Sample Question Paper - 1
CLASS: XII
Session: 2021-22
Mathematics (Code-041)
Term - 1

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections - A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20.
3. Section - B has 20 MCQs, attempt any 16 out of 20
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. There is no negative marking.
6. All questions carry equal marks.

Section A

Attempt any 16 questions

1. The relation R defined on the set A ={1,2, 3, 4, 5} by R = {(a, b) : | $a^2 - b^2$ | < 16}, is given by **[1]**

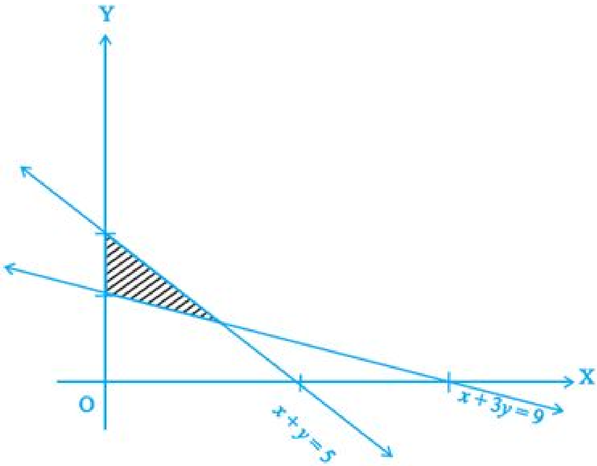
a) {(2, 2), (3, 2), (4, 2), (2, 4)}

b) R = {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3),(2, 2), (3, 2), (4, 2), (2, 4),(3, 3), (5, 4), (3, 4)}

c) none of these

d) {(3, 3), (4, 3), (5, 4), (3, 4)}

2. The feasible region for a LPP is shown in Figure. Find the maximum value of Z = 11x + 7y. **[1]**



a) 49

b) 48

c) 50

d) 47

3. Derivative of cos(sin x) w.r.t. sin x is **[1]**

a) – sin(sin x)cosx

b) $\sec^2\left(\frac{x}{\sqrt{x^2+1}}\right)$

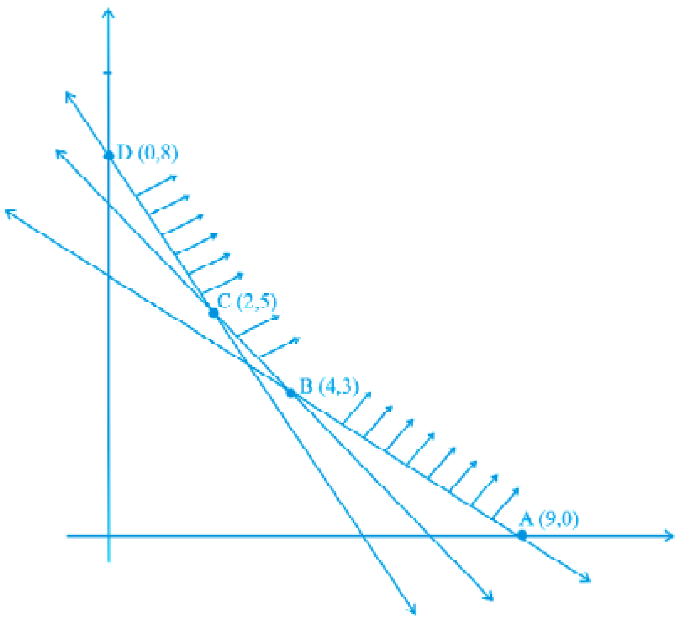
c) – sin(sin x)

d) None of these

4. If $2\begin{bmatrix}3 & 4\\ 5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 10 & 5\end{bmatrix}$ then **[1]**

a) (x = 2, y = 8)
b) (x = 3, y = -6)
c) (x = -3, y = 6)
d) (x = 2, y = -8)

5. Feasible region (shaded) for a LPP is shown in the Figure. Minimum of Z = 4x + 3y occurs at the point **[1]**



a) (4, 3)
b) (9, 0)
c) (0, 8)
d) (2, 5)

6. If y = $\tan^{-1}$ (sec x + tan x) then $\frac{dy}{dx} = ?$ **[1]**

a) None of these
b) $\frac{1}{2}$
c) 1
d) $\frac{-1}{2}$

7. For an invertible square matrix of order 3 with real entries $A^{-1} = A^2$, then det. A = **[1]**

a) $\frac{1}{3}$
b) 3
c) None of these
d) 1

8. The slope of the tangent to the curve x = a (cos θ + θ sin θ),y = a (sin θ – θ cos θ) at any point ‘θ’ is **[1]**

a) tan θ
b) – cot θ
c) – tan θ
d) cot θ

9. The feasible solution for an LPP is shown in Figure. Let Z = 3x – 4y be the objective function. Maximum value of Z occurs at **[1]**

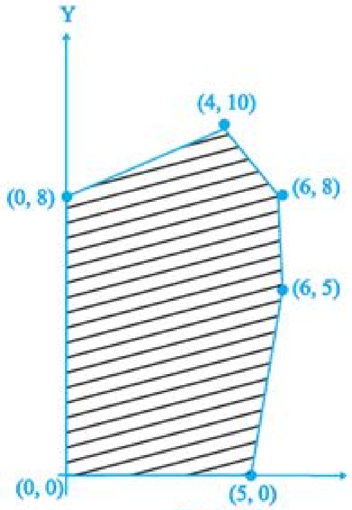


Fig. 12.12

a) (6, 8)
b) (6, 5)
c) (4, 10)
d) (5, 0)

10. If A = $\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$ then [1]

a) only BA is defined
b) only AB is defined
c) AB and BA both are not defined
d) AB and BA both are defined

11. The function f(x) $= \begin{cases} \frac{\sin x}{x} + \cos x, \text{ if } x \neq 0 \\ k \quad, \text{ if } x = 0 \end{cases}$ is continuous at x = 0, then the value of k is [1]

a) 1
b) 3
c) 1.5
d) 2

12. Which of the following statements is correct? [1]

a) A LPP admits unique optimal solution
b) Every LPP admits an optimal solution
c) If an LPP admits two optimal solutions it has an infinite number of optimal solutions
d) The set of all feasible solutions of a LPP is not a converse set

13. If the function f(x) = $kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then [1]

a) k > 3
b) k < 3
c) $k \geq 3$
d) $k \leq 3$

14. If $f(x) = \begin{cases} \frac{1}{1+e^{1x}} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ then f(x) is [1]

a) none of these
b) differentiable but not continuous at x = 0
c) continuous but not differentiable at x = 0
d) continuous as well as differentiable at x = 0

15. The function $f(x) = e^{|x|}$ **[1]**

a) not continuous at x = 0

b) continuous everywhere but not differentiable at x = 0

c) continuous and differentiable everywhere

d) None of these

16. The function f(x) = $\log_e\left(x^3+\sqrt{x^6+1}\right)$ is of the following types: **[1]**

a) even and increasing

b) odd and decreasing

c) even and decreasing

d) odd and increasing

17. The point on the curve y = x^2 - 3x + 2 where tangent is perpendicular to y = x is **[1]**

a) (1, 0)

b) (2, -2)

c) (0, 2)

d) (-1, 6)

18. $\cos^{-1}(\cos \frac{2\pi}{3}) + \sin^{-1}(\sin \frac{2\pi}{3})$ = ? **[1]**

a) π

b) $\frac{\pi}{3}$

c) $\frac{3\pi}{4}$

d) $\frac{4\pi}{3}$

19. If $f(x)=\frac{1-\sin x}{(\pi-2x)^2}$, when $x \neq \frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right)=\lambda$, then f(x) will be continuous function at $x=\frac{\pi}{2}$, where λ = **[1]**

a) 1/4

b) none of these

c) 1/2

d) 1/8

20. Tangents to the curve y= x^3 + 3x at x = -1 and x = 1 are **[1]**

a) intersecting at right angles

b) intersecting at an angle of 45^o.

c) intersecting obliquely but not at an angle of 45^o

d) parallel

Section B

Attempt any 16 questions

21. f : N $\rightarrow$ N : f(x) = x^2 + x + 1 is **[1]**

a) many-one and onto

b) one-one and onto

c) one-one and into

d) many-one and into

22. f(x) = 1 + 2 sin x + 3 $\cos^2$x, 0 $\leq$ x $\leq$ $\frac{2\pi}{3}$ is **[1]**

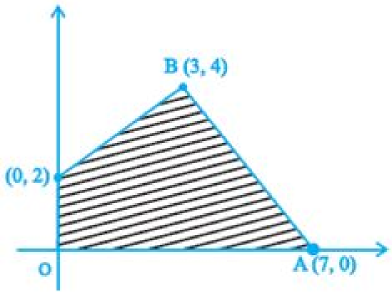
a) Minimum at x = $\frac{\pi}{2}$

b) Maximum at $\sin^{-1}(\frac{1}{6})$

c) Minimum at x = $\frac{\pi}{6}$

d) Maximum at x = $\sin^{-1}(\frac{1}{\sqrt{3}})$

23. Feasible region (shaded) for a LPP is shown in Figure. Maximize Z = 5x + 7y. **[1]**



a) 45

b) 49

c) 47

d) 43

24. The minimum value of $\left(x^2 + \frac{250}{x}\right)$ is **[1]**

a) 25

b) 55

c) 50

d) 75

25. Let $f(x) = |\sin x|$Then **[1]**

a) f is everywhere differentiable

b) f is everywhere continuous but not differentiable at x = (2x + 1)$\frac{\pi}{2}$, x $\in$ Z

c) None of these

d) f is everywhere continuous but not differentiable at $x = n\pi, n \in \mathbf{Z}$

26. The principal value of cosec^{-1} (2) is **[1]**

a) $\frac{2\pi}{3}$

b) $\frac{\pi}{3}$

c) $\frac{5\pi}{6}$

d) $\frac{\pi}{6}$

27. Let $f : R \rightarrow R$ be defined as f (x) = 3x. Choose the correct answer. **[1]**

a) many – one onto

b) neither one – one nor onto

c) one – one but not onto

d) one – one onto

28. The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is **[1]**

a) 0

b) $\frac{10\pi}{3}$

c) $\frac{\pi}{2}$

d) $\frac{5\pi}{3}$

29. The function f(x) = x^x decreases on the interval **[1]**

a) (0, e)

b) (0, 1)

c) (1/e,e)

d) (0 , $\frac{1}{e}$)

30. Which of the following is correct **[1]**

a) Determinant is a square matrix

b) None of these

c) Determinant is a number associated to a square matrix.

d) Determinant is a number associated to a matrix.

31. If $y^{1/n} + y^{-1/n} = 2x$, then $(x^2 - 1)\, y_2 + xy_1 =$ **[1]**

a) 0

b) none of these

c) n^2y

d) $-n^2y$

32. If $f(x)=\begin{cases}\frac{\sin(\cos x)-\cos x}{(\pi-2x)^2} & ,x\neq\frac{\pi}{2}\\ k & ,x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, then k is equal to **[1]**

a) 1
b) -1
c) 0
d) $\frac{1}{2}$

33. In the interval (1, 2), function f (x) = 2 |x - 1| + 3|x - 2| is **[1]**

a) not monotonic
b) constant
c) monotonically increasing
d) monotonically decreasing

34. $\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}$ is equal to **[1]**

a) None of these
b) $\frac{\pi}{2}$
c) $\frac{\pi}{4}$
d) $\frac{3\pi}{4}$

35. If $A=\begin{bmatrix}2x & 0\\ x & x\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$ then x = ?. **[1]**

a) 1
b) -2
c) $\frac{1}{2}$
d) 2

36. The value of objective function is maximum under linear constraints **[1]**

a) at (0, 0)
b) at any vertex of feasible region
c) the vertex which is maximum distance from (0, 0)
d) at the centre of feasible region

37. If $A=\frac{1}{3}\begin{bmatrix}1 & 1 & 2\\ 2 & 1 & -2\\ x & 2 & y\end{bmatrix}$ satisfies $A^TA = I$, then x + y = **[1]**

a) -3
b) none of these
c) 0
d) 3

38. If f(x) = $\frac{1}{4x^2+2x+1}$, then its maximum value is **[1]**

a) 1
b) $\frac{4}{3}$
c) $\frac{3}{4}$
d) $\frac{2}{3}$

39. If $y = ae^{mx} + be^{-mx}$, then y_2 is equal to **[1]**

a) my_1
b) $-m^2y$
c) m^2y
d) None of these

40. Let A = {1, 2, 3, 4, 5, 6}. Which of the following partitions of A correspond to an equivalence relation on A? **[1]**

a) none of these
b) {1, 2, 3}, {3, 4, 5, 6}.
c) {1, 2, }, {3, 4}, {2, 3, 5, 6}
d) {1, 3}, {2, 4, 5}, {6}

Section C

Attempt any 8 questions

41. The value of $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is: **[1]**

a) $\frac{1}{\sqrt{2}}$

b) $\frac{1}{3\sqrt{3}}$

c) $\frac{1}{2\sqrt{2}}$

d) $\frac{1}{\sqrt{3}}$

42. Minimize Z = 50x+60y , subject to constraints $x + 2y \leq 50$, $x + y \geq 30$, $x, y \geq 0$. **[1]**

a) 1800

b) 1550

c) 1700

d) 1200

43. Let $f(x + y) = f(x) + f(y)\ \forall\ x, y \in \mathbf{R}$. Suppose that f (6) = 5 and f ‘ (0) = 1, then f ‘ (6) is equal to **[1]**

a) 1

b) 30

c) None of these

d) 25

44. If A is a non singular matrix and A’ denotes the transpose of A, then **[1]**

a) $|AA'| \neq |A^2|$

b) None of these

c) $|A| + |A'| \neq 0$

d) $|A| \neq |A'|$

45. The void relation (a subset of $A \times A$) on a nonempty set A is: **[1]**

a) Reflexive

b) Transitive and symmetric

c) Only symmetric

d) Only transitive

Question No. 46 to 50 are based on the given text. Read the text carefully and answer the questions:

Manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below:

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
A	10,000	2,000	18,000
B	6,000	20,000	8,000

If the unit Sale price of Pencil, Eraser and Sharpener are ₹2.50, ₹1.50 and ₹1.00 respectively, and unit cost of the above three commodities are ₹2.00, ₹1.00 and ₹0.50 respectively, then,

46. Gross profit in both market: **[1]**

a) ₹23,000

b) ₹30,200

c) ₹32,000

d) ₹20,300

47. Total revenue of market A: **[1]**

a) ₹40,600
b) ₹60,400
c) ₹64,000
d) ₹46,000

48. Total revenue of market B: **[1]**

a) ₹35,000
b) ₹30,500
c) ₹50,300
d) ₹53,000

49. Cost incurred in market A: **[1]**

a) ₹10,300
b) ₹30,100
c) ₹13,000
d) ₹31,000

50. Profit in market A and B respectively are: **[1]**

a) (₹10,000, ₹20,000)
b) (₹51,000, ₹71,000)
c) (₹15,000, ₹17,000)
d) (₹17,000, ₹15,000)

Solution

Section A

1. **(c)** none of these

   Explanation: Given set A = {1, 2, 3, 4, 5} and relation R ={(a, b):| a^2-b^2| < 16}

   According to the condition |a^2-b^2|<16:
   ⇒ R = {(1, 1),(1,2), (2, 1), (1,3), (3, 1), (1,4), (4, 1), (2, 3),(2, 2), (3, 2), (4, 2), (2, 4),(3, 3), (4, 3), (5, 4), (3, 4), (4,4), (5,5)}.Which is the required solution.

2. **(d)** 47
   Explanation:

Corner points	Z = 11x + 7y
(0, 5)	35
(0,3)	21
(3,2)	47

   The maximum value is 47

3. **(c)** – sin(sin x)

   Explanation: Let y = cos(sinx), z = sinx , then , $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{-\sin(\sin x)\cos x}{\cos x} = -\sin(\sin x)$

4. **(d)** (x = 2, y = -8)
   Explanation: $2\begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$
   To solve this problem we will use the comparison that is we will use that all the elements of L.H.S. are equal to R.H.S.
   $= \begin{pmatrix} 6 & 8 \\ 10 & 2x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}$
   $= \begin{pmatrix} 7 & 8+y \\ 10 & 2x+1 \end{pmatrix}$
   Comparing with R.H.S.
   8 + y = 0
   y = -8
   2x + 1 = 5
   2x = 4
   x = 2

5. **(d)** (2, 5)
   Explanation: Z=4x+3y
   1. (0,8)=24
   2.(2,5)=8+15=23
   3.(4,3)=16+9=25
   4. (9,0)=36+0=36
   The correct answer is (2, 5) as it gives the minimum value.

6. **(b)** $\frac{1}{2}$

   Explanation: Given that y = $\tan^{-1}$ (sec x + tan x)
   Hence, $y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$
   Using $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta + \sin^2\theta = 1$

Hence, $y=\tan^{-1}\left(\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right)=\tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}\right)$

$\Rightarrow y=\tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$

Dividing by cos $\frac{x}{2}$ in numerator and denominator, we obtain

$y=\tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

Using $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+\tan x}{1-\tan x}$, we obtain

$y=\tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)=\frac{\pi}{4}+\frac{x}{2}$

Differentiating with respect to x, we

$\frac{dy}{dx}=\frac{1}{2}$

7. **(d)** 1

Explanation: $A^2 = I \Rightarrow A^2A^{-1} = IA^{-1} \Rightarrow A = A^{-1}$ and it is possible only if A is an identity matrix and determinant of the identity matrix is equal to 1

8. **(a)** tan θ

Explanation: Given $x=a(cos\theta+\theta sin\theta),\quad y=a(sin\theta–\theta cos\theta)$

$\frac{dx}{d\theta}=a\left[-sin\theta+\theta.cos\theta+sin\theta\right]=a\theta\cos\theta, \frac{dy}{d\theta}=a\left[cos\theta-(\theta.-\sin\theta+cos\theta)\right]=a\theta\sin\theta$

Slope of the tangent $=\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\theta sin\theta}{a\theta cos\theta}=tan\theta$

9. **(d)** (5, 0)

Explanation:

Corner points	Z = 3x - 4y
(0, 0)	0
(5,0)	15
(6,8)	-14
(6 , 5)	-2
(4,10)	-28
(0,8)	-32

The maximum value occurs at (5,0)

10. **(d)** AB and BA both are defined

Explanation: In given matrix

order of A = 2×3

order of B $=3\times2$

AB will be defined if the number of column in A is equal to the number of rows in B

so, $(A_{2\times3})(B_{3\times2})=AB_{2\times2}$

Similarly $(B_{3\times2})(A_{2\times3})=BA_{3\times3}$

Thus, Both AB and BA are defined.

11. **(d)** 2

Explanation: Since the given function is continuous,

$\therefore$ k = $\lim_{x\to0}\frac{Sinx}{x}+Cosx$

$\Rightarrow$ k = 1 + 1 = 2

12. **(c)** If an LPP admits two optimal solutions it has an infinite number of optimal solutions

Explanation: It is known that the optimal solution of an LPP either exists uniquely, does not exist or exists infinitely. So, If a LPP admits two optimal solution it has an infinite number of optimal solutions

13. **(a)** k > 3

Explanation: f(x) = $kx^3 - 9x^2 + 9x + 3$

f'(x) = $3kx^2 - 18x + 9$

= $3(kx^2 - 6x + 3)$

Given: f(x) is monotonically increasing in every interval.

$\Rightarrow$ f'(x) > 0

$\Rightarrow 3(kx^2 - 6x + 3) > 0$

$\Rightarrow (kx^2 - 6x = 3) > 0$

$\Rightarrow$ K > 0 and $(-6)^2 - 4(k)(3) < 0$ [$\because ax^2 + bx + c > 0$ and D is c < 0]

$\Rightarrow$ k > 0 and $(-6)^2 - 4(k)(3) < 0$

$\Rightarrow$ k > 0 and 36 - 12k < 0

$\Rightarrow$ k > 0 and 12k > 36

$\Rightarrow$ k > 0 and k > 3

$\Rightarrow$ k > 3

14. **(a)** none of these

Explanation: Given that $f(x) = \left\{ \begin{matrix} \frac{1}{1+e^{\frac{1}{x}}}, x \neq 0 \\ 0, x = 0 \end{matrix} \right\}$

Checking continuity at x = 0,

LHL: $\lim\limits_{x \to 0^-} \frac{1}{1+e^{\frac{1}{x}}} = 1$

But f(x = 0) = 0

Hence, function is neither continuous nor differentiable at x = 0

15. **(b)** continuous everywhere but not differentiable at x = 0

Explanation: Let $u(x) = |x|$ and v(x) = e^x

$\therefore$ f(x) = vof(x) = v[u(x)]

$= v|x| = e^{|x|}$

Since, u(x) and v(x) are both continuous functions.

So f(x) is also continuous function but u(x) = |x| is not differentiable at x = 0, whereas v(x) = e^x is differentiable at everywhere.

Hence, f(x) is continous everywhere but not differentiable at x = 0.

16. **(d)** odd and increasing

Explanation: odd and increasing

17. **(a)** (1, 0)

Explanation: We have

y = $x^2 - 3x + 2$

Slope of tangent,

$\frac{dy}{dx} = 2x - 3$

Tangent perpendicular to this line,

Slope of tangent

2x - 3 = -1

$\Rightarrow x = 1$

Now, y = 1 - 3 + 2

$\Rightarrow$ y = 0

Point is (1, 0)

18. **(a)** π

Explanation: The given equation is $\cos^{-1}(\cos \frac{2\pi}{3}) + \sin^{-1}(\sin \frac{2\pi}{3})$

Let us consider $\cos^{-1}(\cos(\frac{2\pi}{3}))$ ($\because$ the principle value of cos lies in the range [0, π] and since $\frac{2\pi}{3} \in [0, \pi]$]

$\Rightarrow \cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

Also, consider $\sin^{-1}(\sin(\frac{2\pi}{3}))$

Since here the principle value of sine lies in range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and since $\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \sin^{-1}(\sin(\frac{2\pi}{3})) = \sin^{-1}(\sin(\pi - \frac{\pi}{3}))$

$= \sin^{-1}(\sin(\frac{\pi}{3}))$

$= \frac{\pi}{3}$

Therefore,

$\cos^{-1}(\cos(\frac{2\pi}{3})) + \sin^{-1}(\sin(\frac{2\pi}{3})) = \frac{2\pi}{3} + \frac{\pi}{3}$

$= \frac{3\pi}{3}$

$= \pi$.

Which is the required solution.

19. **(d)** 1/8

Explanation: If f(x) is continuous at $x = \frac{\pi}{2}$ then

$$\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{(\pi-2x)^2} = f\left(\frac{\pi}{2}\right) \text{ ...(i)}$$

Now lets suppose

$\left(\frac{\pi}{2} - x\right)$ = t, then limit becomes

$$\lim_{t\to 0}\left[\frac{1-\sin\left(\frac{\pi}{2}-t\right)}{(2t)^2}\right] = f\left(\frac{\pi}{2}\right) \qquad \text{[from equation (i)]}$$

$$\Rightarrow \lim_{t\to 0}\left[\frac{1-\cos t}{4t^2}\right] = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{4}\lim_{t\to 0}\left[\frac{2\sin^2\left(\frac{t}{2}\right)}{t^2}\right] = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{4}\lim_{t\to 0}\left[\frac{\frac{2}{4}\sin^2\left(\frac{t}{2}\right)}{\frac{t^2}{4}}\right] = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{8}\lim_{t\to 0}\left[\frac{\sin^2\left(\frac{t}{2}\right)}{\frac{t^2}{4}}\right] = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{8}\lim_{t\to 0}\left[\frac{\sin\left(\frac{t}{2}\right)}{\frac{t}{2}}\right]^2 = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \lambda = \frac{1}{8}$$

20. **(d)** parallel

Explanation: Given $y = x^3 + 3x$

$$\Rightarrow \frac{dy}{dx} = 3x^2 + 3$$

Slope of tangent at x = 1 = 6 and

Slope of tangent at x = -1 = 6

Hence, the two tangents are parallel.

Section B

21. **(c)** one-one and into

Explanation: $f(x) = x^2 + x + 1$

One-one function

Let p, q be two arbitrary elements in N

Then, f(p) = f(q)

$\Rightarrow p^2 + p + 1 = q^2 + q + 1$

$\Rightarrow p^2 - q^2 + p - q = 0$

$\Rightarrow (p - q)(p + q + 1) = 0$

$\Rightarrow$ p = q, p + q + 1 $\neq$ 0 ($\because$ p, q $\in$ N)
When f(p) = f(q), p = q
thus, f(x) is one-one function.
Onto function
For x = 1, f(x) assumes value 3.
As, f(x) cannot assume value less than 3, for x $\in$ N
Thus, f(x) is not onto function. It is into function.

22. **(a)** Minimum at x = $\frac{\pi}{2}$

Explanation: f(x) = 1 + 2 sinx + 3cos^2 x
$\Rightarrow$ f'(x) = 2 cosx - 6 cosx sinx
$\Rightarrow$ f'(x) = 2 cosx - 3 sin2x
to find minima or maxima of the function
2 cosx - 6 cosx sinx = 0
2 cos x (1 - 3sinx) = 0
$\Rightarrow$ cos x = 0 or sinx = $\frac{1}{3}$
x = $\frac{\pi}{2}$ or x = sinx$^{-1}\left(\frac{1}{3}\right)$
f''(x) = - 2 sinx - 6 cos2x
f''$\left(\frac{\pi}{2}\right)$ = 4 > 0
$\Rightarrow$ x = $\frac{\pi}{2}$ is a local minima.
f''$\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$ = - $\left(\frac{2}{3} + 4\sqrt{2}\right)$ < 0
function has maxima at x = sin$^{-1}\left(\frac{1}{3}\right)$

23. **(d)** 43

Explanation:

Corner points	Z = 5x + 7y
O(0,0)	0
B (3,4)	43
A(7,0)	35
C(0,2)	14

Hence the maximum value is 43

24. **(d)** 75

Explanation: f(x) = x^2 + $\frac{250}{x}$
$\Rightarrow$f'(x) = 2x - $\frac{250}{x^2}$
For the local minima or maxima we must have
f'(x) = 0
2x - $\frac{250}{x^2}$ = 0
$\Rightarrow$ x = 5
f''(x) = 2 + $\frac{500}{x^3}$
f''(5) = 2 + $\frac{500}{125}$ > 0
function has minima at x = 5
f(5) = 75

25. **(d)** f is everywhere continuous but not differentiable at $x = n\pi, n \in \mathbf{Z}$

Explanation: We have, $f(x) = |\sin x|$
Let $f(x) = \mathrm{vou}(x) = v[u(x)]$[where, $u(x) = \sin x$ and $v(x) = |x|$]
$= v(\sin x) = |\sin x|$
Where, $u(x)$ and $v(x)$are both continuous.
Hence, $f(x) = vou(x)$is also a continuous function but $v(x)$is not differentiable at x = 0

So, f(x) is not differentiable where $\sin x = 0 \Rightarrow x = n\pi, \quad n \in Z$
Hence, f(x) is continuous everywhere but not differentiable at $x = n\pi, n \in Z$

26. **(d)** $\frac{\pi}{6}$
Explanation: Let the principle value be given by x
Now, let x = $\mathrm{cosec}^{-1}(2)$
$\Rightarrow$ cosec x = 2
$\Rightarrow$ cosec x = cosec $\left(\frac{\pi}{6}\right)$ $\left(\because \cos\left(\frac{\pi}{6}\right) = 2\right)$
$\Rightarrow x = \frac{\pi}{6}$

27. **(d)** one – one onto
Explanation: Injectivity: Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$. Then, $f(x_1) = f(x_2) \Rightarrow 3x_1 = 3x_2$ $\Rightarrow x_1 = x_2$. So, f : R $\rightarrow$ R is one –one.
Surjectivity: Let y $\in$ R, Then $f(x) = y \Rightarrow 3x = y \Rightarrow x = \frac{y}{3}$, Clearly, $\frac{y}{3} \in R$ *for any* $y \in R$ such that $f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$. So, Let f : $R \rightarrow R$ is onto.

28. **(a)** 0
Explanation: $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$
$= \cos^{-1}\left(\cos\left(2\pi - \frac{\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(2\pi - \frac{\pi}{3}\right)\right)$
$= \cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right) + \sin^{-1}\left(-\sin\left(\frac{\pi}{3}\right)\right)$
$= \cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right) - \sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$
$= \frac{\pi}{3} - \frac{\pi}{3}$
= 0

29. **(d)** (0 , $\frac{1}{e}$)
Explanation: (0 , $\frac{1}{e}$)
Let y = x^x
$\Rightarrow \log(y)$ = xlogx
$\Rightarrow \frac{1}{y} \times \frac{dy}{dx}$ = 1 + logx
$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$
Since the function is decreasing,
$\Rightarrow x^x x(1 + |\log x) < 0$
$\Rightarrow 1 + \log x < 0$
$\Rightarrow \log x < -1$
$\Rightarrow x < \frac{1}{e}$
Therefore, function is decreasing on (0,$\frac{1}{e}$)

30. **(c)** Determinant is a number associated to a square matrix.
Explanation: The determinant is an operation that we perform on arranged numbers. A square matrix is a set of arranged numbers. We perform some operations on a matrix and we get a value that value is called as a determinant of that matrix hence a determinant is a number associated to a square matrix.

31. **(c)** n^2y
Explanation: $y^{1/n} + y^{-1/n} = 2x$
Differentiating both sides we get
$\frac{y_1}{n}\left(y^{\frac{1}{n}-1} - y^{\frac{-1}{n}-1}\right) = 2$
$\Rightarrow y_1\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) = 2ny$
Again differentiating both sides we get
$\mathrm{y}_2\left(\mathrm{y}^{\frac{1}{\mathrm{n}}} - \mathrm{y}^{\frac{-1}{\mathrm{n}}}\right) + \frac{\mathrm{y}_1}{\mathrm{n}}\left(\mathrm{y}^{\frac{1}{\mathrm{n}}^{-1}} + \mathrm{y}^{\frac{-1}{\mathrm{n}}-1}\right) = 2\mathrm{ny}_1$
$\Rightarrow ny_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + \frac{y_1^2}{y}\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right) = 2n^2y_1$
$\Rightarrow nyy_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + 2xy_1^2 = 2n^2yy_1$

$\Rightarrow nyy_2 \frac{2ny}{y_1} + 2xy_1^2 = 2n^2yy_1$

$\Rightarrow \frac{n^2y^2y_2}{y_1^2} + xy_1 = n^2y$

$\Rightarrow y_2 \frac{\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right)^2}{4} + xy_1 = n^2y$

$\Rightarrow y_2 \frac{\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right)^2 - 4}{4} + xy_1 = n^2y$

$\Rightarrow y_2 \frac{4x^2-4}{4} + xy_1 = n^2y$

$\Rightarrow (x^2 - 1) y_2 + xy_1 = n^2y$

32. **(c)** 0

Explanation: Since, f is continuous at x = $\frac{\pi}{2}$

$\therefore$ f($\frac{\pi}{2}$) = $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sin(\cos x - \cos x)}{(\pi - 2x)^2}$

i.e. k = $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sin(\cos x - \cos x)}{(\pi - 2x)^2}$

Let x = $\frac{\pi}{2}$ - h,

$\Rightarrow$ k = $\lim\limits_{h \to 0} \frac{\sin(\cos(\frac{\pi}{2} - h) - \cos(\frac{\pi}{2} - h)}{\left(\pi - 2(\frac{\pi}{2} - h)\right)^2}$

= $\lim\limits_{h \to 0} \frac{\sin(\sin\ h) - \sin\ h}{4h^2}$

Using sin x = x - $\frac{x^3}{3!}$ + $\frac{x^5}{5!}$ + ...

$\Rightarrow$ k = $\lim\limits_{h \to 0} \frac{(\sin\ h - \frac{\sin^3 h}{3!} + \frac{\sin^5 h}{5!} ....) - \sin\ h}{4h^2}$

= $\lim\limits_{h \to 0} \left(\frac{-\sin^3 h}{3! \times 4h^2} + \frac{\sin^5 h}{5! \times 4h^2} \cdots\right)$

= 0

$\therefore$ $\lim\limits_{x \to \frac{\pi}{2}}$ f(x) = 0 = k

$\Rightarrow$ k = 0

33. **(d)** monotonically decreasing

Explanation: monotonically decreasing

34. **(c)** $\frac{\pi}{4}$

Explanation: $\tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{3}{4} \Rightarrow \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{2.\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}$

$= \tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{3}{4} \Rightarrow \tan^{-1}\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}.\frac{3}{4}} \Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$

35. **(c)** $\frac{1}{2}$

Explanation: We know that A $\times$ A^{-1} = I

$\begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 2\mathrm{x} \times 1 + 0 \times (-1) & 2\mathrm{x} \times 0 + 0 \times 2 \\ \mathrm{x} \times 1 + \mathrm{x} \times (-1) & \mathrm{x} \times 0 + \mathrm{x} \times 2\mathrm{x} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 2\mathrm{x} & 0 \\ 0 & 2\mathrm{x} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

To satisfy the above condition 2x = 1

$x = \frac{1}{2}$

36. **(b)** at any vertex of feasible region

Explanation: In linear programming problem we substitute the coordinates of vertices of feasible region in the objective function and then we obtain the maximum or minimum value. Therefore, the value of objective function is maximum under linear constraints at any vertex of feasible region.

37. **(b)** none of these

Explanation: We have, $A=\frac{1}{3}\begin{bmatrix}1 & 1 & 2\\ 2 & 1 & -2\\ x & 2 & y\end{bmatrix}$

$\Rightarrow A^T=\frac{1}{3}\begin{bmatrix}1 & 2 & x\\ 1 & 1 & 2\\ 2 & -2 & y\end{bmatrix}$

Now, A^TA = I

$\Rightarrow\begin{bmatrix}x^2+5 & 2x+3 & xy-2\\ 3+2x & 6 & 2y\\ xy-6 & 2y & y^2+8\end{bmatrix}=\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$

The corresponding elements of two equal matrices are not equal.

Thus, the matrix A is not orthogonal.

38. **(b)** $\frac{4}{3}$

Explanation: f(x) = $\frac{1}{4x^2+2x+1}$

$\Rightarrow$ f'(x) = 8x + 2

For local minima or maxima we have

f'(x) = 8x + 2 = 0

$\Rightarrow$ x = $\frac{-1}{4}$

f''(x) = 8 > 0

$\Rightarrow$ function has maxima at x = $\frac{-1}{4}$

$f\left(\frac{-1}{4}\right)=\frac{4}{3}$

39. **(c)** m^2y

Explanation: $y=ae^{mx}+be^{-mx}\Rightarrow y_1=ame^{mx}+(-m)be^{-mx}\Rightarrow y_2=am^2e^{mx}+(m^2)be^{-mx}$ $\Rightarrow y_2=m^2(ae^{mx}+be^{-mx})\Rightarrow y_2=m^2y$

40. **(d)** {1, 3}, {2, 4, 5}, {6}

Explanation: Conditions for the partition sub-sets to be an equivalence relation:

The partition sub-sets must be disjoint i.e.there is no common elements between them

Their union must be equal to the main set (super-set)

Here, the set A={1,2,3,4,5,6},the partition sub-sets {1,3},{2,4,5},{6} are pairwise disjoint and their union i.e. {1,3} U {2,4,5} U {6} = {1,2,3,4,5,6} = A, which is the condition for the partition sub-sets to be an equivalence relation of the set A.

Section C

41. **(c)** $\frac{1}{2\sqrt{2}}$

Explanation: $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let, $\sin^{-1}\frac{\sqrt{63}}{8}=x$

sin x $=\frac{\sqrt{63}}{8}$

cos x $=\sqrt{1-\sin^2 x}$

cos x $=\sqrt{1-\frac{63}{64}}$

cos x $=\frac{1}{8}$

Consider,

$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

$=\sin\left(\frac{1}{4}x\right)$

$= \sqrt{\frac{1-\cos\frac{x}{2}}{2}}$ $\left(\because \sin x = \frac{1-\cos 2x}{2}\right)$

$= \sqrt{\frac{1-\sqrt{\frac{1+\cos x}{2}}}{2}}$ $\left(\because \cos x = \frac{1+\cos 2x}{2}\right)$

$= \sqrt{\frac{1-\sqrt{\frac{1+\frac{1}{8}}{2}}}{2}}$

$= \sqrt{\frac{1-\frac{3}{4}}{2}}$

$= \sqrt{\frac{1}{8}}$

$= \frac{1}{2\sqrt{2}}$

42. **(c)** 1700

Explanation: Here , Maximize Z = 50x+60y , subject to constraints x +2 y ≤ 50 , x + y ≥ 30, x, y ≥ 0.

Corner points	Z = 50x +60 y
P(50 ,0)	2500
Q(0 , 30)	1800
R(10, 20)	1700

Hence the minimum value is 1700

43. **(a)** 1

Explanation: $f'(6) = \lim_{h\to 0} \frac{f(6+h)-f(6)}{h}$ $= \lim_{h\to 0} \frac{f(6+h)-f(6+0)}{h}$

$= \lim_{h\to 0} \frac{f(6)+f(h)-\{f(6)+f(0)\}}{h}$

$= \lim_{h\to 0} \frac{f(h)-f(0)}{h} = f'(0) = 1$

44. **(c)** |A| + |A'| $\neq$ 0

Explanation: Because, the determinant of a matrix and its transpose are always equal that is |A| = |A'|

45. **(b)** Transitive and symmetric

Explanation: The relation { }$\subset$ A $\times$ A on A is surely not reflexive. However, neither symmetry nor transitivity is contradicted. So { } is a transitive and symmetric relation on A.

46. **(c)** ₹32,000

Explanation: ₹32,000

47. **(d)** ₹46,000

Explanation: ₹46,000

48. **(d)** ₹53,000

Explanation: ₹53,000

49. **(d)** ₹31,000

Explanation: ₹31,000

50. **(c)** (₹15,000, ₹17,000)

Explanation: (₹15,000, ₹17,000)