PHYSICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 36

Total Marks : 25

Max. Time : 25 min.

Topics : Circular Motion, Gravitation, Rigid Body Dynamics, Work, Power and Energy, Center of Mass, Electrostatics.

Type of Questions		**M.M., Min.**
Single choice Objective ('–1' negative marking) Q.1 to Q.4	**(12 marks, 12 min.)**	**[12, 12]**
Multiple choice objective ('–1' negative marking) Q.5	**(4 marks, 4 min.)**	**[4, 4]**
Comprehension ('–1' negative marking) Q.6 to Q.8	**(3 marks, 3 min.)**	**[9, 9]**

1. A particle is projected along a horizontal field whose coefficient of friction varies as $\mu = \frac{A}{r^2}$ where r is the distance from the origin in meters and A is a positive constant. The initial distance of the particle is 1 m from the origin and its velocity is radially outwards. The minimum initial velocity at this point so that particle never stops is :

(A) ∞ (B) $2\sqrt{gA}$ (C) $\sqrt{2gA}$ (D) $4\sqrt{gA}$

2. An automobile enters a turn of radius R. If the road is banked at an angle of 45^0 and the coefficient of friction is 1, the minimum and maximum speed with which the automobile can negotiate the turn without skidding is:

(A) $\sqrt{\frac{rg}{2}}$ and $\sqrt{rg}$ (B) $\frac{\sqrt{rg}}{2}$ and $\sqrt{rg}$

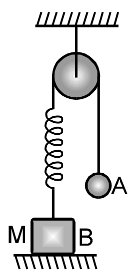
(C) $\frac{\sqrt{rg}}{2}$ and $2\sqrt{rg}$ (D) 0 and infinite

3. A hollow cylinder has mass M, outside radius R_2 and inside radius R_1. Its moment of inertia about an axis parallel to its symmetry axis and tangential to the outer surface is equal to :

(A) $\frac{M}{2}(R_2^2 + R_1^2)$ (B) $\frac{M}{2}(R_2^2 - R_1^2)$

(C) $\frac{M}{4}(R_2 + R_1)^2$ (D) $\frac{M}{2}(3R_2^2 + R_1^2)$

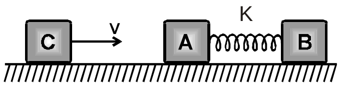
4. In the Figure, the ball A is released from rest when the spring is at its natural length. For the block B, of mass M to leave contact with the ground at some stage, the minimum mass of A must be:



(A) 2 M (B) M

(C) M/2 (D) A function of M and the force constant of the spring.

5. Two blocks A and B each of mass m are connected to a massless spring of natural length L and spring constant K. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length as shown in the figure. A third identical block C also of mass m moves on the floor with speed v along the line joining A and B and collides elastically with A, then :



(A) the K.E. of the A–B system at maximum compression of the spring is zero

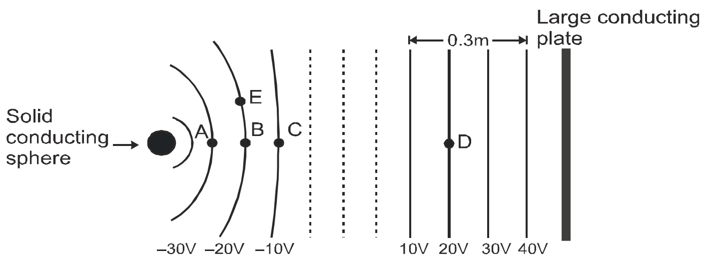
(B) the K.E. of the A–B system at maximum compression of the spring is $mv^2/4$

(C) the maximum compression of the spring is $v\sqrt{(m/K)}$

(D) the maximum compression of the spring is $v\sqrt{(m/2K)}$

COMPREHENSION

The sketch below shows cross-sections of equipotential surfaces between two charged conductors that are shown in solid black. Some points on the equipotential surfaces near the conductors are marked as A,B,C,......... . The arrangement lies in air. (Take ϵ_0 = 8.85 × 10^{-12} $C^2/N\,m^2$]



6. Surface charge density of the plate is equal to

(A) 8.85 × 10^{-10} C/m^2 (B) –8.85 × 10^{-10} C/m^2

(C) 17.7 × 10^{-10} C/m^2 (D) –17.7 × 10^{-10} C/m^2

7. A positive charge is placed at B. When it is released :

(A) no force will be exerted on it. (B) it will move towards A.

(C) it will move towards C. (D) it will move towards E.

8. How much work is required to slowly move a – 1μC charge from E to D ?

(A) 2 × 10^{-5} J (B) –2 × 10^{-5} J

(C) 4 × 10^{-5} J (D) –4 × 10^{-5} J

Answers Key

1.	(C)	**2.**	(D)	**3.**	(D)	**4.**	(C)
5.	(B)(D)	**6.**	(A)	**7.**	(B)	**8.**	(D)

Hints & Solutions

1. **(C)** Work done against friction must equal the initial kinetic energy.

$$\therefore \quad \frac{1}{2}mv^2 = \int_1^{\infty} \mu mg\,dx \quad ; \quad \frac{v^2}{2} = Ag\int_1^{\infty} \frac{1}{x^2}dx \ ;$$

$$\frac{v^2}{2} = Ag\left[-\frac{1}{x}\right]_1^{\infty}$$

$$v^2 = 2gA \qquad \Rightarrow v = \sqrt{2gA}$$

2. F.B.D. for minimum speed (w.r.t. automobile)

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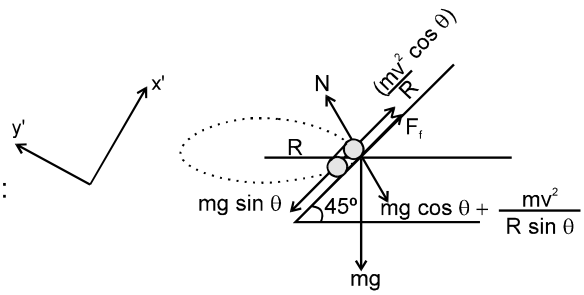


$$\Sigma f_{y'} = N - mg\cos\theta - \frac{mv^2}{R}\sin\theta = 0.$$

$$\Sigma f_{x'} = \frac{mv^2}{R}\cos\theta + \mu N - mg\sin\theta = 0$$

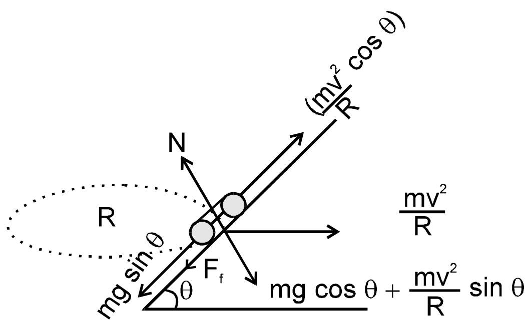
$$\Rightarrow \quad \frac{mv^2}{R}\cos\theta + \mu\left(mg\cos\theta + \frac{mv^2}{R}\right.$$

$$\left.\sin\theta\right) - mg\sin\theta = 0$$

$$\Rightarrow \quad v^2 = \frac{(\mu Rg\cos\theta - Rg\sin\theta)}{(\cos\theta + \mu\sin\theta)}$$

for $\theta = 45^\circ$ and $\mu = 1$:

$$v_{min} = \frac{Rg - Rg}{1+1} = 0$$



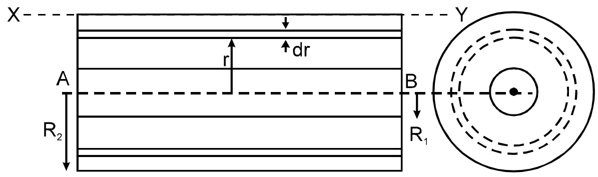
F.B.D for maximum speed (w.r.t. automobile)

$$\Sigma f_{x'} = \frac{mv^2}{R}\cos\theta - mg\sin\theta - \mu(mg\cos\theta$$

$$+ \frac{mv^2}{R}\sin\theta) = 0$$

for $\theta = 45^o$ and $\mu = 1$

$v_{max} = \infty$ (infinite)

3. 

Taking cylindrical element of radius r and thickness dr

$$dm = \frac{M}{\pi(R_2^2 - R_1^2)\ell} \times (2\pi r\,\ell\,dr)$$

$$I_{AB} = \int dI_{e\ell} = \int dmr^2 = \int_{R_1}^{R_2} \frac{2M}{(R_2^2 - R_1^2)}.r^3\,dr$$

$$= \frac{1}{2}m(R_2^2 + R_1^2)$$

Using parallel axis theorem

$$I_{XY} = \frac{1}{2}m(R_2^2 + R_1^2) + MR_2^2$$

4. *Let m be minimum mass of ball.*

Let mass A moves downwards by x.

From conservation of energy,

$$mgx = \frac{1}{2}kx^2$$

$$x = \left(\frac{2mg}{k}\right)$$

For mass M to leave contact with ground,

$$kx = Mg$$

$$K\left(\frac{2mg}{k}\right) = Mg$$

$$m = \frac{M}{2}.$$

5. In elastic collision the velocities are exchanged if masses are same.

$\therefore$ after the collision ;

$V_C = 0$

$V_A = v$

Now the maximum compression will occure when both the masses A and B move with same velocity.

$\therefore$ mv = (m + m) V (for system of A – B and spring)

$\therefore\ V = \frac{v}{2}$

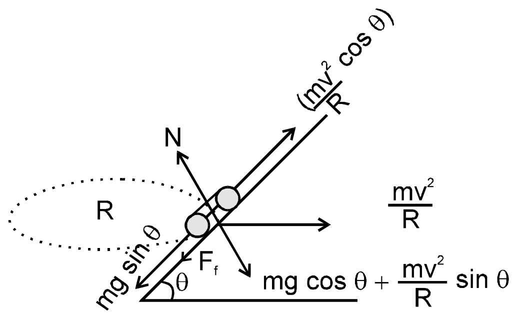
$\therefore$ KE of the A – B system $= \frac{1}{2} \times 2m\left(\frac{v}{2}\right)^2$

$= \frac{mv^2}{4}$

And at the time of maximum compression ;

$\frac{1}{2}mv^2 = \frac{1}{2} \times 2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}K\,X^2max$

$\therefore\ X_{max} = v\sqrt{\frac{m}{2K}}$

6. 

$E = \frac{40-10}{0.3} = 100$ V/m

(near the plate the electric field has to be uniform
$\therefore$ it is almost due to the plate).
For conducting plate

$E = \frac{\sigma}{\epsilon_0}$

$\Rightarrow\ \sigma = \epsilon_0 E$

Therefore :, $\sigma = 8.85 \times 10^{-12} \times 100$

$= 8.85 \times 10^{-10}$ C/m^2

7. Direction of E.F. at B is towards A that will exert force in this direction only, causing the positive charge to move. [$\vec{E}$ is perpendicular to equipotential surface and its direction is from high potential to low potential.]

8. W = q.dV
$= -1 \times 10^{-6}[20 - (-20)]$
$= -4 \times 10^{-5}$ J.