Mathematics

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Question	Very Short Answer (VSA) 1 Mark	Short Answer– I (SA I) 2 Marks	Short Answer– II (SA II) 2 Marks	Long Answer (LA) 5 Marks	Total Marks	% Weightage
1.	**Remembering**	2	2	2	2	20	25%
2.	**Understanding**	2	1	1	4	23	29%
3.	**Application**	2	2	3	1	19	24%
4.	**High Order Thinking Skills**	-	1	4	-	14	17%
5.	**Inferential and Evaluative**	-	-	-	1	4	5%
	Total	6 × 1 = 6	6 × 2 = 12	10 × 3 = 30	8 × 4 = 32	80	100%

Note: one of the LA will be to assess the values inherent in the texts.

Time allowed: 3 hours **Maximum marks: 80**

General Instructions:

(*i*) All question are compulsory.
(*ii*) The question paper consists of 30 question divided into four section A, B, C and D.
(*ii*) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
(*ii*) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(*v*) Use of calculators is not permitted.

SECTION - A

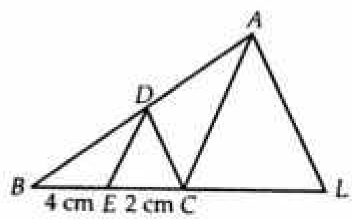
1. If $\frac{p}{q}$ is a rational number ($q \neq 0$). What is condition on q so that the decimal representation of $\frac{p}{q}$ is terminating.

2. For which value of p, the pair of equations $6x + 5y = 4$ and $12x + py = -8$ has no solution?

3. Find the distance between the points ($10 \cos 30°$, 0) and (0, $10 \cos 60°$).

4. Which term of the AP : 121, 117, 113, ... is first negative term?

5. In figure, $CD \parallel LA$ and $DE \parallel AC$. Find CL.



6. What is the nature of roots of the quadratic equation $5x^2 - 2x - 3 = 0$?

SECTION - B

7. The area of a rectangular plot is 528 m^2. The length of the plot is one metre more than twice its breadth and we need to find the length and breadth of the plot. Represent the above situation in the form of quadratic equation.

8. In an AP, the sum of first n terms is $\left(\frac{3n^2}{2} + \frac{5n}{2}\right)$. Find its 25th term.

9. From a circular cylinder of diameter 10 cm and height 12 cm, a conical cavity of the same base radius and of the same height is hollowed out. Find the volume of the remaining solid. [take, $\pi = 3.14$]

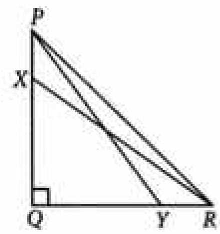
10. If the points A (4, 3) and B (x, 5) are on the circle with centre O (2, 3), then find the value of $x^2 + 5$.

11. A bag contains tickets, numbered 11, 12, 13, ..., 30. A ticket is taken out from the bag at random. Find the probability that the number on the drawn ticket (i) is a multiple of 7(ii) is greater than 15 and a multiple of 5.

12. Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of tanker which can measure the petrol of either tanker in exact number of times.

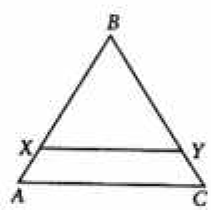
SECTION - C

13. In given figure, PQR is a right triangle, right angled at Q. X and Y are the points on PQ and QR such that $PX : XQ = 1 : 2$ and $QY : YR = 2 : 1$. Prove that $9(PY^2 + XR^2) = 13PR^2$.



OR

In the given figure, in ΔABC, $XY \parallel AC$ and XY divides the ΔABC into two regions such that $ar(\Delta BXY) = 2ar(ACYX)$. Determine $\frac{AX}{AB}$.



14. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p and q are primes.

15. Draw a circle of radius 4 cm. Take two points P and Q on one of its extended diameters, each at a distance of 9 cm from its centre. Draw tangents to the circle from these two points P and Q.

16. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$ then prove that : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

17. If α and β are zeroes of the polynomial $x^2 - p(x + 1) + c$ such that $(\alpha + 1)(\beta + 1) = 0$, then find the value of c.

OR

If α and β are the zeroes of polynomial $x^2 - 2x - 8$, then form a quadratic polynomial whose zeroes are 3α and 3β.

18. An incomplete distribution is given as follows :

Class interval	Frequency
0-10	10
10-20	20
20-30	?
30-40	40
40-50	?
50-60	25
60-70	15

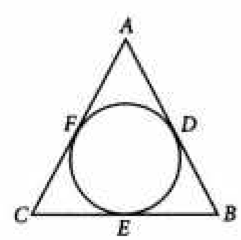
The median value is 35 and the sum of all the frequencies is 170. Using the median formula, fill up the missing frequencies.

OR

The mean of the following frequency distribution is 25.2. Find the missing frequency x.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	8	x	10	11	9

19. A circle is inscribed in a ΔABC having sides $AB = 8$ cm, $BC = 10$ cm and $CA = 12$ cm as shown in figure. Find AD, BE and CF.



20. Two dice are thrown at the same time. Find the probability of getting

(i) same number on both dice.

(ii) different number on both dice.

21. AD is an altitude of an equilateral ΔABC. If AD as base another equilateral triangle ADE is constructed. Find $ar(\Delta ADE) : ar\Delta(ABC)$.

22. A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is its original average speed?

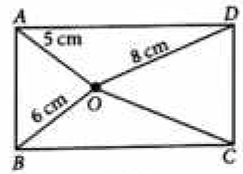
OR

At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.

SECTION - D

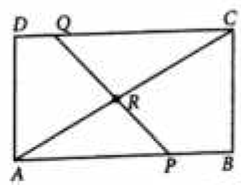
23. Use Euclid's Division Lemma to show that the square of any positive integer is either of the form $3n$ or $3n + 1$ for some integer n.

24. In the given figure, O is any point inside a rectangle $ABCD$ such that $OB = 6$ cm = 6 cm, $OD = 8$ cm and $OA = 5$ cm. Find the length of OC.



OR

In the given figure, $ABCD$ is a parallelogram AB is divided at P and CD at Q, so that $AP : PB = 3 : 2$ and $CQ : QD = 4 : 1$. If PQ meets AC at R, prove that $AR = \frac{3}{7}AC$.

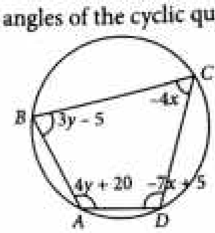


25. The diameter of bottom of a frustum of right circular cone is 10 cm, and that of the top is 6 cm ad height is 5 cm. Find out the total surface area and the volume of the frustum.

OR

A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also, find the total surface area of the toy.

26. $ABCD$ is a cyclic quadrilateral. Find the angles of the cyclic quadrilateral.



27. A sum of ₹ 2000 is invested at 7% simple interest per year. Calculate the interest at the end of each year. Do these interest form an AP? If so, then find the interest at the end of 20 years making use of this fact.

28. Find the missing frequencies and the median for the following distribution, if the mean is 1.46.

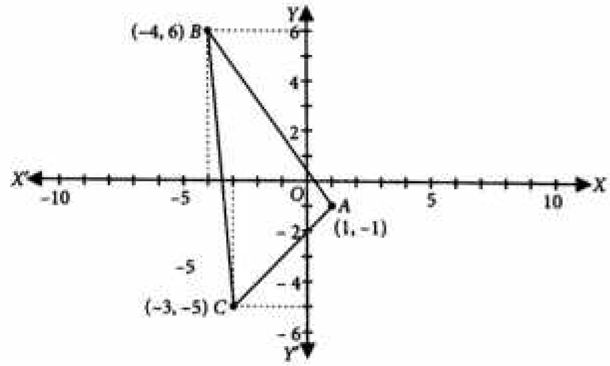
Number of accidents	0	1	2	3	4	5	Total
Number of days (frequency)	46	x	y	25	10	5	200

OR

Following is the cumulative frequency distribution (of less than type) of 1000 persons each of age 20 years and above. Determine the mean age.

Age (in years)	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80
Number of persons	100	220	350	750	950	1000

29. Observe the graph given below and state whether ΔABC is scalene, isosceles or equilateral. Justify your answer. Also, find the area of ΔABC.



30. Due to some technical problems, an aeroplane started late by one hour from its starting point. The pilot decided to increase the speed of the aeroplane by 100 km/h from its usual speed to cover a journey of 1200 km in same time.

(i) Find the usual speed of the aeroplane.

(ii) What values (qualities) of the pilot are represented in the question?

SOLUTION

1. The prime factorisation of q is of the form $2^n \times 5^m$, where n, m are some non-negative integers.

2. Given, pair of equations is

$$6x + 5y = 4 \text{ and } 12x + py = -8$$

Here, $a_1 = 6, b_1 = 5, c_1 = -4$

and $a_2 = 12, b_2 = p, c_2 = 8$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \frac{6}{12} = \frac{5}{p} \neq \frac{-4}{8} \quad \Rightarrow \quad p = 10$$

3. Here, $x_1 = 10\cos 30°, y_1 = 0$ and $x_2 = 0, y_2 = 10\cos 60°$

$\therefore$ Distance between the points

$$= \sqrt{(0 - 10\cos 30°)^2 + (10\cos 60° - 0)^2}$$

$$= \sqrt{(5\sqrt{3})^2 + (5)^2}$$

$$\left[\because \ \cos 30° = \frac{\sqrt{3}}{2} \text{ and } \cos 60° = \frac{1}{2}\right]$$

$$= \sqrt{100} = 10 \text{ units}$$

4. Given, AP is 121, 117, 113, ...

Here, first term, $a = 121$

and common difference, $d = 117 - 121 = -4$

Let, n^{th} term of this AP be the first negative term.

Then, $a_n < 0$

$\Rightarrow \ a + (n - 1)d < 0 \Rightarrow 121 + (n - 1)(-4) < 0$

$\Rightarrow \ 125 - 4n < 0$

$\Rightarrow \ 125 < 4n$ or $4n > 125$

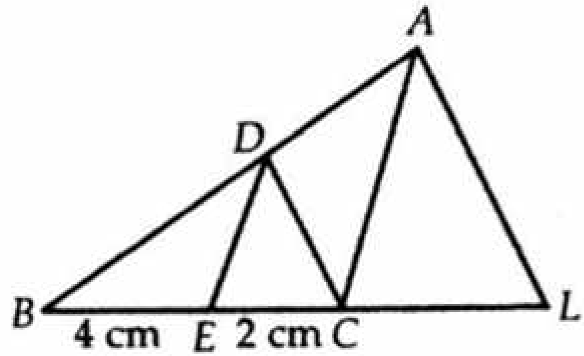
$\Rightarrow \ n > \frac{125}{4} \Rightarrow n > 31\frac{1}{4}$

Least integral value of n is 32

Hence, 32^{th} term of the given AP is the first negative term.

5. Let $CL = x$ cm.

In ΔABC, $DE \parallel AC$



$$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC}$$

[By Thale's Theorem]

$$\Rightarrow \frac{BD}{DA} = \frac{4}{2} \quad \Rightarrow \frac{BD}{DA} = 2 \qquad ...(1)$$

In ΔBLA, $CD \parallel LA$

$$\Rightarrow \frac{BC}{CL} = \frac{BD}{DA}$$ [By basic proportionality theorem]

$$\Rightarrow \frac{4+2}{x} = \frac{BD}{DA} \Rightarrow \frac{6}{x} = \frac{BD}{DA} \qquad ...(2)$$

From (1) and (2), we have

$\frac{6}{x} = 2 \Rightarrow x = 3$. Hence, $CL = 3$ cm.

6. Here, Discriminant $(D) = (-2)^2 - 4\,(5)\,(-3)$

$= 4 + 60 = 64 > 0$

$\therefore$ Roots are real and unequal.

7. Let the breadth of rectangular plot (b) be x m by the given condition, length of rectangular plot (l) $= (2x + 1)\ m$

Area of rectangular plot $= 528\ m^2$ (Given)

i.e., $l \times b = 528$

$\Rightarrow\ (2x + 1)(x) = 528$

$\Rightarrow\ 2x^2 + x = 528$

$\Rightarrow\ 2x^2 + x - 528 = 0$

Therefore, the length and breadth of a plot satisfy the quadratic equation $2x^2 + x - 528 = 0$.

8. $$S_n = \frac{3n^2}{2} + \frac{5n}{2}$$

$$a_{25} = S_{25} - S_{24}$$

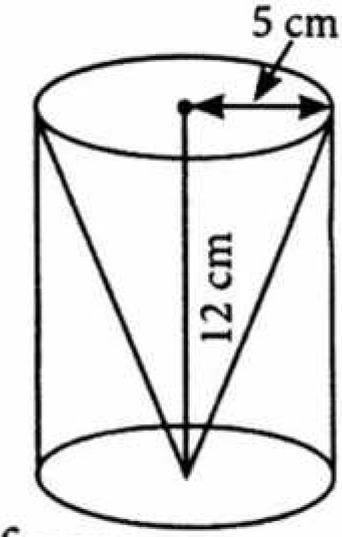
$$\Rightarrow \frac{3}{2}(25)^2 + \frac{5}{2}(25) - \frac{3}{2}(24)^2 - \frac{5}{2}(24)$$

$$\Rightarrow \frac{3}{2}[25)^2 - 24^2] + \frac{5}{2}(25 - 24) \Rightarrow \frac{3}{2} \times 49 + \frac{5}{2}$$

$$\Rightarrow \frac{147 + 5}{2} = \frac{152}{2} = 76$$

9. Given, diameter of the cylinder = 10 cm

$\therefore$ Radius of the cylinder, $r = 5$ cm



Since, the base of the cone and cylinder is same.

So, radius of cone, $r = 5$ cm

Height of cone

= Height of cylinder = h = 12 cm

Now, volume of the remaining solid

= Volume of cylinder – Volume of cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times 3.14 \times (5)^2 \times 12$$

$$= 2 \times 3.14 \times 25 \times 4 = 628 \text{ cm}^3$$

Hence, the volume of the remaining solid is 628 cm^3

10. Since, A (4, 3) and B (x, 5) lie on the circle having centre O (2, 3).

$\therefore \quad OA = OB$ [radii of circle]

$\Rightarrow \quad \sqrt{(4-2)^2+(3-3)^2} = \sqrt{(x-2)^2+(5-3)^2}$

[using distance formula]

$\Rightarrow \quad \sqrt{4} = \sqrt{(x-2)^2+4}$

On squaring both sides, we get

$$4 = (x-2)^2 + 4 \Rightarrow (x-2)^2 = 0$$

$\Rightarrow \quad x - 2 = 0 \Rightarrow x = 2$

Hence, the value of $x^2 + 5$ is 9 units.

11. Total number of tickets in the bag = 20

(i) Number of tickets of multiple of 7 = {14, 21, 28}

i.e., 3

$\therefore \quad P(\text{getting a ticket of multiple of 7}) = \frac{3}{20}$

(ii) Number of tickets greater than 15 and multiple of 5 = { 20, 25, 30} *i.e.*, 3

$\therefore \quad P(\text{greater than 15 and multiple of 5}) = \frac{3}{20}$

12. The maximum capacity of tanker which can measure the petrol of either tanker in exact number of times is HCF of 680 litres and 850 litres.

Now, $680 = 2^3 \times 5 \times 17$

$850 = 2 \times 5^2 \times 17$

$\therefore$ Required HCF of 680 and $850 = 2 \times 5 \times 17$

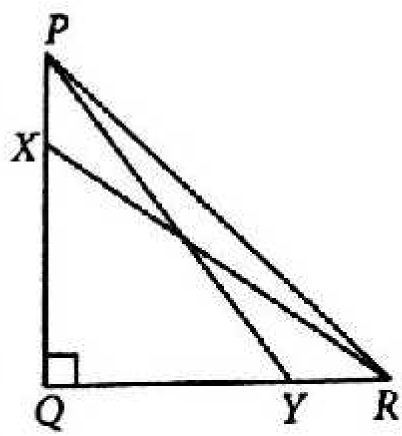
$= 170$ litres.

Hence, the maximum capacity of tanker which can measure the petrol in exact number of times is 170 litres.

13. X divides PQ in ratio $1:2$

$\Rightarrow XQ = 2PX$

$\Rightarrow XQ = \frac{2}{3}PQ$ ...(i)



Also Y divides QR in ratio $2:1$

$\Rightarrow QY = 2YR$

$\Rightarrow QY = \frac{2}{3}QR$ ...(ii)

In right ΔPQY, $PY^2 = PQ^2 + QY^2$

$\Rightarrow PY^2 = PQ^2 + \left(\frac{2}{3}QR\right)^2$

$\Rightarrow PY^2 = PQ^2 + \frac{4}{9}QR^2$

$\Rightarrow\ 9PY^2 = 9PQ^2 + 4QR^2$ ...(iii)

In right ΔXQR, $XR^2 = XQ^2 + QR^2$

$$\Rightarrow\ XR^2 = \left(\frac{2}{3}PQ\right)^2 + QR^2$$

$$\Rightarrow\ XR^2 = \frac{4PQ^2}{9} + QR^2$$

$\Rightarrow\ 9XR^2 = 4PQ^2 + 9QR^2$ ...(iv)

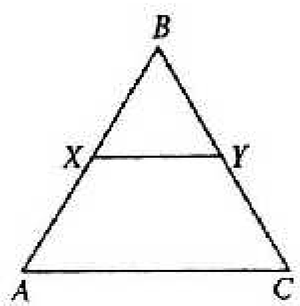
Adding (iii) and (iv), we get

$9PY^2 + 9XR^2 = 13PQ^2 + 13QR^2$

$\Rightarrow\ 9(PY^2 + XR^2) = 13\ (PQ^2 + QR^2)$

$= 13\ PR^2$ $[\because PQ^2 + QR^2 = PR^2]$

OR



$ar(\Delta BXY) = 2ar(ACYX)$

$\Rightarrow\ ar(\Delta BXY) = 2[ar(\Delta BAC) - ar(\Delta BXY)]$

$\Rightarrow\ ar(\Delta BXY) = 2ar(\Delta BAC) - 2ar(\Delta BXY)$

$\Rightarrow\ 3ar(\Delta BXY) = 2ar(\Delta BAC)$

$$\Rightarrow\ \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{2}{3}$$ ...(i)

In ΔBXY and ΔBAC,

$\angle B = \angle B$ [Common]

$\angle BXY = \angle BAC$ [$XY \parallel AC$]

$\therefore \quad \Delta BXY \sim \Delta BAC$ [AA similarly]

$$\Rightarrow \quad \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{BX^2}{BA^2}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{BX^2}{BA^2} \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad \frac{BX}{BA} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow \quad 1 - \frac{BX}{BA} = 1 - \frac{\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{BA - BX}{BA} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{AX}{AB} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}}$$

14. Let us suppose that $\sqrt{p}+\sqrt{q}$ be rational.

Let $\sqrt{p}+\sqrt{q}=r$, where r is rational.

$\therefore \quad \sqrt{p}=r-\sqrt{q}$

Squaring both sides, we have

$p=r^2+q-2r\sqrt{q}$

$\Rightarrow \quad \sqrt{q}=\dfrac{r^2+q-p}{2r}$

Since r is rational.

$\Rightarrow$ r^2 is rational ; q and p are both rationals.

[$\because$ p, q are primes]

$\Rightarrow \quad \dfrac{r^2+q-p}{2r}$ is rational

But $\sqrt{q}$ is irrational. [$\because$ q is prime]

Which contradict our supposition.

Hence, $\sqrt{p} + \sqrt{q}$ is irrational.

15. Steps of Construction :

I. Draw a circle of radius 4 cm with centre O and draw a diameter.

II. Extend its diameter on both sides and cut $OP = OQ = 9$ cm

III. Bisect PO such that M be its mid-point.

IV. Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at A and B.

V. Join PA and PB.

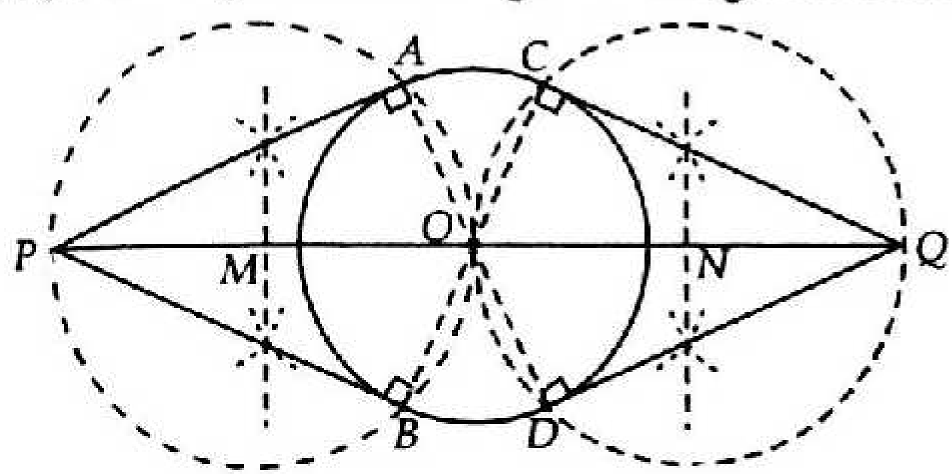
Thus, PA and PB are the two required tangents from P.

VI. Now bisect OQ such that N is its mid point.

VII. Taking N as centre and NO as radius, draw a circle. Let it intersect the given circle at C and D.

VIII. Join QC and QD.

Thus, QC and QD are the required tangents from Q.



16. Here, $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ...(i)

and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$ ...(ii)

Squaring and adding (i) and (ii), we have

$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{x}{a}\cdot\frac{y}{b}\cos\theta\sin\theta$$

$$+\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{x}{a}\cdot\frac{y}{b}\cos\theta\sin\theta = 1+1$$

$$\Rightarrow \frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

Hence proved.

17. Since, α and β are the zeroes of polynomial $x^2 - px - p + c$.

So, sum of zeroes, $\alpha + \beta = P$ ...(i)

and product of zeroes, $\alpha\beta = c - p$ ...(ii)

But $(\alpha + 1)(\beta + 1) = 0$ (Given)

$\therefore \quad \alpha\beta + (\alpha + \beta) + 1 = 0$

$\Rightarrow\ c - p + p + 1 = 0$ [From (i) and (ii)]

$\Rightarrow\ c = -1$

OR

Since, α and β are the zeroes of the polynomial $x^2 - 2x - 8$.

So, $\alpha + \beta = 2$ ...(i)

and $\alpha\beta = -8$ ...(ii)

If 3α and 3β are the zeroes, then the required quadratic polynomial is

$x^2 - (3\alpha + 3\beta)x + (3\alpha)(3\beta) = x^2 - 3(\alpha + \beta)x + 9(\alpha\beta)$

$= x^2 - 3 \times 2 \times x + 9 \times (-8)$ [From (i) and (ii)]

$= x^2 - 6x - 72$

18. Let the frequency of the class 20-30 be f_1 and that of class 40-50 be f_2.

The cumulative frequency table for given distribution is

Class interval	Frequency	Cumulative frequency
0-10	10	10
10-20	20	30
20-30	f_1	$30 + f_1$
30-40	40	$70 + f_1$
40-50	f_2	$70 + f_1 + f_2$
50-60	25	$95 + f_1 + f_2$
60-70	15	$110 + f_1 + f_2$

Here, median = 35

So, the median class is 30-40. Also, $n = 170$

$\Rightarrow\ \frac{n}{2} = 85$

$\therefore\ l = 30, f = 40, cf = 30 + f_1$ and $h = 10$

Now, median $= l + \left\{\frac{\frac{n}{2} - cf}{f}\right\} \times h$

$$\Rightarrow \quad 35 = 30 + \left\{\frac{85 - (30 + f_1)}{40}\right\} \times 10$$

$\Rightarrow \; 35 \times 4 = 120 + (55 - f_1)$

$\Rightarrow \; 140 = 175 - f_1 \Rightarrow f_1 = 35$

Also, $110 + f_1 + f_2 = 170$

[$\because$ sum of all frequencies = 170, given]

$\Rightarrow \; f_1 + f_2 = 60$

$\Rightarrow \; 35 + f_2 = 60$

$\Rightarrow \; f_2 = 60 - 35 = 25$

Hence, the missing frequency of the class 20-30 is 35 and the class 40-50 is 25.

OR

The class marks of the classes are

$x_1 = 5, x_2 = 15, x_3 = 25, x_4 = 35, x_5 = 45$ and their corresponding frequencies are

z

$f_1 = 8, f_2 = x, f_3 = 10, f_4 = 11, f_5 = 9$

$n = \sum f_i = 8 + x + 10 + 11 + 9 = 38 + x$

$\sum f_i x_i = 8 \times 5 + x \times 15 + 10 \times 25 + 11 \times 35 + 9 \times 45$

$= 15\,x + (40 + 250 + 385 + 405) = 15\,x + 1080$

We are given that mean $\bar{x} = 25.2$

$$\Rightarrow \quad \frac{\sum f_i x_i}{n} = 25.2 \Rightarrow \frac{15x + 1080}{x + 38} = \frac{252}{10}$$

$\Rightarrow \; 10 \times (15x + 1080) = 252 \times (x + 38)$

$\Rightarrow \; 150\,x + 10800 = 252\,x + 252 \times 38$

$\Rightarrow \; 252\,x - 150\,x = 10800 - 252 \times 38$

$\Rightarrow \; 102\,x = 1224 \Rightarrow x = 12$

19. We know that , tangents drawn from an external point to a circle are equal in length.

$\therefore \quad AD = AF = x$ cm

$BD = BE = y$ cm

$CE = CF = z$ cm

Given, $AB = 8$ cm

$\Rightarrow \quad AD + BD = 8 \text{ cm} \Rightarrow x + y = 8$ ...(i)

$BC = 10 \text{ cm} \Rightarrow BE + CE = 10 \text{ cm} \Rightarrow y + z = 10$ ...(ii)

and $CA = 12 \text{ cm} \Rightarrow CF + AF = 12$ cm ...(iii)

$\Rightarrow \quad z + x = 12$

On adding (i), (ii) and (iii), we get

$2(x + y + z) = 30$

$\Rightarrow \quad x + y + z = 15$ ...(iv)

On subtracting (ii) from (iv), we get

$x = 15 - 10 = 5$

On subtracting (iii) from (iv), we get

$y = 15 - 12 = 3$

On subtracting (i) from (iv), we get

$z = 15 - 8 = 7$

$\therefore \quad AD = x \text{ cm} = 5$ cm,

$BE = y \text{ cm} = 3$ cm

and $CF = z \text{ cm} = 7$ cm

Hence, the length of AD, BE and CE are 5 cm, 3 cm and 7 cm, respectively.

20. Two dice are thrown at the same time.

So, total number of possible outcomes, $n(S) = 36$

(i) Let E_1 = Event of getting same number on both dice

$= \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$\therefore$ Number of favourable outcomes, $n(E_1) = 6$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) We get different number on both dice.

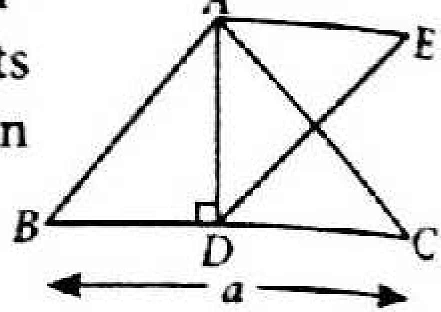
Then, total number of favourable outcomes,

$n(E_2) = 36$ – Total number of favourable outcomes for same number on both dice.

$= 36 - n(E_1) = 36 - 6 = 30$ $\quad [\because n(E_1) = 6]$

$$\therefore \text{ Required probability } = \frac{n(E_2)}{n(S)} = \frac{30}{36} = \frac{5}{6}$$

21. We know that perpendicular drawn from a vertex to its opposite base bisects the base in an equilateral triangle.



$$\therefore \quad BD = \frac{BC}{2} = \frac{a}{2}$$

In ΔADB, $\angle ADB = 90°$

$\therefore \quad AB^2 = AD^2 + BD^2$ [Using Pythagoras theorem]

$$\Rightarrow \quad a^2 = AD^2 + \left(\frac{a}{2}\right)^2$$

$$\Rightarrow \quad AD = \frac{\sqrt{3}a}{2}$$

ΔABC and ΔADE are equilateral triangles and equiangular.

$\therefore \quad \Delta ABC \sim \Delta ADE$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2} = \frac{\left(\frac{\sqrt{3}a}{2}\right)^2}{a^2} = \frac{3}{4}$$

22. Let original average speed of the train be x km/h.

$\therefore \quad \frac{63}{x} + \frac{72}{x+6} = 3$

$\Rightarrow \quad \frac{7}{x} + \frac{8}{x+6} = \frac{3}{9} = \frac{1}{3} \Rightarrow \frac{7(x+6)+8x}{x(x+6)} = \frac{1}{3}$

$\Rightarrow \quad 3(7x + 42 + 8x) = x^2 + 6x \Rightarrow 45x + 126 = x^2 + 6x$

$\Rightarrow \quad x^2 - 39x - 126 = 0$

$\Rightarrow \quad x^2 - 42x + 3x - 126 = 0$

$\Rightarrow \quad x(x - 42) + 3(x - 42) = 0$

$\Rightarrow \quad (x - 42)(x + 3) = 0$

$\Rightarrow$ Either $x - 42 = 0$ or $x + 3 = 0$

$\Rightarrow \quad x = 42$ or $x = -3$

Since x is the average speed of the train, x cannot be negative.

$\therefore \quad x = 42$

OR

Let present age of daughter Nisha be x years.

$\therefore$ Present age of Asha $= (x^2 + 2)$ years.

Now, when Nisha grows to her mother's present age, then her age would be $(x^2 + 2)$ years.

Then, her mother's age would be

$x^2 + 2 + x^2 + 2 - x$ *i.e.*, $2x^2 - x + 4$

Now, as per statement of the question, we obtain

$2x^2 - x + 4 = 10x - 1 \Rightarrow 2x^2 - 11x + 5 = 0$

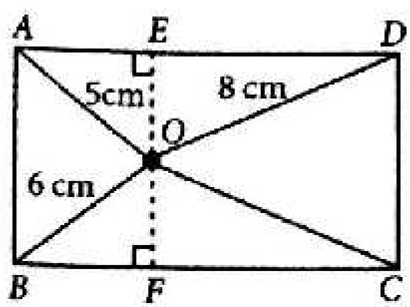
$\Rightarrow \quad (x - 5)(2x - 1) = 0 \Rightarrow$ Either $x - 5 = 0$ or $2x - 1 = 0$

$\Rightarrow \quad x = 5$ or $x = \frac{1}{2}$ (Rejecting)

Hence, present age of Nisha is 5 years and her mother's age is $5^2 + 2$ *i.e.*, 27 years.

23. Let x be any positive integer, then it is of the form $3q$ or $3q + 1$ or $3q + 2$.
Now, squaring each of these, we have
$(3q)^2 = 9q^2 = 3 \times 3q^2 = 3 \times$ some integer
$= 3 \times n$, where n is an integer ... (i)
$(3q + 1)^2 = 9q^2 + 6q + 1 = 3q(3q + 2) + 1$
$= 3 \times (3q^2 + 2q) + 1 = (3 \times$ some integer $) + 1$
$= 3n + 1$, where n is an integer ... (ii)
and $(3q + 2)^2 = 9q^2 + 4 + 12q = 3 \times 3q^2 + 3 \times 4q + 3 + 1$
$= 3(3q^2 + 4q + 1) + 1 = (3 \times$ some integer$) + 1$
$= 3n + 1$, where n is an integer ... (iii)
Thus, from (i), (ii) and (iii), we have square of any positive integer is either of the form $3n$ or $3n + 1$ for some integer n.

24. Given, $OB = 6$ cm, $OD = 8$ cm, $OA = 5$ cm
Let as draw $EOF \parallel AB$ such that $OE \perp AD$ and $OF \perp BC$.



In ΔOFB, by Pythagoras theorem, we get
$OB^2 = OF^2 + BF^2$ $[\angle OFB = 90°]$ ...(i)
In ΔOED, by Pythagoras theorem, we get
$OD^2 = OE^2 + DE^2$ $[\because \angle OED = 90°]$ ...(ii)
On adding eqn (ii) and (ii), we get

$OB^2 + OD^2 = OF^2 + BF^2 + OE^2 + DE^2$

$= OF^2 + AE^2 + OE^2 + CF^2$

[$\because$ $BF = AE$ and $CF = DE$, as $EOF \parallel AB \parallel CD$]

$= (OF^2 + CF^2) + (OE^2 + AE^2)$

$= OC^2 + OA^2$

[$\because$ In ΔOFC, $OF^2 + CF^2 = OC^2$ and in ΔOEA, $OE^2 + AE = OA^2$]

$= OC^2 + 5^2$ [$\because$ $OA = 5$ cm, given]

$\Rightarrow$ $6^2 + 8^2 = OC^2 + 5^2$

[$\because$ $OB = 6$ cm, $OD = 8$ cm, given]

$\Rightarrow$ $36 + 64 = OC^2 + 25$

$\therefore$ $OC^2 = 36 + 64 - 25 = 75$

$\Rightarrow$ $OC = \sqrt{25 \times 3} = 5\sqrt{3}$ cm

OR

Given $ABCD$ is a parallelogram such that

$AP : PB = 3 : 2$

Also, $CQ : QD = 4 : 1$

To Prove $AR = \frac{3}{7}AC.$

Proof : $ABCD$ is a parallelogram, so opposite sides are parallel and equal, *i.e.* $AB||CD$ and $AD||BC$.

In ΔAPR and ΔCQR,

$\angle ARP = \angle CRQ$ [vertically opposite angles]

$\angle RPA = \angle RQC$

[$\because$ alternate angle, $AB||CD$ and PQ is transversal]

$\therefore$ $\Delta APR \sim \angle CQR$

[By AA similarity criterion]

Then, $\frac{AP}{CQ} = \frac{PR}{QR} = \frac{AR}{CR}$ ...(i)

Given, $AP = \frac{3}{5}AB$

[$\because AP : PB = 3 : 2 \Rightarrow AP = \frac{3}{5}\ AB$]

and $CQ = \frac{4}{5}CD = \frac{4}{5}AB$ ...(iii)

[$\because$ $CQ : QD = 4 : 1 \Rightarrow CQ = \frac{4}{5}\ CD$ and $CD = AB$]

On dividing Eq. (ii) by Eq.(iii), we get

$$\frac{AP}{CQ} = \frac{3/5AB}{4/5AB}$$

$$\Rightarrow \frac{AP}{CQ} = \frac{3}{4} \Rightarrow \frac{AR}{CR} = \frac{3}{4}$$ [From Eq.(i)]

$$\Rightarrow \frac{CR}{AR} = \frac{4}{3}$$

$$\Rightarrow \frac{CR}{AR} + 1 = \frac{CR + AR}{AR} = \frac{4}{3} + 1$$

$\Rightarrow \quad \frac{AC}{AR} = \frac{7}{3}$ $\qquad \therefore \; AR = \frac{3}{7} AC$

25. Here, $r = \frac{6}{2} = 3$ cm and $R = \frac{10}{2} = 5$ cm be the radii of the top and bottom of a frustum of a right circular cone.Also, $h = 5$ cm (given) be the height of the frustum of a cone.

$\therefore$ Slant height (l) of the frustum of a cone

$= \sqrt{h^2 + (R - r)^2}$

$= \sqrt{5^2 + (5-3)^2} = \sqrt{29} = 5.38$ cm

Total surface area of a frustum of a cone = curved surface area of a cone + area of top circular section + area of bottom circular section

$= [\pi(R + r)\, l + \pi r^2 + \pi R^2]$

$= \pi[(5 + 3)\, 5.38 + (3)^2 + (5)^2]$

$= 242.22 \text{ cm}^2$

Volume of the frustum of a cone $= \frac{\pi h}{3}[R^2 + r^2 + Rr]$

$= \frac{22}{7} \times \frac{5}{3} \times [(5)^2 + (3)^2 + 5 \times 3]$

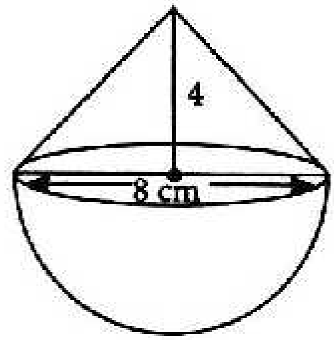
$= \frac{770}{3} \text{cm}^3 = 256.67 \text{ cm}^3$

OR

Let r be the radius of the hemisphere and the cone and h be the height of the cone.

Given, radius of hemisphere

$= \text{radius of cone} = r = \frac{8}{2} = 4\,\text{cm}$



Volume of the toy = Volume of the hemisphere + Volume of the cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times 4^3 + \frac{1}{3} \times \frac{22}{7} \times 4^2 \times 4$ $[\because h = 4\text{ cm}]$

$= \frac{1408}{7}\text{cm}^3$

A cube circumscribes the given solid. Therefore, edge of the cube should be 8 cm.

$\therefore$ Volume of the cube $= 8^3\text{ cm}^3 = 512\text{ cm}^3$

Difference in the volumes of the cube and the toy

$= \left(512 - \frac{1408}{7}\right)\text{cm}^3 = 310.86\,\text{cm}^3$

T.S.A. of the toy = C.S.A. of cone + C.S.A. of hemisphere

$= \pi rl + 2\pi r^2,$

where $l = \sqrt{h^2 + r^2} = \pi r(l + 2r)$

$= \frac{22}{7} \times 4 \times \left(\sqrt{4^2 + 4^2} + 2 \times 4\right)$

$= \frac{22}{7} \times 4 \times [4\sqrt{2} + 8]$

$= \frac{88 \times 4}{7}[\sqrt{2} + 2] = 171.67\,\text{cm}^2$

26. We know that, in a cyclic quadrilateral, the sum of two opposite angles is 180°.

$\therefore$ $\angle B + \angle D = 180°$ and $\angle A + \angle C = 180°$

$\Rightarrow$ $3y - 5 - 7x + 5 = 180°$ and $4y + 20 - 4x = 180°$

$\Rightarrow$ $3y - 7x = 180°$ ...(i)

and $4y - 4x = 160°$

$\Rightarrow$ $y - x = 40°$ ...(ii)

On solving (i) and (ii) we get

$x = -15$ and $y = 15$

On putting the values of x and y, we calculate the angles as

$\angle A = 4y + 20 = 100 + 20 = 120°$,

$\angle B = 3y - 5 = 75 - 5 = 70°$,

$\angle C = -4x = -4(-15) = 60°$

and $\angle D = -7x + 5 = 105 + 5 = 110°$

Hence, the angles are $\angle A = 120°$, $\angle B = 70°$, $\angle C = 60°$ and $\angle D = 110°$.

27. Given, initial money $P = ₹\ 2000$

Rate of interest, $R = 7\%$ per year; Time, $T = 1, 2, 3, 4, ...$

We know that, simple interest is given by the following formula S.I. $= \frac{P \times R \times T}{100}$

$\therefore$ S.I. at the end of 1st year $= \frac{2000 \times 7 \times 1}{100} = ₹\ 140$

S.I. at the end of 2nd year $= \frac{2000 \times 7 \times 2}{100} = ₹\ 280$

S.I. at the end of 3rd year $= \frac{2000 \times 7 \times 3}{100} = ₹\ 420$

Yes, interest form an A.P. with common difference (d) $= 280 - 140 = 420 - 280 = = ₹\ 140$.

$\therefore$ S.I. at the end of 20 years (a_{20}) $= 140 + (20 - 1)\ 14$

$14\ [10 + 19] = 14 \times 29 = 406$.

28. Let the missing frequencies be x and y. Then the given distribution is

Number of accidents (x_i)	Number of days (f_i)	$f_i x_i$
0	46	0
1	x	x
2	y	$2y$
3	25	75
4	10	40
5	5	25

Given, $\Sigma f_i = 200$

$\Rightarrow$ $46 + x + y + 25 + 10 + 5 = 200$

$\Rightarrow$ $x + y = 200 - 46 - 25 - 10 - 5$

$\Rightarrow\ x + y = 114$ ...(i)

and mean = 1.46

$$\Rightarrow \frac{\Sigma f_i x_i}{\Sigma f_i} = 1.46 \Rightarrow \frac{x + 2y + 140}{200} = 1.46$$

$\Rightarrow\ x + 2y = 292 - 140 = 152$ ...(ii)

On solving (i) and (ii), we get

$x = 76$ and $y = 38$

Now, cumulative frequency distribution table is

Number of accidents (x_i)	Number of days (f_i)	Cumulative frequency
0	46	46
1	76	122
2	38	160
3	25	185
4	10	195
5	5	200

We have, $n = 200$

$$\therefore\quad \text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{term} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{term}}{2}$$

$$= \frac{100^{th} \text{ term} + 101^{th} \text{ term}}{2}$$

$\therefore$ Since both 100^{th} and 101^{th} terms so, lie in cumulative frequency table in 122. So, the corresponding value of 122 is 1.

$$\therefore \quad \text{Median} = \frac{1+1}{2} = 1$$

OR

Firstly, we make the frequency distribution of the given data and then proceed to calculate mean by computing class marks (x_i), u_i's and f_iu_i's as follows

Age (in years)	Number of persons (f_i)	Class marks (x_i)	$u_i = \frac{x_i - 45}{10}$	f_iu_i
20-30	100	25	−2	−200
30-40	120	35	−1	−120
40-50	130	45	0	0
50-60	400	55	1	400
60-70	200	65	2	400
70-80	50	75	3	150
Total	$\Sigma f_i = 1000$			$\Sigma f_iu_i =$ 630

$$\text{Mean } (\bar{x}) = \frac{a + h\Sigma f_i u_i}{\Sigma f_i}$$

$$= 45 + \frac{10 \times 630}{1000} = 45 + 6.3 = 51.3$$

29. Given vertices of ΔABC are $A(1, -1)$, $B(-4, 6)$ and $C(-3, -5)$, respectively.

Now, $AB = \sqrt{(-4-1)^2 + (6+1)^2}$ [by distance formula]

$$= \sqrt{(-5)^2 + (7)^2} = \sqrt{25+49} = \sqrt{74} = 8.6 \text{ units}$$

$$BC = \sqrt{(-3+4)^2 + (-5-6)^2}$$

$$= \sqrt{(1)^2 + (-11)^2} = \sqrt{1+121} = \sqrt{122} = 11.01 \text{ units}$$

and $CA = \sqrt{(-3-1)^2 + (-5+1)^2}$

$$= \sqrt{(-4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 5.7 \text{ units}$$

Here, we can see that $AB \neq BC \neq CA$
and $BC^2 \neq AB^2 + CA^2$
So, given triangle is a scalene triangle.
Now, area of ΔABC

$$= \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$$

$$= \frac{1}{2} | 1(6+5) + (-4)(-5+1) + (-3)(-1-6) |$$

$$= \frac{1}{2} | 11 - 4(-4) - 3(-7) | = \frac{1}{2} | 11 + 16 + 21 |$$

$$= \frac{1}{2} | 48 | = \frac{1}{2} \times 48 = 24 \text{ sq units}$$

30. (i) Let the usual speed of the aeroplane be x km/h

Then, the time taken to cover 1200 km $= \frac{1200}{x}$

Now, if plane started late by one hour, then its speed $= (x + 100)$ km/h and time taken to cover 1200 km

$$= \frac{1200}{x+100}$$

According to given condition, we have

$$\frac{1200}{x} - \frac{1200}{x+100} = 1$$

$$\Rightarrow \quad 1200\left[\frac{1}{x} - \frac{1}{x+100}\right] = 1$$

$$\Rightarrow \quad 1200\left[\frac{x+100-x}{x(x+100)}\right] = 1$$

$\Rightarrow$ $120000 = x^2 + 100x \Rightarrow x^2 + 100x - 120000 = 0$

$\Rightarrow$ $x^2 + 400x - 300x - 120000 = 0$

$\Rightarrow$ $x(x + 400) - 300\,(x + 400) = 0$

$\Rightarrow$ $(x + 400)(x - 300) = 0 \Rightarrow x = -400$ or $x = -300$

$\because$ Speed cannot be negative.

$\therefore$ $x = 300$

Hence, usual speed of the aeroplane is 300 km/h.

(ii) The values (qualities) of the pilot represented in this question are leadership and punctuality.