

QNo 1 Find the distance between the following pairs of points:

(i) $(2, 3, 5)$ and $(4, 3, 1)$ (ii) $(-3, 7, 2)$ and $(2, 4, -1)$

(iii) $(-1, 3, -4)$ and $(1, -3, 4)$ (iv) $(2, -1, 3)$ and $(-2, 1, 3)$

Sol. (i) Let d be the distance between given points $(2, 3, 5)$ and $(4, 3, 1)$ Using distance formula

$$\therefore d = \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} = \sqrt{4+0+16} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

(ii) Let d be the distance between $(-3, 7, 2)$ and $(2, 4, -1)$

$$\therefore d = \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2} = \sqrt{25+9+9} = \sqrt{43}$$

(iii) Let d be the distance between the given points $(-1, 3, -4)$ and $(1, -3, 4)$

$$d = \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2} = \sqrt{4+36+64} = \sqrt{104} = \sqrt{4 \times 26} = 2\sqrt{26}$$

(iv) Let d be the distance between the points $(2, -1, 3)$ and $(-2, 1, 3)$

$$d = \sqrt{(-2-2)^2 + (-1-1)^2 + (3-3)^2} = \sqrt{16+0+0} = \sqrt{16} = 4.$$

QNo 2 : Show that the points $(-2, 3, 5)$, $(1, 2, 3)$ and $(7, 0, -1)$ are collinear.

Sol : Given points are $P(-2, 3, 5)$, $Q(1, 2, 3)$ and $R(7, 0, -1)$

$$|PQ| = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{9+1+4} = \sqrt{14}$$

$$|QR| = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{36+4+16} = \sqrt{56} = \sqrt{4 \times 14} = 2\sqrt{14}$$

$$|RP| = \sqrt{(-2-7)^2 + (3-0)^2 + (5+1)^2} = \sqrt{81+9+36} = \sqrt{126} = \sqrt{9 \times 14} = 3\sqrt{14}$$

$$\text{Now } PQ + QR = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14} = RP$$

$\therefore P, Q, R$ are collinear.

QNo 3 : Verify the following :

(i) $(0, 7, -10)$, $(1, 6, -6)$ and $(4, 9, -6)$ are vertices of an isosceles triangle.

(ii) $(0, 7, 10)$, $(1, 6, 6)$ and $(-4, 9, 6)$ are vertices of right angled triangle.

(iii) $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$ and $(2, -3, 4)$ are vertices of a parallelogram.

Sol (i) Let $A(0, 7, -10)$; $B(1, 6, -6)$ and $C(4, 9, -6)$ be given points

$$\therefore AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} = \sqrt{1+1+16} = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

$$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} = \sqrt{9+9+0} = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

$$CA = \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$$

$$\text{Now } AB = BC$$

$\therefore \Delta ABC$ is isosceles.

(i) Sol: Let $A(0, 7, 10)$, $B(-1, 6, 6)$, $C(-4, 9, 6)$ be given points.

$$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} = \sqrt{1+1+16} = \sqrt{18} = \sqrt{2 \times 9} = 3\sqrt{2}$$

$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} = \sqrt{9+9+0} = \sqrt{18} = \sqrt{2 \times 9} = 3\sqrt{2}$$

$$CA = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$$

Now $AB = BC$

$\therefore \Delta ABC$ is isosceles.

$$\text{Also } AB^2 + BC^2 = 18 + 18 = 36$$

$$\therefore AB^2 + BC^2 = CA^2$$

$\therefore \Delta ABC$ is right-angled at B.

(iii) Let $A(-1, 2, 1)$; $B(1, -2, 5)$, $C(4, -7, 8)$, $D(2, -3, 4)$ be given points.

$$AB = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$

$$BC = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2} = \sqrt{9+25+9} = \sqrt{43}$$

$$CD = \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$

$$DA = \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2} = \sqrt{9+25+9} = \sqrt{43}$$

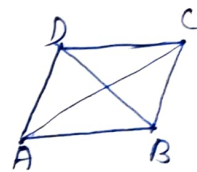
$$AC = \sqrt{(4+1)^2 + (-7-2)^2 + (8-1)^2} = \sqrt{24+81+49} = \sqrt{155}$$

$$BD = \sqrt{(2-1)^2 + (-3+2)^2 + (4-5)^2} = \sqrt{1+1+1} = \sqrt{3}$$

$\therefore AB = CD$, $BC = DA$ and $AC \neq BD$

ie Opposite sides are equal and diagonal are not equal

$\therefore ABCD$ is a parallelogram.



Q No. 4: Find the equation of the set of points which are equidistance from the points $(1, 2, 3)$ and $(3, 2, -1)$

Sol Let $P(x, y, z)$ be equidistance from the points $A(1, 2, 3)$ and $B(3, 2, -1)$

From given condition.

$$PA = PB \text{ or } PA^2 = PB^2$$

$$\therefore (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$$

$$\Rightarrow 4x - 8z = 0 \text{ or } x - 2z = 0$$

which is required equation.

Q No 5 : Find the equation of 'Set of points P, the sum of whose distances from A(4,0,0) and B(-4,0,0) is equal to 10.

Sol : Given points are A(4,0,0) and B(-4,0,0)
Let the coordinates of point P be (x,y,z)
From given condition.

$$PA + PB = 10$$

$$\therefore \sqrt{(x-4)^2 + (y-0)^2 + (z-0)^2} + \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2} = 10$$

$$\therefore \sqrt{x^2 - 8x + 16 + y^2 + z^2} + \sqrt{x^2 + 8x + 16 + y^2 + z^2} = 10$$

$$\text{or } \sqrt{x^2 - 8x + 16 + y^2 + z^2} = 10 - \sqrt{x^2 + 8x + 16 + y^2 + z^2}$$

Squaring both sides

$$x^2 + y^2 + z^2 - 8x + 16 = 100 + x^2 + y^2 + z^2 + 8x + 16 - 20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$-16x - 100 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\therefore 4x + 25 = 5\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

Again Squaring, we get

$$16x^2 + 625 + 200x = 25(x^2 + y^2 + z^2 + 8x + 16)$$

$$\Rightarrow -9x^2 - 25y^2 - 25z^2 = -225$$

$$\Rightarrow 9x^2 + 25y^2 + 25z^2 = 225$$

which is required equation.

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