerator and denominator,

$$\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \frac{\cos 9(1 + \tan 9°)}{\cos 9°(1 - \tan 9°)} \Rightarrow \frac{\tan 45° + \tan 9°}{1 - \tan 45°.\tan 9°} = \tan(45° + 9°) \Rightarrow \tan 54$$

$$\left(\text{using}\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}\text{and}\tan 45^\circ=1\right)$$

Q. 10. Prove that $\dfrac{\cos 8^0-\sin 8^0}{\cos 8^0+\sin 8^0}=\tan 37^0$

Answer : First we will take out cos8° common from both numerator and denominator,

$$\frac{\cos 8^\circ-\sin 8^\circ}{\cos 8^\circ+\sin 8^\circ}=\frac{\cos 8^\circ(1-\tan 8^\circ)}{\cos 8^\circ(1+\tan 8^\circ)}\Rightarrow\frac{\tan 45^\circ-\tan 8^\circ}{1+\tan 45^\circ.\tan 8^\circ}=\tan(45^\circ-8^\circ)\Rightarrow\tan 37^\circ$$

$$[\text{using}\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x.\tan y}\text{and}\tan 45^\circ=1$$

Q. 11. Prove that $\dfrac{\cos(\pi+\theta)\cos(-\theta)}{\cos(\pi-\theta)\cos\left(\dfrac{\pi}{2}+\theta\right)}=-\cot\theta$

Answer :

$$\frac{\cos(\pi+\theta).\cos(-\theta)}{\cos(\pi-\theta).\cos\left(\frac{\pi}{2}+\theta\right)}=\frac{-\cos\theta.\cos\theta}{-\cos\theta.-\sin\theta}$$

$$\Rightarrow\frac{\cos\theta}{-\sin\theta}=-\cot\theta$$

$$\left(\text{Using}\cos(\pi-\theta)=-\cos\theta\text{and}\cos\left(\frac{\pi}{2}-\theta\right)=-\sin\theta,\cos(-\theta)=-\cos\theta\right)$$

$$(\text{In III quadrant}\cos x\text{ is negative},\cos(\pi+\theta)=-\cos\theta)$$

Q. 12. Prove that

$$\frac{\cos\theta}{\sin(90^0+\theta)}+\frac{\sin(-\theta)}{\sin(180^0+\theta)}-\frac{\tan(90^0+\theta)}{\cot\theta}=3$$

Answer : Using sin(90° + θ) = cosθ and sin(- θ) = sinθ,tan(90° + θ) = - cotθ

Sin(180° + θ) = - sinθ(III quadrant sinx is negative)

$$\frac{\cos\theta}{\sin\left(90°+\ \theta\right)}+\frac{\sin\left(-\theta\right)}{\sin\left(180°+\ \theta\right)}-\frac{\tan\left(90°+\ \theta\right)}{\cot\theta}=\frac{\cos\theta}{\cos\theta}+\frac{-\sin\theta}{-\sin\theta}-\frac{-\cot\theta}{\cot\theta}$$

$$=1+\left(1\right)-\left(-1\right)\Rightarrow 1+1+1=3$$

Q. 13. Prove that

$$\frac{\sin(180^0+\theta)\cos(90^0+\theta)\tan(270^0-\theta)\cot(360^0-\theta)}{\sin(360^0-\theta)\cos(360^0+\theta)\mathrm{cosec}(-\theta)\sin(270^0+\theta)}=1$$

Answer : Using cos(90° + θ) = - sinθ(I quadrant cosx is positive

cosec(- θ) = - cosecθ

tan(270° - θ) = tan(180° + 90° - θ) = tan(90° - θ) = cotθ

(III quadrant tanx is positive)

Similarly sin(270° + θ) = - cosθ (IV quadrant sinx is negative

cot(360° - θ) = cotθ(IV quadrant cotx is negative)

$$=\frac{\sin\left(180°+\ \theta\right).\cos\left(90°+\ \theta\right).\tan\left(270°-\theta\right).\cot\left(360°-\theta\right)}{\sin\left(360°-\theta\right).\cos\left(360°-\theta\right).\mathrm{cosec}\left(-\theta\right).\sin\left(270°+\ \theta\right)}$$

$$=\frac{-\sin\theta.-\sin\theta.\cot\theta.-\cot\theta}{-\sin\theta.\cos\theta.-\mathrm{cosec}\theta.-\cos\theta}$$

$$=\ \cot\theta.\tan\theta.\cot\theta.\tan\theta\Rightarrow 1$$

Q. 14. If θ and Φ lie in the first quadrant such that $\sin\theta = \frac{8}{17} \text{ and } \cos\phi = \frac{12}{13}$**, find the values of**

(i) sin (θ - Φ)
(ii) cos (θ - Φ)
(iii) tan (θ - Φ)

Answer : $\text{Given } \sin\theta = \frac{8}{17} \text{and } \cos\phi = \frac{12}{13}$

$$\cos\theta = \sqrt{\left(1-\sin^2\theta\right)} \Rightarrow \sqrt{(1-\left(\frac{8}{17}\right)^2} = \sqrt{\left(\frac{289-84}{289}\right)} \Rightarrow \sqrt{\left(\frac{225}{289}\right)} = \frac{15}{17}$$

$$\sin\phi = \sqrt{\left(1-\left(\frac{12}{13}\right)^2\right)} \Rightarrow \sqrt{\left(\frac{169-144}{169}\right)} = \sqrt{\left(\frac{25}{169}\right)} \Rightarrow \frac{5}{13}$$

(i) sin(θ - Φ) = sinθcosΦ + cosθsinΦ

$$= \frac{8}{17}.\frac{12}{13} + \frac{15}{17}.\frac{5}{13} \Rightarrow \frac{96+75}{221} = \frac{171}{221}$$

(ii) cos(θ - Φ) = cosθ.cosΦ + sinθ.sinΦ

$$= \frac{15}{17}.\frac{12}{13} + \frac{8}{17}.\frac{5}{13} \Rightarrow \frac{180+40}{221} = \frac{220}{221}$$

(iii) We will first find out the Values of tanθ and tanΦ,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{8/17}{15/17} = \frac{8}{15} \text{antan}\phi = \frac{\sin\phi}{\cos\phi} \Rightarrow \frac{5/13}{12/13} = \frac{5}{12}$$

$$\tan(\theta - \Phi) = \tan\left(\theta - \phi\right) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta.\tan\phi} \Rightarrow \frac{\frac{8}{15} - \frac{5}{12}}{1 + \frac{8}{15}.\frac{5}{12}}$$

Q. 15. If x and y are acute such that $\sin x = \frac{1}{\sqrt{5}}$ and $\sin y = \frac{1}{\sqrt{10}}$**, prove that** $(x + y) = \frac{\pi}{4}$

Answer : $\text{Given} \sin x = \frac{1}{\sqrt{5}} \text{ and} \sin y = \frac{1}{\sqrt{10}}$,

Now we will calculate value of cos x and cosy

$$\cos x = \sqrt{\left(1 - \sin^2 x\right)} \Rightarrow \sqrt{\left(1 - \left(\frac{1}{\sqrt{5}}\right)^2\right)} = \sqrt{\left(\frac{5-1}{5}\right)} \Rightarrow \sqrt{\left(\frac{4}{5}\right)} = \frac{2}{\sqrt{5}}$$

$$\cos y = \sqrt{\left(1 - \sin x^2\right)} \Rightarrow \sqrt{\left(1 - \left(\frac{1}{\sqrt{10}}\right)^2\right)} = \sqrt{\left(\frac{10-1}{10}\right)} \Rightarrow \sqrt{\left(\frac{9}{10}\right)} = \frac{3}{\sqrt{10}}$$

Sin(x + y) = sinx.cosy + cosx.siny

$$= \frac{1}{\sqrt{5}} . \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} . \frac{1}{\sqrt{10}} \Rightarrow \frac{3+2}{\sqrt{50}} = \frac{5}{5\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin(x + y) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x + y = \frac{\pi}{4}$$

Q. 16. If x and y are acute angles such that $\cos x = \frac{13}{14}$ and $\cos y = \frac{1}{7}$**, prove that** $(x - y) = -\frac{\pi}{3}$**.**

Answer : $\text{Given} \cos x = \frac{13}{14} \text{ and} \cos y = \frac{1}{7}$

Now we will calculate value of sinx and siny

$$\sin x = \sqrt{(1-\cos^2 x)} \Rightarrow \sqrt{\left(1-\left(\frac{13}{14}\right)^2\right)} = \sqrt{\left(\frac{196-169}{196}\right)} \Rightarrow \sqrt{\left(\frac{27}{196}\right)} = \frac{3\sqrt{3}}{14}$$

$$\sin y = \sqrt{(1-\cos^2 y)} \Rightarrow \sqrt{\left(1-\left(\frac{1}{7}\right)^2\right)} = \sqrt{\left(\frac{49-1}{49}\right)} \Rightarrow \sqrt{\left(\frac{48}{49}\right)} = \frac{4\sqrt{3}}{7}$$

Hence,

Cos(x - y) = cosx.cosy + sinx.siny

$$= \frac{13}{14}.\frac{1}{7} + \frac{3\sqrt{3}}{14}.\frac{4\sqrt{3}}{7} \Rightarrow \frac{13+36}{98} = \frac{49}{98}$$

$$\cos(x-y) = \frac{1}{2}$$

$$x - y = \frac{\pi}{3}$$

Q. 17. If $\sin x = \frac{12}{3}$ and $\sin y = \frac{4}{5}$**, where**

$\frac{\pi}{2} < x < \pi$ and $0 < y < \frac{\pi}{2}$ **, find the values of**
(i) sin (x + y)
(ii) cos (x + y)
(iii) tan (x – y)

Answer : Given $\sin x = \frac{12}{13}$ and $\sin y = \frac{4}{5}$,

Here we will find values of cosx and cosy

$$\cos x = \sqrt{(1-\sin^2 x)} \Rightarrow \sqrt{\left(1-\left(\frac{12}{13}\right)^2\right)} = \sqrt{\left(\frac{169-144}{169}\right)} \Rightarrow \sqrt{\left(\frac{25}{169}\right)} = \frac{5}{13}$$

$$\cos y = \sqrt{\left(1-\sin^2 y\right)} \Rightarrow \sqrt{\left(1-\left(\frac{4}{5}\right)^2\right)} = \sqrt{\left(\frac{25-16}{25}\right)} \Rightarrow \sqrt{\left(\frac{9}{25}\right)} = \frac{3}{5}$$

(i) sin(x + y) = sinx.cosy + cosx.siny

$$\Rightarrow \frac{12}{13}.\frac{3}{5} + \frac{5}{13}.\frac{4}{5} \Rightarrow \frac{36+20}{65} = \frac{56}{65}$$

(ii) cos(x + y) = cosx.cosy + sinx.siny

$$= \frac{5}{13}.\frac{3}{5} + \frac{12}{13}.\frac{4}{5} \Rightarrow \frac{15+48}{65} = \frac{63}{65}$$

(iii) Here first we will calculate value of tanx and tany,

$$\tan x = \frac{\sin x}{\cos x} \Rightarrow \frac{12/13}{5/13} = \frac{5}{12} \text{ and } \tan y = \frac{\sin y}{\cos y} \Rightarrow \frac{4/5}{3/5} = \frac{4}{3}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x.\tan y} \Rightarrow \frac{\frac{5}{12}-\frac{4}{3}}{1+\frac{5}{12.}.\frac{4}{3}} = \frac{\frac{5-16}{12}}{\frac{36+20}{36}} \Rightarrow \frac{\frac{-11}{12}}{\frac{56}{36}} = \frac{-33}{56}$$

Q. 18.

If $\cos x = \frac{3}{5}$ and $\cos y = \frac{-24}{25}$, where $\frac{3\pi}{2} < x < 2\pi$ and $\pi < y < \frac{3\pi}{2}$, find the values of

(i) sin (x + y)
(ii) cos (x – y)
(iii) tan (x + y)

Answer : Given $\cos x = \frac{3}{5}$ and $\cos y = \frac{-24}{25}$

We will first find out value of sinx and siny,

$$\sin x = \sqrt{\left(1-\cos^2 x\right)} \Rightarrow \sqrt{\left(1-\left(\frac{3}{5}\right)^2\right)} = \sqrt{\left(\frac{25-9}{25}\right)} \Rightarrow \sqrt{\left(\frac{16}{25}\right)} = \frac{4}{5}$$

$$\sin y = \sqrt{\left(1-\cos^2 y\right)} \Rightarrow \sqrt{\left(1-\left(\frac{-24}{25}\right)^2\right)} = \sqrt{\left(\frac{625-576}{625}\right)} \Rightarrow \sqrt{\left(\frac{49}{625}\right)} = \frac{7}{25}$$

(i) sin(x + y) = sinx.cosy + cosx.siny

$$= \frac{4}{5}.\frac{-24}{25} + \frac{3}{5}.\frac{7}{25} \Rightarrow \frac{-96+21}{125} = \frac{-75}{125}$$

$$= \frac{-3}{5}$$

(ii) cos(x - y) = cosx.cosy + sinx.siny

$$= \frac{3}{5}.\frac{-24}{25} + \frac{4}{5}.\frac{7}{25} \Rightarrow \frac{-72+28}{125} = \frac{-44}{125}$$

(iii) Here first we will calculate value of tanx and tany,

$$\tan x = \frac{\sin x}{\cos x} \Rightarrow \frac{4/5}{3/5} = \frac{4}{3} \text{ and } \tan y = \frac{\sin y}{\cos y} \Rightarrow \frac{7/25}{-24/25} = \frac{7}{-24}$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x.\tan y} \Rightarrow \frac{\frac{4}{3}+\frac{-7}{24}}{1+\frac{4}{3.}.\frac{-7}{24}} = \frac{\frac{32-7}{24}}{\frac{72-28}{72}} \Rightarrow \frac{\frac{25}{24}}{\frac{44}{72}} = \frac{75}{44}$$

Q. 19. Prove that

(i) $\cos\left(\frac{\pi}{3}+x\right) = \frac{1}{2}(\cos x - \sqrt{3}\sin x)$

(ii) $$\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cos x$$

(iii) $$\frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{4}+x\right)=\frac{1}{2}(\cos x-\sin x)$$

(iv) $$\cos x+\cos\left(\frac{2\pi}{3}+x\right)+\cos\left(\frac{2\pi}{3}-x\right)=0$$

Answer : (i) $$\cos\left(\frac{\pi}{3}+x\right) = \cos\frac{\pi}{3}.\cos x-\sin\frac{\pi}{3}.\sin x$$

$$\Rightarrow \frac{1}{2}.\cos x-\frac{\sqrt{3}}{2}.\sin x = \frac{1}{2}\left(\cos x-\sqrt{3}\sin x\right)$$

(ii) $$\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}-x\right)$$

$$= \sin\frac{\pi}{4}.\cos x+\cos\frac{\pi}{4}.\sin x+\sin\frac{\pi}{4}.\cos x-\cos\frac{\pi}{4}.\sin x$$

$$= 2.\sin\frac{\pi}{4}.\cos x \Rightarrow 2.\frac{1}{\sqrt{2}}.\cos x = \sqrt{2}.\cos x$$

(iii) $$\frac{1}{\sqrt{2}}.\cos\left(\frac{\pi}{4}+x\right) = \frac{1}{\sqrt{2}}.\left(\cos\frac{\pi}{4}.\cos x-\sin\frac{\pi}{4}.\sin x\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}.\cos x-\frac{1}{\sqrt{2}}.\sin x\right) = \frac{1}{2}\left(\cos x-\sin x\right)$$

(iv) $$\cos x+\cos\left(\frac{2\pi}{3}+x\right)+\cos\left(\frac{2\pi}{3}-x\right)$$

$$= \cos x + \cos\frac{2\pi}{3}.\cos x - \sin\frac{2\pi}{3}.\sin x + \cos\frac{2\pi}{3}.\cos x + \sin\frac{2\pi}{3}.\sin x$$

$$= \cos x + 2.\cos\left(\pi - \frac{\pi}{3}\right).\cos x$$

$$= \cos x + 2.\left(-\frac{1}{2}\right).\cos x$$

$$= \cos x - \cos x \Rightarrow 0$$

Q. 20. Prove that

(i) $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12} = \frac{1}{2}$

(ii) $2\cos\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}$

(iii) $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{(2+\sqrt{3})}{2}$

Answer : (i) $2\sin\frac{5\pi}{12}.\sin\frac{\pi}{12} = -\left(\cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)\right)$

………[Using –2sinx.siny = cos(x + y)–cos (x–y)]

$$= -\left(\cos\frac{6\pi}{12} - \cos\frac{4\pi}{12}\right)$$

$$= -\left(\cos\frac{\pi}{2} - \cos\frac{\pi}{3}\right) \Rightarrow -\left(0 - \frac{1}{2}\right) = \frac{1}{2}$$

(ii) $2\cos\frac{5\pi}{12}.\cos\frac{\pi}{12} = \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) + \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$

………..[using 2cosx.cosy = cos(x + y) + cos(x–y)]

$$= \cos\frac{6\pi}{12} + \cos\frac{4\pi}{12} \Rightarrow \cos\frac{\pi}{2} + \cos\frac{\pi}{3} = 0 + \frac{1}{2}$$

$$= \frac{1}{2}$$

(iii) $$2\sin\frac{5\pi}{12}.\cos\frac{\pi}{12} = \sin\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) + \sin\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$$

...[Using2sinx.cosy = sin(x + y) + sin(x–y)]

$$= \sin\frac{6\pi}{12} + \sin\frac{4\pi}{12} \Rightarrow \sin\frac{\pi}{2} + \sin\frac{\pi}{3}$$

$$= 1 + \frac{\sqrt{3}}{2} \Rightarrow \frac{2 + \sqrt{3}}{2}$$

Exercise 15C

Q. 1. Prove that

$\sin(150^0 + x) + \sin(150^0 - x) = \cos x$

Answer : In this question the following formula will be used:

Sin(A +B)= sinA cos B + cosA sinB

Sin(A - B)= sinA cos B - cosA sinB

$= \sin150^\circ \cos x + \cos 150^\circ \sin x + \sin150^\circ \cos x - \cos150^\circ \sin x$

$= 2\sin150^\circ \cos x$

$= 2\sin(90^\circ + 60^\circ)\cos x$

$= 2\cos60^\circ \cos x$

$= 2 \times \frac{1}{2}$ cosx

= cosx

Q. 2. Prove that

cos x + cos (120^0 – x) + cos (120^0 + x) = 0

Answer : In this question the following formulas will be used:

cos (A + B) = cosAcosB – sinAsinB

cos (A - B) = cosAcosB+ sinAsinB

= cos x + cos 120^0 cosx – sin120sinx + cos 120^0cosx+sin120sinx

= cosx + 2cos120 cosx

= cosx + 2cos (90 + 30) cosx

= cosx + 2 (-sin30) cosx

$= \cos x - 2 \times \frac{1}{2}$ cosx

$= \cos x - \cos x$

= 0.

Q. 3. Prove that

$$\sin\left(x - \frac{\pi}{6}\right) + \cos\left(x - \frac{\pi}{3}\right) = \sqrt{3}\sin x$$

Answer : In this question the following formulas will be used:

sin (A - B) = sinA cos B - cosA sinB

cos (A - B) = cosAcosB+ sinAsinB

$= \sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6} + \cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3}$

$= \sin x \times \frac{\sqrt{3}}{2} - \cos x \times \frac{1}{2} + \cos x \times \frac{1}{2} + \sin x \times \frac{\sqrt{3}}{2}$

$= \sin x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{\sqrt{3}}{2}$

$= \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) \sin x$

$= \sqrt{3} \sin x$.

Q. 4. Prove that

$$\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$$

Answer : In this question the following formulas will be used:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan\left(\frac{\pi}{4} + x\right) = \frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \tan x}$$

$$= \frac{1 + \tan x}{1 - \tan x} \because \tan\frac{\pi}{4} = 1$$

Q. 5. Prove that

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan x}{1 + \tan x}$$

Answer : In this question the following formulas will be used:

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \tan x}$$

$= \frac{1-\tan x}{1+\tan x} \because \tan\frac{\pi}{4} = 1$

Q. 6. Express each of the following as a product.

1. sin 10x + sin 6x
2. sin 7x – sin 3x
3. cos 7x + cos 5x
4. cos2x – cos 4x

Answer :

$$1.\ \sin 10x + \sin 6x = 2\sin\frac{10x+6x}{2}\cos\frac{10x-sinx}{2}$$

$$=2\sin\frac{18x}{2}\cos\frac{4x}{2}$$

$$=2\sin 9x\cos 2x$$

Using,

sin(A +B)= sinA cos B + cosA sinB

$$2.\ \sin 7x - \sin 3x = 2\cos\frac{7x+3x}{2}\sin\frac{7x-3x}{2}$$

$$= 2\cos\frac{10x}{2}\sin\frac{4x}{2}$$

= 2cos5x sin2x

Using,

sin(A - B)= sinA cos B - cosA sinB

$3. \cos 7x + \cos 5x = 2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}$

$= 2\cos\frac{12x}{2}\cos\frac{2x}{2}$

$= 2\cos 6x \cos x$

Using,

cos (A + B) = cosAcosB – sinAsinB

$4. \cos 2x - \cos 4x = -2\sin\frac{2x+4x}{2}\sin\frac{2x-4x}{2}$

$= -2\sin\frac{6x}{2}\sin\frac{-2x}{2}$

= 2sin3x sinx

Using,

cos (A - B) = cosAcosB+ sinAsinB

Q. 7. Express each of the following as an algebraic sum of sines or cosines :

(i) 2sin 6x cos 4x
(ii) 2cos 5x din 3x
(iii) 2cos 7x cos 3x
(iv) 2sin 8x sin 2x

Answer : (i) 2sin 6x cos 4x = sin (6x+4x) + sin (6x-4x)

= sin 10x + sin 2x

Using,

2sinAcosB = sin (A+ B) + sin (A - B)

(ii) 2cos 5x sin3x = sin (5x + 3x) – sin (5x – 3x)

= sin8x – sin2x

Using,

2cosAsinB = sin(A + B) – sin (A - B)

(iii) 2cos7xcos3x = cos (7x+3x) + cos (7x – 3x)

= cos10x + cos 4x

Using,

2cosAcosB = cos (A+ B) + cos (A - B)

(iv) 2sin8 x sin2 x = cos (8x - 2x) – cos (8x + 2x)

= cos6x – cos10x

Using,

2sinAsinB = cos (A - B) – cos (A+ B)

Q. 8. Prove that

$$\frac{\sin x + \sin 3x}{\cos x - \cos 3x} = \cot x$$

Answer :

$$\frac{\sin x + \sin 3x}{\cos x - \cos 3x}$$

$$=\frac{2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}}{-2\sin\frac{x+3x}{2}\sin\frac{x-3x}{2}}$$

$$=\frac{2\sin\frac{4x}{2}\cos\frac{2x}{2}}{2\sin\frac{4x}{2}\sin\frac{2x}{2}}$$

$$=\frac{\cos x}{\sin x}$$

$= \cot x$

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 9. Prove that

$$\frac{\sin 7x - \sin 5x}{\cos 7x + \cos 5x} = \tan x$$

Answer :

$$\frac{\sin 7x - \sin 5x}{\cos 7x + \cos 5x}$$

$$=\frac{2\cos\frac{7x+5x}{2}\sin\frac{7x-5x}{2}}{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}}$$

$$=\frac{2\cos 6x\sin x}{2\cos 6x\cos x}$$

$$=\frac{\sin x}{\cos x}$$

= tanx

Using the formula,

sinA - sinB = $2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

cosA + cosB = $2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 10. Prove that

$$\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$$

Answer :

$$\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$$

$$=\frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$$

$$=\frac{2\sin 4x\cos x}{2\cos 4x\cos x}$$

= tan4x

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 11. Prove that

$$\frac{\cos 9x - \cos 5x}{\cos 17x - \sin 3x} = \frac{-\sin 2x}{\cos 10x}$$

Answer :

$=\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$=\frac{-2\sin\frac{9x+5x}{2}\sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}$

$=\frac{-2\sin 7x \sin 2x}{2\cos 10x \sin 7x}$

$=\frac{-\sin 2x}{\cos 10x}$

Using the formula,

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 12. Prove that

$$\frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \tan 3x$$

Answer :

$= \frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x}$

$= \frac{(\sin 5x + \sin x) + \sin 3x}{(\cos 5x + \cos x) + \cos 3x}$

$= \frac{2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2} + \sin 3x}{2\cos\frac{5x+x}{2}\cos\frac{5x-x}{2} + \cos 3x}$

$= \frac{2\sin 3x \cos x + \sin 3x}{2\cos 3x \cos x + \cos 3x}$

$= \frac{\sin 3x(2\cos x + 1)}{\cos 3x(2\cos x + 1)}$

= tan3x.

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 13. Prove that

$$\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$$

Answer :

$$=\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$$

$$=\frac{2\sin\frac{7x+5x}{2}\cos\frac{7x-5x}{2}+2\sin\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}+2\cos\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}$$

$$=\frac{2\sin 6x\cos x+2\sin 6x\cos 3x}{2\cos 6x\cos x+2\cos 6x\cos 3x}$$

$$=\frac{2\sin 6x(\cos x+\cos 3x)}{2\cos 6x(\cos x+\cos 3x)}$$

$$=\frac{\sin 6x}{\cos 6x}$$

$$=\tan 6x$$

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 14. Prove that

cot 4x (sin 5x + sin 3x) = cot x (sin 5x – sin 3x)

Answer : L.H.S

cot 4x (sin 5x + sin3x)

$= \cot 4x \ (2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2})$

$= \cot 4x \ (2 \sin 4x \cos x)$

$= \frac{\cos 4x}{\sin 4x}(2 \sin 4x \cos x)$

$= 2\cos 4x \cos x$

R.H.S

$\cot x \ (\sin 5x - \sin 3x)$

$= \cot x \ (2\cos\frac{5x+3x}{2}\sin\frac{5x-3x}{2})$

$= \cot x\ (2\cos 4x \sin x)$

$= \frac{\cos x}{\sin x}(2\cos 4x \sin x)$

$= 2\cos 4x \cos x$

L.H.S=R.H.S

Hence, proved.

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2} \cos\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2} \sin\frac{A-B}{2}$

Q. 15. Prove that

(sin 3x + sin x) sin x + (cos 3x – cos x) cos x = 0

Answer : = (sin 3x + sin x) sin x + (cos 3x – cos x) cos x

$= (2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2})\ \sin x + (-2\sin\ \frac{3x+x}{2}\sin\frac{3x-x}{2})\ \cos x$

= (2sin2x cosx) sinx-(2sin2x sinx) cosx

= 0.

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 16. Prove that

$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$

Answer : $= (\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$=\left(-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}\right)^2 + \left(2\cos\frac{x+y}{2}\sin\frac{x-y}{2}\right)^2$

$=4\sin^2\left(\frac{x-y}{2}\right)\left(\sin^2\left(\frac{x-y}{2}\right)+\cos^2\left(\frac{x-y}{2}\right)\right)$

$=4\sin^2\left(\frac{x-y}{2}\right)$

Using the formula,

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 17. Prove that

$$\frac{\sin 2x - \sin 2y}{\cos 2y - \cos 2x} = \cot(x+y)$$

Answer :

$$=\frac{\sin 2x-\sin 2y}{\cos 2y-\cos 2x}$$

$$=\frac{2\cos\frac{2x+2y}{2}\sin\frac{2x-2y}{2}}{-2\sin\frac{2x+2y}{2}\sin\frac{2y-2x}{2}}$$

$$=\frac{\cos(x+y)\sin(x-y)}{\sin(x+y)\sin(x-y)}$$

$$=\frac{\cos(x+y)}{\sin(x+y)}$$

$$=\cot(x+y)$$

Using the formula,

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 18. Prove that

$$\frac{\cos x+\cos y}{\cos y-\cos x}=\cot\left(\frac{x+y}{2}\right)\cot\left(\frac{x-y}{2}\right)$$

Answer :

$$=\frac{\cos x-\cos y}{\cos y-\cos x}$$

$$=\frac{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{-2\sin\frac{x+y}{2}\sin\frac{y-x}{2}}$$

$$=\frac{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{2\sin\frac{x+y}{2}\sin\frac{x-y}{2}}$$

$$=\frac{\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{\sin\frac{x+y}{2}\sin\frac{x-y}{2}}$$

$$=\cot\frac{x+y}{2}\cot\frac{x-y}{2}$$

Using the formula,

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

Q. 19. Prove that

$$\frac{\sin x+\sin y}{\sin x-\sin y}=\tan\left(\frac{x+y}{2}\right)\cot\left(\frac{x-y}{2}\right)$$

Answer :

$=\frac{\sin x+\sin y}{\sin x-\sin y}$

$=\frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}$

$=\tan\frac{x+y}{2}\cot\frac{x-y}{2}$

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

Q. 20. Prove that

$$\sin 3x + \sin 2x - \sin x = 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$$

Answer : =Sin3x+sin2x-sinx

= (sin3x- sinx)+sin2x

$= \left(2\cos\frac{3x+x}{2}\sin\frac{3x-2x}{2}\right) + \sin 2x$

= 2cos2xsinx +sin2x

= 2cos2xsinx + 2sinxcosx

= 2sinx (cos2x + cosx)

$= 2\sin x \left(2\cos\frac{2x+x}{2}\cos\frac{2x-x}{2}\right)$

$= 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Using the formula,

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

Q. 21. Prove that

$$\frac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x \sin x + \cos 6x \cos x} = \tan 2x$$

Answer :

$$= \frac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x \sin x + \cos 6x \cos x}$$

$$= \frac{2\cos 4x \sin 3x - 2\cos 2x \sin x}{2\sin 4x \sin x + 2\cos 6x \cos x}$$

$$= \frac{\sin(4x+3x) - \sin(4x-3x) - \{\sin(2x+x) - \sin(2x-x)\}}{\cos(4x-x) - \cos(4x+x) + \cos(6x+x) + \cos(6x-x)}$$

$$= \frac{\sin 7x + \sin x - \sin 3x + \sin x}{\cos 3x - \cos 5x + \cos 7x + \cos 5x}$$

$$= \frac{\sin 7x - \sin 3x}{\cos 3x + \cos 7x}$$

$$= \frac{2\cos\frac{7x+3x}{2}\sin\frac{7x-3x}{2}}{2\cos\frac{7x+3x}{2}\cos\frac{7x-3x}{2}}$$

Using the formulas,

2cosAsinB = sin (A + B) – sin (A - B)

2cosAcosB = cos (A + B) + cos (A - B)

2sinAsinB = cos (A - B) – cos (A + B)

Q. 22. Prove that

$$\frac{\cos 2x \sin x + \cos 6x \sin 3x}{\sin 2x \sin x + \sin 6x \sin 3x} = \cot 5x$$

Answer :

$$= \frac{\cos 2x \sin x + \cos 6x \sin 3x}{\sin 2x \sin x + \sin 6x \sin 3x}$$

$$= \frac{2\cos 2x \sin x + 2\cos 6x \sin 3x}{2\sin 2x \sin x + 2\sin 6x \sin 3x}$$

$$= \frac{\sin(2x+x) - \sin(2x-x) + \{\sin(6x+3x) - \sin(6x-3x)\}}{\cos(2x-x) - \cos(2x+x) + \cos(6x-3x) - \cos(6x+3x)}$$

$$= \frac{\sin 3x - \sin x + \sin 9x - \sin 3x}{\cos x - \cos 3x + \cos 3x - \cos 9x}$$

$$= \frac{\sin 9x - \sin x}{\cos x - \cos 9x}$$

$$= \frac{2\cos\frac{9x+x}{2}\sin\frac{9x-x}{2}}{-2\sin\frac{x+9x}{2}\sin\frac{x-9x}{2}}$$

$$=\frac{2\cos\frac{9x+x}{2}\sin\frac{9x-x}{2}}{2\sin\frac{x+9x}{2}\sin\frac{9x-x}{2}}$$

$$=\frac{\cos 5x \sin 4x}{\sin 5x \cos 4x}$$

$$=\cot 5x$$

Using the formulas,

2cosAsinB = sin (A + B) – sin (A - B)

2sinAsinB = cos (A - B) – cos (A + B)

Q. 23. Prove that

$$\sin 10^0 \sin 30^0 \sin 50^0 \sin 70^0 = \frac{1}{16}$$

Answer : L.H.S

$=\sin 10° \sin 30° \sin 50° \sin 70°$

$=\frac{1}{2}(2\sin 70° \sin 10°) \sin 50° \frac{1}{2}$

$=\frac{1}{4}\{\cos(70° - 10°) - \cos(70° + 10°)\} \sin 50°$

$=\frac{1}{4}\{\cos 60° \sin 50° - \cos 80° \sin 50°\}$

$=\frac{1}{4}\{\frac{1}{2} \sin 50° - \cos 80° \sin 50°\}\}$

$=\frac{1}{8}\{\sin 50° - 2\cos 80° \sin 50°\}$

$=\frac{1}{8}\{\sin 50° - (\sin(80° + 50°) - \sin(80° - 50°)\}$

$=\frac{1}{8}\{\sin 50° - \sin 130° + \sin 30°\}$

$=\frac{1}{8}\{\sin 50° - \sin 130° + \frac{1}{2}\}$

$=\frac{1}{8}\{\sin 50° - \sin(180° - 50°) + \frac{1}{2}\}$

$=\frac{1}{8}\{\sin 50° - \sin 50° + \frac{1}{2}\}$

$=\frac{1}{16}$

=R.H.S

Q. 24. Prove that

$$\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$$

Answer : L.H.S

$$=\frac{1}{2}(2\sin 80° \sin 20°) \sin 40° \frac{\sqrt{3}}{2}$$

$$=\frac{\sqrt{3}}{4}\{\cos(80° - 20°) - \cos(80° + 20°)\} \sin 40°$$

$$=\frac{\sqrt{3}}{4}\{\cos 60° \sin 40° - \cos 100° \sin 40°\}$$

$$=\frac{\sqrt{3}}{4}\{\frac{1}{2} \sin 40° - \cos 100° \sin 40°\}\}$$

$$=\frac{\sqrt{3}}{8}\{\sin 40° - 2\cos 100° \sin 40°\}$$

$$=\frac{\sqrt{3}}{8}\{\sin 40° - (\sin(100° + 40°) - \sin(100° - 40°)\}$$

$$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin 140° + \sin 60°\}$$

$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin 140° + \frac{\sqrt{3}}{2}\}$

$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin(180° - 40°) + \frac{\sqrt{3}}{2}\}$

$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin 40° + \frac{\sqrt{3}}{2}\}$

$=\frac{3}{16}$

=R.H.S

Q. 25. Prove that

$$\cos 10^0 \cos 30^0 \cos 50^0 \cos 70^0 = \frac{3}{16}$$

Answer : L.H.S

$$=\cos 10° \cos 30° \cos 50° \cos 70°$$

$$=\frac{1}{2}(2\cos 70° \cos 10°) \cos 50° \frac{\sqrt{3}}{2}$$

$$=\frac{\sqrt{3}}{4}\{\cos(70° + 10°) + \cos(70° - 10°)\} \cos 50°$$

$$=\frac{\sqrt{3}}{4}\{\cos 80° \cos 50° + \cos 60° \cos 50°\}$$

$$=\frac{\sqrt{3}}{4}\{cos 80° \cos 50° + \frac{1}{2} \cos 50°\}\}$$

$$=\frac{\sqrt{3}}{8}\{2 \cos 80° \cos 50° + \cos 50°\}$$

$$=\frac{\sqrt{3}}{8}\{(\cos(80° + 50°) - \cos(80° - 50°) + \cos 50°\}$$

$$=\frac{\sqrt{3}}{8}\{\cos 130° - \cos 30° + \cos 50°\}$$

$$=\frac{\sqrt{3}}{8}\{\cos 130° - \cos 50° + cos 30°\}$$

$$=\frac{\sqrt{3}}{8}\{\cos(180° - 50°) - \cos(50°) + \frac{\sqrt{3}}{2}\}$$

$$=\frac{\sqrt{3}}{8}\{\cos 50° - \cos 50° + \frac{\sqrt{3}}{2}\}$$

$$=\frac{3}{16}$$

Q. 26. If $\cos x + \cos y = \frac{1}{3}$ and $\sin x + \sin y = \frac{1}{4}$, **prove that** $\tan\left(\frac{x+y}{2}\right) = \frac{3}{4}$

Answer :

cosx+cosy = $\frac{1}{3}$ --------- i

sinx+siny = $\frac{1}{4}$ ----------ii

dividing ii by I we get,

$\Rightarrow \frac{\sin x+\sin y}{\cos x+\cos y} = \frac{\frac{1}{4}}{\frac{1}{3}}$

$\Rightarrow \frac{\sin x+\sin y}{\cos x+\cos y} = \frac{3}{4}$

$\Rightarrow \frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}} = \frac{3}{4}$

$\Rightarrow \tan(\frac{x+y}{2}) = \frac{3}{4}$

Using the formula,

sinA + sinB = 2sin$\frac{A+B}{2}$ cos$\frac{A-B}{2}$

cosA + cosB = 2cos$\frac{A+B}{2}$cos$\frac{A-B}{2}$

Q. 27. A. Prove that

$$2\cos 45^0 \cos 15^0 = \frac{\sqrt{3}+1}{2}$$

Answer : L.H.S

$$= 2\cos 45° \cos 15°$$

$$=2\cos 45° \cos(45° - 30°)$$

$$=2\frac{1}{\sqrt{2}} (\cos 45° \cos 30° + \sin 45° \sin 30°)$$

$$=\sqrt{2}(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2})$$

$$=\sqrt{2}(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}})$$

$$=\sqrt{2}(\frac{\sqrt{3+1}}{2\sqrt{2}})$$

$$=\frac{\sqrt{3}+1}{\sqrt{2}}$$

Q. 27. B. Prove that

$$2\sin 75^0 \sin 15^0 = \frac{1}{2}$$

Answer : L.H.S

$= 2\sin 75° \sin 15°$

$= 2\sin(45° + 30°)\sin(45° - 30°)$

$= \cos(45° - 30° - 45° - 30°) - \cos(45° + 30° + 45° - 30°)$

$= \cos(-60°)$ - $\cos 90°$

$= \cos 60° - 0$

$= \frac{1}{2}$

Q. 27. C. Prove that

$$\cos 15^0 - \sin 15 = \frac{1}{\sqrt{2}}$$

Answer : L.H.S

$\Rightarrow \cos 15° - \sin 15°$

$\Rightarrow \cos(45° - 30°) - \sin(45° - 30°)$

$\Rightarrow (\cos 45° \cos 30° + \sin 45° \sin 30°)$ - $(\sin 45° \cos 30° - \cos 45° \sin 30°)$

$\Rightarrow (\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}) - (\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2})$

$\Rightarrow \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$$\Rightarrow \frac{2}{2\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}$$

Exercise 15D

Q. 1. A. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$**, find the values of**

sin 2x

Answer : Given: $\sin x = \frac{\sqrt{5}}{3}$

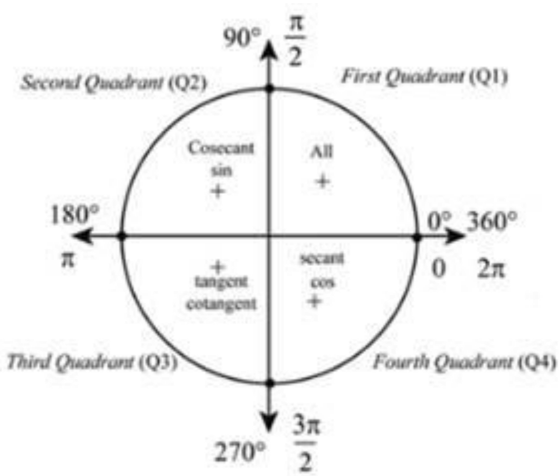
To find: sin2x

We know that,

sin2x = 2 sinx cosx …(i)

Here, we don’t have the value of cos x. So, firstly we have to find the value of cosx

We know that,



$\sin^2 x + \cos^2 x = 1$

Putting the values, we get

$$\left(\frac{\sqrt{5}}{3}\right)^2 + \cos^2 x = 1$$

$$\Rightarrow \frac{5}{9} + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \frac{5}{9}$$

$$\Rightarrow \cos^2 x = \frac{9-5}{9}$$

$$\Rightarrow \cos^2 x = \frac{4}{9}$$

$$\Rightarrow \cos x = \sqrt{\frac{4}{9}}$$

$$\Rightarrow \cos x = \pm\frac{2}{3}$$

It is given that $0 < x < \frac{\pi}{2}$

$$\Rightarrow \cos x = \frac{2}{3}$$

Putting the value of sinx and cosx in eq. (i), we get

sin2x = 2sinx cosx

$$\sin 2x = 2 \times \frac{\sqrt{5}}{3} \times \frac{2}{3}$$

$$\therefore \sin 2x = \frac{4\sqrt{5}}{9}$$

Q. 1. B. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$**, find the values of**

cos 2x

Answer :

Given: $\sin x = \frac{\sqrt{5}}{3}$

To find: cos2x

We know that,

cos 2x = 1 – $2\sin^2 x$

Putting the value, we get

$$\cos 2x = 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2$$

$$\cos 2x = 1 - 2 \times \frac{5}{9}$$

$$\cos 2x = 1 - \frac{10}{9}$$

$$\cos 2x = \frac{9-10}{9}$$

$$\therefore \cos 2x = -\frac{1}{9}$$

Q. 1. C. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$**, find the values of**

tan 2x

Answer : To find: tan2x

From part (i) and (ii), we have

$$\sin 2x = \frac{4\sqrt{5}}{9}$$

And $\cos 2x = -\frac{1}{9}$

We know that,

$$\tan x = \frac{\sin x}{\cos x}$$

Replacing x by 2x, we get

$$\tan 2x = \frac{\sin 2x}{\cos 2x}$$

Putting the values of sin 2x and cos 2x, we get

$$\tan 2x = \frac{\frac{4\sqrt{5}}{9}}{-\frac{1}{9}}$$

$$\tan 2x = \frac{4\sqrt{5}}{9} \times (-9)$$

∴ tan 2x = -4√5

Q. 2. A. If $\cos x = \frac{-3}{5} \text{ and } \pi < x < \frac{3\pi}{2}$ **, find the values of**

sin 2x

Answer :

Given: $\cos x = \frac{-3}{5}$

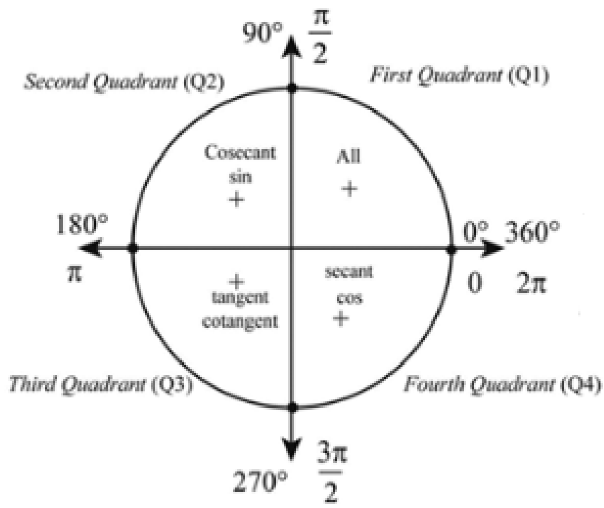
To find: sin2x

We know that,

sin2x = 2 sinx cosx …(i)

Here, we don’t have the value of sin x. So, firstly we have to find the value of sinx

We know that,



$\cos^2 x + \sin^2 x = 1$

Putting the values, we get

$\left(-\frac{3}{5}\right)^2 + \sin^2 x = 1$

$\Rightarrow \frac{9}{25} + \sin^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \frac{9}{25}$

$\Rightarrow \sin^2 x = \frac{25-9}{25}$

$\Rightarrow \sin^2 x = \frac{16}{25}$

$\Rightarrow \sin x = \sqrt{\frac{16}{25}}$

$\Rightarrow \sin x = \pm\frac{4}{5}$

It is given that $\pi < x < \frac{3\pi}{2}$

$\Rightarrow \sin x = -\frac{4}{5}$

Putting the value of sinx and cosx in eq. (i), we get

sin2x = 2sinx cosx

$$\sin 2x = 2 \times \left(-\frac{4}{5}\right) \times \left(-\frac{3}{5}\right)$$

$$\therefore \sin 2x = \frac{24}{25}$$

Q. 2. B. If $\cos x = \frac{-3}{5} \text{ and } \pi < x < \frac{3\pi}{2}$ **, find the values of**

cos 2x

Answer :

Given: $\cos x = \frac{-3}{5}$

To find: cos2x

We know that,

cos 2x = $2\cos^2 x$ – 1

Putting the value, we get

$$\cos 2x = 2\left(-\frac{3}{5}\right)^2 - 1$$

$$\cos 2x = 2 \times \frac{9}{25} - 1$$

$$\cos 2x = \frac{18}{25} - 1$$

$$\cos 2x = \frac{18-25}{25}$$

$$\therefore \cos 2x = -\frac{7}{25}$$

Q. 2. C. If $\cos x = \frac{-3}{5} \text{ and } \pi < x < \frac{3\pi}{2}$**, find the values of**

tan 2x

Answer : To find: tan2x

From part (i) and (ii), we have

$$\sin 2x = \frac{24}{25}$$

$$\text{and } \cos 2x = -\frac{7}{25}$$

We know that,

$$\tan x = \frac{\sin x}{\cos x}$$

Replacing x by 2x, we get

$$\tan 2x = \frac{\sin 2x}{\cos 2x}$$

Putting the values of sin 2x and cos 2x, we get

$$\tan 2x = \frac{\frac{24}{25}}{-\frac{7}{25}}$$

$$\tan 2x = \frac{24}{25} \times \left(-\frac{25}{7}\right)$$

$$\therefore \tan 2x = -\frac{24}{7}$$

Q. 3. A. If $\tan x = \frac{-5}{12} \text{ and } \frac{\pi}{2} < x < \pi$**, find the values of**

sin 2x

Answer :

Given: $\tan x = -\frac{5}{12}$

To find: sin 2x

We know that,

$$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$$

Putting the values, we get

$$\sin 2x = \frac{2\times\left(-\frac{5}{12}\right)}{1+\left(-\frac{5}{12}\right)^2}$$

$$\sin 2x = \frac{-\frac{5}{6}}{1+\frac{25}{144}}$$

$$\sin 2x = \frac{-5}{6\left(\frac{144+25}{144}\right)}$$

$$\sin 2x = \frac{-5\times144}{6\times169}$$

$$\sin 2x = \frac{-5\times24}{169}$$

$$\sin 2x = -\frac{120}{169}$$

Q. 3. B. If $\tan x = \frac{-5}{12} \text{ and } \frac{\pi}{2} < x < \pi$ **, find the values of**

cos 2x

Answer :

Given: $\tan x = -\frac{5}{12}$

To find: cos 2x

We know that,

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

Putting the values, we get

$$\cos 2x = \frac{1-\left(-\frac{5}{12}\right)^2}{1+\left(-\frac{5}{12}\right)^2}$$

$$\cos 2x = \frac{1-\frac{25}{144}}{1+\frac{25}{144}}$$

$$\cos 2x = \frac{\frac{144-25}{144}}{\left(\frac{144+25}{144}\right)}$$

$$\cos 2x = \frac{\frac{119}{144}}{\frac{169}{144}}$$

$$\cos 2x = \frac{119}{169}$$

Q. 3. C. If $\tan x = \frac{-5}{12} \text{ and } \frac{\pi}{2} < x < \pi$**, find the values of**

tan 2x

Answer :

Given: $\tan x = -\frac{5}{12}$

To find: tan 2x

We know that,

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Putting the values, we get

$$\tan 2x = \frac{2\times\left(-\frac{5}{12}\right)}{1-\left(-\frac{5}{12}\right)^2}$$

$$\tan 2x = \frac{-\frac{5}{6}}{1-\frac{25}{144}}$$

$$\tan 2x = \frac{-5}{6\left(\frac{144-25}{144}\right)}$$

$$\tan 2x = \frac{-5\times 144}{6\times 119}$$

$$\tan 2x = \frac{-5\times 24}{119}$$

$$\tan 2x = -\frac{120}{119}$$

Q. 4. A. If Sin X = $\frac{1}{6}$, find the value of sin 3x.

Answer : Sin X = $\frac{1}{6}$

Given: Sin X = $\frac{1}{6}$

To find: sin 3x

We know that,

sin 3x = 3 sinx – $\sin^3$x

Putting the values, we get

$$\sin 3x = 3 \times \left(\frac{1}{6}\right) - \left(\frac{1}{6}\right)^3$$

$$\sin 3x = \frac{1}{6}\left[3 - \left(\frac{1}{6}\right)^2\right]$$

$$\sin 3x = \frac{1}{6}\left[3 - \frac{1}{36}\right]$$

$$\sin 3x = \frac{1}{6}\left[\frac{108-1}{36}\right]$$

$$\sin 3x = \frac{107}{216}$$

Q. 4. B. If Cos X = $\frac{-1}{2}$, find the value of cos 3x.

Answer : Given: Cos X = $\frac{-1}{2}$

To find: cos 3x

We know that,

cos 3x = $4\cos^3 x$ – 3 cosx

Putting the values, we get

$$\cos 3x = 4 \times \left(-\frac{1}{2}\right)^3 - 3 \times \left(-\frac{1}{2}\right)$$

$$\cos 3x = 4 \times \left(-\frac{1}{8}\right) + \frac{3}{2}$$

$$\cos 3x = \left[-\frac{1}{2} + \frac{3}{2}\right]$$

$$\cos 3x = \left[\frac{-1+3}{2}\right]$$

$$\cos 3x = \frac{2}{2}$$

cos 3x = 1

Q. 5. Prove that

$$\frac{\cos\ 2x}{\cos\ x - \sin\ x} = \cos\ x + \sin\ x$$

Answer :

To Prove: $\frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$

Taking LHS,

$$= \frac{\cos 2x}{\cos x - \sin x}$$

$= \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$ [$\because \cos 2x = \cos^2 x - \sin^2 x$]

Using, $(a^2 - b^2) = (a - b)(a + b)$

$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x - \sin x)}$

= cos x + sin x

= RHS

∴ LHS = RHS

Q. 6. Prove that

$$\frac{\sin 2x}{1 + \cos 2x} = \tan x$$

Answer : To Prove: $\frac{\sin 2x}{1 + \cos 2x} = \tan x$

Taking LHS,

$= \frac{\sin 2x}{1 + \cos 2x}$

$= \frac{2 \sin x \cos x}{1 + \cos 2x}$ [$\because \sin 2x = 2 \sin x \cos x$]

$= \frac{2 \sin x \cos x}{2 \cos^2 x}$ [$\because 1 + \cos 2x = 2 \cos^2 x$]

$= \frac{\sin x}{\cos x}$

= tan x $\left[\because \tan \theta = \frac{\sin \theta}{\cos \theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 7. Prove that

$$\frac{\sin\ 2x}{1-\cos\ 2x}=\cot x$$

Answer :

To Prove: $\frac{\sin 2x}{1-\cos 2x}=\tan x$

Taking LHS,

$=\frac{\sin 2x}{1-\cos 2x}$

$=\frac{2\sin x\cos x}{1-\cos 2x}$ [∵ sin 2x = 2 sinx cosx]

$=\frac{2\sin x\cos x}{2\sin^2 x}$ [∵ 1 – cos 2x = 2 $\sin^2 x$]

$=\frac{\cos x}{\sin x}$

$=\cot x\ \left[\because \cot\theta=\frac{\cos\theta}{\sin\theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 8. Prove that

$$\frac{\tan\ 2x}{1+\sec\ 2x}=\tan x$$

Answer :

To Prove: $\frac{\tan 2x}{1+\sec 2x} = \tan x$

Taking LHS,

$$= \frac{\frac{\sin 2x}{\cos 2x}}{1+\frac{1}{\cos 2x}} \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \;\&\sec\theta = \frac{1}{\cos\theta}\right]$$

$$= \frac{\sin 2x}{\cos 2x\left(\frac{\cos 2x+1}{\cos 2x}\right)}$$

$$= \frac{\sin 2x}{1+\cos 2x}$$

$$= \frac{2\sin x\cos x}{1+\cos 2x} [\because \sin 2x = 2 \sin x \cos x]$$

$$= \frac{2\sin x\cos x}{2\cos^2 x} [\because 1 + \cos 2x = 2\cos^2 x]$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

= RHS

∴ LHS = RHS

Hence Proved

Q. 9. Prove that

sin 2x(tan x + cot x) = 2

Answer : To Prove: sin 2x(tan x + cot x) = 2

Taking LHS,

sin 2x(tan x + cot x)

We know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}\ \&\ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \sin 2x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \sin 2x\left(\frac{\sin x(\sin x) + \cos x(\cos x)}{\cos x \sin x}\right)$$

$$= \sin 2x\left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)$$

We know that,

sin 2x = 2 sinx cosx

$$= 2\sin x\cos x\left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)$$

= $2(\sin^2 x + \cos^2 x)$

= 2 × 1 [∵ $\cos^2\theta + \sin^2\theta = 1$]

= 2

= RHS

∴ LHS = RHS

Hence Proved

Q. 10. Prove that

cosec 2x + cot 2x = cot x

Answer : To Prove: cosec 2x + cot 2x = cot x

Taking LHS,

= cosec 2x + cot 2x …(i)

We know that,

$$\operatorname{cosec} x = \frac{1}{\sin x} \ \& \cot x = \frac{\cos x}{\sin x}$$

Replacing x by 2x, we get

$$\operatorname{cosec} 2x = \frac{1}{\sin 2x} \ \& \cot 2x = \frac{\cos 2x}{\sin 2x}$$

So, eq. (i) becomes

$$= \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$$

$$= \frac{1+\cos 2x}{\sin 2x}$$

$= \frac{2\cos^2 x}{\sin 2x}$ [∵ 1 + cos 2x = 2 $\cos^2$x]

$= \frac{2\cos^2 x}{2\sin x \cos x}$ [∵ sin 2x = 2 sinx cosx]

$$= \frac{\cos x}{\sin x}$$

= cot x $\left[\because \cot x = \frac{\cos x}{\sin x}\right]$

= RHS

Hence Proved

Q. 11. Prove that

cos 2x + 2sin^{2}x = 1

Answer :

To Prove: cos 2x + 2sin^{2}x = 1

Taking LHS,

$= \cos 2x + 2\sin^2 x$

$= (2\cos^2 x - 1) + 2\sin^2 x$ [$\because 1 + \cos 2x = 2\cos^2 x$]

$= 2(\cos^2 x + \sin^2 x) - 1$

$= 2(1) - 1$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= 2 - 1$

$= 1$

= RHS

∴ LHS = RHS

Hence Proved

Q. 12. Prove that

$(\sin x - \cos x)^2 = 1 - \sin 2x$

Answer : To Prove: $(\sin x - \cos x)^2 = 1 - \sin 2x$

Taking LHS,

$= (\sin x - \cos x)^2$

Using,

$(a - b)^2 = (a^2 + b^2 - 2ab)$

$= \sin^2 x + \cos^2 x - 2\sin x \cos x$

$= (\sin^2 x + \cos^2 x) - 2\sin x \cos x$

$= 1 - 2\sin x \cos x$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= 1 - \sin 2x$ [$\because \sin 2x = 2\sin x \cos x$]

= RHS

∴ LHS = RHS

Hence Proved

Q. 13. Prove that

cot x – 2cot 2x = tan x

Answer : To Prove: cot x – 2cot 2x = tan x

Taking LHS,

= cot x – 2cot 2x …(i)

We know that,

$$\cot x = \frac{\cos x}{\sin x}$$

Replacing x by 2x, we get

$$\cot 2x = \frac{\cos 2x}{\sin 2x}$$

So, eq. (i) becomes

$$= \frac{\cos x}{\sin x} - 2\left(\frac{\cos 2x}{\sin 2x}\right)$$

$$= \frac{\cos x}{\sin x} - 2\left(\frac{\cos 2x}{2\sin x \cos x}\right)$$ [∵ sin 2x = 2 sinx cosx]

$$= \frac{\cos x}{\sin x} - \left(\frac{\cos 2x}{\sin x \cos x}\right)$$

$$= \frac{\cos x(\cos x) - \cos 2x}{\sin x \cos x}$$

$$= \frac{\cos^2 x - \cos 2x}{\sin x \cos x}$$

$$= \frac{\cos^2 x - [2\cos^2 x - 1]}{\sin x \cos x}$$ [∵ 1 + cos 2x = 2 $\cos^2 x$]

$$= \frac{\cos^2 x - 2\cos^2 x + 1}{\sin x \cos x}$$

$$= \frac{-\cos^2 x + 1}{\sin x \cos x}$$

$= \frac{1-\cos^2 x}{\sin x \cos x}$

$= \frac{\cos^2 x+\sin^2 x-\cos^2 x}{\sin x \cos x}$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= \frac{\sin^2 x}{\sin x \cos x}$

$= \frac{\sin x}{\cos x}$

= tan x $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 14. Prove that

$$(\cos^4 x + \sin^4 x) = \frac{1}{2}(2 - \sin^2 2x)$$

Answer :

To Prove: $\cos^4 x + \sin^4 x = \frac{1}{2}(2 - \sin^2 2x)$

Taking LHS,

$= \cos^4 x + \sin^4 x$

Adding and subtracting $2\sin^2 x \cos^2 x$, we get

$= \cos^4 x + \sin^4 x + 2\sin^2 x \cos^2 x - 2\sin^2 x \cos^2 x$

We know that,

$a^2 + b^2 + 2ab = (a + b)^2$

$= (\cos^2 x + \sin^2 x) - 2\sin^2 x \cos^2 x$

$= (1) – 2\sin^2x \cos^2x$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= 1 – 2\sin^2x \cos^2x$

Multiply and divide by 2, we get

$$= \frac{1}{2}[2 \times (1 - 2\sin^2 x \cos^2 x)]$$

$$= \frac{1}{2}[2 - 4\sin^2 x \cos^2 x]$$

$$= \frac{1}{2}[2 - (2\sin x \cos x)^2]$$

$$= \frac{1}{2}[2 - (\sin 2x)^2]$$ [$\because \sin 2x = 2 \sin x \cos x$]

$$= \frac{1}{2}(2 - \sin^2 2x)$$

= RHS

∴ LHS = RHS

Hence Proved

Q. 15. Prove that

$$\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \frac{1}{2}(2 + \sin 2x)$$

Answer :

To Prove: $\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \frac{1}{2}(2 + \sin 2x)$

Taking LHS,

$$= \frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} \ldots(i)$$

We know that,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So, $\cos^3x - \sin^3x = (\cos x - \sin x)(\cos^2x + \cos x \sin x + \sin^2x)$

So, eq. (i) becomes

$$= \frac{(\cos x - \sin x)(\cos^2 x + \cos x \sin x + \sin^2 x)}{\cos x - \sin x}$$

$= \cos^2x + \cos x \sin x + \sin^2x$

$= (\cos^2x + \sin^2x) + \cos x \sin x$

$= (1) + \cos x \sin x$ [$\because \cos^2 \theta + \sin^2 \theta = 1$]

$= 1 + \cos x \sin x$

Multiply and Divide by 2, we get

$$= \frac{1}{2}[2 \times (1 + \cos x \sin x)]$$

$$= \frac{1}{2}[2 + 2 \sin x \cos x]$$

$$= \frac{1}{2}[2 + \sin 2x]$$ [$\because \sin 2x = 2 \sin x \cos x$]

= RHS

∴ LHS = RHS

Hence Proved

Q. 16. Prove that

$$\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$$

Answer :

To prove: $\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$

Taking LHS,

$= \frac{1-\cos 2x+\sin x}{\sin 2x+\cos x}$

$= \frac{(1-\cos 2x)+\sin x}{\sin 2x+\cos x}$

We know that,

1 – cos 2x = 2 $\sin^2$x & sin 2x = 2 sinx cosx

$= \frac{2\sin^2 x+\sin x}{2\sin x\cos x+\cos x}$

Taking sinx common from the numerator and cosx from the denominator

$= \frac{\sin x(2\sin x+1)}{\cos x(2\sin x+1)}$

$= \frac{\sin x}{\cos x}$

= tan x $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 17. Prove that

$$\cos x\cos 2x\cos 4x\cos 8x = \frac{\sin 16x}{16\sin x}$$

Answer :

To Prove: $\cos x\cos 2x\cos 4x\cos 8x = \frac{\sin 16x}{16\sin x}$

Taking LHS,

= cosx cos2x cos4x cos8x

Multiply and divide by 2sinx, we get

$$= \frac{1}{2\sin x}[2\sin x\cos x\cos 2x\cos 4x\cos 8x]$$

$$= \frac{1}{2\sin x}[(2\sin x\cos x)\cos 2x\cos 4x\cos 8x]$$

$$= \frac{1}{2\sin x}[\sin 2x\cos 2x\cos 4x\cos 8x]$$ [∵ sin 2x = 2 sinx cosx]

Multiply and divide by 2, we get

$$= \frac{1}{2\times 2\sin x}[(2\sin 2x\cos 2x)\cos 4x\cos 8x]$$

We know that,

sin 2x = 2 sinx cosx

Replacing x by 2x, we get

sin 2(2x) = 2 sin(2x) cos(2x)

or sin 4x = 2 sin 2x cos 2x

$$= \frac{1}{4\sin x}[\sin 4x\cos 4x\cos 8x]$$

Multiply and divide by 2, we get

$$= \frac{1}{2\times 4\sin x}[2\sin 4x\cos 4x\cos 8x]$$

We know that,

sin 2x = 2 sinx cosx

Replacing x by 4x, we get

sin 2(4x) = 2 sin(4x) cos(4x)

or sin 8x = 2 sin 4x cos 4x

$$= \frac{1}{8\sin x}[\sin 8x\cos 8x]$$

Multiply and divide by 2, we get

$$=\frac{1}{2\times 8\sin x}[2\sin 8x\cos 8x]$$

We know that,

sin 2x = 2 sinx cosx

Replacing x by 8x, we get

sin 2(8x) = 2 sin(8x) cos(8x)

or sin 16x = 2 sin 8x cos 8x

$$=\frac{1}{16\sin x}[\sin 16x]$$

= RHS

∴ LHS = RHS

Hence Proved

Q. 18. A. Prove that

$$2\sin 22\frac{1}{2}^{0}\cos 22\frac{1}{2}^{0}=\frac{1}{\sqrt{2}}$$

Answer :

To Prove: $2\sin 22\frac{1}{2}^{\circ}\cos 22\frac{1}{2}^{\circ}=\frac{1}{\sqrt{2}}$

Taking LHS,

$$=2\sin 22\frac{1}{2}^{\circ}\cos 22\frac{1}{2}^{\circ}\ \ldots\text{(i)}$$

We know that,

2sinx cosx = sin 2x

Here, $x=22\frac{1}{2}=\frac{45}{2}$

So, eq. (i) become

$= \sin 2\left(\frac{45}{2}\right)$

= sin 45°

$= \frac{1}{\sqrt{2}}\left[\because \sin(45^\circ) = \frac{1}{\sqrt{2}}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 18. B. Prove that

$$2\cos^2 15^0 - 1 = \frac{\sqrt{3}}{2}$$

Answer :

To Prove: $2\cos^2 15^\circ - 1 = \frac{\sqrt{3}}{2}$

Taking LHS,

= 2 $\cos^2$ 15° - 1 …(i)

We know that,

1 + cos 2x = 2 $\cos^2$x

Here, x = 15°

So, eq. (i) become

= [1 + cos 2(15°)] – 1

= 1 + cos 30° - 1

$= \cos 30^\circ \left[\because \cos(30^\circ) = \frac{\sqrt{3}}{2}\right]$

$= \frac{\sqrt{3}}{2}$

= RHS

∴ LHS = RHS

Hence Proved

Q. 18. C. Prove that

$$8\cos^3 20^0 - 6\cos 20^0 = 1$$

Answer : To Prove: $8 \cos^3 20° - 6 \cos 20° = 1$

Taking LHS,

$= 8 \cos^3 20° - 6 \cos 20°$

Taking 2 common, we get

$= 2(4 \cos^3 20° - 3 \cos 20°)$ …(i)

We know that,

$\cos 3x = 4\cos^3 x – 3 \cos x$

Here, x = 20°

So, eq. (i) becomes

= 2[cos 3(20°)]

= 2[cos 60°]

$= 2 \times \frac{1}{2} \left[\because \cos(60°) = \frac{1}{2}\right]$

= 1

= RHS

∴ LHS = RHS

Hence Proved

Q. 18. D. Prove that

$$3\sin 40° - \sin^3 40° = \frac{\sqrt{3}}{2}$$

Answer :

To prove: $3\sin 40° - \sin^3 40° = \frac{\sqrt{3}}{2}$

Taking LHS,

= 3 sin 40° - $\sin^3$ 40° …(i)

We know that,

sin 3x = 3 sinx – $\sin^3 x$

Here, x = 40°

So, eq. (i) becomes

= sin 3(40°)

= sin 120°

= sin (180° - 60°)

= sin 60° [∵sin (180° - θ) = sin θ]

$= \frac{\sqrt{3}}{2}\left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

= RHS

∴ LHS = RHS

Hence Proved

Q. 19. A. Prove that

$$\sin^2 24^0 - \sin^2 6^0 = \frac{(\sqrt{5}-1)}{8}$$

Answer :

To Prove: $\sin^2 24° - \sin^2 6° = \frac{\sqrt{5}-1}{8}$

Taking LHS,

= $\sin^2 24°$ - $\sin^2 6°$

We know that,

$\sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B)$

= sin(24°+ 6°) sin(24° - 6°)

= sin 30° sin 18° …(i)

Now, we will find the value of sin 18°

Let x = 18°

so, 5x = 90°

Now, we can write

2x + 3x = 90°

so 2x = 90° - 3x

Now taking sin both the sides, we get

sin2x = sin(90° - 3x)

sin2x = cos3x [as we know, sin(90°- 3x) = Cos3x]

We know that,

sin2x = 2sinxcosx

Cos3x = $4\cos^3 x$ - 3cosx

2sinxcosx = $4\cos^3 x$ - 3cosx

⇒ 2sinxcosx - $4\cos^3 x$ + 3cosx = 0

⇒ cosx (2sinx - $4\cos^2 x$ + 3) = 0

Now dividing both side by cosx we get,

$2\sin x - 4\cos^2 x + 3 = 0$

We know that,

$\cos^2 x + \sin^2 x = 1$

or $\cos^2 x = 1 - \sin^2 x$

$\Rightarrow 2\sin x - 4(1 - \sin^2 x) + 3 = 0$

$\Rightarrow 2\sin x - 4 + 4\sin^2 x + 3 = 0$

$\Rightarrow 2\sin x + 4\sin^2 x - 1 = 0$

We can write it as,

$4\sin^2 x + 2\sin x - 1 = 0$

Now applying formula

Here, $ax^2 + bx + c = 0$

So, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Now applying it in the equation

$$\sin x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2}$$

$$\sin x = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$\sin x = \frac{(-2 \pm 2.\sqrt{5})}{8}$$

$$\sin x = \frac{2\left(-1 \pm \sqrt{5}\right)}{8}$$

$$\sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin 18° = \frac{-1 \pm \sqrt{5}}{4}$$

Now sin 18° is positive, as 18° lies in first quadrant.

$$\therefore \sin 18° = \frac{\sqrt{5}-1}{4}$$

Putting the value in eq. (i), we get

= sin 30° sin 18°

$$= \frac{1}{2} \times \frac{\sqrt{5}-1}{4}$$

$$= \frac{\sqrt{5}-1}{8}$$

= RHS

∴ LHS = RHS

Hence Proved

Q. 19. B. Prove that

$$\sin^2 72^0 - \cos^2 30^0 = \frac{(\sqrt{5}-1)}{8}$$

Answer :

To Prove: $\sin^2 72° - \cos^2 30° = \frac{\sqrt{5}-1}{8}$

Taking LHS,

= $\sin^2 72°$ - $\cos^2 30°$

$= \sin^2(90° - 18°) - \cos^2 30°$

$= \cos^2 18° - \cos^2 30°$ …(i)

Here, we don’t know the value of cos 18°. So, we have to find the value of cos 18°

Let x = 18°

so, 5x = 90°

Now, we can write

2x + 3x = 90°

so 2x = 90° - 3x

Now taking sin both the sides, we get

sin2x = sin(90° - 3x)

sin2x = cos3x [as we know, sin(90°- 3x) = Cos3x]

We know that,

sin2x = 2sinxcosx

$\text{Cos}3x = 4\cos^3 x - 3\cos x$

$2\sin x\cos x = 4\cos^3 x - 3\cos x$

$\Rightarrow 2\sin x\cos x - 4\cos^3 x + 3\cos x = 0$

$\Rightarrow \cos x\,(2\sin x - 4\cos^2 x + 3) = 0$

Now dividing both side by cosx we get,

$2\sin x - 4\cos^2 x + 3 = 0$

We know that,

$\cos^2 x + \sin^2 x = 1$

or $\cos^2 x = 1 – \sin^2 x$

$\Rightarrow 2\sin x – 4(1 – \sin^2 x) + 3 = 0$

⇒ $2\sin x - 4 + 4\sin^2 x + 3 = 0$

⇒ $2\sin x + 4\sin^2 x - 1 = 0$

We can write it as,

$4\sin^2 x + 2\sin x - 1 = 0$

Now applying formula

Here, $ax^2 + bx + c = 0$

So, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

now applying it in the equation

$$\sin x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2}$$

$$\sin x = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$\sin x = \frac{(-2 \pm 2.\sqrt{5})}{8}$$

$$\sin x = \frac{2(-1 \pm \sqrt{5})}{8}$$

$$\sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin 18° = \frac{-1 \pm \sqrt{5}}{4}$$

Now sin 18° is positive, as 18° lies in first quadrant.

$\therefore \sin 18° = \frac{\sqrt{5}-1}{4}$

Now, we know that

$\cos^2 x + \sin^2 x = 1$

or $\cos x = \sqrt{1 - \sin^2 x}$

$\therefore \cos 18° = \sqrt{1 - \sin^2 18°}$

$$\Rightarrow \cos 18° = \sqrt{1 - \left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$\Rightarrow \cos 18° = \sqrt{\frac{16-(5+1-2\sqrt{5})}{16}}$$

$$\Rightarrow \cos 18° = \sqrt{\frac{16-6+2\sqrt{5}}{16}}$$

$$\Rightarrow \cos 18° = \frac{1}{4}\sqrt{10+2\sqrt{5}}$$

Putting the value in eq. (i), we get

$= \cos^2 18° - \cos^2 30°$

$$= \left(\frac{1}{4}\sqrt{10+2\sqrt{5}}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \quad \left[\because \cos 30° = \frac{\sqrt{3}}{2}\right]$$

$$= \frac{1}{16}\left(10+2\sqrt{5}\right) - \frac{3}{4}$$

$$= \frac{10+2\sqrt{5}-12}{16}$$

$$= \frac{2\sqrt{5}-2}{16}$$

$$= \frac{2(\sqrt{5}-1)}{16}$$

$= \frac{\sqrt{5}-1}{8}$

= RHS

∴ LHS = RHS

Hence Proved

Q. 20. Prove that tan 6⁰ tan 42⁰ tan 66⁰ tan 78⁰ = 1

Answer : To Prove: tan 6° tan 42° tan 66° tan 78° = 1

Taking LHS,

= tan 6° tan 42° tan 66° tan 78°

Multiply and divide by tan 54° tan 18°

$$= \frac{\tan 6° \tan 42° \tan 66° \tan 78°}{\tan 54° \tan 18°} \times \tan 54° \tan 18°$$

$$= \frac{(\tan 6° \tan 54° \tan 66°)(\tan 18° \tan 42° \tan 72°)}{\tan 54° \tan 18°} \quad \ldots(i)$$

We know that,

tan x tan(60° – x) tan (60° + x) = tan 3x

In first x = 6°
tan 6° tan (60° – 6°) tan (60° + 6°)
= tan 6° tan 54° tan 66°
and
In second x = 18°
tan 18° tan (60° – 18°) tan (60° + 18°)
= tan 18° tan 42° tan 78°

So, eq. (i) becomes

$$= \frac{[\tan 3(6°)][\tan 3(18°)]}{\tan 54° \tan 18°}$$

$$= \frac{\tan 18° \tan 54°}{\tan 54° \tan 18°}$$

= 1

= RHS

∴ LHS = RHS

Hence Proved

Q. 21. If tan $\theta = \frac{a}{b}$, prove that $a\sin 2\theta + b\cos 2\theta = b$

Answer : Given: $\theta = \frac{a}{b}$

To Prove: a sin 2θ + b cos 2θ = b

Given: $\theta = \frac{a}{b}$

We know that,

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a}{b}$$

By Pythagoras Theorem,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (a)^2 + (b)^2 = (H)^2$

$\Rightarrow a^2 + b^2 = (H)^2$

$$\Rightarrow H = \sqrt{a^2 + b^2}$$

So,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{\sqrt{a^2+b^2}}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{b}{\sqrt{a^2+b^2}}$$

Taking LHS,

= a sin 2θ + b cos 2θ

We know that,

$\sin 2\theta = 2 \sin \theta \cos \theta$

and $\cos 2\theta = 1 - 2 \sin^2\theta$

$= a(2 \sin \theta \cos \theta) + b(1 - 2 \sin^2\theta)$

Putting the values of $\sin\theta$ and $\cos\theta$, we get

$$= a \times 2 \times \frac{a}{\sqrt{a^2+b^2}} \times \frac{b}{\sqrt{a^2+b^2}} + b\left[1 - 2 \times \left(\frac{a}{\sqrt{a^2+b^2}}\right)^2\right]$$

$$= \frac{2a^2b}{a^2+b^2} + b\left[1 - 2 \times \frac{a^2}{a^2+b^2}\right]$$

$$= \frac{2a^2b}{a^2+b^2} + b - \frac{2a^2b}{a^2+b^2}$$

$= b$

$= \text{RHS}$

$\therefore$ LHS = RHS

Hence Proved

Exercise 15E

Q. 1.

If $\sin x = \frac{\sqrt{5}}{3}$ and $\frac{\pi}{2} < x < \pi$, **find the values of**

(i) $\sin\frac{x}{2}$ **(ii)** $\cos\frac{x}{2}$

(iii) $\tan\frac{x}{2}$

Answer : Given: $\sin x = \frac{\sqrt{5}}{3}$ and $\frac{\pi}{2} < x < \pi$ i.e, x lies in the Quadrant II .

To Find: i) $\sin\frac{x}{2}$ ii) $\cos\frac{x}{2}$ iii) $\tan\frac{x}{2}$

Now, since $\sin x = \frac{\sqrt{5}}{3}$

We know that $\cos x = \pm\sqrt{1-\sin^2 x}$

$\cos x = \pm\sqrt{1-(\frac{\sqrt{5}}{3})^2}$

$\cos x = \pm\sqrt{1-\frac{5}{9}}$

$\cos x = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

since cos x is negative in II quadrant, hence $\cos x = -\frac{2}{3}$

i) $\sin\frac{x}{2}$

Formula used:

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Now, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-(\frac{-2}{3})}{2}} = \pm\sqrt{\frac{\frac{5}{3}}{2}} = \pm\sqrt{\frac{5}{6}}$

Since sinx is positive in II quadrant, hence $\sin\frac{x}{2} = \sqrt{\frac{5}{6}}$

ii) $\cos\frac{x}{2}$

Formula used:

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

now, cos $\frac{x}{2}$ = $\pm\sqrt{\frac{1+(\frac{-2}{3})}{2}}$ = = $\pm\sqrt{\frac{\frac{1}{3}}{2}}$ = = $\pm\sqrt{\frac{1}{6}}$

since cosx is negative in II quadrant, hence cos $\frac{x}{2}$ = $-\frac{1}{\sqrt{6}}$

iii) tan $\frac{x}{2}$

Formula used:

tan x = $\frac{\sin x}{\cos x}$

hence, tan $\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{\frac{5}{6}}}{-\frac{1}{\sqrt{6}}} = \frac{\sqrt{5}}{\sqrt{6}} \times \frac{\sqrt{6}}{-1} = -\sqrt{5}$

Here, tan x is negative in II quadrant.

Q. 2.

If $\cos x = \frac{-3}{5}$ and $\frac{\pi}{2} < x < \pi$, **find the values of**

(i) $\sin\frac{x}{2}$ **(ii)** $\cos\frac{x}{2}$

(iii) $\tan\frac{x}{2}$

Answer :

Given: cos x = = $-\frac{3}{5}$ and $\frac{\pi}{2}$<x< π.i.e, x lies in II quadrant

To Find: i)sin $\frac{x}{2}$ ii)cos $\frac{x}{2}$ iii)tan $\frac{x}{2}$

i) sin $\frac{x}{2}$

Formula used:

sin $\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Now, sin $\frac{x}{2} = \pm\sqrt{\frac{1-(\frac{-3}{5})}{2}} = \pm\sqrt{\frac{\frac{8}{5}}{2}} = \pm\frac{2}{\sqrt{5}}$

Since sinx is positive in II quadrant, hence sin $\frac{x}{2} = \frac{2}{\sqrt{5}}$

ii) $\cos\frac{x}{2}$

Formula used:

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

now, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+(\frac{-3}{5})}{2}} = = \pm\sqrt{\frac{\frac{2}{5}}{2}} = = \pm\sqrt{\frac{1}{5}}$

since cosx is negative in II quadrant, hence $\cos\frac{x}{2} = -\frac{1}{\sqrt{5}}$

iii) $\tan\frac{x}{2}$

Formula used:

$\tan x = \frac{\sin x}{\cos x}$

hence, $\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{2}{\sqrt{5}}}{-\frac{1}{\sqrt{5}}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{-1} = -2$

Here, tanx is negative in II quadrant.

Q. 3. If Sin X = $\frac{-1}{2}$ and X lies in Quadrant IV, find the values of

(i) Sin $\frac{X}{2}$

(ii) Cos $\frac{X}{2}$

(iii) tan $\frac{X}{2}$

Answer :

Given: sin x = $\frac{-1}{2}$ and x lies in Quadrant IV.

To Find: i)sin $\frac{x}{2}$ ii)cos $\frac{x}{2}$ iii)tan $\frac{x}{2}$

Now, since sin x = $\frac{-1}{2}$

We know that cos x = $\pm\sqrt{1 - \sin^2 x}$

cos x = $\pm\sqrt{1 - (\frac{-1}{2})^2}$

cos x = $\pm\sqrt{1 - \frac{1}{4}}$

cos x = $\pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

since cos x is positive in IV quadrant, hence cos x = $\frac{\sqrt{3}}{2}$

i) sin $\frac{x}{2}$

Formula used:

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

Now, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-(\frac{\sqrt{3}}{2})}{2}} = \pm\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = = \pm\frac{\sqrt{2-\sqrt{3}}}{2}$

Since sinx is negative in IV quadrant, hence $\sin\frac{x}{2} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

ii)cos $\frac{x}{2}$

Formula used:

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

now, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+(\frac{\sqrt{3}}{2})}{2}} = = \pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = = \pm\frac{\sqrt{2+\sqrt{3}}}{2}$

since cosx is positive in IV quadrant, hence $\cos\frac{x}{2} = \frac{\sqrt{2+\sqrt{3}}}{2}$

iii)$\tan\frac{x}{2}$

Formula used:

$\tan x = \frac{\sin x}{\cos x}$

Q. 4. If Cos $\frac{X}{2} = \frac{12}{13}$ and X lies in Quadrant I, find the values of

(i) sin x
(ii) cos x
(iii) cot x

Answer : Given: $\cos\frac{X}{2} = \frac{12}{13}$ and x lies in Quadrant I i.e, All the trigonometric ratios are positive in I quadrant

To Find: (i) sin x ii) cos x iii) cot x

(i) sin x

Formula used:

We have, Sin x = $\sqrt{1-\cos^2 x}$

We know that, cos $\frac{x}{2}$ = $\sqrt{\frac{1+\cos x}{2}}$ (∵cos x is positive in I quadrant)

⇒ $2\cos^2\frac{x}{2} - 1 = \cos x$

⇒ $2\times\left(\frac{12}{13}\right)^2 - 1 = \cos x$

⇒ $2\times\left(\frac{144}{169}\right) - 1 = \cos x$

⇒ $\cos x = \frac{119}{169}$

Since, Sin x = $\sqrt{1-\cos^2 x}$

⇒ Sin x = $\sqrt{1-(\frac{119}{169})^2}$

⇒ Sin x = $\frac{120}{169}$

Hence, we have Sin x = $\frac{120}{169}$.

ii)cos x

Formula used:

We know that, cos $\frac{x}{2}$ = $\sqrt{\frac{1+\cos x}{2}}$ (∵cos x is positive in I quadrant)

⇒ $2\cos^2\frac{x}{2}$ – 1 = cos x

$\Rightarrow 2\times\left(\frac{12}{13}\right)^2 - 1 = \cos x$

$\Rightarrow 2\times\left(\frac{144}{169}\right) - 1 = \cos x$

$\Rightarrow \cos x = \frac{119}{169}$

iii) cot x

Formula used:

$\cot x = \frac{\cos x}{\sin x}$

$\cot x = \frac{\frac{119}{169}}{\frac{120}{169}} = \frac{119}{169} \times \frac{169}{120} = \frac{119}{120}$

Hence, we have $\cot x = \frac{119}{120}$

Q. 5. **If** $\sin x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$**, find the value of** $\tan\frac{x}{2}$**.**

Answer : Given: $\sin x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$ i.e, x lies in Quadrant I and all the trigonometric ratios are positive in quadrant I.

To Find: $\tan\frac{x}{2}$

Formula used:

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$

Now, $\cos x = \sqrt{1-\sin^2 x}$ ($\because$cos x is positive in I quadrant)

$\Rightarrow \cos x = \sqrt{1-(\frac{3}{5})^2} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

Since, $\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{\frac{3}{5}}{1+\frac{4}{5}} = \frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$

Hence, $\tan\frac{x}{2} = \frac{1}{3}$

Q. 6. Prove that

$$\cot\frac{x}{2} - \tan\frac{x}{2} = 2\cot x$$

Answer :

To Prove: $\cot\frac{x}{2} - \tan\frac{x}{2} = 2\cot x$

Proof: Consider L.H.S,

$\cot\frac{x}{2} - \tan\frac{x}{2} = \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} - \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$= \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{\cos x}{\sin\frac{x}{2}\cos\frac{x}{2}}$ $(\because \cos^2 x - \sin^2 x = \cos 2x)$

$\Rightarrow (\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x)$

Here multiply and divide L.H.S by 2

$= \frac{2\cos x}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{2\cos x}{\sin x}$ $(\because 2\sin x\cos x = \sin 2x)$

$\Rightarrow (2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x)$

$\cot \quad - \tan\frac{x}{2} = 2\cot x$ = R.H.S

$\therefore$L.H.S = R.H.S, Hence proved

Q. 7. Prove that

$$\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)=\tan x+\sec x$$

Answer : To Prove: $\tan(\frac{\pi}{4}+\frac{\pi}{2})$ = tan x + sec x

Proof: Consider L.H.S,

$$\tan(\frac{\pi}{4}+\frac{x}{2}) = \frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}} \quad (\because \text{this is of the form } \tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y})$$

$$= \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} = \frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

Multiply and divide L.H.S by $\cos\frac{x}{2}+\sin\frac{x}{2}$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} \times \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$$

$$= \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$$

$$= \frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x} \quad (\because \cos^2\frac{x}{2}-\sin^2\frac{x}{2} = \cos x)$$

$$= \frac{1+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$$

$= \frac{1+\sin x}{\cos x}$ $(\because 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x)$

$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$\tan(\frac{\pi}{4} + \frac{x}{2}) = \sec x + \tan x = $ R.H.S

$\because$ L.H.S = R.H.S, Hence proved

Q. 8. Prove that

$$\sqrt{\frac{1+\sin x}{1-\sin x}} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

Answer :

To Prove: $\sqrt{\frac{1+\sin x}{1-\sin x}} = \tan(\frac{\pi}{4} + \frac{x}{2})$

Proof: Consider, L.H.S = $\sqrt{\frac{1+\sin x}{1-\sin x}}$

Multiply and divide L.H.S by $\sqrt{1 + \sin x}$

$= \sqrt{\frac{1+\sin x}{1-\sin x}} \times \frac{\sqrt{1+\sin x}}{\sqrt{1+\sin x}} = \frac{1+\sin x}{\sqrt{1-\sin^2 x}}$

$= \frac{1+\sin x}{\cos x} = \frac{1+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$ $(\because 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x)$

$= \frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$ $(\because \cos^2 x + \sin^2 x = 1)$

$$= \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} \times \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}} \quad (\because x^2 + y^2 = (x+y)(x-y))$$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

Multiply and divide the above with cos $\frac{x}{2}$

$$= \frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$

Here, since tan$\frac{\pi}{4}$ = 1

Here, since tan$\frac{\pi}{4}$ = 1

$$\sqrt{\frac{1+\sin x}{1-\sin x}} = \frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}} = \tan\left(\frac{\pi}{4}+\frac{x}{2}\right) = \text{R.H.S}$$

Since, L.H.S = R.H.S, Hence proved.

Q. 9. Prove that

$$\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=2\sec x$$

Answer :

To prove: $\tan(\frac{\pi}{4}+\frac{x}{2}) + \tan(\frac{\pi}{4}-\frac{x}{2}) = 2\sec x$

Proof: Consider, L.H.S $= \tan(\frac{\pi}{4}+\frac{x}{2}) + \tan(\frac{\pi}{4}-\frac{x}{2})$

$$\tan(\frac{\pi}{4}+\frac{x}{2}) + \tan(\frac{\pi}{4}-\frac{x}{2}) = \frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}} + \frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{1+\tan\frac{\pi}{4}\tan\frac{x}{2}}$$

$$(\because \tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y} \text{ and } \tan(x-y) = \frac{\tan x-\tan y}{1+\tan x\tan y})$$

$$= \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} + \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$$

$$= \frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}} + \frac{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$$

$$= \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2+(\cos\frac{x}{2}-\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$$

By Expanding the numerator we get,

$$= \frac{2}{\cos x} \left(\because \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x\right)$$

$\tan(\frac{\pi}{4} + \frac{x}{2}) + \tan(\frac{\pi}{4} - \frac{x}{2}) = 2\sec x$ = R.H.S

since L.H.S = R.H.S, Hence proved.

Q. 10. Prove that

$$\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$$

Answer :

To Prove: $\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$

Proof: consider, L.H.S = $\frac{\sin x}{1+\cos x}$

$\frac{\sin x}{1+\cos x} = \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{1+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$ $\left(\because \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x \text{ and } 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x\right)$

$= \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$ $\left(\because \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 1\right)$

$= \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{2\cos^2\frac{x}{2}} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \tan\frac{x}{2}$

$\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$ = R.H.S

Since L.H.S = R.H.S, Hence proved.