

INTEGRALS / EXERCISE 7.4 (NCERT)

Integrals of some Particular functions:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

$$3. \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$4. \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$5. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$6. \int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + C$$

QNo1 $\int \frac{3x^2}{x^6 + 1} dx = \int \frac{3x^2}{(x^3)^2 + 1} dx$

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$$\therefore I = \int \frac{dt}{t^2 + 1} = \int \frac{dt}{t^2 + 1^2} = \frac{1}{1} \tan^{-1} \frac{t}{1} + C$$

$$= \tan^{-1}(x^3) + C$$

QNo2 $\int \frac{1}{\sqrt{1+4x^2}} dx = \int \frac{1}{\sqrt{4\left(\frac{1}{4} + x^2\right)}} = \frac{1}{2} \int \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 + x^2}} dx$

$$= \frac{1}{2} \log \left(x + \sqrt{x^2 + \left(\frac{1}{2}\right)^2} \right) + C$$

$$= \frac{1}{2} \log \left(2x + \sqrt{4x^2 + 1} \right) + C' \quad \text{where } C' = C - \frac{1}{2} \log 2.$$

QNo3

$$\int \frac{1}{\sqrt{(2-x)^2+1}} dx = \int \frac{1}{\sqrt{(x-2)^2+1}} dx$$

$$\left[\because y^2 = (-y)^2 = y^2 \right]^2$$

$$= \log \left| (x-2) + \sqrt{(x-2)^2+1} \right| + C$$

QNo4

$$\int \frac{1}{\sqrt{9-25x^2}} dx = \int \frac{1}{\sqrt{25(\frac{9}{25}-x^2)}} dx$$

$$= \frac{1}{5} \int \frac{1}{\sqrt{\left(\frac{3}{5}\right)^2 - (x)^2}} dx = \frac{1}{5} \sin^{-1} \frac{x}{3/5} + C$$

$$= \frac{1}{5} \sin^{-1} \frac{5x}{3} + C$$

QNo5

$$\int \frac{3x}{1+2x^4} dx.$$

Sol.

$$\text{Put } x^2 = t \Rightarrow 2x dx = dt.$$

$$\therefore \int \frac{3 \cdot \frac{dt}{2}}{1+2t^2} = \frac{3}{2} \int \frac{1}{2\left(\frac{1}{2}+t^2\right)} dt$$

$$= \frac{3}{4} \int \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2 + (t)^2} dt = \frac{3}{4} \frac{1}{\frac{1}{\sqrt{2}}} \tan^{-1} \frac{t}{1/\sqrt{2}} + C$$

$$= \frac{3}{2\sqrt{2}} \tan^{-1}(\sqrt{2}x^2) + C$$

QNo6

$$\int \frac{x^2}{1-x^6} dx = \int \frac{x^2}{1-(x^3)^2} dx$$

$$\text{Put } x^3 = t \Rightarrow 3x^2 dx = dt$$

$$\therefore \int \frac{dt/3}{1-t^2} = \frac{1}{3} \int \frac{1}{1-t^2} dt$$

$$= \frac{1}{3} \cdot \frac{1}{2x} \log \left| \frac{1+t}{1-t} \right| + C$$

$$= \frac{1}{6} \log \left| \frac{1+x^3}{1-x^3} \right| + C$$

QNo 7

$$\int \frac{x-1}{\sqrt{x^2-1}} dx = \int \frac{x}{\sqrt{x^2-1}} dx - \int \frac{1}{\sqrt{x^2-1}} dx$$

Sol. $I = \int \frac{1}{2} (x^2-1)^{-\frac{1}{2}} \cdot 2x dx - \int \frac{1}{\sqrt{(x)^2-(1)^2}} dx$

$$= \frac{1}{2} \frac{(x^2-1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \log |x + \sqrt{x^2-1}| + C$$

$$= \frac{\sqrt{x^2-1}}{1} - \log |x + \sqrt{x^2-1}| + C$$

QNo 8

$$\int \frac{x^2}{\sqrt{x^6+a^6}} dx = \int \frac{x^2}{\sqrt{(x^3)^2+(a^3)^2}} dx$$

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$$\therefore I = \int \frac{dt/3}{\sqrt{t^2+(a^3)^2}} = \frac{1}{3} \log |t + \sqrt{t^2+a^6}| + C$$

$$= \frac{1}{3} \log |x^3 + \sqrt{x^6+a^6}| + C$$

QNo 9

$$\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$$

Sol.

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{t^2+4}} = \int \frac{dt}{\sqrt{t^2+2^2}} = \log |t + \sqrt{t^2+4}| + C$$

$$= \log |\tan x + \sqrt{\tan^2 x + 4}| + C$$

QNo 10

$$\int \frac{1}{\sqrt{x^2+2x+2}} dx = \int \frac{1}{\sqrt{x^2+2x+1+1}} dx = \int \frac{1}{\sqrt{(x+1)^2+1}} dx$$

$$= \log |(x+1) + \sqrt{(x+1)^2+1}| + C$$

$$= \log |x+1 + \sqrt{x^2+2x+2}| + C$$

QNo 11.
$$\int \frac{1}{9x^2+6x+5} dx = \frac{1}{9} \int \frac{1}{x^2 + \frac{6}{9}x + \frac{5}{9}} dx$$

$$= \frac{1}{9} \int \frac{1}{x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 + \frac{5}{9}} dx$$

$$= \frac{1}{9} \int \frac{1}{\left(x + \frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} dx = \frac{1}{9} \cdot \frac{1}{\frac{2}{3}} \tan^{-1} \frac{x + \frac{1}{3}}{\frac{2}{3}} + C$$

$$= \frac{1}{6} \tan^{-1} \left(\frac{3x+1}{2} \right) + C$$

QNo 12

$$\int \frac{1}{\sqrt{7-6x-x^2}} dx = \int \frac{1}{\sqrt{7-(x^2+6x)}} dx$$

$$= \int \frac{1}{\sqrt{7-(x^2+6x+(3)^2-(3)^2)}} dx = \int \frac{1}{\sqrt{7+9-(x+3)^2}} dx$$

$$= \int \frac{1}{\sqrt{(4)^2-(x+3)^2}} dx = \sin^{-1} \frac{x+3}{4} + C$$

QNo 13

$$\int \frac{1}{\sqrt{(x-1)(x-2)}} dx = \int \frac{1}{\sqrt{x^2-3x+2}} dx = \int \frac{1}{\sqrt{x^2-3x+\frac{9}{4}-\frac{9}{4}+2}} dx$$

$$= \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$$

$$= \log \left| x - \frac{3}{2} + \sqrt{\left(x-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right| + C$$

$$= \log \left| x - \frac{3}{2} + \sqrt{x^2-3x+2} \right| + C$$

QNo 14

$$\int \frac{1}{\sqrt{8+3x-x^2}} dx = \int \frac{1}{\sqrt{8-(x^2-3x)}} dx$$

$$= \int \frac{1}{\sqrt{8-(x^2-3x+\frac{9}{4}-\frac{9}{4})}} dx$$

$$= \int \frac{1}{\sqrt{\left(8+\frac{9}{4}\right) - \left(x-\frac{3}{2}\right)^2}} dx = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - \left(x-\frac{3}{2}\right)^2}} dx$$

$$= \sin^{-1} \left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}} \right) = \sin^{-1} \left(\frac{2x-3}{\sqrt{41}} \right) + C$$

QNo 15

$$\begin{aligned}\int \frac{1}{\sqrt{(x-a)(x-b)}} dx &= \int \frac{1}{\sqrt{x^2 - (a+b)x + ab}} dx \\&= \int \frac{1}{\sqrt{x^2 - (a+b)x + \left(\frac{a+b}{2}\right)^2 - \left(\frac{a+b}{2}\right)^2 + ab}} dx \\&= \int \frac{1}{\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2} dx \\&= \log \left| x - \frac{a+b}{2} + \sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2} \right| + C \\&= \log \left| \frac{2x-a-b}{2} + \sqrt{x^2 - (a+b)x + ab} \right| + C\end{aligned}$$

QNo 16

$$\begin{aligned}\int \frac{4x+1}{\sqrt{2x^2+x-3}} dx &= \int (2x^2+x-3)^{-\frac{1}{2}} \cdot \frac{d}{dx} (2x^2+x-3) dx \\&= \frac{(2x^2+x-3)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C \quad \left((f(x))^n f'(x) \text{ type} \right) \\&= 2 \sqrt{2x^2+x-3} + C\end{aligned}$$

QNo 17

$$\begin{aligned}\int \frac{x+2}{\sqrt{x^2-1}} dx &= \int \left(\frac{x}{\sqrt{x^2-1}} + \frac{2}{\sqrt{x^2-1}} \right) dx \\&= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx + 2 \int \frac{1}{\sqrt{x^2-1}} dx \\&= \frac{1}{2} \frac{(x^2-1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + 2 \log |x + \sqrt{x^2-1}| + C \\&= \sqrt{x^2-1} + 2 \log |x + \sqrt{x^2-1}| + C\end{aligned}$$

QNo 18

$$\begin{aligned}\int \frac{5x-2}{1+2x+3x^2} dx &= 5 \int \frac{x - \frac{2}{5}}{3x^2+2x+1} dx \\&= \frac{5}{6} \int \frac{6(x - \frac{2}{5})}{3x^2+2x+1} dx = \frac{5}{6} \int \frac{6x+2-2-\frac{12}{5}}{3x^2+2x+1} dx\end{aligned}$$

$$= \frac{5}{6} \int \frac{6x+2}{3x^2+2x+1} dx - \frac{5}{6} \times \frac{22}{5} \int \frac{dx}{3(x^2+\frac{2}{3}x+\frac{1}{3})}$$

$$= \frac{5}{6} \log |3x^2+2x+1| - \frac{11}{3} \times \frac{1}{3} \int \frac{dx}{x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}+\frac{1}{3}}$$

$$= \frac{5}{6} \log |3x^2+2x+1| - \frac{11}{9} \int \frac{dx}{(x+\frac{1}{3})^2 - (\frac{\sqrt{2}}{3})^2}$$

$$= \frac{5}{6} \log |3x^2+2x+1| - \frac{11}{9} \cdot \frac{1}{\sqrt{2}/3} \cdot \tan^{-1} \frac{x+\frac{1}{3}}{\sqrt{2}/3} + C$$

$$= \frac{5}{6} \log |1+2x+3x^2| - \frac{11}{3\sqrt{2}} \tan^{-1} \left(\frac{3x+1}{\sqrt{2}} \right) + C$$

QNo 19

$$\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx = \int \frac{6x+7}{\sqrt{x^2-9x+20}} dx = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}} dx$$

$$= 3 \int (x^2-9x+20)^{-\frac{1}{2}} (2x+9) dx + 34 \int \frac{1}{\sqrt{x^2-9x+20}} dx$$

$$= 3 \frac{(x^2-9x+20)^{-\frac{1}{2}+1}}{\frac{1}{2}} + 34 \int \frac{1}{\sqrt{x^2-9x+(\frac{9}{2})^2-(\frac{9}{2})^2+20}} dx$$

$$= 6\sqrt{x^2-9x+20} + 34 \int \frac{1}{\sqrt{(x-\frac{9}{2})^2-(\frac{1}{2})^2}} dx$$

$$= 6\sqrt{x^2-9x+20} + 34 \log \left| x - \frac{9}{2} + \sqrt{(x-\frac{9}{2})^2 - \frac{1}{4}} \right| + C$$

$$= 6\sqrt{x^2-9x+20} + 34 \log \left| \frac{2x-9}{2} + \sqrt{x^2-9x+20} \right| + C$$

QNo 20.

$$\int \frac{x+2}{\sqrt{4x-x^2}} dx = \int \frac{-\frac{1}{2}(-2x+4)+4}{\sqrt{4x-x^2}} dx$$

$$= -\frac{1}{2} \int (4x-x^2)^{-\frac{1}{2}} (-2x+4) + 4 \int \frac{dx}{\sqrt{4x-x^2+4-4}}$$

$$= -\frac{1}{2} \frac{(4x-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + 4 \int \frac{dx}{\sqrt{4-(x-2)^2}}$$

$$= -\sqrt{4x-x^2} + 4 \sin^{-1}\left(\frac{x-2}{2}\right) + C.$$

QNo 21 $\int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \int \frac{\frac{1}{2}(2x+2)+1}{\sqrt{x^2+2x+3}} dx.$

$$= \frac{1}{2} \int \frac{2x+2}{\sqrt{x^2+2x+3}} + \int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \frac{1}{2} \int (x^2+2x+3)^{-1/2} (2x+2) dx + \int \frac{dx}{\sqrt{x^2+2x+1+2}}$$

$$= \frac{1}{2} \frac{(x^2+2x+3)^{1/2}}{1/2} + \int \frac{dx}{\sqrt{(x+1)^2+2}}$$

$$= \sqrt{x^2+2x+3} + \log |x+1+\sqrt{(x+1)^2+2}| + C$$

QNo 22

$$\int \frac{x+3}{x^2-2x-5} dx = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5} dx.$$

$$= \frac{1}{2} \int \frac{2x-2}{x^2-2x-5} dx + 4 \int \frac{1}{(x^2-2x+1)-6} dx$$

$$= \frac{1}{2} \log |x^2-2x-5| + 4 \int \frac{1}{(x-1)^2 - (\sqrt{6})^2} dx.$$

$$= \frac{1}{2} \log |x^2-2x-5| + 4 \left(\frac{1}{2\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| \right) + C$$

$$= \frac{1}{2} \log |x^2-2x-5| + \frac{2}{\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| + C$$

QNo 23

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx = \int \frac{\frac{5}{2}(2x+4) + (3-10)}{\sqrt{x^2+4x+10}} dx.$$

$$= \frac{5}{2} \int (x^2+4x+10)^{-1/2} (2x+4) dx - 7 \int \frac{dx}{\sqrt{x^2+4x+4+6}}$$

$$= \frac{5}{2} \frac{(x^2+4x+10)^{1/2}}{1/2} - 7 \int \frac{dx}{(x+2)^2 + (\sqrt{6})^2}$$

$$= 5\sqrt{x^2+4x+10} - 7 \log |x+2+\sqrt{(x+2)^2+(\sqrt{6})^2}| + C$$

$$= 5\sqrt{x^2+4x+10} - 7 \log |x+2+\sqrt{x^2+4x+10}| + C.$$

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QNo 24 . $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{x^2+2x+1+1} = \int \frac{dx}{(x+1)^2+1^2}$

$$= \frac{1}{1} \tan^{-1}\left(\frac{x+1}{1}\right) + C = \tan^{-1}(x+1) + C.$$

$\therefore$ (B) is correct option.

QNo 25 . $\int \frac{dx}{\sqrt{9x-4x^2}} = \frac{1}{\sqrt{4}} \int \frac{dx}{\sqrt{\frac{9}{4}x-x^2}} = \frac{1}{2} \int \frac{dx}{\sqrt{-\left(x^2-\frac{9}{4}x+\left(\frac{9}{8}\right)^2\right)+\left(\frac{9}{8}\right)^2}}$

$$= \frac{1}{2} \int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2 - \left(x-\frac{9}{8}\right)^2}}$$

$$= \frac{1}{2} \sin^{-1}\left(\frac{x-\frac{9}{8}}{9/8}\right) + C$$

$$= \frac{1}{2} \sin^{-1}\left(\frac{8x-9}{9}\right) + C$$

$\therefore$ (B) is correct option.

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