Chapter 03

Working Stress Method (WSM) of Design

3.1 Introduction

As discussed in the previous chapters, there are many design philosophies for the design of reinforced concrete members. In this chapter, we will discuss the **Working Stress Method** of design which is the most traditional method of design. With the recent advances in the understanding of the behavior of materials (concrete and steel), now we have more rational methods of design. Due to this reason only, most design codes in the world, have dispensed **Working Stress Method**. In the Indian code also i.e. **IS 456: 2000, Working Stress Method** has been put at annexure and major focus is on the recent **Limit State Method** of design.

3.2 Proceeding from Bending Moments to Flexural Stresses

From the principles of Structural Analysis, we arrive at bending moments, shear forces, reactions, axial forces etc. In the theory of bending, we are mainly concerned with the bending moments. The bending moment at any section of the beam produce normal stress (compressive and tensile). The theory of flexure for homogeneous materials is the part of Basic Structural Analysis or Solid Mechanics.

3.3 Analysis of Composite/Non-homogeneous Sections

The principles of the theory of flexure as studied in Solid Mechanics is NOT applicable for **non-homogeneous** materials. Reinforced concrete is non-homogeneous and thus the results developed in the theory of flexure cannot be applied directly to reinforced concrete.

3.4 Stress-strain Distribution

One of the fundamental assumptions of theory of flexure is **"plane sections remain plane before and after the bending"**. This assumption is true only if there exists a perfect bond between the two materials in order to act as an integral unit without any possibility of slip in between.

Thus, there will be a linear strain variation. The two materials bonded together and located at the same distance from the neutral axis will have same amount of strain (ε) in both of them. The corresponding stress in the respective materials will be $f_1 = E_1 \varepsilon$ and $f_2 = E_2 \varepsilon$. Where, E_1 and E_2 are modulus of elasticity of the two materials.

From strain compatibility,

$$\varepsilon_1 = \varepsilon_2$$

$$\frac{f_1}{E_1} = \frac{f_2}{E_2}$$

$$f_2 = \frac{E_2}{E_1} f_1 = m f_1 \qquad \text{where, } m = \frac{E_2}{E_1} = \text{modular ratio}$$

3.5 Transformed Section

In order to analyse of composite materials by the use of the linear elastic principles of Structural Analysis, it is necessary to transform the composite section into a single homogeneous section. This is made possible by the concept of **modular ratio (m)**.

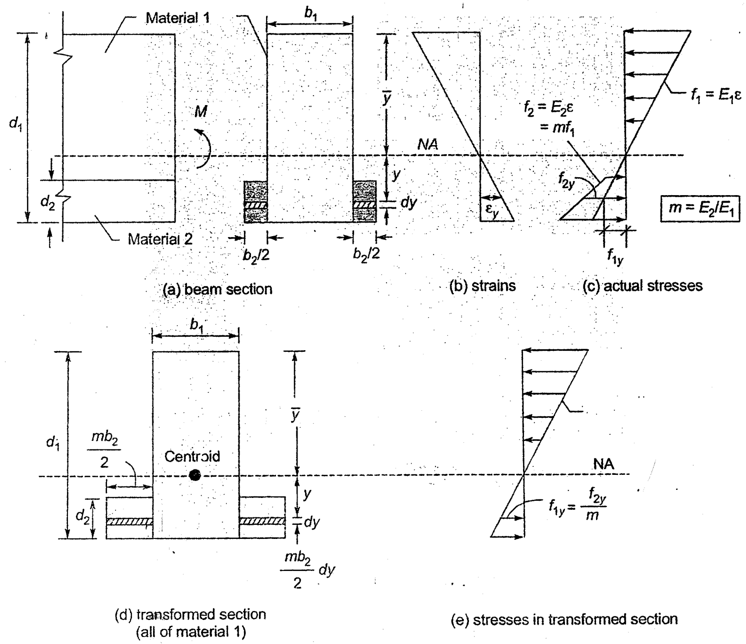


Fig. 3.1 *Transformed Section*

Let there be an infinitesimal element in material 2 located at a distance 'y' from the neutral axis.

Thus

$$dF_2 = f_2(b_2 \cdot dy)$$

But

$$f_2 = \left(\frac{E_2}{E_1}\right) f_1 = m f_1 \text{ as proved earlier}$$

Therefore $dF_2 = mf_1(b_2 \cdot dy) = f_1(mb_2 \cdot dy)$

Thus material 2 can be transformed to materials 1 by replacing original width b_2 by 'mb_2', where m is the modular ratio $\left(= \frac{E_2}{E_1}\right)$. The term **width** implies dimension parallel to the neutral axis.

NOTE: In the transformed section, the magnitude of resultant forces, their direction and line of action does not change.

In the transformed section, as the material is homogeneous (of one material only), the principles of linear elastic analysis is applicable.

3.6 Modular Ratio

In case of working stress method of analysis, the composite section of reinforced concrete is transformed into an equivalent section of single material of concrete. For this, we need to define modular ratio which is the ratio of elastic modulus of steel to concrete.

However, the short term modulus of elasticity of concrete does not take into account the long term effects of creep and shrinkage and thus it is not considered in defining the modular ratio (m). However, partly taking into account the long term effects of creep and shrinkage, **Cl. B-1.3 of IS 456: 2000** defines modular ratio (m) as:

$$\boxed{m = \frac{280}{3\sigma_{cbc}}}$$

or $$m\sigma_{cbc} = \frac{280}{3} = 93.33 = \text{constant}$$

where, σ_{cbc} = Permissible stress in concrete in bending compression

Table 3.1: *Values of σ_{cbc} and m for different grades of concrete*

Grade of Concrete	σ_{cbc} (MPa)	Modular Ratio (m)
M15	5	18.67
M20	7	13.33
M25	8.5	10.98
M30	10	9.33
M35	11.5	8.11
M40	13	7.18
M45	14.5	6.44
M50	16	5.83

3.6.1 A Note on Modular Ratio (m)

The two different materials of reinforced concrete i.e., the concrete and the steel are transformed into a single material using the modular ratio (m). The modular ratio (m) is expressed as:

$$m = \text{Modular ratio} = \frac{\text{Modulus of elasticity of steel}}{\text{Modulus of elasticity of concrete}} = \frac{E_s}{E_c}$$

The modulus of elasticity of steel = $E_s = 2 \times 10^5$ N/mm²

The short-term modulus of elasticity of concrete $E_c = 5000\sqrt{f_{ck}}$

Above expression does not take into account the effect of creep and shrinkage and other long term effects.

Thus, $$m = \frac{2\times10^5}{5000\sqrt{f_{ck}}}$$

But it is given as $$m = \frac{280}{3\sigma_{cbc}}$$

(this partially takes into account the long-term effects of creep and shrinkage)

3.7 Transformed Area of Reinforcement-Tension Steel

The area of tension reinforcement steel (A_{st}) is converted to an equivalent concrete area as mA_{st}. **Cl. B-2.1.1 of IS 456: 2000** states that this transformation is applicable not only for flexural members but also for direct tension members. The corresponding stress in steel 'f_{st}' is given from the equivalent transformed concrete stress f_c (at the level of tension steel) as $f_{st} = mf_c$.

3.8 Transformed Area of Reinforcement-Compression Steel

The modular ratio for compression steel (eg. Steel in columns, compression steel in doubly reinforced beams) is greater than that for tension steel. This is due to the long term effects of creep and shrinkage of concrete along with non-linearity in material behavior at higher stress levels results in much higher compressive strains in compression steel rather than those indicated by linear elastic theory. Thus **IS 456: 2000** recommends transformed area of compression steel to be equal to 1.5 mA_{sc} and NOT mA_{sc}. The corresponding stress in compression steel f_{sc} is given as $f_{sc} = 1.5\ mf_c$. Where, f_c is the corresponding stress in equivalent transformed concrete.

3.9 Cracking Moment

The very first crack in the extreme tension fibre of a beam appears when the stress reaches the value of **modulus of rupture of concrete (f_{cr})**. Assuming a linear stress-strain relationship for concrete in compression and tension with the same modulus of elasticity of concrete, the corresponding **cracking moment** is given by:

$$\text{Cracking moment } (M_{cr}) = f_{cr}\frac{I_T}{y_T}$$

where, I_T = Second moment of area or moment of inertia of transformed concrete about neutral axis

y_T = Distance of extreme tension fibre from neutral axis

When the concrete beam is very lightly loaded such that the applied moment (M) is less than the cracking moment (M_{cr}) then section is **uncracked section**, and both concrete and steel takes part in resisting tension.

3.10 Behaviour of Reinforced Concrete in Flexure

When a reinforced concrete beam is loaded, it behaves differently according to the stage of loading that is being superimposed on the beam. In general, this behavior can be studied right from *uncracked phase to the* final ultimate stage that leads to collapse because of flexural resistance capacity of the section is being exceeded.

3.10.1 The Uncracked Phase in Reinforced Concrete

Initially, there is no externally applied load on the beam. Now gradually, as the beam is being loaded, the corresponding moment at a particular section in the beam increases. When this applied moment at any section of the beam (M) is less than the cracking moment (M_{cr}), then the maximum tensile stress f_T (at the extreme fibre of beam) is less than the flexural tensile strength of concrete f_{cr}. This phase is called as **uncracked phase** and the whole section takes part (i.e. effective) in resisting the applied moment M.

The limiting case of **uncracked phase** occurs when the applied moment (M) becomes equal to the **cracking moment (M_{cr})**.

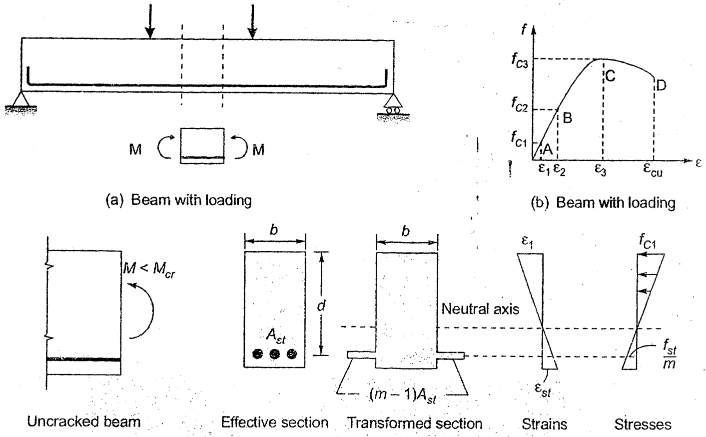


Fig. 3.2 *Uncracked phase of concrete*

3.10.2 The Cracked Phase in Reinforced Concrete

As the externally applied moment M (due to external loading) exceeds the **cracking moment** M_{cr}, the tensile stress in the concrete exceeds the **flexural tensile strength of concrete.** At this stage, **cracks get initiated** from the bottom most fibre (in tension) of concrete. As the loading is increased further, these cracks widen and they start propagating towards the neutral axis of the beam. As the concrete has cracked, it is not effective in resisting the tension anymore. The effective area of concrete gets reduced. The tension which was resisted by concrete just before the commencement of crack is now gets transferred to the reinforcing steel at the location of cracked section of the beam. Any further increase in the loading is now entirely resisted by the reinforcing steel in tension side of the section.

Now tension in the steel increases suddenly from the value of zero. This leads to increase in the tensile strain in the steel bars at the cracked section. This large increase of strain at the level of reinforcing steel results in shifting the neutral axis upwards.

It is to be noted that, because cracks appear in concrete at very low tensile stresses, it is quite conservative to ignore the tensile strength of concrete. Cl. B-1.3(b) of IS 456: 2000 states that all tensile stresses are taken up by the reinforcement and none by the concrete, except otherwise specifically permitted.

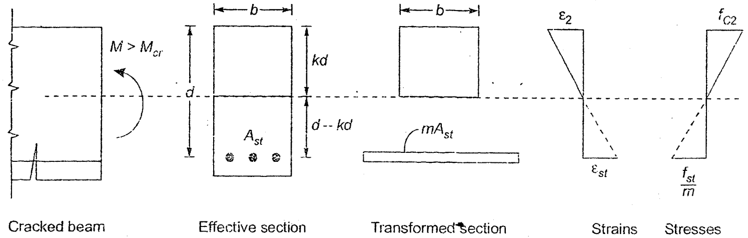


Fig. 3.3 *Cracked phase of concrete (with linear stress distribution)*

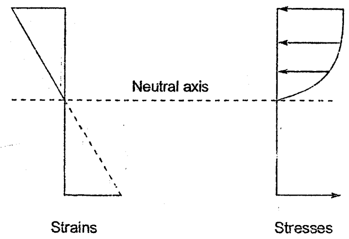


Fig. 3.4 *Cracked concrete with nonlinear stress distribution*

3.11 Location of Reinforcing Bars in Beam Section

In order to effectively resist the flexural tension due to the applied moment, the reinforcing bars must be placed as far from the neutral axis as possible provided the requirements of minimum cover to reinforcing bars and spacing of bars are satisfied.

3.12 Usefulness of Concrete in Tension Side

The concrete in the tension side is not at all useless but it serves many other important functions which are given below:

1. It holds the reinforcing bars in place.
2. It takes part in resisting the shear and torsion.
3. It increases the flexural stiffness of the beam i.e. it increases second moment of area (I) and thereby reduces deflections.
4. It protects the steel from fire and corrosion.

3.13 Permissible Stresses in Concrete and Steel

3.13.1 Permissible Stresses in Concrete in Tension

Cl. B-2.1.1 of IS 456: 2000 specifies permissible stress in direct tension for different grades of concrete. Although full tension is to be taken by reinforcement only, the actual tensile stress in concrete shall not exceed the respective permissible stresses in order to prevent cracks in concrete. The **factor of safety of concrete** in direct tension ranges from 8.5 to 9.5.

Concrete grade	M10	M15	M20	M25	M30	M35	M40	M45	M50
Tensile stress (N/mm^2)	1.2	2.0	2.8	3.2	3.6	4.0	4.4	4.8	5.2

3.13.2 Permissible Stresses in Concrete in Compression

Table 21 of IS 456: 2000 gives the values of permissible stresses in concrete in direct compression, bending compression.

Table 3.2: Permissible Stresses in Concrete (N/mm²)

Concrete grade	Permissible stress in compression	
	Bending (σ_{cbc})	Direct (σ_{cc})
M10	3.0	2.5
M15	5.0	4.0
M20	7.0	5.0
M25	8.5	6.0
M30	10.0	8.0
M35	11.5	9.0
M40	13.0	10.0
M45	14.5	11.0
M50	16.0	12.0

The factor of safety of concrete in bending compression, direct compression are taken as 3, 4 respectively.

3.13.3 Permissible Stress in Steel

Table 22 of IS 456: 2000 specifies permissible stresses in steel reinforcement for various steel grades, bar diameters and types of stress.

In the above table, FOS for steel = 1.8. (This is much lower than concrete due to better quality control during the production of steel)

Table 3.3: Permissible Stresses in Steel (N/mm²)

Type of stress	Mild steel (Fe 250)	HYSD (Fe 415)
Tension		
(i) Bar dia upto 20 mm	140	230
(ii) Bar dia greater than 20 mm	130	230
Compression	130	190

3.13.4 Increase in Permissible Stresses

Cl. B-2.3 of IS 456: 2000 recommends an increase in the permissible stresses in concrete and steel given in Table 21 and Table 22 up to a limit of 33.33%. This increase in permissible stresses is made where stresses due to wind loading, seismic forces, temperature loads, shrinkage effects etc. are combined with those due to dead loads, live loads and impact loads. Wind and seismic forces needn't be considered simultaneously.

3.13.5 Significance of Permissible Stress of Concrete in Direct Tension (σ_{td})

Concrete in not assumed to take any tension, the actual tensile stress in concrete is always tried to keep below the permissible stress of concrete in direct tension to avoid cracks in concrete. As stated earlier the FOS of concrete in direct tension ranges from 8.5 to 9.5.

3.14 Assumptions in the analysis of beams by working stress method (at service loads)

The working stress method is based on elastic theory of analysis and following assumptions are made as per Cl. B-1.3 of IS 456: 2000:

1. Plane sections remain plane before and after the bending.
2. All tensile stresses are taken up by steel reinforcement only.
3. Under working loads, the stress strain relationship of steel and concrete is linear.
4. The modular ratio (m) has the value $280/3\sigma_{cbc}$. Where, σ_{cbc} is the permissible compressive stress in concrete in flexure in N/mm² as per Table 21 of IS 456: 2000.

3.15 Design of Reinforced Concrete Structures

Design of reinforced concrete structures started purely on the basis of an empirical approach. After that linear elastic theory was proposed in which concrete and steel were assumed to be elastic. It was further assumed that the load deflection relationship is linear and obeys Hooke's law. This method is called as **working stress method** since the loads for design of structures are service loads or the working loads. However the failure of structure occurs at a much higher load. The ratio of failure load to working load is called as **factor of safety**.

i.e.,

$$FOS = \frac{\text{Failure load}}{\text{Working load}}$$

Thus stresses in concrete and steel are allowed not to exceed a particular value called as **permissible stresses**. As per Cl. B-2 of IS 456: 2000, permissible stress are determined by dividing the characteristics values by factor of safety i.e.,

$$\text{Permissible stress} = \frac{\text{Characteristic strength}}{\text{Factor of safety (FOS)}}$$

The value of factor of safety depends on:

(i) the grade/type of material (ii) the type of stress

For example: Permissible stress of concrete (M 20) in bending compression

$$\sigma_{cbc} = \frac{\text{Characteristic strength of M20 concrete}}{\text{FOS of concrete in bending compression}}$$

$$= \frac{20}{3} = 6.67 \simeq 7 \text{ N/mm}^2$$

Where, the subscript in σ_{cbc} implies "concrete in bending compression (*cbc*)"

3.16 Singly Reinforced Sections

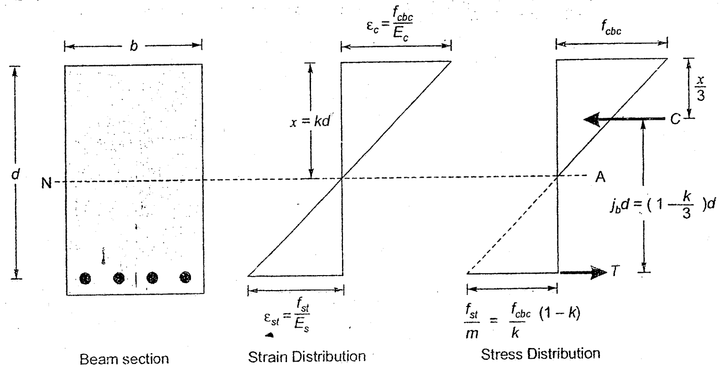


Fig. 3.5 *Singly reinforced beam section*

The above figure represents strain and stress distribution in a beam section subjected to pure flexure.

where, b = width of beam section

d = Effective depth of beam section

f_{cbc} = Actual stress in concrete in bending compression at the top fibre $\not> \sigma_{cbc}$

f_{st} = Actual stress in steel at the level of centroid of steel reinforcement $\not> \sigma_{st}$

$x = kd$ = depth of neutral axis from the top fibre.

k = Neutral axis factor

$jd = \left(1-\frac{k}{3}\right)d$ = Lever arm i.e., distance between the lines of action of compression (C) and tension (T)

Here, stress at the level of centroid of steel reinforcement is $\frac{f_{st}}{m}$ due to transformation of steel into an equivalent area of concrete (= mA_{st})

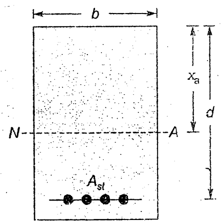
Natural Axis

Actual depths of NA is calculated by equating the moment of area on both sides of NA.

Equating moment of area

$$bx_a \cdot \frac{x_a}{2} = mA_{st}(d-x_a)$$

$$\frac{bx_a^2}{2} = mA_{st}(d-x_a)$$



Solving above equation will give x_a.

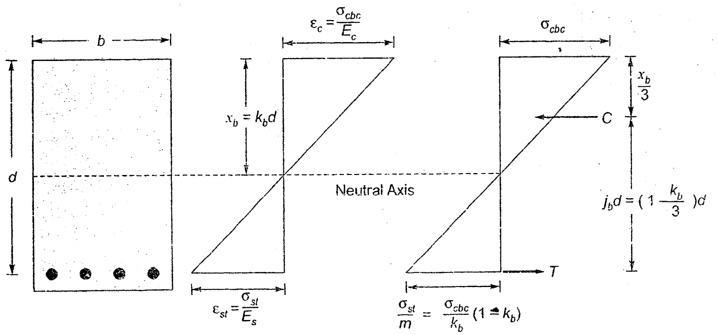
Actual depth of NA is used for

(i) Calculating stress in the section due to any given moment.

(ii) Calculating MOR of the section

3.16.1 Singly Reinforced Balanced Section

In balanced section, both f_{cbc} and f_{st} reach their permissible values of σ_{cbc} and σ_{st} respectively at the same time.



Section Strain Distribution Stress Distribution

From stress distribution diagram,

$$\frac{\sigma_{st}}{m} = \frac{\sigma_{cbc}}{k_b}(1-k_b), \quad \text{but} \quad m = \frac{280}{3\sigma_{cbc}}$$

$$\therefore \quad \frac{\sigma_{st} \cdot 3\sigma_{cbc}}{280} = \frac{\sigma_{cbc}}{k_b}(1-k_b)$$

$$\Rightarrow \quad k_b = \frac{280}{3\sigma_{st}+280} = \text{Neutral axis factor for balanced section.}$$

Thus the neutral axis for balanced section i.e., $x_b = k_b d$ is known as critical depth of NA.

Also Lever arm = $j_b d = \left(1-\frac{k_b}{3}\right)d$

Total compressive force = $C = \frac{1}{2}\sigma_{cbc}bx_b = \frac{1}{2}\sigma_{cbc}bk_b d$ (It acts at a distance of $x_b/3$ from top)

Total tensile force = $T = \sigma_{st}A_{st}$

Moment of resistance of balanced section

$$\therefore M_b \text{(with respect to compression)} = C(j_b d) = \frac{1}{2}\sigma_{cbc}bk_b d\left(1-\frac{k_b}{3}\right)d = \frac{1}{2}\sigma_{cbc}k_b j_b(d^2 b)$$

$$\text{and } M_b \text{(with respect to tension)} = T(j_b d) = \sigma_{st}A_{st}(j_b d) = \left(\frac{p_{t\,bal}}{100}\right)b\sigma_{st}j_b d^2$$

where, $p_{t\,bal} = \left(\frac{A_{st}}{bd}\right)100$ = Percentage of tensile steel for balanced section.

Moment of resistance is also expressed as,

$$M_b = R_b bd^2 \qquad \text{where, } R_b = \frac{1}{2}\sigma_{cbc}k_b j_b = \left(\frac{p_{t\,bal}}{100}\right)\sigma_{st}j_b$$

and lever arm factor $j_b = \left(1-\frac{k_b}{3}\right)$

Also $C = T$

For balanced section, $C = T$ gives,

$$A_{st}\sigma_{st} = \left(\frac{\sigma_{cbc}}{2}\right)bk_b d$$

$$\Rightarrow \quad \boxed{\left(\frac{A_{st}}{bd}\right) = \frac{\sigma_{cbc}k_b}{2\sigma_{st}}}$$

$$\Rightarrow \quad p_{t\,bal} = \left(\frac{A_{st}}{bd}\right)100 = \left(\frac{50\sigma_{cbc}k_b}{\sigma_{st}}\right)$$

3.16.2 Under Reinforced Section-Singly Reinforced

Owing to the fact that bar diameters available cannot meet the requirement of steel area required and thus it is not possible to design a balanced reinforced section. Then percentage of steel provided in beams is always kept below the value of '$p_{t\,bal}$' i.e., percentage of tension steel for balanced section.

Salient Features of Under-reinforced Sections

(i) $x_a < x_b$

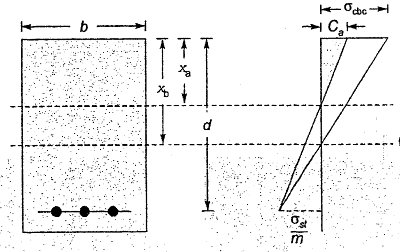
(ii) Failure of beam is due to failure of reinforcement therefore failure will be ductile.

(iii) Stresses in concrete are always less than the maximum permissible value.

(iv) This type of failure is preferred.

Calculation of MOR

$x_a < x_b,\ C_a < \sigma_{cbc},\ t_a = \sigma_{st}$



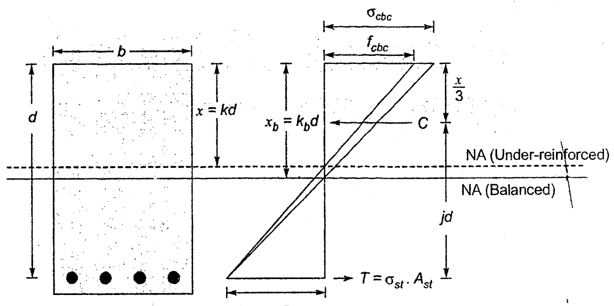
From compression side

$$MR = \frac{1}{2}Bx_aC_a\left(d - \frac{x_a}{3}\right)$$

where $$C_a = \frac{x_a\sigma_{st}}{(d - x_a)^m}$$

For tension side

$$MR = \sigma_{st}A_{st}\left(d - \frac{x_a}{3}\right)$$



$$\frac{\sigma_{st}}{m} = \frac{\sigma_{cbc}}{k_b}(1 - k_b)$$

$$= \frac{f_{cbc}}{k}(1 - k)$$

Equating the moment of compression area and tension area about NA

$$bkd\left(\frac{kd}{2}\right) = m\left(\frac{p_{tbal}}{100} \times bd\right)(d - kd)$$

$$\Rightarrow \quad k^2 + \left(\frac{p_tm}{50}\right)k - \left(\frac{p_tm}{50}\right) = 0$$

$$\therefore \quad k = -\left(\frac{p_tm}{100}\right) + \left[\left(\frac{p_tm}{100}\right)^2 + \left(\frac{p_tm}{50}\right)\right]^{1/2}$$

(Ignoring the negative value as 'k' cannot be negative)

Moment of resistance of under-reinforced section is,

$$M = C(\text{lever arm}) = T(\text{lever arm})$$

$$= \sigma_{st}A_{st}d\left(1 - \frac{k}{3}\right) = \left(\frac{p_tbd^2}{100}\right)\sigma_{st}\left(1 - \frac{k}{3}\right) = Rbd^2$$

where $$R = \left(\frac{p_t}{100}\right)\sigma_{st}\left(1 - \frac{k}{3}\right) = \text{Moment of resistance factor}$$

The actual stress in concrete in the top fibre (f_{cbc}) will not reach σ_{cbc} in an under-reinforced section and f_{cbc} is determined by equating compressive force of concrete (C) to tensile force of steel (T) i.e.,

$$C = T \qquad \text{where, } C = \left(\frac{1}{2}\right)f_{cbc}bkd$$

and $$T = \sigma_{st}A_{st}$$

Here the actual stress in steel 'f_{st}' will reach σ_{st}.

Thus $$\left(\frac{1}{2}\right)f_{cbc}bkd = \sigma_{st}A_{st} \quad \Rightarrow \quad f_{cbc} = \frac{2\sigma_{st}A_{st}}{bkd}$$

Expressing A_{st} as, $$A_{st} = \frac{p_t}{100}bd$$

We have, $$f_{cbc} = \frac{2\sigma_{st}}{bkd}\left(\frac{p_t}{100}bd\right) = \frac{\sigma_{st}p_t}{50k}$$

3.16.3 Over-reinforced Sections

Salient feature of over-reinforcement section:

(i) $x_a < x_b$

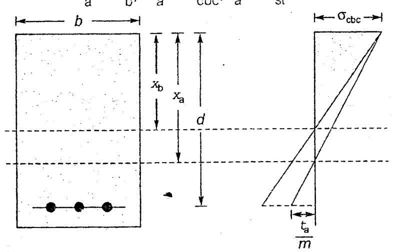
(ii) Failure of beam is due to failure of concrete therefore failure will be prittle failure.

(iii) Stress in steel is always than the maximum permissible value.

(iv) This type of sections are not preferred due to brittle failure.

Calculation of MOR

$x_a > x_b,\ C_a = \sigma_{cbc},\ t_a < \sigma_{st}$



From compression side

$$MR = \frac{1}{2} b x_a \sigma_{cbc}\left(d - \frac{x_a}{3}\right)$$

From tension side

$$MR = t_a A_{st}\left(d - \frac{x_a}{3}\right)$$

where $\quad t_a = \dfrac{(d - x_a) m \sigma_{cbc}}{x_a}$

3.17 Doubly Reinforced Beam Section

A doubly reinforced beam is required whenever size of beam is fixed and beam has to resist higher moment (i.e., more than that of MOR of singly reinforced balanced section).

- Modular ratio for compression steel = 1.5 m
- Critical depth of NA will be same as that of singly reinforced section.

3.17.1 Calculation of NA

$$\frac{b x_a^2}{2} + 1.5 m A_{sc}(x_a - d') - A_{sc}(x_a - d') = m A_{st}(d - x_a)$$

$$\frac{b x_a^2}{2} + (1.5m - 1) A_{sc}(x_a - d') = m A_{st}(d - x_a)$$

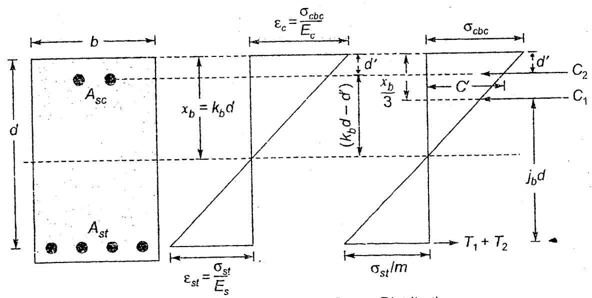
3.17.2 Calculation of MOR

MR = Moment resisted by concrete above NA + Moment resisted by compression reinforcement

$MR = C_1 (LA)_1 + C_2 (LA)_2$

$$MR = b x_a \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m - 1) A_{sc} C'(d - d_c)$$

NOTE: Hanger bars must not be confused with compression reinforcement bars. Hanger bars are provided to hold stirrups.



Section Strain Distribution Stress Distribution

3.17.3 Detailed Analysis of Doubly Reinforced Balanced Beam

$$M = M_b + M'$$

where, M' = Additional moment to be resisted by compression steel

$$A_{st} = A_{st1} + A_{st2}$$

where, A_{st2} = Additional tensile steel for compression steel (A_{sc})

Modular ratio for compression steel is taken as 1.5 m.

C_1 and T_1 are compressive and tensile forces corresponding to balanced section and

Now $\quad C = T$

$\Rightarrow \quad C_1 + C_2 = T_1 + T_2$

But $\quad C_1 = T_1;\ C_2 = T_2$

But $\quad T_2 = (p_t - p_{t\,bal})\dfrac{bd}{100}\sigma_{st}$

and $\quad C_2(k_b d - d') = C' \times (1.5m - 1) A_{sc}(k_b d - d') \qquad \left[C' = \dfrac{\sigma_{cbc}}{k_b d}(k_b d - d')\right]$

$\therefore \quad C_2(k_b d - d') = \dfrac{\sigma_{cbc}}{k_b d}(k_b d - d')(1.5m - 1) A_{sc}(k_b d - d')$

$$\Rightarrow \quad C_2 = \frac{A_{sc}(1.5m - 1)\sigma_{cbc}(k_b d - d')}{k_b d}$$

Additional moment, $\quad (M') = C_2(d - d') = \left(\dfrac{p_c bd}{100}\right)(1.5m - 1)\sigma_{cbc}\dfrac{(k_b d - d')}{k_b d}(d - d')$

$$= \left(\frac{p_c}{100}\right)(1.5m - 1)\sigma_{cbc}\left(1 - \frac{d'}{k_b d}\right)\left(1 - \frac{d'}{d}\right) bd^2$$

Also, $\quad M' = T_2(d - d')$

$$= (p_t - p_{t\,bal})\frac{bd}{100}\sigma_{st}(d - d') = \frac{(p_t - p_{t\,bal})}{100}\sigma_{st}\left(1 - \frac{d'}{d}\right) bd^2$$

From $\quad C_2 = T_2$ we have,

$$p_c(1.5m - 1)\sigma_{cbc}\left(1 - \frac{d'}{k_b d}\right) = (p_t - p_{t\,bal})\sigma_{st}$$

$\therefore \quad$ Total moment (M) $= M_b + M' = M_b + \dfrac{(p_t - p_{t\,lim})}{100}\sigma_{st}\left(1 - \dfrac{d'}{d}\right) bd^2$

$$A_{st} = A_{st1} + A_{st2} = \left(\frac{p_{t\,bal}}{100}\right) bd + \frac{M'}{\sigma_{st}(d - d')}$$

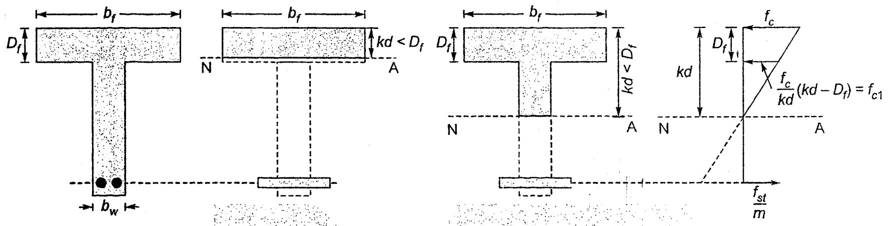
The compression steel (A_{sc}) is expressed in terms of additional tensile reinforcement (A_{st2}) as:

$$\frac{A_{sc}}{A_{st2}} = \frac{p_c}{p_t - p_{t\,bal}} = \frac{\sigma_{st}}{\sigma_{cbc}(1.5m - 1)\left(1 - \dfrac{d'}{kd}\right)}$$

Table 'M' of SP-16 gives the values of $\dfrac{A_{sc}}{A_{st2}}$ for different values of $\left(\dfrac{d'}{d}\right)$ and σ_{cbc} with true values of σ_{st} i.e., σ_{st} = 140 N/mm² and σ_{st} = 230 N/mm².

SP-16 gives values of p_t and p_c for various $\left(\frac{d'}{d}\right)$ values and different values of $\frac{M}{bd^2}$ for four concrete grades and two steel grades.

3.18 Singly Reinforced Flanged Section



(a) Section (b) NA lies in flange (c) NA lies in web (d) Stress distribution

The neutral axis may lie either in the flange ($kd \leq D_f$) or in the web ($kd > D_f$). When neutral axis (NA) lies in flange ($kd \leq D_f$), the flanged beam behaves like a rectangular beam only (since concrete on tension side of NA assumed to be ineffective) of width b_f i.e., width of flange and effective depth 'd'. Thus, this case can be analysed just like singly reinforced rectangular beams as described in earlier sections by just replacing 'b' by 'b_f'.

In other case, when NA lies in web ($kd > D_f$), the area of concrete in compression consists of flange area and a part of web area above the NA. The exact location of NA is determined by cracked section analysis of the transformed section. Equating moment of compression and tension area of cross-section about NA.

$$(b_f - b_w)D_f\left(kd - \frac{D_f}{2}\right) + b_w(kd)\left(\frac{kd}{2}\right) = mA_{st}(d - kd)$$

The above equation is true only if $kd > D_f$.

Now the concrete in compression consists of two opposite rectangular areas viz. $b_f(kd)$ and $[-(b_f - b_w)(kd - D_f)]$. Thus from $C = T$, we have

$$C = T$$

$$\Rightarrow \quad \frac{1}{2}(f_c)b_f(kd) - \frac{1}{2}f_{c1}(b_f - b_w)(kd - D_f) = f_{st}A_{st}$$

Here, f_{c1} is obtained from similar triangles as

$$f_{c1} = \frac{f_c}{kd}(kd - D_f)$$

Also moment about the tension steel is given by,

$$M = \frac{1}{2}(f_c)b_f(kd)\left(d - \frac{kd}{3}\right) - \frac{1}{2}f_{c1}(b_f - b_w)(kd - D_f)\left[(d - D_f) - \frac{(kd - D_f)}{3}\right]$$

As derived in earlier sections,

$$k_b = \frac{280}{280 + 3\sigma_{st}}$$

By comparing lever arm factor (k) with (k_b), the type of section can determined.

If $k < k_b$ then section is under-reinforced and $f_{st} = \sigma_{st}$ and $f_c < \sigma_{cbc}$

If $k > k_b$ then section is over-reinforced and $f_c = \sigma_{cbc}$ and $f_{st} < \sigma_{st}$

3.19 Doubly Reinforced Flanged Section

If neutral axis (NA) lies in flange ($kd \leq D_f$), then the section can be analysed just like doubly reinforced rectangular beam section of size ($b_f \times d$).

If NA lies in web ($kd > D_f$) then the section is analysed as per the following procedure. Equating moment of compression and tension area of cross-section about NA.

$$(b_f - b_w)D_f\left(kd - \frac{D_f}{2}\right) + b_w(kd)\left(\frac{kd}{2}\right) + (1.5m - 1)A_{sc}(kd - d') = mA_{st}(d - kd)$$

The net compressive force in concrete is given just as described in earlier section as:

$$C_c = \frac{f_c}{2}(b_f)(kd) - \frac{1}{2}(f_c)(b_f - b_w)\frac{(kd - D_f)^2}{kd} + (1.5m - 1)A_{sc}f_c\left(\frac{kd - d'}{kd}\right)$$

Moment of resistance of beam section is given by,

$$M = \left[\frac{1}{2}(f_c)b_f(kd)\left(d - \frac{kd}{3}\right) - \frac{1}{2}(f_c)(b_f - b_w)\frac{(kd - D_f)^2}{2}\left(d - D_f - \frac{kd - D_f}{3}\right)\right] + (1.5m - 1)A_{sc}f_c\frac{(kd - d')}{kd}(d - d')$$

3.20 Limitations of WSM of Design

Use of WSM of design is not only limited to concrete structures but earlier it was used for the design of timber and steel structures also. However, it has the following drawbacks:

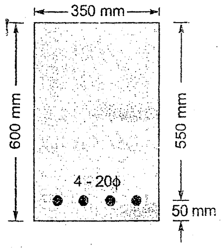
1. Assumption of linear elastic behaviour of concrete does not hold good. Also, the assumption of stresses in concrete and steel within the permissible limits is not true due to long term effects of creep and shrinkage which are not taken into account.
2. Actual factor of safety is not known precisely.
3. Uncertainties of various types of loads acting simultaneously is not being considered in this method.
4. It is NOT advantageous to use high strength deformed bars as compression reinforcement since the permissible stresses are relatively low and is independent of the grade of steel use as compression reinforcement.
5. In case where large moments are encountered, the area of compression steel (A_{sc}) may even exceed the area of tension steel (A_{st}).

Thus, WSM has been gradually replaced by LSM of design. In **IS 456: 2000**, the WSM has been placed in **Annexure - B** and gives greater emphasis on LSM of design. **Cl. 18.2.2 of IS 456: 2000** recommends to use WSM, where LSM cannot be easily applied. (e.g. in water retaining structures)

NOTE: Design of water retaining structures and tension structures are not covered by IS 456: 2000.

Example 3.1. A reinforced concrete beam of size 350 mm × 600 mm (effective cover of 50 mm) is made up of M 20 concrete and reinforced with 4-20 bars of Fe 415 steel. Calculate the cracking moment of the beam and stresses due to an applied moment of 55 kNm. (Take 'm' for M 20 concrete as 13.33)

Solution:



$$\text{Modular ratio } (m) = 13.33$$

$$\text{Modulus of rupture of M 20 concrete } (f_{cr}) = 0.7\sqrt{f_{ck}}$$

$$= 0.7\sqrt{20} = 3.13 \text{ N/mm}^2$$

$$\text{Section modulus } (Z) = \frac{bD^2}{6}$$

$$= \frac{350 \times 600^2}{6} = 21 \times 10^6 \text{ m}^3$$

$\therefore$ Cracking moment (M_{cr}) (using gross area) $= f_{cr}.Z = 3.13\,(21 \times 10^6)$ Nmm = 65.73 kNm

Transformed Section

$$\text{Area of tension steel } (A_{st}) = 4\frac{\pi}{4}(20)^2 = 1256.64 \text{ mm}^2$$

Now transformed area (A_T) = concrete area + transformed steel area

$$= (A_g - A_{st}) + mA_{st}$$
$$= A_g + (m-1)A_{st}$$
$$= bD + (m-1)A_{st}$$
$$= 350 \times 600 + (13.33 - 1)\,1256.64$$
$$= 225.5 \times 10^3 \text{ mm}^2$$

Depth of NA (y)

Taking moment of transformed area about the top edge

$$A_T y = (bD)\frac{D}{2} + (m-1)A_{st}d$$

$$\Rightarrow \quad y = \frac{\frac{(350)(600)^2}{2} + (13.33-1)1256.64(550)}{225.5 \times 10^3}$$

$$= 317.17 \text{ mm}$$

Thus distance of NA from topmost compression fibre = 317.17 mm = y_c

and distance of NA from tension steel = 550 − 317.17 = 232.83 mm = y_s

distance of NA from bottom most tension fibre = 600 − 317.17 = 282.83 mm = y_t

$\therefore$ Second moment of area/MOI of transformed section

$$= I_T = \frac{by_c^3}{3} + \frac{by_t^3}{3} + (m-1)A_{st}y_s^2$$

$$= \frac{350 \times 317.17^3}{3} + \frac{350 \times 282.83^3}{3} + (13.33-1)1256.64(232.83)^2$$

$$= 3722.4 \times 10^6 + 2639.5 \times 10^6 + 840 \times 10^6$$

$$= 7201.9 \times 10^6 \text{ mm}^4$$

$$\therefore \quad \text{Cracking moment } (M_{cr}) = f_{cr}\frac{I_T}{y_T} = 3.13 \times \frac{7201.9 \times 10^6}{282.83} \text{Nmm} = 79.7 \text{ kNm}$$

Thus cracking moment from gross area (= 65.73 kNm) is under estimated as compared to cracking moment using transformed area (= 79.7 kNm)

Stresses due to applied moment of 55 kNm

$$\text{Applied moment } (M) = 55 \text{ kNm} < M_{cr}$$

Thus uncracked section analysis can be done.

$\therefore$ Maximum compressive stress in concrete

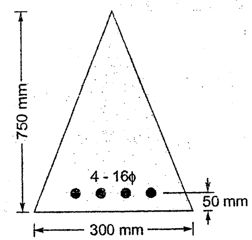
$$= f_c = \frac{M}{I_T}y_c = \frac{55 \times 10^6}{7201.9 \times 10^6} \times 317.17 = 2.42 \text{ N/mm}^2$$

$$\text{Maximum tensile stress in concrete} = \frac{M}{I_T}y_T = f_c\left(\frac{y_T}{y_c}\right) = 2.42\frac{(282.83)}{317.17}$$

$$= 2.16 \text{ N/mm}^2 < f_{cr} (= 3.13 \text{ N/mm}^2)$$

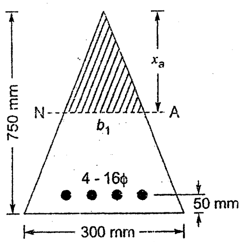
$$\text{Maximum tensile stress in steel} = f_{st} = mf_c\left(\frac{y_s}{y_c}\right) = 13.33(2.42)\frac{232.83}{317.17} = 23.68 \text{ N/mm}^2$$

Example 3.2 Calculate moment of resistance (MR) of the section as shown in figure below. Concrete grade is M20 and steel is Fe415.



Solution:

$\sigma_{cbc} = 7$ N/mm², $m = 13$, $\sigma_{st} = 230$ N/mm² and $d = 700$ mm



$$\text{Limiting depth of neutral axis,} \quad x_c = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} \times d = 198.4 \text{ mm}$$

Actual depth of neutral axis, x_a will be calculated from similar triangle as,

$$\frac{b_1}{x_a} = \frac{300}{750}$$

$$b_1 = 0.4\,x_a$$

Equating moment of areas of both the sides about NA

$$\frac{1}{2}b_1 x_a \frac{x_a}{3} = mA_{st}(d-x_a)$$

$$\frac{1}{2}0.4x_a\frac{x_a^2}{3} = 13\times 804.24(700-x_a) \qquad \left[\because A_{st} + \frac{\pi}{4}\times(16)^2\times 4 = 804.24\ \text{mm}^2\right]$$

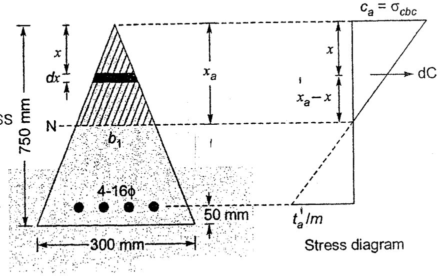
$$x_a = 371.48\ \text{mm} \approx 372\ \text{mm}$$

$$b_1 = 0.4\times 372\ \text{mm} = 148.8\ \text{mm}$$

$$x_a > x_c$$

It is an over reinforcement section.

$$\therefore \qquad t_a < \sigma_{st}$$



Let us consider an elementary strip of thickness 'dx' at a distance x from top.

Width of strip = $b_x = 0.40x$

Compressive force in strip = $dC = b_x dx C_x$

Now, from similar triangle in stress diagram

$$\frac{C_x}{(x_a-x)} = \frac{7}{x_a}$$

$$C_x = \frac{(x_a-x)}{x_a}\times 7 = \frac{372-x}{372}\times 7 = \frac{372-x}{53.14}$$

Moment of resistance of this elementary strip

$$dM_R = dC\times \text{lever arm} = dC(d-x) = b_x dx\left(\frac{372-x}{53.14}\right)\times(700-x)$$

∴Moment of resistance of the section

$$M_R = \int_0^{x_a} dM_R = \frac{1}{53.14}\int_0^{x_a} 0.4x(700-x)(372-x)dx = 33.19\ \text{kNm}$$

Example 3.3 Design a simply supported reinforced concrete beam using WSM of an effective span of 7.0 m. The beam is subjected to a live load of 42 kN/m. Width of the beam is 350 mm. Use M30 concrete and Fe500 steel.

Solution:

Given: Effective span (l) = 7.0 m

Width of the beam (b) = 350 mm

Assuming an initial depth of the beam as $\frac{\text{span}}{10} = \frac{7000}{10} = 700$ mm

Self weight of the beam = 0.7 × 0.35 × 25 kN/m = 6.12 kN/m

Live load = 42 kN/m (given)

Total load = 42 + 6.12 kN/m = 48.12 kN/m

Maximum bending moment for simply supported beam (M)

$$= \frac{wl^2}{8} = \frac{48.12\times 7^2}{8}\text{kNm} = 294.77\ \text{kNm}$$

Depth of the beam required from bending moment consideration

$$d = \sqrt{\frac{M}{Qb}}$$

Where, $Q = \frac{1}{2}cjk$

$$k = \frac{mc}{mc+t} = \frac{9\times 10}{9\times 10+275} = 0.246$$

$$j = 1-\frac{k}{3} = 0.918$$

$$Q = \frac{1}{2}\times 10\times 0.918\times 0.246 = 1.129 \simeq 1.13$$

$$\therefore \qquad d = \sqrt{\frac{294.76\times 10^6}{1.13\times 350}}\ \text{mm} = 863.3\ \text{mm}$$

D = 863.3 + 50 mm = 913.3 mm (Assuming effective cover 50 mm)

Adopt D = 950 mm

∴ Self weight of the beam = 0.95 × 0.35 × 25 kN/m = 8.31 kN/m

Total load = 42 + 8.31 kN/m = 50.31 kN/m

$$M = \frac{wl^2}{8} = 50.31\times\frac{7^2}{8} = 308.16\ \text{kNm}$$

$$d = \sqrt{\frac{308.16\times 10^6}{1.13\times 350}}\ \text{mm} = 882.7\ \text{mm}$$

Overall depth of the beam (D) = 882.7 + 50 mm = 932.7 mm < 950 mm (OK)

∴ Provide D = 950 mm, d = 900 mm

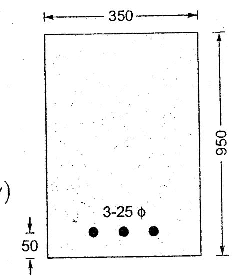
and b = 350 mm

Area of steel required

$$A_{st} = \frac{M}{\sigma_{st}\left(d-\frac{x_c}{3}\right)} = \frac{M}{\sigma_{st}(jd)}$$

$$= \frac{308.16\times 10^6}{275(0.918\times 900)} = 1356.31\ \text{mm}^2$$

Using 25 mm dia. bars, no. of bars required = $\frac{1356.31}{\frac{\pi}{4}\times 25^2} = 2.76 \simeq 3$ (say)



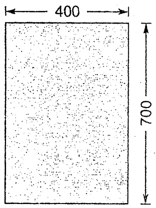
Provide 3-25 ϕ bars as tension reinforcement.

Example 3.4 An RCC rectangular beam of size 400 × 700 mm overall depth is to be designed for BM of (i) 120 kNm (ii) 260 kNm. Calculate area of steel required for above moments if M 25 concrete and Fe 415 steel are used. Calculate steel required just for balanced section. Assume effective cover as 50 mm.

Solution:

Given: M 25 and Fe 415

$$B = 400 \text{ mm}, \quad D = 700 \text{ mm}$$



Assume effective cover = 50 mm

$\therefore$ Effective depth, $d = 700 - 50 = 650$ mm

$$\sigma_{st} = 230 \text{ N/mm}^2, \sigma_{cbc} = 8.5 \text{ N/mm}^2, m = 11$$

Step-1: Moment of resistance of singly reinforced balanced section

$$Q = \frac{1}{2}cjk$$

$$k = \frac{mc}{mc+t} = \frac{11\times8.5}{11\times8.5\times230} = 0.289$$

$$J = 1-\frac{k}{3} = 0.903$$

$$Q = \frac{1}{2}\times8.5\times0.903\times0.289 = 1.109$$

$$MR = QBd^2 = 1.109 \times 400 \times 650^2 = 187.49 \text{ kNm}$$

Step-2: Area of steel for balanced section

$$A_{st} = \frac{MR_{bal}}{\sigma_{st}\times jd} = \frac{187.4\times10^6}{230\times0.903\times656} = 1389 \text{ mm}^2$$

Step-3: Design for BM = 120 kNm

$$BM < MR_{bal}$$

So we need a under reinforced section

Condition $\quad x_a < x_c,\ C_a < \sigma_{cbc},\ t_a = \sigma_{st} = 230 \text{ N/mm}^2$

Equating $\quad BM = MR$ (from tension side)

$$BM = \sigma_{st}A_{st}\left(d-\frac{x_a}{3}\right)$$

$$120\times10^6 = 230\times A_{st}\left(650-\frac{x_a}{3}\right) \quad \text{...(i)}$$

From compression side

$$BM = B\times x_a\times\frac{C_a}{2}\left(d-\frac{x_a}{3}\right)$$

$$120\times10^6 = 400\times x_a\times\frac{C_a}{2}\left(650-\frac{x_a}{3}\right) \quad \text{...(ii)}$$

$$\frac{C_a}{x_a} = \frac{t_a/m}{(d-x_a)}$$

$$C_a = \frac{t_a\times x_a}{m(d-x_a)} = \frac{230\times x_a}{11(650-x_a)} \quad \text{...(iii)}$$

From eq. (ii) and (iii),

$$120\times10^6 = 200\times x_a\times\frac{230\times x_a}{11(650-x_a)}\times\left(650-\frac{x_a}{3}\right)$$

$$(120\times10^6)\times11\,(650-x_a) = 200\times230\times x_a^2\left(650-\frac{x_a}{3}\right)$$

$$-1.32\times10^9 x_1 + 8.58\times10^{11} = 29.9\times10^6\times x_a^2 - 15333.34\,x_a^3$$

Solving by trial and error method

$$x_a = 154.2 \text{ mm}$$

Area of steel required

$$A_{st} = \frac{BM}{\sigma_{st}\left(650-\frac{x_a}{3}\right)} = \frac{120\times10^6}{230\times\left(650-\frac{154.2}{3}\right)} = 871.58 \text{ mm}^2$$

If balanced section formula is used

$$A_{st} = \frac{BM}{\sigma_{st}.(jd)} = \frac{120\times10^6}{230\times(0.903\times650)} = 888.8 \text{ mm}^2$$

In under reinforced section we can use the formula of balanced section because this difference is negligible.

NOTE: For beams, balanced section formula can be used for under reinforced section with some minor error.

Step-4: Design for $\quad BM = 260$ kNm

$$BM > MR_{bal}$$

So we need either an over reinforced section or a doubly reinforced section

Condition for over reinforced section

$$C_a = \sigma_{cbc}$$

$$x_a > x_c$$

$$t_a < \sigma_{st}$$

$$BM = MR$$

$$260\times10^6 = bx_a\frac{C_a}{2}\left(d-\frac{x_a}{3}\right) = 400\times x_a\times\frac{8.5}{2}\left(650-\frac{x_a}{3}\right)$$

$$152.94\times10^3 = 650x_a - \frac{x_a^2}{3}(0.33x_a^2)$$

$$x_a = 273.71 \text{ mm}$$

$$\Rightarrow \quad 260\times10^6 = t_a\times A_{st}\left(d-\frac{x_a}{3}\right)$$

$$A_{st} = \frac{260\times10^6}{t_a\times\left(650-\frac{273.71}{3}\right)}$$

Now, $$\frac{C_a}{x_a} = \frac{t_a/m}{d-x_a}$$

$$t_a = \frac{C_am(d-x_a)}{x_a} = \frac{8.5\times11\times(650-273.71)}{273.71} = 128.54 \text{ N/mm}^2$$

$$A_{st} = \frac{260\times10^6}{128.54\times\left(650-\frac{273.71}{3}\right)} = 3619 \text{ mm}^2$$

If balanced section formula is used

$$A_{st} = \frac{260\times10^6}{\sigma_{st}\times jd} = \frac{260\times10^6}{230\times6.903\times650} = 1925 \text{ mm}^2$$

This formula should never be used for over reinforced section.

Example 3.5 The size of a rectangular beam is 400 × 650 mm. Area of steel used

(i) 3 Nos – 16 mm ϕ bar

(ii) 6 Nos. – 25 mm ϕ bar

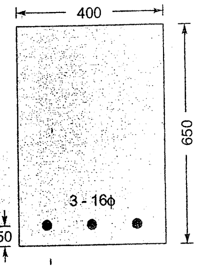
M 30 concrete and Fe 415 steel are used

Calculate MR of the section in two cases and area of steel required for the balanced section also. Effective cover is 50 mm.

Solution:

For Case (i) 3 Nos. 16 mm ϕ bar

$b = 400$ mm, $D = 650$ mm and $d = 650 - 50 = 600$ mm



$$A_{st} = 3 \times \frac{\pi}{4} \times 16^2 = 603.2 \text{ mm}^2$$

For M 30, $m = 9$, $C = 10$, $t = 230$

1. Critical depth of NA

$$x_c = kd = \frac{mc}{t + mc} \times d = \frac{9 \times 10}{230 + 9 \times 10} \times 600$$

$$= 168.75 \text{ mm}$$

2. Actual depth of NA
   Equating moment of area

$$\frac{bx_a^2}{2} = mA_{st}(d - x_a)$$

$$\frac{400 \times x_a^2}{2} = 9 \times 603.2\,(600 - x_a)$$

$$200\,x_a^2 = 3257280 - 5428.8\,x_a$$

$$x_a = 114.76 \text{ mm}$$

3. $x_a < x_c$, It is under reinforced

$$C_a < \sigma_{cbc}$$

$$t_a = \sigma_{st} = 230 \text{ N/mm}^2$$

4. Compressive stress in concrete

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} = \frac{\sigma_{st}/m}{d - x_a}$$

$$C_a = \left(\frac{\sigma_{st}/m}{d - x_a}\right)x_a = \frac{114.76}{(600 - 114.76)} \times \frac{230}{9} = 6.04 \text{ N/mm}^2$$

From compression side

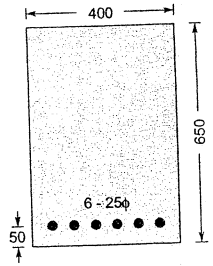
$$MR = bx_a \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) = 400 \times 114.76 \times \frac{6.04}{2}\left(600 - \frac{114.76}{3}\right) \times \frac{1}{10^6}$$

$$MR = 77.87 \times 10^6 = 77.87 \text{ kNm}$$

From tension side

$$MR = \sigma_{st} \cdot A_{st}\left(d - \frac{x_a}{3}\right) = 230 \times 603.2 \times \left(600 - \frac{114.76}{3}\right) \times \frac{1}{10^6} = 77.9 \text{ kNm}$$

For Case (ii) 6 Nos. 25 mmϕ bar



$$A_{st} = \frac{\pi}{4} \times 25^2 \times 6 = 2945.24 \text{ mm}^2$$

1. The value of critical depth of NA (x_c) = 168.75 mm
2. Actual depth of NA

$$b\frac{x_a^2}{2} = mA_{st}(x_a)$$

$$400\frac{x_a^2}{2} = 9 \times 2945.24\,(600 - x_a)$$

$$x_a = 223.41 \text{ mm}$$

3. $x_a > x_c$, It is an over reinforced section

$$C_a = \sigma_{cbc} = 10 \text{ N/mm}^2$$

$$t_a < \sigma_{st}$$

4. From compression side

$$MR = \frac{1}{2} bx_a C_a \times \left(d - \frac{x_a}{3}\right) = \frac{1}{2} \times 400 \times 223.41 \times 10\left(600 - \frac{223.41}{31}\right) \times \frac{1}{10^6}$$

$$MR = 234.8 \text{ kNm}$$

5. Tensile stress in steel

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a}$$

$$t_a = \frac{(d - x_a)\sigma_{cbc} m}{x_a} = \frac{(600 - 223.41) \times 10 \times 9}{223.41} = 151.7 \text{ N/mm}^2$$

From tension side

$$MR = t_a A_{st}\left(d - \frac{x_a}{3}\right) = 151.7 \times 2945.24 \times \left(600 - \frac{223.4}{3}\right) \times \frac{1}{10^6}$$

$$MR = 234.84 \text{ kNm}$$

For the case of balanced section

$$x_a = x_c$$

$$\sigma_{cbc} = C$$

$$t = \sigma_{st}$$

$$\Rightarrow \quad MR = bx_a \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) = 400 \times 168.75 \times \frac{10}{2}\left(600 - \frac{168.75}{3}\right)\frac{1}{10^6}$$

$$MR = 183.52 \text{ kNm}$$

Area of steel for balanced section

$$MR = t_a A_{st}\left(d - \frac{x_a}{3}\right)$$

$$183.52 \times 10^6 = 230 A_{st}\left(600 - \frac{168.75}{3}\right)$$

$$797936.4128 = 600 A_{st} - \frac{168.75}{3} A_{st}$$

$A_{st} = 1467.47\ mm^2$

Second method for area of steel

$C = T$ (valid in both WSM and LSM)

$$bx_a \frac{C_a}{2} = t_a . A_{st}$$

$$A_{st} = \frac{bx_a C_a}{2t_a} = \frac{400 \times 168.75 \times 10}{2 \times 230} = 1467.47\ mm^2$$

Third method for area of steel : Equating moment of area on both sides of neutral axis

$$b\frac{x_a^2}{2} = mA_{st}(d - x_a)$$

$$b\frac{x_c^2}{2} = mA_{st}(d - x_c)$$

(for balanced section)

$$A_{st} = \frac{bx_c^2}{2m(d - x_c)}$$

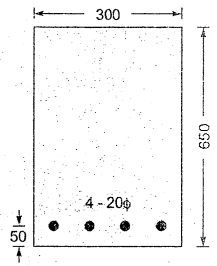
$$= 1467.4\ mm^2$$

Value	Under Reinforced	Balanced	Over Reinforced
Size	400 × 650 mm	400 × 650 mm	400 × 650 mm
A_{st}	603.2 mm^2	1467.69 mm^2	2945.24 mm^2
X_v	x_a = 114.76 mm	x_c = 168.75 mm	x_a = 223.4 mm
M_u	77.9 kNm	183.50 kNm	234.8 kNm
C_a	6.04 N/mm^2	10 N/mm^2	10 N/mm^2
t_a	$t = \sigma_{st}$ = 230 N/mm^2	$t = \sigma_{st}$ = 230 N/mm^2	t_a = 151.7 $N/mm^2 < \sigma_{st}$
Lever Arm	561.74 mm	543.75 mm	525.53 mm

Example 3.6 **A rectangular beam of the size 300 × 650 mm is reinforced with 4 no. 20 mmφ bars. Calculate stresses developed in steel and concrete due to BM of (i) 90 kNm (ii) 210 kNm (modular ratio method WSM). Modular ratio (m) = 15. Assume effective cover 50 mm.**

Solution:

$$\text{Area} = \frac{\pi}{4} \times 20^2 \times 4 = 1256.63\ mm^2$$



(i) For BM of 90 kNm

1. Actual depth of NA

$$b\frac{x_a^2}{2} = mA_{st}(d - x_a)$$

$$300\frac{x_a^2}{2} = 15 \times 4 \times \frac{\pi}{4}(20)^2(600 - x_a)$$

$$15x_a^2 = 11309670 - 18849.45x_a$$

$$x_a = 218.85\ mm$$

2. From compression side

$$BM = MR$$

$$90 \times 10^6 = bx_a \frac{C_a}{2}\left(d - \frac{x_a}{3}\right)$$

$$90 \times 10^6 = 300 \times 218.85 \times \frac{C_a}{2}\left(600 - \frac{218.85}{3}\right)$$

$$C_a = 5.2\ N/mm^2$$

From tension side

$$BM = t_a \times A_{st}\left(d - \frac{x_a}{3}\right)$$

$$90 \times 10^6 = t_a \times 1256.64 \times \left(600 - \frac{218.85}{3}\right)$$

$$t_a = 135.9\ N/mm^2$$

2nd method for t_a

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a}$$

$$t_a = \frac{mC_a(d - x_a)}{x_a} = 135.84\ N/mm^2$$

For BM of 210 kNm linear interpolation can be done for stress in concrete and steel.

For 90 kNm moment, concrete stress = 5.2 N/mm^2

∴ For 210 kNm moment, concrete stress = $\frac{5.2}{90} \times 210 = 12.13\ N/mm^2$

Similarly stress in steel = $\frac{135.9}{90} \times 210 = 317.1\ N/mm^2$

Example 3.7 **A simply supported rectangular beam of size 400 × 650 mm is to be provided for a simply supported span of 6.0 m. The beam has to support a live load of 55 kN/m (excluding self weight). Calculate area of steel required using WSM. Use M 30 concrete and Fe 500 steel. Effective cover = 50 mm.**

Solution:

$\sigma_{st} = 275\ N/mm^2$, $m = 9$, $\sigma_{cbc} = 10\ N/mm^2$, $b = 400$ mm, $D = 650$ mm and $d = 600$ mm

Span = 6 m

Live load = 55 kN/m

Self weight = 0.4 × 0.65 × 1 × 25 = 6.5 kN/m

Total load = 55 + 6.5 = 61.5 kN/m

1. Maximum Bending moment = $\frac{wl^2}{8} = \frac{61.5 \times 6^2}{8} = 276.75$ kNm

$$k = \frac{mc}{mc + t} = \frac{9 \times 10}{9 \times 10 + 275} = 0.246$$

$$j = 1 - \frac{k}{3} = 0.917$$

$$C = 10$$

$$Q = \frac{1}{2} \times cjk = \frac{1}{2} \times 10 \times 0.917 \times 0.246 = 1.128$$

2. Moment of resistance of singly reinforced balanced section

$MR_1 = Qbd^2 = 1.128 \times 400 \times 600^2 = 162.432$ kNm

Total BM > MR_1 (so we need a doubly reinforced section)

3. $$A_{st1} = \frac{MR_1}{\sigma_{st}(jd)} = \frac{162.432 \times 10^6}{275 \times (0.917 \times 200)} = 1073.5\ mm^2$$

4. Remaining BM, $MR_2 = BM - MR_1 = 276.75 - 162.6 = 114.15$ kNm

5. $$A_{st2} = \frac{MR_2}{\sigma_{st}(d - d_c)} = \frac{114.15 \times 10^6}{275 \times (600 - 50)} = 754.7\ mm^2$$

6. Total $A_{st} = A_{st_1} + A_{st2} = 1073.5 + 754.7 = 1828.2\ mm^2$

7. Value of compressive stress in concrete at the level of compression steel

$$C' = \frac{(x_a - d_c)}{x_a} \times C_a \qquad (\because C_a = 10)$$

$$= \frac{(x_c - d_c)}{x_c} \times C_a \qquad (\because x_c = kd)$$

$$= \frac{(0.246 \times 600 - 50)}{0.246 \times 600} \times 10 = 6.612 \text{ N/mm}^2$$

8. $$A_{sc} = \frac{MR_2}{(1.5m - 1)C'(d - d_c)} = \frac{114.15 \times 10^6}{(1.5 \times 9 - 1)6.612 \times (600 - 50)}$$

$$= 2511 \text{ mm}^2$$

Alternate formula for A_{sc}

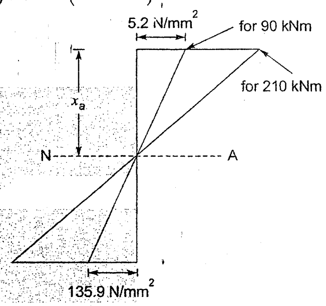
$$A_{sc} = (1.5m - 1)A_{sc}(x_a - d_c)$$

$$= mA_{st2}(d - x_a)$$

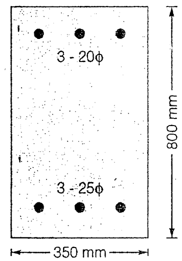
$$A_{sc} = \frac{mA_{st_2}(d - x_a)}{(1.5 \times 9 - 1)(x_a - d_c)}$$

$$= \frac{9 \times 754.7 \times (600 - 147.6)}{(1.5 \times 9 - 1) \times (147.6 - 50)}$$

$$= 2518 \text{ mm}^2$$



Example 3.8 Calculate the moment of resistance of a doubly reinforced beam as shown in figure using WSM. Use M 25 concrete and Fe 500 steel. Assume effective cover 50 mm.



Solution:

Given:

Width of the beam (b) = 350 mm

Overall depth of the beam (D) = 800 mm

Effective cover = 50 mm

Effective depth of the beam (d) = 800 – 50 mm = 750 mm

$$\text{Area of tension steel } (A_{st}) = 3 \times \frac{\pi}{4} \times 25^2 = 1472.62 \text{ mm}^2$$

$$\text{Area of compression steel } (A_{sc}) = 3 \times \frac{\pi}{4} \times 20^2 = 942.5 \text{ mm}^2$$

$$\text{Design coefficient, } k = \frac{mC}{mC + t} = \frac{11 \times 8.5}{11 \times 8.5 + 275} = 0.254$$

Limiting depth of neutral axis, $x_c = kd = 0.254 \times 750 \text{ mm} = 190.5 \text{ mm}$

Actual depth of neutral axis (x_a)

$$b\frac{x_a^2}{2} + (1.5m - 1)A_{sc}(x_a - d_c) = mA_{st}(d - x_a)$$

$$\frac{350}{2}x_a^2 + (1.5 \times 11 - 1) \times 942.5(x_a - 50) = 11 \times 1472.62(750 - x_a)$$

$$x_a = 197.19 \text{ mm} > x_c (= 190.5 \text{ mm})$$

∴ Section is over reinforced

$$C_a = \sigma_{cbc} = 8.5 \text{ N/mm}^2$$

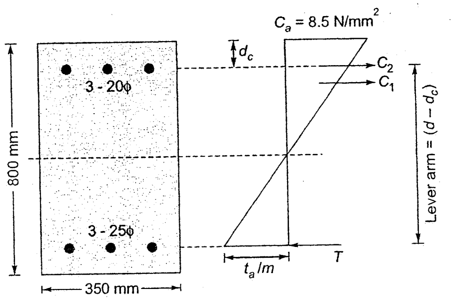
and $$t_a < \sigma_{st}$$

$$\frac{C'}{x_a - d_c} = \frac{C_a}{x_a}$$

$$C' = \frac{x_a - d_c}{x_a} \times C_a$$

$$= \frac{197.19 - 50}{197.19} \times 8.5$$

$$= 6.345 \text{ N/mm}^2$$



Moment of resistance

$$M_R = bx_a\frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m - 1)A_{sc}C'(d - d_c)$$

$$= 350 \times 197.19 \times \frac{8.5}{2}\left(750 - \frac{197.19}{3}\right) + (1.5 \times 11 - 1) \times 942.47 \times 6.344(750 - 50)$$

$$= 266.5 \text{ kNm}$$

Objective Brain Teasers

Q.1 Match the following

A. FOS of concrete in bending compression — I. 4

B. FOS of concrete in direct compression — II. 8.5 to 9.5

C. FOS of concrete in direct tension — III. 3

(a) A-I, B-II, C-III (b) A-I, B-III, C-II

(c) A-II, B-I, C-III (d) A-III, B-I, C-II

Q.2 The ratio of working load to ultimate load is equal to

(a) FOS (b) $(FOS)^{-1}$

(c) Load factor (d) None of the above

Q.3 As per IS 456: 2000 provisions, stresses in concrete and steel can be increased upto ________ when the structure is subjected to wind loading and seismic forces.

(a) 25% (b) 50%

(c) $33\frac{1}{3}$% (d) 10%

Q.4 The factor of safety for stress in steel is taken as

(a) 1.15 (b) 1.3

(c) 1.8 (d) 2.0

Q.5 Analysis of flanged beams can be done just like rectangular beam when

(a) $kd \le D_f$ (b) $kd > D_f$

(c) $kd \le b_f$ (d) $kd > b_f$

Q.6 Which of the following is true?

The modular ratio (m) expressed as $m = \frac{280}{3\sigma_{cbc}}$

(a) does not take into account the long term effects of creep and shrinkage

(b) partially takes into account the long term effects of creep and shrinkage

(c) is the ratio of modulus of elasticity of concrete to steel

(d) (b) and (c)

Q.7 The product '$m\sigma_{cbc}$' is

(a) a real variable (b) a complex variable

(c) a complex constant (d) a real constant

Q.8 Cracking moment is useful while analysing a beam section as

(a) cracked section

(b) partially cracked section

(c) uncracked section

(d) plastic section

Q.9 Which of the following statement is false?
The concrete on tension side of neutral axis is useful because

(a) it resists the applied moment in cracked phase of concrete
(b) it holds the reinforcing bars in place
(c) it prevents the bars from corrosion
(d) option (b) and (c)

Q.10 For concrete members subjected to axial compression, the permissible shear stress in concrete shall be increased by a factor of

(a) $1-\frac{5P}{A_g f_{ck}}$ (b) $1+\frac{5P}{A_g f_{ck}}$

(c) $1+\frac{3P}{A_g f_{ck}}$ (d) $\frac{5P}{A_g f_{ck}}$

Q.11 In WSM of design, modular ratio (M) is defined as $280/3\sigma_{cbc}$. The extent of creep allowance taken in this definition of modular ratio (M) is

(a) Partial compensation
(b) No compensation
(c) Full compensation
(d) Insufficient data

Answers

1. (d) 2. (b) 3. (c) 4. (c) 5. (a)
6. (b) 7. (d) 8. (c) 9. (d) 10. (b)
11. (a)

Hints:

2. (b)

$$FOS = \frac{\text{ultimate load}}{\text{working load}}$$

6. (b)

$$m = \frac{E_s}{E_c}$$

7. (d)

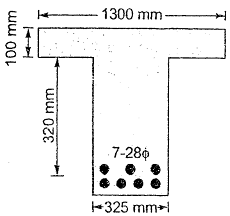
$$m\sigma_{cbc} = \frac{280}{3} = \text{real constant}$$

Conventional Practice Questions

Q.1 A beam caries a uniformly distributed service load of 38 kN/m including self weight on a simply supported span of 7.0 m. The beam width is 300 mm and effective depth is 655 mm and is reinforced with 4-25ϕ Fe 415 bars. Determine the stresses developed in concrete and steel at service load. Use M 20 concrete.

Ans. [10.4 N/mm^2, 209 N/mm^2]

Q.2 A T-beam section is as shown below. Using M20 concrete and Fe415 steel, determine the stresses developed in concrete and steel under a service moment of 150 kN-m.

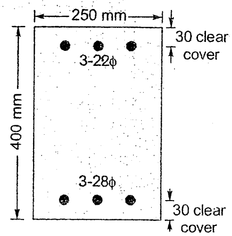


Ans. [4.3 N/mm^2, 93 N/mm^2]

Q.3 In Q.2 above, the allowable moment capacity of the beam at service load is _______

Ans. [244 kNm]

Q.4 A doubly reinforced beam section is as shown. Using M20 concrete and Fe415 steel, calculate the stresses in concrete and steel at a service moment of 125 kNm.



Ans. [11.7 N/mm^2, 170 N/mm^2, 218 N/mm^2]

Q.5 In Q.4 above, what is the allowable moment capacity of the section?

Ans. [74.5 kNm]