04

HCF and LCM

HCF (Highest Common Factor)

HCF of two or more than two numbers is the product of highest or maximum number of common factors in the given numbers. It is also known as Greatest Common Divisor (GCD).

There are two methods to find the HCF, which are described as follows.

1. Prime Factorisation Method

To find the HCF using Prime factorisation, *following steps are taken*

Step I Break the given numbers into prime factors.

Step II Find the product of all the prime factors common to all the numbers.

Step III This product is the required HCF.

e.g. If we are to determine the HCF of 4, 8, 12.

First find the factors.

$$\begin{array}{c|c} 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$\therefore$ $4 = 2 \times 2$ $\quad 8 = 2 \times 2 \times 2$ $\quad 12 = 2 \times 2 \times 3$

$\therefore$ HCF = Product of the highest number of common factors in 4, 8, 12, i.e. $2 \times 2 = 4$

Example 1 Find the greatest number that will exactly divide 200 and 320. (by Prime factorisation method)

(a) 50 (b) 120

(c) 40 (d) 60

Sol. *(c)* $200 = 2 \times 2 \times 2 \times 5 \times 5$

$320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$

So, required number = 40

2. Division Method

To find the HCF using division method, *following steps are taken*

Step I Divide the larger number by smaller number.

Step II Divide the divisor by the remainder.

Step III Repeat Step II till the remainder becomes zero. Last divisor is the required HCF.

e.g. To determine the HCF of 72, 60, 96.

Start division method with the smaller two numbers and proceed as shown below, till the remainder became zero.

$$\begin{array}{l} 60)\ 72\ (1 \\ \quad\ \underline{60} \\ \quad\ 12)\ 60\ (5 \\ \qquad\quad \underline{60} \\ \qquad\quad \underline{\times} \end{array}$$

Since, there are more than two numbers, we will repeat the whole process with 12 as the divisor and 96 as the dividend. The last divisor will then be the required HCF of the three numbers.

$$\begin{array}{l} 12)\ 96\ (8 \\ \qquad \underline{96} \\ \qquad \underline{\times} \end{array}$$

$\therefore$ HCF of 60, 72 and 96 = 12

Example 2 Find the HCF of 26 and 455. (By division Method)

(a) 13 (b) 26 (c) 9 (d) 11

Sol. *(a)*

```
26)455(17
   26
   195
   182
    13)26(2
       26
       ×
```

$\therefore$ Required HCF = 13

LCM (Lowest Common Multiple)

LCM of two or more than two numbers is the product of the highest powers of all the prime factors that occurs in these numbers. In the other words, LCM of given numbers is the smallest number which is exactly divisible by each of them. It is also known as Lowest Common Dividend (LCD).

There are two methods to find the LCM, which are described as follows.

1. Prime Factorisation Method

In this method, following steps are used to find the LCM of given numbers.

Step I Break the given numbers into their prime factors.

Step II Find the product of highest powers of all the factors, which occur in the given numbers.

Step III This product is the required LCM.

e.g. To determine the LCM of 6, 8 and 12, find the prime factors of each number.

$$6 = 2 \times 3 = 2^1 \times 3^1$$
$$8 = 2 \times 2 \times 2 = 2^3$$
$$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$$

Here, highest powers of the factors 2 and 3 are 3 and 1, respectively.

$\therefore$ Required LCM $= 2^3 \times 3^1 = 8 \times 3 = 24$

Example 3 Find the LCM of 8, 12 and 15. (By prime factorisation Method)

(a) 140 (b) 130 (c) 120 (d) 125

Sol. *(c)* Factors of 8 $= 2 \times 2 \times 2$

Factors of 12 $= 2 \times 2 \times 3$

Factors of 15 $= 3 \times 5$

$\therefore$ Required LCM $= 2 \times 2 \times 2 \times 3 \times 5 = 120$

2. Division Method

In this method, following steps are used to find the LCM of the given numbers.

Step I Write down the given numbers in a line, separating them by commas.

Step II Divide by anyone of the prime numbers which exactly divides atleast two of the given numbers.

Step III Write down the quotients and the undivided numbers in the line below the first.

Step IV Repeat this process until you get a line of numbers which are prime to one-another.

Step V The product of all the divisors and the numbers in the last line is the required LCM.

e.g. LCM of 16, 24, 36 and 54 is determined as follows

2	16, 24, 36, 54
2	8, 12, 18, 27
3	4, 6, 9, 27
3	4, 2, 3, 9
2	4, 2, 1, 3
	2, 1, 1, 3

$\therefore$ Required LCM $= 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 3 = 432$

Example 4 What will be the LCM of 15, 24, 32 and 45?

(a) 1540 (b) 1640
(c) 1440 (d) 1340

Sol. *(c)* LCM of 15, 24, 32 and 45 is calculated as

2	15, 24, 32, 45
2	15, 12, 16, 45
2	15, 6, 8, 45
3	15, 3, 4, 45
5	5, 1, 4, 15
2	1, 1, 4, 3
	1, 1, 2, 3

$\therefore$ Required LCM $= 2 \times 2 \times 2 \times 3 \times 5 \times 2 \times 2 \times 3$

$= 1440$

LCM and HCF of Fractions

LCM of fractions = $\dfrac{\text{LCM of numerator}}{\text{HCF of denominator}}$

HCF of fractions = $\dfrac{\text{HCF of numerator}}{\text{LCM of denominator}}$

HCF × LCM = Product of numbers

Example 5 Find the LCM of $3\frac{1}{4}, \frac{3}{4}, 9\frac{3}{5}$ and $4\frac{6}{9}$.

(a) 4368 (b) 4598
(c) 4366 (d) 4359

Sol. *(a)* $4\frac{6}{9} = 4\frac{2}{3} = \frac{14}{3}, 9\frac{3}{5} = \frac{48}{5}, 3\frac{1}{4} = \frac{13}{4}$ and $\frac{3}{4}$

LCM of numerators 14, 48, 13 and 3

2	14, 48, 13, 3
3	7, 24, 13, 3
	7, 8, 13, 1

$\therefore$ LCM $= 2 \times 3 \times 7 \times 8 \times 13 \times 1 = 4368$

HCF of denominators 3, 5, 4, 4 = 1

We can clearly see that there is no factor which is common to all

$$\text{LCM} = \frac{\text{LCM of numerators}}{\text{HCF of denominator}} = \frac{4368}{1}$$

$\therefore$ Required LCM = 4368

Example 6 Find the HCF of $3\frac{24}{46}$ and $4\frac{18}{92}$.

(a) $\frac{2}{45}$ (b) $\frac{1}{46}$
(c) $\frac{5}{36}$ (d) $\frac{1}{35}$

Sol. *(b)* $3\frac{24}{46} = 3\frac{12}{23} = \frac{81}{23}$

$4\frac{18}{92} = 4\frac{9}{46} = \frac{193}{46}$

HCF of numerators 81 and 193 = 1

LCM of denominators 23 and 46

2	23, 46
23	23, 23
	1, 1

LCM of 23 and 46 = $2 \times 23 = 46$

$\therefore$ Required HCF $= \dfrac{\text{HCF of numerators}}{\text{LCM of denominators}} = \dfrac{1}{46}$

Example 7 The product of two numbers is 144 and the HCF of two numbers is 8. Find the LCM of the numbers.

(a) 18 (b) 15
(c) 19 (d) 12

Sol. *(a)* Product of numbers = LCM × HCF

$$\text{LCM} = \frac{\text{Product of numbers}}{\text{HCF}}$$

$\Rightarrow$ $\text{LCM} = \frac{144}{8} = 18$

Example 8 Find the greatest number of 4 digits such that they are exactly divisible by 12, 15, 20 and 35.

(a) 9880 (b) 9660 (c) 9980 (d) 9600

Sol. *(b)*

2	12, 15, 20, 35
2	6, 15, 10, 35
3	3, 15, 5, 35
5	1, 5, 5, 35
7	1, 1, 1, 7
	1, 1, 1, 1

LCM of 12, 15, 20 and 35
$= 2 \times 2 \times 3 \times 5 \times 7 = 420$

Greatest 4 digit number = 9999

Then,
```
420) 9999 (23
     840
     ----
     1599
     1260
     ----
      339
```

$\therefore$ Required number = 9999 − 339 = 9660.

Example 9 Find the least number which when divided by 27, 35, 45 and 49 leaves the remainder 6 in each case.

(a) 6615 (b) 6621 (c) 6651 (d) 6561

Sol. *(b)*

3	27, 35, 45, 49
3	9, 35, 15, 49
3	3, 35, 5, 49
5	1, 35, 5, 49
7	1, 7, 1, 49
7	1, 1, 1, 7
	1, 1, 1, 1

LCM of 27, 35, 45 and 49
$= 3 \times 3 \times 3 \times 5 \times 7 \times 7 = 6615$

$\therefore$ Required number = 6615 + 6 = 6621.

Practice Exercise

1. Find the HCF of 160, 165 and 305.
(a) 4 (b) 5 (c) 3 (d) 6

2. Find HCF of 3 kg and 400 g.
(a) 425 g (b) 625 g (c) 400 g (d) 200 g

3. Find the LCM of 28, 35, 56 and 84.
(a) 840 (b) 980 (c) 850 (d) 950

4. LCM of 24, 36 and 42.
(a) 504 (b) 144 (c) 196 (d) 256

5. HCF of the reciprocals of the fractions $\frac{3}{4}, \frac{9}{10}$ and $\frac{15}{16}$ is
(a) $\frac{3}{15}$ (b) $\frac{3}{80}$
(c) $\frac{1}{15}$ (d) $\frac{2}{45}$

6. Find the greatest number that will divide 1375 and 4935 exactly.
(a) 6 (b) 5 (c) 8 (d) 4

7. Find the least number which is divided by 12, 15, 20 and 25 exactly.
(a) 500 (b) 200
(c) 300 (d) 400

8. Five bells beginning together toll at interval of 4s, 5s, 7s, 8s and 10 s, respectively. After what interval of time will they toll again together?
(a) 2 min 40 s (b) 4 min 40 s
(c) 3 min 15 s (d) 5 min

9. Find the greatest possible length of a scale to measure exactly 6 m 25 cm, 9 m 50 cm and 12 m 25 cm.
(a) 25 cm (b) 50 cm (c) 75 cm (d) 65 cm

10. The HCF of two numbers is 38 and their LCM is 98154. If one of the number is 1558. Find the other number.
(a) 2304 (b) 2401 (c) 2290 (d) 2394

11. Find the smallest 4 digit number such that when it is divided by 12, 18, 21 and 28. It leaves remainder 3 in each case.
(a) 1101 (b) 1000 (c) 1011 (d) 1110

12. The HCF and the LCM of two numbers are 13 and 1989, respectively. If one of the numbers is 117, then find out the other number.
(a) 221 (b) 225 (c) 211 (d) 215

13. Find the smallest number which when divided by 25, 40, 60 leaves remainder 7 in each case.
(a) 600 (b) 607 (c) 508 (d) 916

14. The greatest number which divides 204 and 714 exactly is 34. Find the smallest number which is divisible by 204 and 714.
(a) 4200 (b) 4284 (c) 4004 (d) 4080

15. Find the least possible 5-digit number, which when divided by 10, 12, 16 and 18 leaves remainder 27.
(a) 10107 (b) 11007 (c) 11700 (d) 10170

Answers

1	(b)	2	(d)	3	(a)	4	(a)	5	(d)	6	(b)	7	(c)	8	(b)	9	(a)	10	(d)
11	(c)	12	(a)	13	(b)	14	(b)	15	(a)										

Hints & Solutions

1. HCF of 160, 165 and 305 is

```
160)165(1              5)305(61
    160                  30
    ---                  --
      5)160(32            5
        15                5
        --               --
         10               ×
         10
         --
          ×
```

∴ HCF = 5

Alternate Method

$160 = 2 \times 2 \times 2 \times 2 \times 2 \times 5$

$165 = 3 \times 5 \times 11$

$305 = 5 \times 61$

HCF of 160, 165 and 305 $= 1 \times 5 = 5$

∴ HCF = 5

2. HCF of 3 kg = 3 × 1000 = 3000 g and 400 g is [∵ 1 kg = 1000 g]

```
400)3000(7
    2800
    ----
     200)400(2
         400
         ---
          ×
```

∴ HCF of 3kg and 400g = 200 g

Alternate Method

$3000 = \boxed{2 \times 2 \times 2} \times 3 \times \boxed{5 \times 5} \times 5$

$400 = \boxed{2 \times 2 \times 2} \times 2 \times \boxed{5 \times 5}$

HCF of 3000 and 400 $= 2 \times 2 \times 2 \times 5 \times 5 = 200$

So, HCF of 3kg and 400g = 200 g

3. LCM of 28, 35, 56 and 84 is

2	28, 35, 56, 84
2	14, 35, 28, 42
2	7, 35, 14, 21
3	7, 35, 7, 21
5	7, 35, 7, 7
7	7, 7, 7, 7
	1, 1, 1, 1

∴ LCM of 28,35,56 and 84 $= 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$

4. LCM of 24,36 and 42 is

2	24, 36, 42
2	12, 18, 21
3	6, 9, 21
	2, 3, 7

LCM of 24, 36 and 42 $= 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 504$

5. Reciprocal of fractions $= \frac{4}{3}, \frac{10}{9}, \frac{16}{15}$

HCF of numerator = HCF of 4, 10 and 16 = 2

LCM of denominator = LCM of 3, 9, 15 = 45

Hence, HCF of fraction =

$$\frac{\text{HCF of numerator}}{\text{LCM of denominator}} = \frac{2}{45}$$

6. HCF of 1375 and 4935 is

```
1375)4935(3
     4125
     ----
      810)1375(1
           810
           ---
           565)810(1
               565
               ---
               245)565(2
                   490
                   ---
                    75)245(3
                       225
                       ---
                        20)75(3
                           60
                           --
                           15)20(1
                              15
                              --
                               5)15(3
                                 15
                                 --
                                  ×
```

∴ Required HCF = 5.

Alternate Method

$1375 = 5 \times \boxed{5} \times 5 \times 11$

$4935 = 3 \times \boxed{5} \times 329$

HCF of 1375 and 4935 $= 5 \times 1 = 5$

∴Required number is 5.

7. LCM of 12, 15, 20 and 25 is

2	12, 15, 20, 25
2	6, 15, 10, 25
3	3, 15, 5, 25
5	1, 5, 5, 25
5	1, 1, 1, 5
	1, 1, 1, 1

∴ LCM $= 2 \times 2 \times 3 \times 5 \times 5 = 300$

300 is the least number which is exactly divided by 12, 15, 20 and 25.

8. LCM of 4, 5, 7, 8 and 10 is

2	4, 5, 7, 8, 10
2	2, 5, 7, 4, 5
2	1, 5, 7, 2, 5
5	1, 5, 7, 1, 5
7	1, 1, 7, 1, 1
	1, 1, 1, 1, 1

LCM $= 2 \times 2 \times 2 \times 5 \times 7 = 280$

$\therefore \quad 280 \text{ s} = \dfrac{280}{60} = 4 \text{ min } 40 \text{ s}$

After 4 min and 40 s bells will toll together.

9. 6 m 25 cm = 625 cm = $6 \times 100 + 25 = 625$

9 m 50 cm = 950 cm

$= 9 \times 100 + 50 = 950$

[$\because$ 1 m = 100 cm]

12 m 25 cm = $12 \times 100 + 25 = 1225$

$625 = 5 \times \boxed{5 \times 5} \times 5$

$950 = 2 \times \boxed{5 \times 5} \times 19$

$1225 = \boxed{5 \times 5} \times 7 \times 7$

HCF of 625, 950 and 1225 $= 5 \times 5 = 25$

$\therefore$ The greatest possible length of scale to measure is 25 cm.

10. Second number $= \dfrac{\text{HCF} \times \text{LCM}}{\text{First number}} = \dfrac{38 \times 98154}{1558}$

$= 2394$

11. LCM of 12, 18, 21 and 28 = 252.

Smallest 4-digit number = 1000.

So,

$$\begin{array}{r|l|l} 252 & 1000 & 3 \\ & \underline{756} & \\ & \underline{244} & \end{array}$$

$\therefore$ The required number

$= 1000 + (252 - 244) + 3 = 1000 + 8 + 3 = 1011$

12. Second number $= \dfrac{13 \times 1989}{117} = \dfrac{25857}{117} = 221$

13.

5	25, 40, 60
2	5, 8, 12
2	5, 4, 6
	5, 2, 3

LCM $= 5 \times 2 \times 2 \times 5 \times 2 \times 3 = 600$

$\therefore$ Required number $= 600 + 7 = 607$

14. First number = 204

Second number = 714

HCF = 34

$\because$ Product of numbers = HCF $\times$ LCM

$\Rightarrow \quad 204 \times 714 = 34 \times \text{LCM}$

$\Rightarrow \quad \text{LCM} = \dfrac{145656}{34}$

$\therefore \quad \text{LCM} = 4284$

15. LCM of 10, 12, 16 and 18 = 720

Smallest 5-digit number = 10000

On, dividing 10000 by 720

$$\begin{array}{r|l|l} 720 & 10000 & 13 \\ & \underline{720} & \\ & 2800 & \\ & \underline{2160} & \\ & 640 & \end{array}$$

$\therefore$ Required number $= 10000 + (720 - 640) + 27$

$= 10000 + 80 + 27 = 10107$

Try Yourself

1) What will be the HCF of $(2 \times 3 \times 7 \times 9)$, $(2 \times 3 \times 9 \times 11)$ and $(2 \times 3 \times 4 \times 5)$?

(a) $2 \times 3 \times 7$ (b) $2 \times 3 \times 9$

(c) 2×3 (d) $2 \times 7 \times 9 \times 11$

2) LCM of 24, 36 and 48.

(a) 288 (b) 144 (c) 196 (d) 256

3) Find LCM of ₹ 2.40 and ₹ 3.20.

(a) ₹ 24 (b) ₹ 32 (c) ₹ 9.60 (d) ₹ 64

4) LCM of the reciprocals of the fractions $\dfrac{5}{4}, \dfrac{3}{10}$ and $\dfrac{13}{16}$ is

(a) 3/15 (b) 80 (c) 15 (d) 30

5) Find the smallest possible length of scale to measure exactly 64 cm, 80 cm and 96 cm.

(a) 9.60 m (b) 9.60 cm

(c) 7.20 m (d) 7.20 cm

6) Find the maximum number of students among whom 429 mangoes and 715 oranges can be equally distributed.

(a) 23 (b) 69 (c) 129 (d) 143

7) Find a number which is divisible by 15, 18 and 25 and is perfect square.

(a) 100 (b) 200 (c) 300 (d) 900

8) The LCM of two numbers is 630 and their HCF is 9. If one number is 90, then find the other number.

(a) 53 (b) 63 (c) 93 (d) 73

9) The HCF of two numbers is 96 and their LCM is 1296. If one number is 864. Find the other number.

(a) 64 (b) 148 (c) 144 (d) 196

10) A number of three digits when divided by 2, 5, 9, 11 leaves remainder 1 in each case. The number is

(a) 981 (b) 983 (c) 991 (d) 997

Answers

1	*(c)*	2	*(b)*	3	*(c)*	4	*(b)*	5	*(a)*
6	*(d)*	7	*(d)*	8	*(b)*	9	*(c)*	10	*(c)*